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Erratum: “Low vibration laboratory with a single-stage vibration isolation for microscopy applications” [Rev. Sci. Instrum. **88**, 023703 (2017)]

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In our publication entitled “Low vibration laboratory with a single-stage vibration isolation for microscopy applications”¹ we have erroneously labeled the measured *spectrum* N_{spec} of the vibrations as the *spectral density* $N_{\text{PSD}} \equiv N_v$ of the vibrations in our laboratory. Due to this, the measured vibration levels (spectral densities) shown in Figs. 3 and 5–7 have to be multiplied by a factor of 5.9. The corrected figures are shown in Figs. 1–5. Note that the magenta curve in Fig. 3 of Ref. 1 is not shifted because it was measured with a bandwidth of 1 Hz.

While experts in spectral analysis will be aware of the difference between a *spectrum* and the *spectral density*, scientists using spectral analysis only occasionally may not be aware of the difference between both. Thus we discuss, in the following, this difference and how the two can be converted into each other. Moreover, we discuss in which circumstances the one or the other should be used, and how the calibration of a spectrum or a spectral density is experimentally verified.

Nowadays, when spectrum analyzer instruments (with all the calibration steps already included) are less frequently used in favor of analog to digital conversion of the measured signal followed by a subsequent software discrete Fourier transform (DFT), the correct calibration is no more “included” by the spectrum analyzer hardware. Since the discrete Fourier transform just transforms n numbers to n new numbers, the user has to take care about the necessary calibration steps, and errors may occur in this calibration. While this is straightforward in principle, it involves a number of non-trivial details. Here, we also include the description of an experimental calibration procedure which gives an easy cross-check for the correct calibration of the spectrum or spectral density. We realized that the above mentioned information is not easily found in the literature, and thus this information is presented compactly here.

If a continuous signal $S(t)$ is sampled with a sampling frequency f_{sample} , this signal is represented as a discrete time series $S(k/f_{\text{sample}})$. The discrete Fourier transform (DFT) of a time series of length n is defined as

$$\hat{S}(m) = \sum_{k=0}^{n-1} S(k/f_{\text{sample}}) e^{-2\pi i k m / n}, \quad (1)$$

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with $m = 0, \dots, n-1$. The power spectral density (termed PSD or N_{PSD}^2) is proportional to the absolute square of the discrete Fourier transform (DFT).² If we do not consider windowing yet (i.e., consider a rectangular window),^{3,4} the single-sided PSD results as

$$N_{\text{PSD}}^2(m) = \frac{2}{f_{\text{sample}} n} |\hat{S}(m)|^2, \quad m = 0, \dots, n/2. \quad (2)$$

Note that other definitions of the DFT than the one in Eq. (1) result in other factors in Eq. (2).³ The spectral density N_{PSD} is the square root of the power spectral density N_{PSD}^2 .

For a continuous signal, the power spectral density of a signal is related, via Parseval’s identity, to the root mean square (RMS) S_{RMS} of the signal as

$$S_{\text{RMS}}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T S^2(t) dt \equiv \langle S^2(t) \rangle = \int_0^\infty N_{\text{PSD}}^2(f) df. \quad (3)$$

If the signal is a discrete time series, the continuous quantities $S(t)$ and $N_{\text{PSD}}(f)$ translate to discrete values as

$$S(t) \leftrightarrow S(k/f_{\text{sample}}) \text{ and } N_{\text{PSD}}(f) \leftrightarrow N_{\text{PSD}}(m f_{\text{res}}), \quad (4)$$

respectively, with $k = 0, \dots, n-1$, and $m = 0, \dots, n/2$. The width of the n frequency bins of the DFT is given by $f_{\text{res}} = f_{\text{sample}}/n$.³ For a discrete signal, Eq. (3) translates to

$$S_{\text{RMS}}^2 = \frac{1}{T} \sum_{k=0}^{n-1} S^2(k/f_{\text{sample}}) / f_{\text{sample}} = \sum_{m=0}^{n/2} N_{\text{PSD}}^2(m f_{\text{res}}) f_{\text{res}}. \quad (5)$$

In the following, we consider two simple examples for the power spectral density: a constant power spectral density [Fig. 5(a)] and a power spectral density of a tonal sinusoidal signal [Fig. 5(b)].

Spectral density: If the power spectral density of the signal is considered within a certain frequency bandwidth $B = f_2 - f_1$ between f_1 and f_2 [as indicated by the blue shaded area in Fig. 5(a)], the power spectral density is zero outside the range of the bandwidth B . We assume further that $f_{\text{res}} \ll B$, which is usually the case. If the power spectral density is constant for the j bins between f_1 and f_2 , the N_{PSD}^2 in Eq. (5) can be written in front of the sum and the sum yields $j \cdot f_{\text{res}} = B$. Thus for

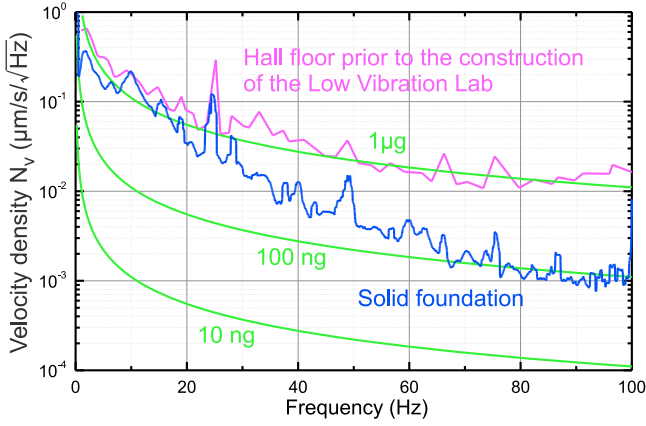


FIG. 1. Corresponds to Fig. 3 in Ref. 1. Spectral density of the RMS vibration velocity (vertical component) $N_v(f)$ on the floor of the building before any construction has been started (magenta line) compared to the RMS vibration level on the solid 200 ton solid foundation (blue line). For comparison, $N_v(f)$ curves (RMS) corresponding to different fractions of the gravitational acceleration g are shown.

a constant PSD, Eq. (5) results in

$$N_{\text{PSD}}^2 = \frac{S_{\text{RMS}}^2}{B}. \quad (6)$$

If, for example, the signal is a voltage, e.g., of RMS amplitude $S_{\text{RMS}} = 1$ V and $B = 100$ Hz, a spectral density of $N_{\text{PSD}} = 0.1$ V/ $\sqrt{\text{Hz}}$ results. In this case, the (power) spectral density is independent of the width of the frequency bins. This case is desirable as the value of the (power) spectral density has a significance independent of the width of the frequency bins, i.e., independent of details of the sampling.

For the case of a tonal (sinusoidal) signal, the situation is different. For the sake of simplicity, we consider cases without spectral leakage present.⁴ Then the tonal signal is usually located within a single non-zero frequency bin of width f_{res} at a frequency f_i , as shown in Fig. 5(b). In this case, only one term of the sum in Eq. (5) survives and the (power) spectral density of this bin depends (undesirably) on the width of the

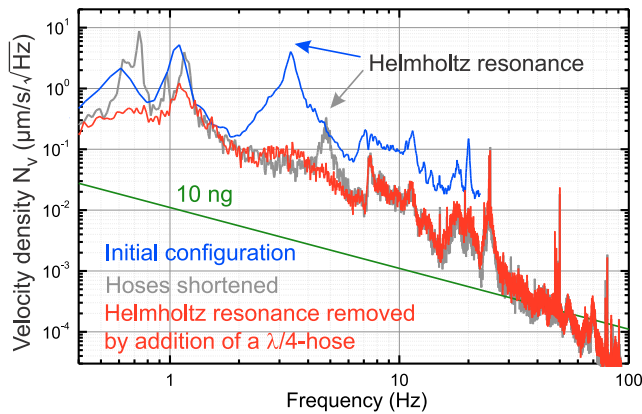


FIG. 2. Corresponds to Fig. 5 in Ref. 1. Spectral density of the RMS vibration velocity (vertical component) for different stages of the optimization of the vibration isolation. The initial state (blue) is characterized by a large peak at about 3 Hz due to the Helmholtz resonance. After shortening of the hoses, the Helmholtz resonance shifted to about 5 Hz (gray). The Helmholtz resonance has been removed by destructive interference of standing waves using an additional $\lambda/4$ -hose (red).

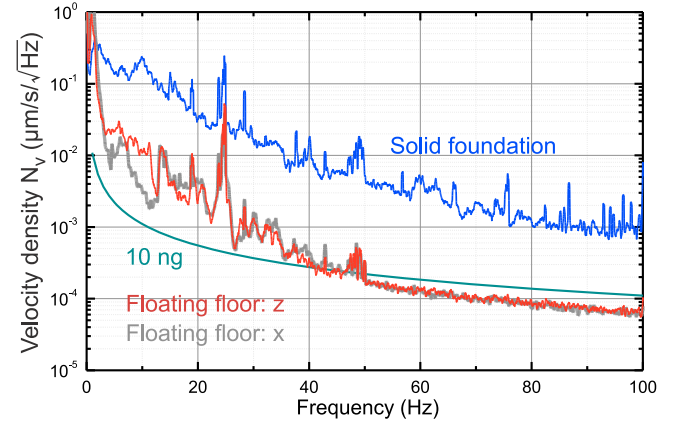


FIG. 3. Corresponds to Fig. 6 in Ref. 1. Spectral density of the velocity (RMS) measured on the floating floor in the final configuration. The vertical direction is shown in red, while the horizontal direction one is shown in gray. The 10 ng line is shown in green.

frequency bins f_{res} as

$$N_{\text{PSD}}^2(f_{\text{res}}) = \frac{S_{\text{RMS}}^2}{f_{\text{res}}}. \quad (7)$$

This means that, for instance, a tonal signal with an RMS amplitude of 1 V results in different values for the (power) spectral density, depending on the width of the frequency bins f_{res} , as also shown in Fig. 5(b) for two different values of the frequency bin width f_{res} and f'_{res} , respectively. This dependence of the value of the (power) spectral density on the frequency bin width f_{res} , which depends on the particular length of the time series used for the DFT and the particular sampling rate, is of course undesirable. Thus the value of the (power) spectral density for a tonal signal has no unique significance without the knowledge of some details on the sampling process, such as the sampling rate f_{sample} and the length n of the DFT.

Spectrum: A different quantity, the power spectrum N_{spec}^2 or the spectrum N_{spec} , defined as

$$N_{\text{spec}}^2 \equiv N_{\text{PSD}}^2 \cdot f_{\text{res}} \quad (8)$$

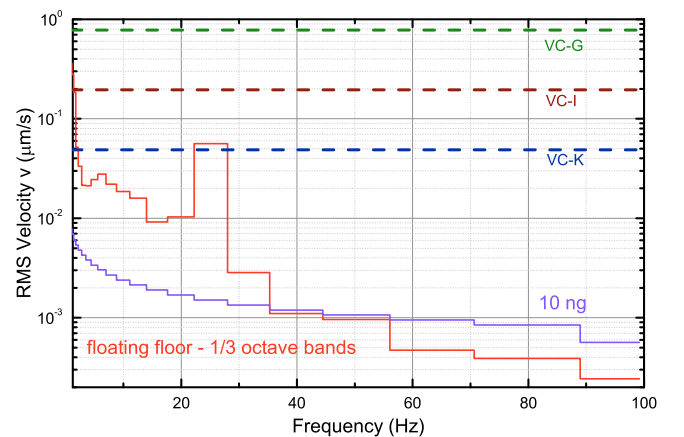


FIG. 4. Corresponds to Fig. 7 in Ref. 1. One-third octave band analysis of the RMS velocity v in the z -direction measured on the floating floor in the final configuration. The red curve is calculated from the corresponding curve in Fig. 3. Standards for vibration isolation of buildings (vibration criteria, VC) are given for comparison (following IEST-RP-CC024), as well as the 10 ng line (in terms of one-third octave bands).

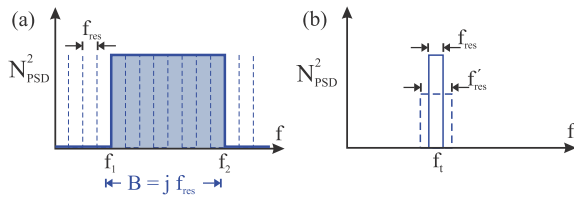


FIG. 5. (a) Case of a constant power spectral density within B (blue shaded area). The DFT representation of the power spectral density has j (same) values with a frequency bin width of $f_{\text{res}} = f_{\text{sample}}/n$. In this case, the power spectral density is independent of the width of the frequency bin of the DFT, f_{res} [cf. Eq. (6)], while the power spectrum depends on the bin width [cf. Eq. (10)]. (b) Power spectral density of a tonal signal (sinusoidal), which has a nonvanishing value only in one frequency bin. In this case, the DFT representation of the power spectral density depends on the frequency bin width f_{res} [cf. Eq. (7)], while the power spectrum is independent of the bin width [cf. Eq. (9)].

avoids this disadvantage. When inserting Eq. (7), valid for a tonal signal, into Eq. (8), the (power) spectrum of a tonal signal turns out to be independent of the width of the frequency bin as

$$N_{\text{spec}}^2 = S_{\text{RMS}}^2. \quad (9)$$

As Eq. (9) shows, the value of the spectrum is equal to the RMS amplitude of the tonal (sinusoidal) signal, $N_{\text{spec}} = S_{\text{RMS}}$ (e.g., 1 V).

However, undesirably for a signal of constant (power) spectral density, the spectrum N_{spec} depends on the width of the frequency bin f_{res} , as evident when inserting Eq. (6) into Eq. (8), resulting in

$$N_{\text{spec}}^2 = \frac{S_{\text{RMS}}^2}{B} f_{\text{res}}. \quad (10)$$

In conclusion, neither the (power) spectral density nor the (power) spectrum delivers a value which is independent of the frequency bin for a tonal signal as well as for a signal with constant PSD (representative of a broad band signal with a relatively flat PSD). A solution of this dilemma would be to choose the width of the frequency bin $f_{\text{res}} = 1$ Hz so that both the spectral density and the spectrum have the same numeric value (but still different units, e.g., V/ $\sqrt{\text{Hz}}$ and V, respectively). However, if low frequencies approaching 1 Hz and below are of interest, a frequency bin width of 1 Hz is too wide.

So far, we have not considered the windowing in the DFT,⁴ which means we have so far implicitly considered a rectangular window function. When applying other window functions in the DFT, a quantity named “normalized equivalent noise bandwidth” (NENBW) can be defined and Eq. (8) is extended with $f_{\text{res}}^{\text{eff}}$ to

$$N_{\text{spec}}^2 \equiv N_{\text{PSD}}^2 \cdot f_{\text{res}}^{\text{eff}} = N_{\text{PSD}}^2 \cdot f_{\text{res}} \cdot \text{NENBW} \quad (11)$$

and values of NENBW for different windows are shown in Table I.³ In order to present the complete information of a spectral analysis, both the (power) spectral density and the (power) spectrum have to be presented or one of them and $f_{\text{res}}^{\text{eff}}$.

In the signal processing from the time series of the signal to the spectral density or spectrum, several proportionality factors are involved due to the use of, for example, either RMS amplitude or peak amplitude, either two-sided spectrum or

TABLE I. “Normalized equivalent noise bandwidth” (NENBW) for different windows.³

	Hanning	Flat top HP	Welch
NENBW	1.5	3.4279	1.2

single-sided spectrum, either natural frequency PSD or angular frequency PSD, or due to different window types, etc. So one has to consider all these factors carefully. Complementarily also an experimental calibration of the spectral density or spectrum is very desirable and will be considered in the following.

A tonal signal, e.g., from a signal generator can be used to calibrate the spectrum or the spectral density. According to Eq. (9), the RMS signal amplitude of a tonal signal corresponds directly to the amplitude of the spectrum N_{spec} , independent of the width of a frequency bin. For the calibration of the power spectral density using a tonal signal, the effective width of a frequency bin enters. According to Eq. (7) (extended to $f_{\text{res}}^{\text{eff}}$), the RMS signal amplitude of a tonal signal S_{RMS}^2 has to be divided by $f_{\text{res}}^{\text{eff}}$ in order to obtain the power spectral density.

Alternatively to a calibration with a tonal signal, a signal of constant power spectral density (white noise) and known amplitude can be used for the calibration. Such a signal is provided, for instance, by the Johnson-Nyquist noise (thermal noise) of a resistor R as voltage source of known RMS voltage

$$U_{\text{JN}}^{\text{RMS}} = \sqrt{4 k_B T R B} \quad (12)$$

and constant PSD (white noise spectrum). The bandwidth B in Eq. (12) corresponds to the effective width of a frequency bin of the DFT as $B = f_{\text{res}}^{\text{eff}}$. In order to obtain a reasonably large voltage, a resistor of large resistance should be used ($R \geq 500$ k Ω) and considered in parallel with the input resistance of the measurement device.

In conclusion, if a case of a spectral analysis includes tonal peaks, as well as (locally) constant regions as a function of frequency, both the spectrum and the spectral density are required in order to deliver quantitative results for tonal peaks and broad band regions. The tonal peaks are represented quantitatively in the spectrum (e.g., in volts), while constant regions are represented quantitatively in the spectral density (e.g., as V/ $\sqrt{\text{Hz}}$). The spectrum and spectral density can be converted into each other by the proportionality factor

$$\sqrt{f_{\text{res}}^{\text{eff}}} = \sqrt{(f_{\text{sample}}/n) \cdot \text{NENBW}}.$$

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¹B. Voigtländer, P. Coenen, V. Cherepanov, P. Borgens, Th. Duden, and F. S. Tautz, *Rev. Sci. Instrum.* **88**, 023703 (2017).

²C. W. de Silva, *Vibration: Fundamentals and Practice* (Taylor and Francis—CRC Press, London, 2006).

³See <http://pubman.mpg.de/pubman/item/escidoc:152164:1/component/escidoc:152163/395068.pdf> for more information on spectrum, spectral density estimation and DFT.

⁴R. G. Lyons, *Understanding Digital Signal Processing* (Pearson Education, 2013).