Error analysis of gate-based quantum computers with transmon qubits

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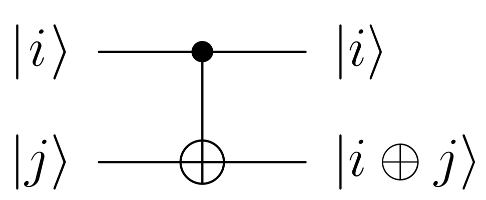
Gate-based quantum computing

A quantum computer contains a set of two-level systems called qubits. Each qubit can be in a complex superposition of the computational states $|0\rangle$ and $|1\rangle$. At each step in the computation, gates transform the qubits.

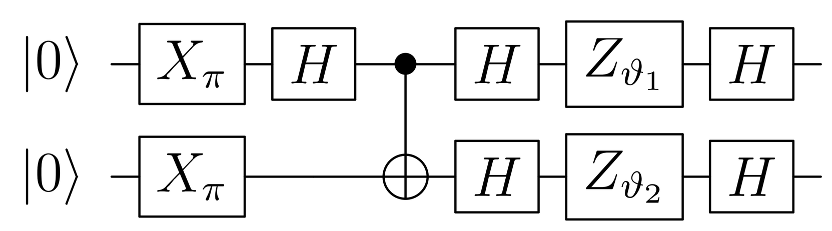
Examples for single-qubit gates:

$|0\rangle$	X_π	$|1\rangle$	$|1\rangle$	X_π	$|0\rangle$
$|0\rangle$	$X_{\pi/2}$	$\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$	$|1\rangle$	$X_{\pi/2}$	$\frac{-i|0\rangle + |1\rangle}{\sqrt{2}}$
$|0\rangle$	Z_ϑ	$|0\rangle$	$|1\rangle$	Z_ϑ	$e^{i\vartheta}|1\rangle$
$|0\rangle$	H	$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$	$|1\rangle$	H	$\frac{|0\rangle - |1\rangle}{\sqrt{2}}$

The two-qubit controlled-NOT (CNOT) gate is a conditional operation to entangle two qubits.



The computation can be expressed as a quantum circuit:



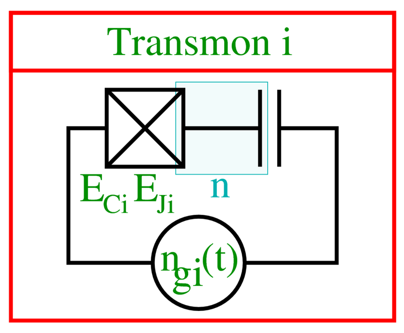
At the end, a measurement of the qubits produces a bit string by projecting each qubit to $|0\rangle$ or $|1\rangle$.

Transmon qubit architecture

The architecture of the transmon quantum computer is defined by the system Hamiltonian

$$H = H_{\mathrm{CPB}} + H_{\mathrm{Res}} + H_{\mathrm{CC}}$$

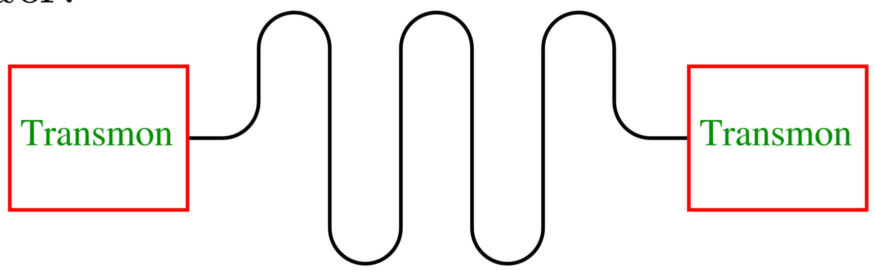
The qubits are given by the lowest eigenstates of Cooper Pair Boxes (CPBs) in the transmon regime [1]:



$$H_{\mathrm{CPB}} = \sum_{i=1}^{N_{\mathrm{Tr}}} \left[E_{Ci}(\hat{n}_i - n_{gi}(t))^2 - E_{Ji}\cos\hat{\varphi}_i \right]$$

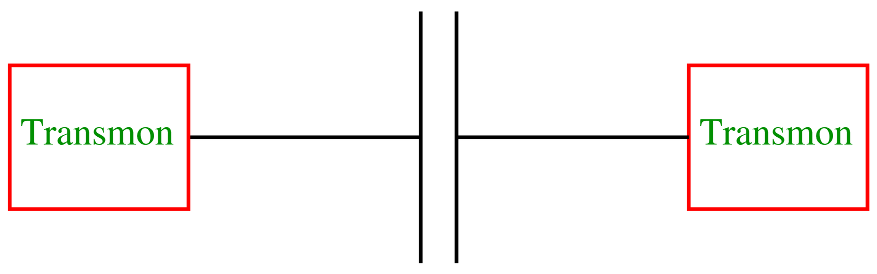
One way of coupling transmons is based on a transmission line resonator, modeled as a harmonic oscillator:

$$H_{\mathrm{Res}} = \omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^{N_{\mathrm{Tr}}} g_i \hat{n}_i (\hat{a} + \hat{a}^\dagger)$$



Another way of coupling transmons is based on a capacitive electrostatic interaction:

$$H_{\mathrm{CC}} = \sum_{1 \le i < j \le N_{\mathrm{Tr}}} E_{Ci,Cj} \hat{n}_i \hat{n}_j$$



Quantum gates are implemented by microwave voltage pulses:

$$n_{gi}(t) = \sum_j \Omega_{ij}(t)\cos(\omega_{ij}t - \gamma_{ij})$$

Transmon	$E_{Ci}/2\pi$	$E_{Ji}/2\pi$	$\omega_i/2\pi$	$\omega_r/2\pi$	$g_i/2\pi$
1	1.204	13.349	5.350	7	0.07
2	1.204	12.292	5.120	7	0.07

Simulation method

The time-dependent Schrödinger equation (TDSE)

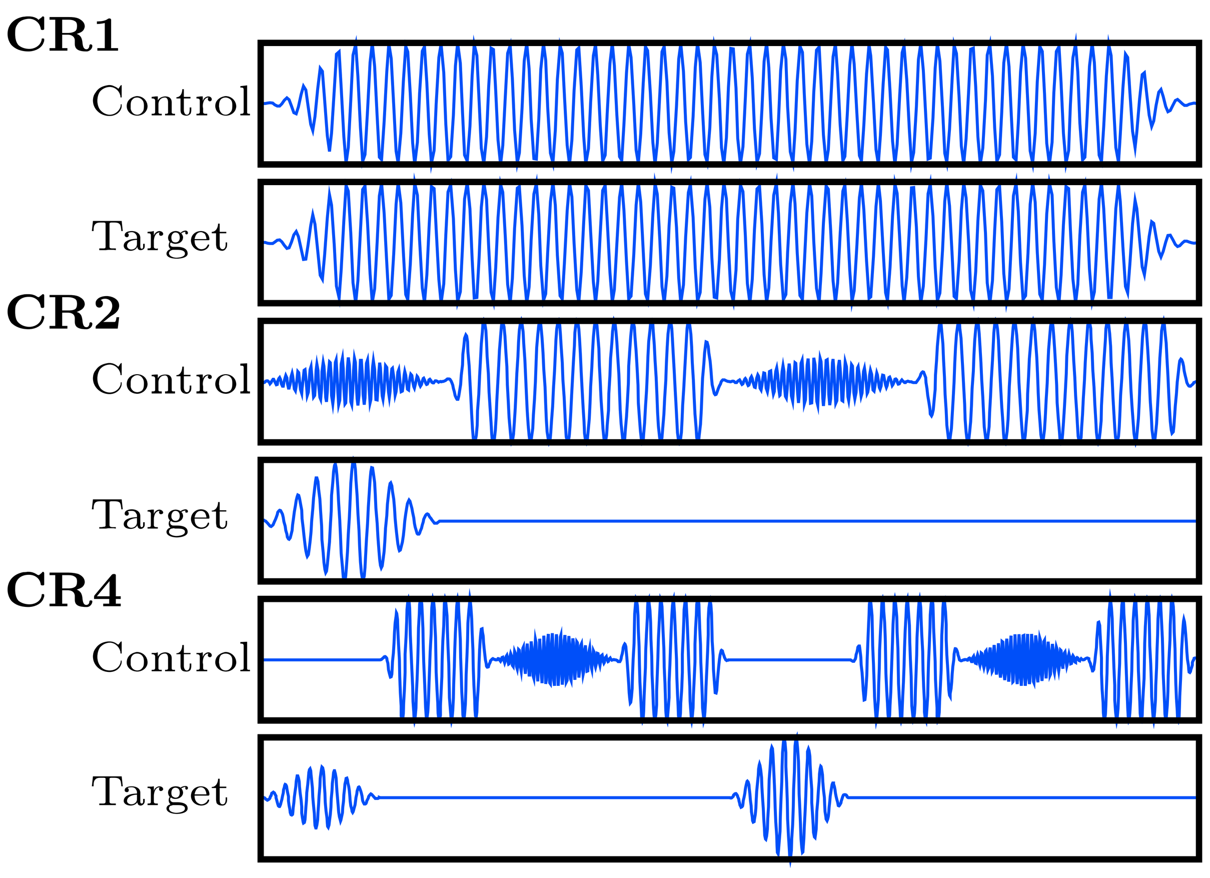
$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$$

is solved numerically using a Suzuki-Trotter product-formula algorithm [2] for the time-evolution operator:

$$\mathcal{U}(\tau) = e^{-i\tau(H_1 + \ldots + H_K)} \approx e^{-i\tau H_1} \cdots e^{-i\tau H_K}$$

The goal is to find a pulse $n_{gi}(t)$ so that $\mathcal{U}(t)$ implements a certain quantum gate on the qubits. We use the Nelder-Mead algorithm to optimize the parameters of the pulse.

The CNOT gate is implemented in three different versions based on cross-resonance (CR) pulses [3].



Gate-error metrics

Projection of the time-evolution operator $\mathcal{U}(t)$ on the qubit subspace gives the matrix M. Ideally, this matrix should be equal to the unitary quantum gate U.

$$\mathcal{G}_{ac}(|\psi\rangle\langle\psi|) = M|\psi\rangle\langle\psi|M^\dagger$$
$$\mathcal{G}_{id}(|\psi\rangle\langle\psi|) = U|\psi\rangle\langle\psi|U^\dagger$$

Average gate fidelity [4]

$$F_{\mathrm{avg}} = \int \mathrm{d}|\psi\rangle\ \langle\psi|\,\mathcal{G}_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\,|\psi\rangle$$

Diamond error rate [5]

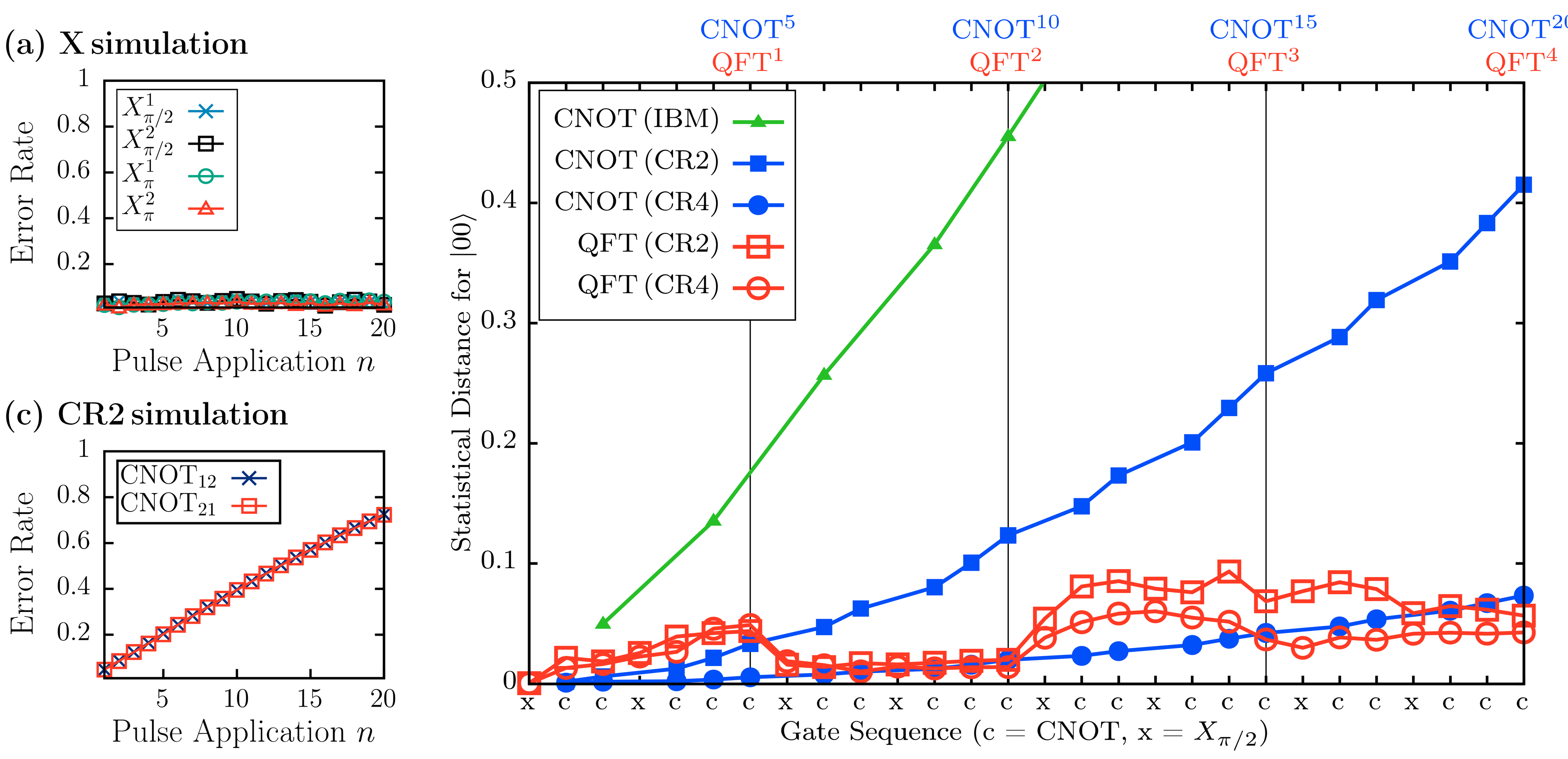
$$\eta_\diamond = \frac{1}{2}\left\|\mathcal{G}_{ac}\circ\mathcal{G}_{id}^{-1} - \mathbb{1}\right\|_\diamond$$

Unitarity [6]

$$u = \frac{d}{d-1}\int \mathrm{d}|\psi\rangle\ \mathrm{Tr}\left[\mathcal{G}'_{ac}(|\psi\rangle\langle\psi|)^\dagger \mathcal{G}'_{ac}(|\psi\rangle\langle\psi|)\right]$$

Simulation results

Gate	F_{avg}	$\eta_\diamond$	u
$X^1_{\pi/2}$	0.9946	0.027	0.990
$X^2_{\pi/2}$	0.9942	0.028	0.989
X^1_{π}	0.9949	0.020	0.990
X^2_{π}	0.9943	0.023	0.989
$\mathbf{CR1}_{12}$	0.9842	0.029	0.969
$\mathbf{CR1}_{21}$	0.9951	0.033	0.991
$\mathbf{CR2}_{12}$	0.9943	0.048	0.991
$\mathbf{CR2}_{21}$	0.9947	0.048	0.992
$\mathbf{CR4}_{12}$	0.9934	0.049	0.989
$\mathbf{CR4}_{21}$	0.9946	0.044	0.991



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Conclusion: The gate metrics of the optimized pulses are nearly perfect and agree with experimental achievements [3]. However, in repeated applications or actual quantum circuits, the gates suffer from systematic errors. These can be observed in experiments [7,8]. Although the gate fidelity and other metrics indicate them, they cannot replace the information of how well and how often a certain gate may be used in a quantum computation [9].