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Institut für Kernphysik

**Analysis of Nuclear Excited States
The Escape and Spreading Widths**

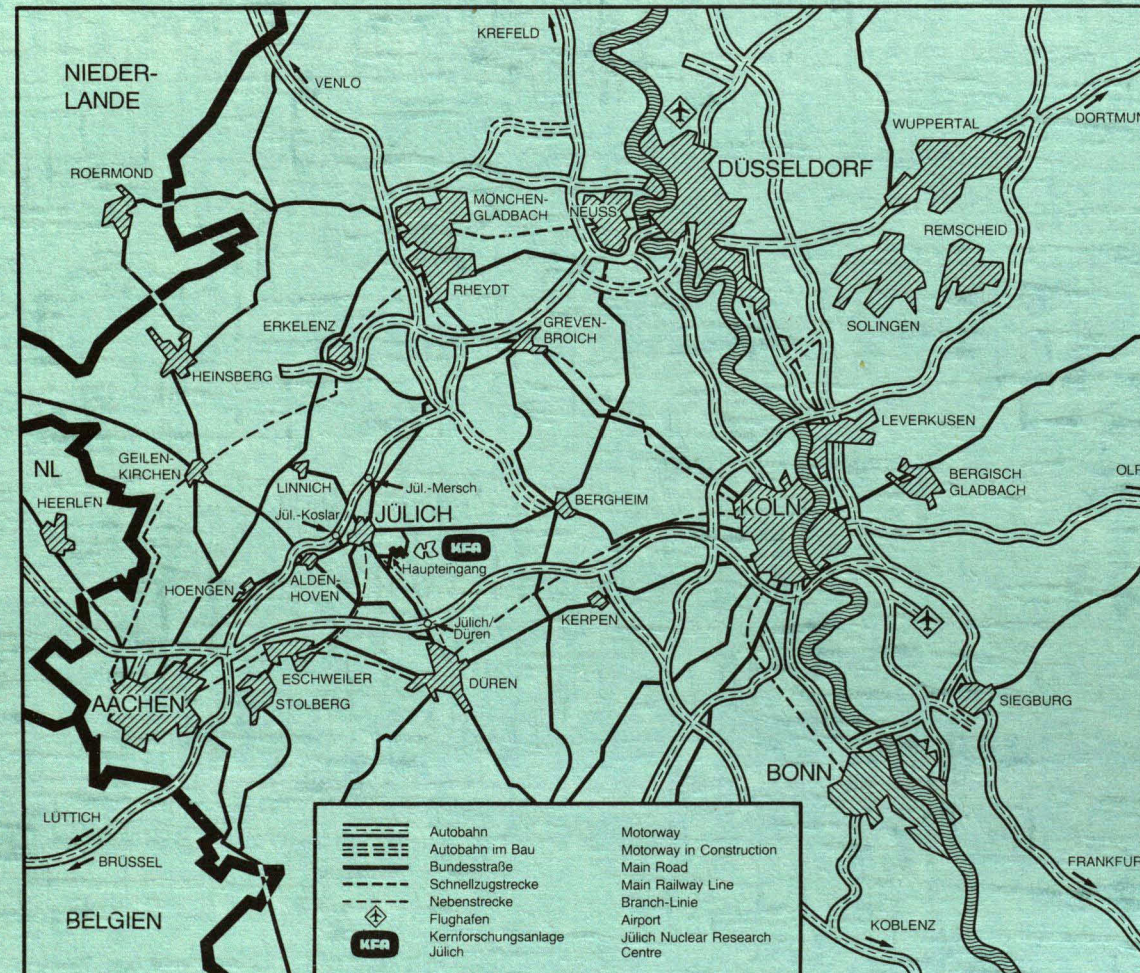
by

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**Analysis of Nuclear Excited States
The Escape and Spreading Widths**

by

R. de Haro

D 5 (Diss. Uni. Bonn)

In Memoriam:

G.P. Aygadoux

To

my parents Raphael and Branca,
the Great Bunch
... and Uli!

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1. Introduction

One of the branches of nuclear structure research that showed a very fast development in the last decade was that of the study of giant resonances.

The idea and concept of giant resonances is to be tributed to Migdal (Mi44), who predicted a resonance behaviour for nuclei. Such a resonance was observed for the first time by Baldwin and Klaiber in 1947 (BK47) as a dipole mode in photo-induced reactions. The search for such modes was then intensified and soon one saw that such a dipolar resonance was to be found over almost the whole periodic table (cf. (Ha79), (SW81) for recent reviews).

Although theoretically predicted, no giant resonance with multipolarity other than the dipole was found until 1971. Pitthan and Walcher (PW71, PW72) and Fukuda and Torizuka (FT71) were then able, from inelastic electron scattering, to get definite indications of a non-dipolar giant resonance, later identified to be the giant quadrupole resonance.

Once more, intense experimental research followed and, for a large variety of nuclei, with different kinds of nuclear reactions (p, p'), (α, α'), (e, e'), ($^3\text{He}, ^3\text{He}'$), heavy ions and photoabsorption, one observed quadrupolar strength (cf. (Wo79), (SW82)). Shortly after that one observed experimentally monopole strength (MM75, YR77) and, recently, indications of octupolar strength have been found (MR80, CC80, YK80) as well as of an isoscalar dipole compression mode (MR80, DM82).

A similar progress is to be registered in the theoretical description of the giant resonances. Shortly after the first indication of Baldwin and Klaiber (BK47), Goldhaber and Teller (GT48) explained the giant dipole resonance as a coherent (collective) motion of protons against neutrons.

A series of phenomenological (macroscopic) models, based in a liquid drop picture has been proposed since then to describe the giant resonances, and the most important are quoted in section 8 of this work.

Although these models are able to describe the global features of the giant resonances, they fail to describe their fine structure, as well as any feature directly connected to the internal (microscopic) structure of the state, such as width, splitting of peaks and even shell structure effects.

If one aims a deep understanding of the giant resonances, one must start from a fully microscopic theory and proceed the analysis step by step.

The first (successful) attempt in this direction was the particle-hole theory of Brown and Bolsterli (BB59). Here the state of the system corresponding to the giant resonance was described as a superposition of p-h excitations (using the schematic model). They were able to show that there exists a collective state with a large transition probability to the ground state and strongly shifted from its unperturbed p-h energy. A clear feature was that due to the nature of interaction (attractive for the isoscalar and repulsive for the isovector branch) isoscalar states should lie generally lower than isovector ones of the same multipolarity. The schematic model is a simplified version of the random phase approximation (RPA). The basic assumption is that a separable residual interaction may be used. This prevents a detailed comparison with experimental data. More realistic RPA calculations require (i) a particle-hole interaction ideally derived from a meson-exchange picture, (ii) a complete single particle space which should contain a correct treatment of the continuum.

As precursor and also simultaneously to the development of the RPA some other methods have been proposed, also based on a p-h structure of the excitation.

The so-called Feshbach formalism (Fe58, Fe62) was based on a suitable decomposition of the space of p-h excitations in different subspaces via projection operators. That way one aimed to get a simple dynamics in each subspace and also to reduce the model subspaces enough in order to allow numerical calculations (Sh63a, Sh63b, EF63, WS72).

The coupled channel calculations played a great role among the microscopic theories being the first to be used more systematically for studying excited states.

The basis of the method is to derive a set of coupled integro-differential equations in the p-h space that must be solved simultaneously (BH77, BA71, MS67, MS69, BR77).

The large drawback of the calculation is, on one side, the fact that one works in configuration space, what leads to very large sets of equations. On the other hand, the numerical work is very intensive, making the method very slow. Both factors make that the coupled channel method must be restricted to light nuclei in small model spaces.

Very soon one recognized the RPA method to be specially suitable for describing and analyzing not only the low-lying excited states of the nucleus but also the giant resonances.

Experimentally it was seen that giant resonances have a characteristic smooth behaviour as a function of mass number. Nowadays one is able to track down e.g. giant dipole and giant quadrupole resonances over a large range of the periodic table. The properties of the giant resonances vary smoothly with the mass number.

Therefore it is interesting to have a microscopic theory which is able to be applied to a large variety of nuclei allowing us to extract conclusions about the microscopic reasons for such a variation.

In order to be able to do that, one must have a suitable formulation, like the RPA, and a well established interaction which can be applied all over the range to be studied.

This is achieved in the frame of the theory of finite Fermi systems (SW77), which is an extension of the Landau-Migdal theory (Mi65).

The idea in the FFS theory is to go beyond the single particle theory considering the polarization effects of the nucleus, treating directly with quasi particles and using also a renormalized or effective interaction.

Going, therefore, beyond the original ansatz for the effective force of Landau and Migdal (La57a, La57b, La59, Mi59, Mi65) a group achieved to determine a good set of parameters for the effective interaction. The parameters were determined by a fit to experimental data in the lead region (^{208}Pb and neighbours). Here excitation energies, transition probabilities and mean radii were fitted simultaneously to the data (SW77, RB73, BS73, SZ74, RS73, RS74).

In section 5 we discuss the interaction used.

The use of effective single-particle energies and realistic (Woods-Saxon) wave functions prescribed by the finite Fermi system theory yielded very improved results for low- and high-lying states compared with those of the first calculations with FFS theory (Ka72, BK75).

The calculations quoted above were realized in a discretized space and reproduced quite well location and strength of states. However, experimentally known giant resonances have a width (typically of the order of some MeV) which cannot be reproduced there.

Microscopically one can understand the width of a state to have two origins: firstly the single particle continuum has, itself, a width, the so-called escape width, which describes the possibility of decay by emitting a particle from a continuum level. One constructs a collective state from the single particle states through the action of the residual interaction. Due to the contribution of each partial width the final (collective) excited state shall gain also an escape width. One usually denotes the escape width by $\Gamma^\dagger$.

On the other hand, the single particle states can also couple to collective nuclear vibrations (phonons) splitting in a number of fragments with reduced single particle strength. The distribution of fragments shows a slightly different centroid with respect to the original single particle energy and one says that the single particle state acquired a width, the so-called spreading width, denoted $\Gamma^\ddagger$.

In order to describe the escape width one must treat the single particle continuum in its full complexity. This means, one allows the single particle wave functions to be functions not only of the spatial coordinate but also of the energy (as it is indeed the case).

Such a treatment has been made by many authors with different approaches.

In the early works, the coupled channel method has been used. Here one essentially solves, in configuration space, a set of coupled integral equations. The drawback of the method lies in its direct dependence on the

number of possible p-h pairs. Therefore, applications were restricted to light nuclei or, for medium size nuclei, only to those states allowing a small number of p-h pairs. In any case, drastic limitations in the model space were needed (BGa65, BGb65, BH67, RM67, MS67, MS69).

A formalism based on a Green's function formulation in coordinate space has been proposed and applied (BT75, SB75). There, the problems with the number of p-h pairs were avoided but, on the other hand, since one was constrained to use a mesh of points in coordinate space, one was restricted to very low resolutions in order to make the problem numerically workable. Furthermore, the theory as formulated by the authors is constrained to the use of zero range forces.

Birkholz and Krewald worked out a method based on a series expansion of the energy dependence of the RPA amplitudes, which was a fast method but presented the usual problems connected to very large matrices for heavy nuclei (Bi71, Bi72, KB74, KS74).

In this work we present a new method which allows us to get rid of the difficulties quoted above, using basically the fact that the whole nuclear information is contained in the density matrix and expanding this quantity in Fourier-Bessel series. The presentation of the method and the formalism are to be found in sections 2 and 3.

Using this method, the so-called Fourier-Bessel RPA, one is able to tackle a number of current problems. One of the most important questions we want to elucidate is that of the existence (or not) of concentrated strength of large multipolarity.

This question, many times asked, has been studied for the lowest multipolarities but, due to the numerical difficulties connected with the large p-h spaces involved it was, until now, a fully open problem for the higher multipolarities.

Since we own now a very fast method, based on a sound interaction that may be globally used, we shall present our analysis of this problem in the case of ^{16}O in section 10.1 and of ^{208}Pb in sections 10.2, 11.1 and 11.3.

An interesting point in virtue of the extremely stringent experimental results is that of the description of the transition densities for low-lying states. Here one can firstly have a test of the theory and secondly, through its description, get insight into the nuclear structure. Among the many open problems here we discuss in section 8 the transition densities for the low-lying states in ^{208}Pb , especially the lowest 3^- state, which none of the available theoretical descriptions can account for.

Since the discovery of the giant monopole resonance (MM75) one wonders whether one can have other compression modes other than the breathing mode.

Recently, sound indications of a further compression mode were found (MR80, DM82), leaving open the possibility of having even further modes (multipolarities higher than one). We investigate this possibility as well as give our theoretical contribution to the definitive identification of the quoted modes experimentally seen.

For that we develop a technique based on the comparison of the nuclear response (calculated with the continuum Fourier-Bessel RPA) to different operators. The method is shown in section 11.2 and the analysis of the different multipolarities is made in section 11.3.

All these points made until now are discussed in the frame of the Fourier-Bessel RPA with full continuum. This automatically describes the escape width of the states.

However, as quoted before, in many cases the spreading width may play a central role. One believes that the spreading width must be very well described if one takes into account two particle-two hole contributions, since the higher contributions (3p-3h, 4p-4h, etc.) should be energetically disfavoured.

Many attempts have been made until now to take into account the most important contributions of 2p-2h correlations. The early works (Sa62, FS62, TU64, IU65, Sc71, Sc76) had to perform drastic truncations of the p-h space or treated the problem in perturbation theory (even without account of the Pauli principle) being, therefore, hardly realistic.

Using the core coupling RPA, Dehesa et al. were able to carry out calculations with discretized states for a series of nuclei from ^{12}C to ^{208}Pb . The splitting of the resonances by the coupling of single particle states could be obtained (DK77, De77, DS77), with fair agreement with experiments. In comparison to the earlier theories, the core coupling RPA represented already a significative advance. Here one worked explicitly with the RPA phonons when considering the possible couplings, what largely reduces the excitation space one must deal with.

However, the method still requires a choice of the most important phonons to be coupled (the lowest-lying) and a further one in the 2p-2h matrix by taking only the most important elements.

Recently a formalism for the calculation of the polarization propagator has been developed (WM82, WF82), based on some simpler approaches (BM75).

Here, one is able to find a closed formulation for the p-h polarization propagator in terms of the mass operator.

The advantage of the method is that it allows us to deal with very large p-h space, permitting a good description of the width, location and energy shift of the giant resonances.

We extend this method in this work in order to treat the continuum contributions. Our aim is a realistic description of the effects of 2p-2h contribution to the width of the states.

Among the current problems we want to deal with are the correct description of the location and width of the giant resonances and an analysis of the concentration of the quadrupole strength in ^{208}Pb . The controversy here comes from the contradictory results for the strength obtained from hadron scattering (MR80, CC81, YK81) and those of inelastic electron scattering (Ri80, KM81, Ri82).

The formalism is developed in section 4 and the analysis is carried out in sections 12 and 13.

Conclusions and appreciations are made in section 14.

CHAPTER I - THEORY

2. Fourier-Bessel RPA Formalism - One Particle-One Hole Level

There is a number of different approaches used to derive the usual RPA equations, starting with the pioneering works of Sawada (Sa57), Brown and Bolsterli (BB59) and Gillet (Gi64) to the more recent derivations, e.g. small vibrations around an equilibrium density with linearization of the TDHF equations (Ro70, GS82).

A very suitable formalism for us is that of the Green's functions, cf. (SW77), firstly because of its transparency, secondly because we are then able to work directly with the density matrix.

Let us start writing down the particle-hole residual interaction (whatever it is) in coordinate space:

$$\hat{V} = \frac{1}{2} \sum_{\lambda\lambda'} \sum_{\mu\mu'} \int d^2x \int d^3x' \hat{\Psi}_{\lambda}^+(\vec{x}) \hat{\Psi}_{\mu}^+(\vec{x}') \langle \lambda\mu | V(\vec{x}, \vec{x}') | \lambda'\mu' \rangle \cdot \hat{\Psi}_{\mu'}(\vec{x}') \hat{\Psi}_{\lambda'}(\vec{x}) \quad (2.1)$$

Here the summation runs over the spin (and isospin) indices λ and μ ; the components of the Fermi field operators satisfy the anticommutation relations:

$$\begin{aligned} \{\hat{\Psi}_{\alpha}(\vec{x}), \hat{\Psi}_{\beta}^+(\vec{x}')\} &= \delta_{\alpha\beta} \delta(\vec{x}-\vec{x}') \\ \{\hat{\Psi}_{\alpha}(\vec{x}), \hat{\Psi}_{\beta}(\vec{x}')\} &= \{\hat{\Psi}_{\alpha}^+(\vec{x}), \hat{\Psi}_{\beta}^+(\vec{x}')\} = 0 \end{aligned} \quad (2.2)$$

The linear response of the nucleus to an external field is determined by the polarization propagator (FW71)

$$i \Pi(\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4; t', t)_{\alpha\beta; \alpha'\beta'} := \langle \Psi_0 | T [\hat{\Psi}_{\beta}^+(\vec{x}_2 t') \hat{\Psi}_{\alpha}(\vec{x}_1 t') \hat{\Psi}_{\alpha'}^+(\vec{x}_3 t) \hat{\Psi}_{\beta'}(\vec{x}_4 t)] | \Psi_0 \rangle \quad (2.3)$$

Here $|\Psi_0\rangle$ denotes the exact ground state. The interacting field operators are

$$\hat{\Psi}_{\alpha}(\vec{x}t) = e^{iH_0 t} \hat{\Psi}_{\alpha}(\vec{x}) e^{-iH_0 t}, \quad (2.4)$$

where H_0 means the unperturbed Hamiltonian. The usual summation of all ring diagrams and a Fourier transformation leads to the following expression for the RPA polarization propagator $\Pi^{(1)}$:

$$\begin{aligned} \Pi^{(1)}(\vec{x}_1, \vec{x}_2; \vec{x}_3, \vec{x}_4; \omega)_{\alpha\beta, \alpha'\beta'} &= \Pi^{(0)}(\vec{x}_1, \vec{x}_2; \vec{x}_3, \vec{x}_4; \omega)_{\alpha\beta, \alpha'\beta'} \\ &+ \sum_{\lambda\lambda'} \int d^3x \int d^3x' \{ \Pi^0(\vec{x}_1, \vec{x}_2; \vec{x}', \vec{x}'; \omega)_{\alpha\beta, \mu\mu'} \langle \lambda\mu | V(\vec{x}\vec{x}') | \lambda'\mu' \rangle \cdot \Pi^{(1)}(\vec{x}, \vec{x}'; \vec{x}_3, \vec{x}_4; \omega)_{\lambda'\lambda; \alpha'\beta'} \\ &- \Pi^0(\vec{x}_1, \vec{x}_2; \vec{x}\vec{x}'; \omega)_{\alpha\beta; \lambda\mu} \langle \lambda\mu | V(\vec{x}\vec{x}') | \lambda'\mu' \rangle \Pi^{(1)}(\vec{x}, \vec{x}'; \vec{x}_3, \vec{x}_4; \omega)_{\lambda'\mu; \alpha'\beta'} \} \end{aligned} \quad (2.5)$$

The unperturbed polarization propagator Π^0 has the representation

$$\begin{aligned} \Pi^0(\vec{x}_1, \vec{x}_2; \vec{x}_3, \vec{x}_4; \omega)_{\alpha\beta, \alpha'\beta'} &= \sum_n \left\{ \frac{\langle 0 | \hat{\Psi}_{\beta}^+(\vec{x}_2) \hat{\Psi}_{\alpha}(\vec{x}_1) | n \rangle \langle n | \hat{\Psi}_{\alpha'}^+(\vec{x}_3) \hat{\Psi}_{\beta'}(\vec{x}_4) | 0 \rangle}{\omega - (E_n - E_0) + i\eta} \right. \\ &\left. - \frac{\langle 0 | \hat{\Psi}_{\alpha'}^+(\vec{x}_3) \hat{\Psi}_{\beta'}(\vec{x}_4) | n \rangle \langle n | \hat{\Psi}_{\beta}^+(\vec{x}_2) \hat{\Psi}_{\alpha}(\vec{x}_1) | 0 \rangle}{\omega + (E_n - E_0) - i\eta} \right\}, \end{aligned} \quad (2.6)$$

where $|n\rangle$ is a complete set of unperturbed intermediate states, i.e. all possible particle-hole excitations; $(E_n - E_0) = \epsilon_p - \epsilon_h$ is the unperturbed particle-hole energy difference. A sum over discrete states and an integral over continuum states is understood. The dressed polarization propagator $\Pi^{(1)}$ has an analogous representation with $|n\rangle = |E_{J\pi}^{(N)}\rangle$ referring to the excited states of the nucleus, and $(E_n - E_0) = E_{J\pi}^{(N)}$ denoting the corresponding excitation energy of the N-th state. Obviously $|E_{J\pi}^{(N)}\rangle$ can be a continuum state, in which case the index (N) becomes redundant.

In order to describe the response of the nucleus to an external field, only the imaginary part of the polarization propagator is required. The imaginary part of $\Pi^{(1)}$ is proportional to $\delta(\omega - E_{J\pi}^{(N)})$, whereas the imaginary part of $\Pi^{(0)}$ is proportional to $\delta(\omega - (\epsilon_p - \epsilon_h))$. The residual interaction, however,

introduces an energy shift of the excited state of the nucleus with respect to a corresponding particle-hole configuration. When taking the imaginary part of eq. (5), the contribution due to the first $\Pi^{(0)}$ on the right hand side consequently vanishes in the discrete case. In the continuum case one gets:

$$\begin{aligned}
 \langle \Psi_0 | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | E_{J^\pi}^{(N)} \rangle &= \sum_{\lambda\lambda'} \sum_{\mu\mu'} \int d^3x \int d^3x' \{ \\
 &\sum_{ph} \int d\epsilon_p \operatorname{Re} \left[\frac{\langle 0 | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | ph \rangle \langle ph | \hat{\Psi}_\mu^+(\vec{x}') \hat{\Psi}_\mu(\vec{x}') | 0 \rangle}{\omega - (\epsilon_p - \epsilon_h) + i\eta} \right. \\
 &\quad \left. - \frac{\langle 0 | \hat{\Psi}_\mu^+(\vec{x}') \hat{\Psi}_\mu(\vec{x}') | ph \rangle \langle ph | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | 0 \rangle}{\omega + (\epsilon_p - \epsilon_h) - i\eta} \right] \\
 &\cdot \langle \lambda\mu | V(\vec{x}\vec{x}') | \lambda'\mu' \rangle \langle \Psi_0 | \hat{\Psi}_\lambda^+(\vec{x}) \hat{\Psi}_\lambda(\vec{x}) | E_{J^\pi}^{(N)} \rangle \\
 &- \sum_{ph} \int d\epsilon_p \operatorname{Re} \left[\frac{\langle 0 | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | ph \rangle \langle ph | \hat{\Psi}_\lambda^+(\vec{x}) \hat{\Psi}_\mu(\vec{x}') | 0 \rangle}{\omega - (\epsilon_p - \epsilon_h) + i\eta} \right. \\
 &\quad \left. - \frac{\langle 0 | \hat{\Psi}_\lambda^+(\vec{x}) \hat{\Psi}_\mu(\vec{x}') | ph \rangle \langle ph | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | 0 \rangle}{\omega + (\epsilon_p - \epsilon_h) - i\eta} \right] \\
 &\cdot \langle \lambda\mu | V(\vec{x}\vec{x}') | \lambda'\mu' \rangle \langle \Psi_0 | \hat{\Psi}_\mu^+(\vec{x}') \hat{\Psi}_\lambda(\vec{x}) | E_{J^\pi}^{(N)} \rangle \} \\
 &+ \sum_{ph} \int d\epsilon_p \langle 0 | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | ph \rangle \delta(\omega - (\epsilon_p - \epsilon_h)) D_{ph}(\epsilon_p) .
 \end{aligned} \tag{2.7}$$

This equation expresses the random-phase approximation in terms of the unknown nonlocal transition densities $\langle \Psi_0 | \hat{\Psi}_\beta^+(\vec{x}_2) \hat{\Psi}_\alpha(\vec{x}_1) | E_{J^\pi}^{(N)} \rangle$. The coefficients $D_{ph}(\epsilon_p)$ have to be determined from the boundary conditions.

The important feature of this equation is that it expresses the RPA equations solely in terms of a (nonlocal) transition density $\rho(\vec{r}_1, \vec{r}_2)$. The non-locality of ρ comes from the possible exchange terms of the (finite range) interaction. If one works with a zero-range interaction one is able to express the

RPA equations only in terms of local transition densities. Note that the developments in refs. (SB75, BT75) are restricted to zero-range interactions, e.g. of the Skyrme type, which allows to put $x_1=x_2$ and $x_3=x_4$ in eqs. (2.3) and (2.5). In this case one may solve eq. (2.5) by a matrix inversion in coordinate space (SB75, BT75). The present approach is based on the fact that the knowledge of the transition density matrix is sufficient to describe the linear response of the nucleus.

That eq. (2.7) is identical to the RPA equations must follow from the fact that we took 1p-1h superpositions to describe our complete set of excited states $|n\rangle$. However, this can also be proved in the inverse way, just starting from the equations of motion, with a specific ansatz (B166) to treat the poles of the RPA amplitudes (HK80).

Although eq. (2.7) contains the whole RPA information, one may be interested, for comparison, in regaining the usual RPA equations in shell model representation. The continuum RPA equations in configuration space (KB74, KS74, RK82a, RK82b) can be obtained from eq. (2.7) by inserting explicitly a representation of the excited state such as

$$|E_{J\pi}^{(N)}\rangle = \sum_{ph} \int_0^\infty d\epsilon_p \left[X_{ph}^{(N)}(\epsilon) (a_p^\dagger(\epsilon) b_h^\dagger)_{J\pi} + Y_{ph}^{(N)} (b_h a_p)_{J\pi} \right] |\Psi_0\rangle \quad (2.8)$$

where $a_p^\dagger(\epsilon)$ and $b_h^\dagger$ are the creation operators for particles and holes obeying the anticommutators

$$\{a_p(\epsilon), a_{p'}^\dagger(\epsilon')\} = \delta_{pp'} \delta(\epsilon-\epsilon') \quad (2.9)$$

and $\{b_h, b_{h'}^\dagger\} = \delta_{hh'}$

On the other hand, the expansion coefficients X_{ph} and Y_{ph} of eq. (2.8) can be obtained from the transition density matrices of eq. (2.7) by using the orthogonality of the single particle basis:

$$X_{ph}^{(N)}(\epsilon) = \int d^3x_2 \psi_{-h}(\vec{x}_2)(-)^{j_h+m_h} \int d^3x_1 \psi_p^+(\epsilon, \vec{x}_1) \langle 0 | \hat{\psi}^+(\vec{x}_2) \hat{\psi}(\vec{x}_1) | E_{J\pi}^{(N)} \rangle \quad (2.10)$$

$$Y_{ph}^{(N)}(\epsilon) = \int d^3x_2 \psi_p(\epsilon, \vec{x}_2) \int d^3x_1 \psi_{-h}^+(\vec{x}_1)(-)^{j_h+m_h} \langle 0 | \hat{\psi}^+(\vec{x}_2) \hat{\psi}(\vec{x}_1) | E_{J\pi}^{(N)} \rangle$$

For the Landau-Migdal force, which is an effective residual short range particle hole interaction (discussed in detail in section 5) we obtain

$$\hat{X}_{ph}^{\tau}(\epsilon) = - \int dr r^2 \sum_{\tau} \left[h_{\tau\tau}(r) C_{ph}^{\tau}(r, \epsilon) \rho^{\tau'}(r) + h_{\tau\tau}(r) F_{ph}^{\tau}(r, \epsilon) \sigma^{\tau'}(r) \right] \quad (2.11)$$

$$Y_{ph}^{\tau}(\epsilon) = \frac{-1}{\epsilon - \epsilon_h + E} \int dr r^2 \sum_{\tau} \left[h_{\tau\tau}(r) C_{hp}^{\tau}(r, \epsilon) \rho^{\tau'}(r) + h_{\tau\tau}(r) F_{hp}^{\tau}(r, \epsilon) \sigma^{\tau'}(r) \right] \quad (2.12)$$

where $\hat{X}_{ph}^{\tau}(\epsilon)$ denotes the regular part of $X_{ph}^{\tau}(\epsilon)$ and we made use of:

$$V(\vec{x}, \vec{x}') = C \delta(\vec{x} - \vec{x}') \left[f_0(\rho(\frac{\vec{x} + \vec{x}'}{2})) + f'_0(\rho(\frac{\vec{x} + \vec{x}'}{2})) \tau \cdot \tau' + g_0 \vec{\sigma} \cdot \vec{\sigma}' + g'_0 \vec{\sigma} \cdot \vec{\sigma}' \tau \cdot \tau' \right] \quad (2.13)$$

For this interaction, only a central and a spin transition density are required, which in a spherical basis read explicitly ($[j_p] = \sqrt{2j_p+1}$):

$$\sum_{\lambda} \langle 0 | \hat{\psi}_{\lambda}^+(\vec{x}) \hat{\psi}_{\lambda}(\vec{x}) | (ph)JM \rangle = \frac{1}{2} (1+(-)^{\ell_p+\ell_h+J}) (-)^{3/2+j_p+J} \begin{pmatrix} j_p & j_h & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} R_p(x) R_h(x) Y_{JM}(\hat{x}),$$

and

$$\sum_{\lambda\lambda'} \langle 0 | \hat{\psi}_{\lambda}^+(\vec{x}) (\vec{\sigma}_q)_{\lambda\lambda'} \hat{\psi}_{\lambda'}(\vec{x}) | (ph)JM \rangle = \sum_{L\mu} (-)^{\ell_h+1+\mu} \sqrt{\frac{6}{4\pi}} \begin{pmatrix} j_p & j_h & J \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} \ell_p & \ell_h & L \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \ell_p & \ell_h & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & 1 & J \\ -\mu & -q & -M \end{pmatrix} \quad (2.14)$$

$$R_p(x) R_h(x) Y_{L, M+q}(\hat{x}).$$

Putting these expressions into eq. (2.7), one obtains the following two conditions:

$$\rho_{\tau}(r) = \int dt t^2 \sum_{\tau} \left[G_{\tau}(r,t) h_{\tau\tau}(t) \rho_{\tau}(t) + H_{\tau}(r,t) h'_{\tau\tau}(t) \sigma_{\tau}(t) \right] + Q_{\tau}(r, \epsilon_h + E) \quad (2.15)$$

$$\sigma_{\tau}(r) = \int dt t^2 \sum_{\tau} \left[H_{\tau}(r,t) h_{\tau\tau}(t) \rho_{\tau}(t) + U_{\tau}(r,t) h'_{\tau\tau}(t) \sigma_{\tau}(t) \right] + W_{\tau}(r, \epsilon_h + E) \quad (2.16)$$

Here the index τ runs over protons and neutrons. The quantities $\rho_{\tau}(r)$ and $\sigma_{\tau}(r)$ refer to the central and spin density and are given in detail by

$$\rho^{\tau'}(r) = \sum_{p'h'} \int d\epsilon' \left[X_{p'h'}^{\tau'}(\epsilon') + (-1)^J Y_{p'h'}^{\tau'}(\epsilon') \right] C_{p'h'}^{\tau'}(r, \epsilon') \quad (2.17)$$

$$\sigma^{\tau'}(r) = \sum_{p'h'} \int d\epsilon' \left[X_{p'h'}^{\tau'}(\epsilon') + (-1)^{\ell_{p'} + \ell_{h'} + 1} Y_{p'h'}^{\tau'}(\epsilon') \right] F_{p'h'}^{\tau'}(r, \epsilon') \quad (2.18)$$

$$C_{ph}^{\tau}(r, \epsilon) = \frac{(-1)^{k_p}}{\sqrt{4\pi}} \sqrt{k_p k_h} \begin{pmatrix} j_p & j_h & J \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} R_p(r, \epsilon) R_h(r) \quad (2.19)$$

$$F_{ph}^{\tau}(r, \epsilon) = \frac{(-1)^{k_p + k_h + \ell_p + 1}}{\sqrt{4\pi}} \sqrt{k_p k_h} \begin{pmatrix} j_p & j_h & J \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix} R_p(r, \epsilon) R_h(r) \quad (2.20)$$

Here the abbreviation $k = j + \frac{1}{2}$ has been used.

The propagators read explicitly, with $\oint$ denoting the principle value integral:

$$G_{\tau}(r,t) = \sum_{ph} \left[\oint \frac{d\epsilon' C_{\tau}^{ph}(r, \epsilon') C_{\tau}^{ph}(t, \epsilon')}{\epsilon' - \epsilon_h - E} + \oint \frac{d\epsilon' C_{\tau}^{ph}(r, \epsilon') C_{\tau}^{ph}(t, \epsilon')}{\epsilon' - \epsilon_h + E} \right] \quad (2.21)$$

$$H_{\tau}(r,t) = \sum_{ph} \left[\oint \frac{d\epsilon' C_{\tau}^{ph}(r, \epsilon') F_{\tau}^{ph}(t, \epsilon')}{\epsilon' - \epsilon_h - E} + \oint \frac{d\epsilon' C_{\tau}^{ph}(r, \epsilon') F_{\tau}^{ph}(t, \epsilon')}{\epsilon' - \epsilon_h + E} \right] \quad (2.22)$$

$$U_{\tau}(r,t) = \sum_{ph} \left[\oint \frac{d\epsilon' F_{\tau}^{ph}(r, \epsilon') F_{\tau}^{ph}(t, \epsilon')}{\epsilon' - \epsilon_h - E} + \oint \frac{d\epsilon' F_{\tau}^{ph}(r, \epsilon') F_{\tau}^{ph}(t, \epsilon')}{\epsilon' - \epsilon_h + E} \right] \quad (2.23)$$

The interaction is represented by

$$\begin{aligned} h'_{\tau\tau} &= 4(g_0 + \tau_3 \tau'_3 g'_0) \\ h_{\tau\tau} &= 2 \left[(f_0 + \tau_3 \tau'_3 f'_0 + g_0 + \tau_3 \tau'_3 g'_0) + P_J \left[(f_0 + \tau_3 \tau'_3 f'_0) - (g_0 + \tau_3 \tau'_3 g'_0) \right] \right] \\ P_J &= (-1)^{\ell_p + \ell_h + J} \end{aligned} \quad (2.24)$$

Finally, the inhomogeneous terms are

$$Q_{\tau}(r, \epsilon_h + E) = - \sum_{ph} C_{\tau}^{ph}(r, \epsilon_h + E) D^{ph}(\epsilon_h + E) \quad (2.25)$$

$$W_{\tau}(r, \epsilon_h + E) = - \sum_{ph} F_{\tau}^{ph}(r, \epsilon_h + E) D^{ph}(\epsilon_h + E) \quad (2.26)$$

The choice of the quantities D^{ph} will be discussed below.

Since both $\rho(r)$ and $\sigma(r)$ involve a product of a particle and a hole wave function, these quantities vanish asymptotically. This fact has been used in the analysis of inelastic electron scattering experiments, where the transition densities and transition currents were expanded on a finite interval ranging from $r=0$ to some suitably chosen cut-off radius R into a Fourier-Bessel series of the form (HK79a, FL72, He80)

$$\rho_J(r) = \sum_{\mu=1} A_{\mu} j_J(q_{\mu} r) \quad (2.27)$$

for $0 \leq r \leq R$.

$j_J(q_{\mu} r)$ denotes the spherical Bessel function of order J ; the q_{μ} 's are determined by the zeros of the Bessel function $j_J(q_{\mu} R) = 0$ and A_{μ} are the expansion coefficients.

The Fourier-Bessel expansion of the transition density was found to converge very fast. In recent analysis of inelastic electron scattering data less than $M=15$ expansion coefficients were used (BH73, FN67, HK79b). Therefore we expand the theoretical transition densities $\rho(r)$ and $\sigma(r)$ also into Fourier-Bessel series. The same can be done for the functions $C_{\tau}^{ph}(\epsilon, r)$ and $F_{\tau}^{ph}(\epsilon, r)$ which show a similar r -dependence. Introducing these expansions in equations (2.18) and (2.19) we obtain:

$$A_{\mu}^{\tau} = \sum_{\omega, \eta, \tau'} \left[G_{\mu\omega}^{\tau} T_{\omega\eta}^{\tau\tau'} A_{\eta}^{\tau'} + H_{\mu\omega}^{\tau} S_{\omega\eta}^{\tau\tau'} B_{\eta}^{\tau'} \right] - Q_{\mu}^{\tau} \quad (2.28)$$

$$B_{\mu}^{\tau} = \sum_{\omega, \eta, \tau'} \left[H_{\mu\omega}^{\tau} T_{\omega\eta}^{\tau\tau'} A_{\eta}^{\tau'} + U_{\mu\omega}^{\tau} S_{\omega\eta}^{\tau\tau'} B_{\eta}^{\tau'} \right] - W_{\mu}^{\tau} \quad (2.29)$$

The new quantities are defined by:

$$\sigma_{\tau}(r) = \sum_{\mu} B_{\mu}^{\tau} j_J(q_{\mu} r) \quad (2.30)$$

$$Q_{\tau}(r) = \sum_{\mu} Q_{\mu}^{\tau} j_J(q_{\mu} r) \quad (2.31)$$

$$W_{\tau}(r) = \sum_{\mu} W_{\mu}^{\tau} j_J(q_{\mu} r) \quad (2.32)$$

The propagators G, H and U have also been expanded for both coordinates (r and t) acquiring indices μ and ν but conserving their structure (eqs. (2.21), (2.22), and (2.23)). Furthermore we define:

$$T_{\omega\eta}^{\tau\tau'} = - \int h_{\tau\tau'}(r) j_J(q_{\omega} r) j_J(q_{\eta} r) r^2 dr \quad (2.33)$$

and

$$S_{\omega\eta}^{\tau\tau'} = - \int h'_{\tau\tau'}(r) j_J(q_{\omega} r) j_J(q_{\eta} r) r^2 dr \quad (2.34)$$

Since τ and τ' run over proton (π) and neutron (ν) we can write explicitly the continuum RPA equations in matrix form:

$$\begin{pmatrix} G^{\pi} T^{\pi\pi} - 1 & G^{\pi} T^{\pi\nu} & H^{\pi} S^{\pi\pi} & H^{\pi} S^{\pi\nu} \\ G^{\nu} T^{\nu\pi} & G^{\nu} T^{\nu\nu} - 1 & H^{\nu} S^{\nu\pi} & H^{\nu} S^{\nu\nu} \\ H^{\pi} T^{\pi\pi} & H^{\pi} T^{\pi\nu} & U^{\pi} S^{\pi\pi} - 1 & U^{\pi} S^{\pi\nu} \\ H^{\nu} T^{\nu\pi} & H^{\nu} T^{\nu\nu} & U^{\pi} S^{\nu\pi} & U^{\nu} S^{\nu\nu} - 1 \end{pmatrix} \begin{pmatrix} A^{\pi} \\ A^{\nu} \\ B^{\pi} \\ B^{\nu} \end{pmatrix} = \begin{pmatrix} Q^{\pi} \\ Q^{\nu} \\ W^{\pi} \\ W^{\nu} \end{pmatrix} \quad (2.35)$$

We note that the dependence on the excitation energy E is contained in the propagators G(E), H(E) and U(E), in the form factors A(E) and B(E) and also in the right hand side of the equations, Q(E) and W(E).

This last matrix equation expresses the set of RPA equations, but now in terms of the coefficients of the Fourier-Bessel expansion of the scalar and spin transition densities given by eq. (2.14).

Eq. (2.35) still depends on the functions $D_{ph}(\epsilon_p = \epsilon_h + E)$ through the right hand side. These functions (Q and W) are until now arbitrary. Their determination follows from the asymptotic conditions.

The procedure used to get the RPA solutions is to make a set of suitable Ansätze for D_{ph} , $\{D_{ph}^{(1)}, D_{ph}^{(2)}, \dots, D_{ph}^{(W)}\}$, in order to attain a set of solutions of the system (2.35). Using this set of solutions we impose then the asymptotic conditions.

3. Asymptotics

According to eq. (2.8) any excited state of the nucleus can be written as

$$\Psi^J(r, E) = \sum_{ph} \int d\varepsilon \left[X_{ph}^J(E) (a_p^\dagger(\varepsilon) b_h^\dagger)_J |0\rangle + Y_{ph}^J(\varepsilon) (b_h a_p(\varepsilon))_J |0\rangle \right] \quad (3.1)$$

Physically, the fact that a particle lies in the continuum means that there is a probability of finding this particle at infinity. In this case we can consider a system composed of the residual core of (A-1) particles and the one considered particle. Therefore, in order to study the asymptotic behaviour of the nuclear wave function one lets one particle coordinate go to infinity and takes a so-called surface function $\varphi_{ph}(A-1)$ to describe the (A-1)-core (MW69).

Since asymptotically no holes can be created, the backgoing graph term in eq. (3.1) does not contribute to the asymptotic wave function. We can then write, in coordinate space

$$\Psi_{\text{asyp}}^J(r, \varepsilon) = \sum_{ph} \int d\varepsilon \left[X_{ph}^J(\varepsilon) \varphi_{ph}(A-1) R_p(r, \varepsilon) \right] \quad (3.2)$$

Here $\varphi_{ph}(A-1)$ denotes the minor of $R_p(r)$ in the nuclear Slater determinant.

The asymptotic behaviour for the particle radial wave function is:

$$R_p(r, k) \sim \sin \left[k_p r - \eta_0 - \frac{\ell\pi}{2} + \delta_\ell \right] \quad (3.3)$$

where η_0 is the Coulomb phase and δ_ℓ the phase shift.

Performing the integrations one obtains (B166):

$$\begin{aligned} \Psi_{\text{asyp}}(r, k) = \sum_{ph} \varepsilon_{ph}(k_p^0) & \left(D_{ph}(k_p^0) \sin \left[k_p^0 r - \frac{\ell\pi}{2} + \delta_\ell - \eta_0 \right] \right. \\ & \left. + \pi \tilde{X}_{ph}(k_p^0) \cos \left[k_p^0 r - \frac{\ell\pi}{2} + \delta_\ell - \eta_0 \right] \right) \end{aligned} \quad (3.4)$$

where ε_{ph} stands for the product $\left[\frac{2\mu}{\pi k_p^2}\right]^{1/2} \varphi_c(A-1)$, μ being the particle's reduced mass and $k_p^0 = k_p(\varepsilon_p = \varepsilon_h + E)$ the particle wave number. The function $D_{ph}(k_p^0)$ is until now arbitrary and for each choice of its values we get a different RPA solution and consequently a different total wave function $\psi_{choice}(r, k)$. By choosing

$$D_{ph}^{p'h'} = \delta_{ph,p'h'} \quad (3.5)$$

we specify a basis for the right hand side and correspondingly a basis set for the wave function $\psi_{asymp}^d(r, k)$. The wave function obeying the correct asymptotic conditions shall be given by a superposition of such solutions ψ_{asymp}^d

$$\begin{aligned} \psi_{asymp}^d(r, k) = \sum_c \varepsilon_c(k_c^0) \left(D_c^d(k_c^0) \sin \left[k_c^0 r - \frac{\pi}{2} + \delta_c - \eta_0 \right] \right. \\ \left. + \pi \tilde{\chi}_c^d(k_c^0) \cos \left[k_c^0 r - \frac{\pi}{2} + \delta_c - \eta_0 \right] \right) \end{aligned} \quad (3.6)$$

or, in terms of incoming (I_c) and outgoing (O_c) waves:

$$\begin{aligned} \psi_{asymp}^d(r, k) = \sum_c \frac{i}{2} \left(e^{-i\delta_c} \left[\delta_{dc} - i\pi \tilde{\chi}_c^d(k_c^0) \right] I_c(k_c^0 r) \right. \\ \left. - e^{i\delta_c} \left[\delta_{dc} + i\pi \tilde{\chi}_c^d(k_c^0) \right] O_c(k_c^0 r) \right) \end{aligned} \quad (3.7)$$

Imposing on the solution the required asymptotic form

$$\psi_{asymp}^d(r, k) \sim I_d(k_d^0 r) + \sum_c S_c^d O_c(k_c^0 r) \quad (3.8)$$

we get a new basis $\{\varphi(r)\}$ which satisfies the correct boundary conditions,

$$\varphi_c^d(r, k) = I_c^d - e^{2i\delta_c} \sum_{de} \left[(1 - i\pi \tilde{\chi}_c^d)^{-1} \right]_{cd} \cdot \left[(1 + i\pi \tilde{\chi}_c^d) \right]_{de} O_e^d, \quad (3.9)$$

where

$$\phi^c(r, k) = \frac{2}{i} e^{i\delta_c} \sum_d \left[(1 - i\pi \tilde{X})^{-1} \right]_{cd} \psi^d(r, k) \quad (3.10)$$

$$S_d^c = e^{2i\delta_c} \sum_e \left[(1 - i\pi \tilde{X})^{-1} \right]_{ce} \left[(1 + i\pi \tilde{X}) \right]_{ed} \quad (3.11)$$

1 denotes the unity matrix, $\tilde{X}$ the matrix composed of all the $\tilde{X}_{cd}$ over open channels and S_{dc} is an element of the scattering matrix. From its form we can already recognize $-\tilde{X}$ to be the reaction matrix which is hermitian,

$$K = -\tilde{X}^* . \quad (3.12)$$

Thus, obtaining eq. (3.10), we just derived a relation between the wave functions directly obtained by solving the RPA equations with the choice defined by eq. (3.5) and those with the correct asymptotic conditions. In the same way we can now express the transition densities which obey the correct asymptotic conditions.

By definition

$$\rho_J(r) = \langle \psi | \hat{\rho} | 0 \rangle \quad (3.13)$$

Using eq. (3.10) we get

$$\rho_J(r) = \langle \psi | \hat{\rho} | 0 \rangle \cdot \left[(1 - i\pi \tilde{X})^{-1} \right] \cdot \Delta^- \quad (3.14)$$

with

$$(\Delta^-)_{cd} = e^{i\delta_c} \delta_{cd} \quad (3.15)$$

Therefore

$$\rho_J(r) = \tilde{\rho}_J(r) \left[(1 - i\pi \tilde{X})^{-1} \right] \cdot \Delta^- \quad (3.16)$$

Here the vector notation merely indicates the whole set of possible channels, e.g. $\{\varphi_{c_1}, \varphi_{c_2}, \varphi_{c_3}, \dots, \varphi_{c_K}\}$ and $\tilde{\rho}_J(r)$ is defined by

$$\tilde{\rho}_J(r) = \sum_{\mu} \tilde{A}_{\mu} j_J(q_{\mu} r) \quad (3.17)$$

$\tilde{A}_{\mu}$ are the form factors which are the solutions of eq. (2.35) with the choice defined by eq. (3.5).

In the same way we can express the form factors with the correct asymptotic conditions as

$$A_{\mu} = \tilde{A}_{\mu} \left[(1 + i\pi \tilde{X})^{-1} \right] \cdot \tilde{A}^{-1} \quad (3.18)$$

and

$$B_{\mu} = \tilde{B}_{\mu} \left[(1 + i\pi \tilde{X})^{-1} \right] \cdot \tilde{A}^{-1} \quad (3.19)$$

Eqs. (3.10), (3.18) and (3.19) allow us to express any physical observable in terms of the quantities with the tilde. This is a great advantage concerning the numerical applications.

The set $\{\varphi^c\}$ of wave functions with the correct asymptotic conditions describes the excited states of the nucleus lying in the continuum in the frame of the 1p-1h RPA. This set is already normalized because the imposition of the correct asymptotic conditions in the continuum case is equivalent to determining the normalization. However, among all excited states of the nucleus we shall also find those which are bound. For these states we still need to define a normalization. In order to find it, consider the norm of an arbitrary state $\langle \psi^{J,n} |$:

$$\langle \psi^{J,n} | \psi^{J,n} \rangle = N^{-2} \quad (3.20)$$

Substituting here the definition of an excited state (eq. (2.8)) we get

$$N^{-2} = \langle 0 | \sum_{cc'} \int d\epsilon d\epsilon' \left[\left[X_c^*(\epsilon) X_{c'}(\epsilon') - Y_c^*(\epsilon) Y_{c'}(\epsilon') \right] \right. \\ \left. \left[\zeta_c(\epsilon) \zeta_{c'}^*(\epsilon') - \zeta_c^*(\epsilon) \zeta_{c'}(\epsilon') \right] \right] | 0 \rangle \quad (3.21)$$

where

$$\zeta_c^+(\epsilon) = (a_p^+ b_h^+)_{Jc} \quad \text{and} \quad \zeta_c(\epsilon) = (b_h a_p)_{Jc} \quad (3.22)$$

Using the quasi boson approximation, for particles in the continuum we have:

$$[\zeta_c^+(\epsilon), \zeta_{c'}^+(\epsilon')] = \delta_{cc'} \delta(\epsilon - \epsilon') \quad (3.23)$$

and for bound particles:

$$[\zeta_c^+, \zeta_{c'}] = \delta_{cc'} \quad (3.24)$$

we finally obtain

$$\langle \Psi^{J,n} | \Psi^{J,n} \rangle = \sum_{\substack{\text{bound} \\ \text{states}}} (X_c^2 - Y_c^2) + \sum_{\substack{\text{continuum} \\ \text{states}}} \int d\epsilon [X_c^2(\epsilon) - Y_c^2(\epsilon)] \quad (3.25)$$

This defines the norm for the bound states:

$$N = \frac{1}{\sqrt{\langle \Psi^{J,n} | \Psi^{J,n} \rangle}} \quad (3.26)$$

4. Two Particle-Two Hole Level

4.1 Overview

The theoretical developments presented so far are based on coherent superpositions of one particle-one hole states:

$$Q_n^+ = \int_0^\infty d\epsilon \sum_{ij} \left[\chi_{ij}^{(n)}(\epsilon) (a_i^+(\epsilon) b_j^+) + \gamma_{ij}^{(n)}(\epsilon) (b_j a_i(\epsilon)) \right] \quad (4.1)$$

If one wants to go further, one must take into account 2p-2h contributions (Ro70, Sa62, FS62, TU64). This can be made by a direct generalization of eq. (4.1) as:

$$Q_n^+(2p-2h) = \sum_{\alpha} \eta_{\alpha}^{(n)} Q_{\alpha}^+ + \sum_{\alpha\beta} \eta_{\alpha\beta}^{(n)} Q_{\alpha}^+ Q_{\beta}^+ \quad (4.2)$$

However, when doing this, one raises the dimension of the RPA matrix by many orders of magnitude. In order to be able to make calculations, one is obliged to make drastic truncations in the p-h space.

A more suitable formalism has been developed by Dehesa et al. (DK77, De77), the so-called core coupling RPA. The basic idea is to work in a basis of quasi particles. Quasi particles are here defined as a dressed single particle, this means as a single particle containing many 2p-1h admixtures.

Here, the operator that creates an excited state can be expressed in terms of quasi particle-quasi hole amplitudes (X_{mi}, Y_{mi}) and quasi particle and quasi hole creation operators α_m^+ and β_i^+ , respectively, as:

$$Q_n^+(2p-2h) = \sum_{mi} \chi_{mi}^{(n)} (\alpha_m^+ \beta_i^+) + \gamma_{mi}^{(n)} (\beta_i \alpha_m) \quad (4.3)$$

and

$$\alpha_m^+ = c_m a_m^+ + \sum_{\mu\nu} c_{\mu\nu}^{(m)} Q_{\mu}^+(1p-1h) a_{\nu}^+ \quad (4.4)$$

$$\beta_i^+ = c_i b_i^+ + \sum_{\mu\nu} c_{\mu\nu}^{(i)} Q_\mu^+ (1p-1h) b_\nu^+ \quad (4.5)$$

The main advantage of this formalism relative to the older ones is that it treats the phonon as an entity (taking the RPA solutions in 1p-1h level as an input for the problem). For these components one constructs the new set $\{a^+ \otimes Q^+\}$, creating a $(1p-1h + 2p-2h)$ matrix which has to be diagonalized.

This procedure reduces the number of particle hole states to be treated by many orders of magnitude making the problem numerically handable.

As an illustration one can quote from (De77) that in ^{208}Pb taking all important low-lying phonons (2^+ , 3^- , 4^+ , 5^-) one ends with a p-h matrix whose dimension is less than 400 in all practical applications.

Dehesa et al. took into account with their ansatz for the RPA state, eq. (4.3), the so-called self-interaction graphs for particles and holes (cf. fig. 36), but not those corresponding to the induced interaction (fig. 36).

Although this may be a good approximation for higher multipolarities, it may present problems for the monopole (and dipole states), when calculating the location and width of the fragmented state. The reason is that the contribution of the induced interaction to the mass operator (as it can be seen in electron gas) should have a sign contrary to that of the selfenergy part, leading to a (partial) cancellation.

The reason why it should be more significant for lower multipolarities lies simply in the Clebsh-Gordan coefficient associated to the induced interaction term, which becomes always smaller the larger the multipolarity is.

A further drawback is the inclusion only of the lowest phonons. In several cases it might be also a reasonable approximation, but one should expect for

the higher lying states a significant contribution coming from the giant resonances.

The formalism of Dehesa et al. can give us a reasonably good description of the spreading width. On the other hand, in order to have a good description of the experimental widths, one must take into account the continuum contributions. One shall see that the higher the state lies the more important is the role played by the escape width. Therefore one should expect that the experimental width can only be reproduced by a 2p-2h calculation that includes the continuum contribution.

The problem is that although numerically feasible the core coupling RPA represents already a mighty numerical effort and inclusion of continuum in its formalism is still an open problem. The problem is partly due to the explicit construction of a 2p-2h wave function.

However, if one is interested only in the location (centroid) and width of a state, one can think of developing a formalism specially appropriate for the calculation of these quantities.

The width of a state (and also its shift concerning the unperturbed one) is determined by its mass operator. Therefore, if one is able to calculate the mass operator, even without having the explicit knowledge of the wave functions, one is able to get the information needed (WM82).

4.2 Two Particle-Two Hole Formalism

The width and also the energy shift of a state can be determined from its mass operator without having the explicit knowledge of the 2p-2h wave functions.

Many calculations have been made in that direction with varying degree of complexity (BV79, SS77, BB78, WM82, WF82) and in somewhat different contexts.

In all those calculations a discretized continuum has been taken. However, one knows that especially for those higher lying states, like the giant resonances, a correct treatment of the continuum may bring large corrections (cf. sections 10 and 11) and be essential to a successful comparison with experiments.

In the following we shall extend the formalism developed by Wambach et al. (WM82, WF82) to include the full continuum. We firstly review shortly the basis of theory.

We start from the full nuclear polarization propagator given by eq. (2.3) and, considering that one uses a short range interaction, we can write it in terms of only two spatial coordinates:

$$i \pi(\vec{x}, \vec{x}') = \langle \psi_0 | T \{ \hat{\psi}^\dagger(\vec{x}) \hat{\psi}(\vec{x}) \hat{\psi}^\dagger(\vec{x}') \hat{\psi}(\vec{x}') \} | \psi_0 \rangle \quad (4.6)$$

which can be alternatively written in the shell model representation using

$$\hat{\psi}^\dagger(\vec{x}) = \sum_{\alpha} \varphi_{\alpha}^*(\vec{x}) a_{\alpha}^{\dagger}(t) \quad (4.7)$$

with $a_{\alpha}^{\dagger}$ a creation operator in Heisenberg representation and φ_{α}^* denotes a Hartree-Fock wave function. We then obtain:

$$\pi_{\alpha\beta\gamma\delta}(t, t') = \langle 0 | T \{ a_{\alpha}^{\dagger}(t) a_{\beta}(t) a_{\gamma}^{\dagger}(t') a_{\delta}(t') \} | 0 \rangle \quad (4.8)$$

and

$$i \pi(\vec{X}, \vec{X}') = \sum_{\alpha\beta\gamma\delta} \varphi_{\alpha}^{+}(\vec{X}) \varphi_{\beta}(\vec{X}) \varphi_{\gamma}(\vec{X}')^{+} \varphi_{\delta}(\vec{X}') i \pi_{\alpha\beta\gamma\delta}(t, t') \quad (4.9)$$

Furthermore we define the Fourier transform of eq. (4.8) with respect to $t-t'$ (since π is a function only of the time differences) to be

$$\pi_{\alpha\beta\gamma\delta}(t-t') = (2\pi)^{-1} \int d\omega e^{-i\omega(t-t')} \pi_{\alpha\beta\gamma\delta}(\omega) \quad (4.10)$$

Similarly to the Dyson equation that one can write down for the single particle Green's function one can write a Bethe-Salpeter equation for the particle-hole polarization propagator (cf. FW71, chapter 15) in the following (matrix) form:

$$\pi(\omega) = \pi^0(\omega) + \pi^0(\omega) K(\omega) \pi(\omega) \quad (4.11)$$

We made the assumption that the proper particle-hole kernel K depends only on ω , otherwise one would have an additional integration over the frequency (Cz61, Sc71, Sc76, RS80).

We want now to use our knowledge of the solutions of the 1p-1h RPA in order to single out the contributions coming from the 1p-1h $\leftrightarrow$ 2p-2h space.

As seen in section 2, the Bethe-Salpeter eq. (4.11), where $K(\omega) = V_0$ (independent of ω), reduces identically to the FB-RPA equations. Therefore we use as an Ansatz for the ph-irreducible interaction:

$$K(\omega) = V + \Delta K(\omega) \quad (4.12)$$

V obeys now the following Bethe-Salpeter equation:

$$\pi^{(1)}(\omega) = \pi^{(0)}(\omega) + \pi^{(0)}(\omega) V \pi^{(1)}(\omega) \quad (4.13)$$

Here V is taken to be the residual interaction, $\pi^{(0)}$ is the shell model propagator, and the solutions of eq. (4.13) are therefore the 1p-1h RPA solutions already found.

With a little algebra one shows that eq. (4.11) can now be written as

$$\pi(\omega) = \pi^{(1)}(\omega) + \pi^{(1)}(\omega) \Delta K(\omega) \pi(\omega) \quad (4.14)$$

Note that until now no approximation has been made; eqs. (4.13) and (4.14) together are identically equal to eq. (4.11).

We can now write down the p-h propagator in the spectral representation (FW71)

$$\pi_{\alpha\beta\gamma\delta}^{(1)}(\omega) \equiv \pi_{\alpha\beta\gamma\delta}^{(RPA)}(\omega) = \int dE \frac{X_{\alpha\beta}(E) X_{\delta\gamma}^*(E)}{\omega - E} - \frac{X_{\delta\gamma}(E) X_{\alpha\beta}^*(E)}{\omega + E} \quad (4.15)$$

Here the integral over E is to be understood also as a sum over the discrete and integral over the continuum eigenstates of the exact nuclear hamiltonian $\{E, E_1, \dots, E_N\}$

$$X_{\alpha\beta}(E) = \begin{cases} X_{\alpha\beta}(E) & \alpha > \epsilon_F \quad \beta < \epsilon_F \\ Y_{\alpha\beta}(E) & \alpha < \epsilon_F \quad \beta > \epsilon_F \end{cases} \quad (4.16)$$

Eq. (4.15) is actually a shorthand way of writing the continuum dependence which we shall treat in detail later.

We can write down the first iteration of eq. (4.14):

$$\begin{aligned} \pi^{(1)}(\omega) \Delta K(\omega) \pi^{(1)}(\omega) &= \int dE dE' \sum_{\substack{\alpha, \beta, \\ \gamma, \delta}} X_{\alpha\beta}(E) \cdot \frac{1}{\omega - E} \\ &\cdot \left[X_{\beta\alpha}^+(E) \cdot \Delta K_{\alpha, \beta, \gamma, \delta}(\omega) \frac{1}{\omega - E'} X_{\gamma, \delta}(E') \right] X_{\delta\gamma}^*(E') \end{aligned} \quad (4.17)$$

+ terms coming from the ground state correlations.

The next higher term in the iteration of eq. (4.14) shall then be:

$$\begin{aligned} & \pi_{\nu}^{(1)}(\omega) \Delta K(\omega) \pi^{(1)}(\omega) \Delta K(\omega) \pi^{(1)}(\omega) \\ &= \sum_{\substack{\alpha, \beta, \gamma, \delta, \\ \mu, \nu, \eta, \varphi}} \int dE dE' dE'' \chi_{\alpha\beta}(E) \frac{1}{\omega-E} \left[\chi_{\beta\alpha}(E) \Delta K_{\alpha\beta\gamma\delta}(\omega) \right. \\ & \quad \left. \cdot \frac{1}{\omega-E'} \chi_{\gamma\delta}(E') \chi_{\mu\nu}(E') \Delta K_{\mu\nu\eta\varphi}(\omega) \frac{1}{\omega-E''} \chi_{\eta\varphi}(E'') \right] \chi_{\delta\gamma}(E'') \end{aligned} \quad (4.18)$$

+ g.s. correlations.

Let us keep only the diagonal part of $\Delta K(\omega)$ in the RPA eigenstates $|E\rangle$. The term in brackets is exactly the contribution corresponding to the collective mass operator (to the iteration order considered) and can be seen to build up a series, symbolically written as:

$$\sum_{N=1}^{\infty} [(N)] = 1 + \frac{\chi \Delta K \chi}{\omega-E} + \left(\frac{\chi \Delta K \chi}{\omega-E} \right)^2 + \left(\frac{\chi \Delta K \chi}{\omega-E} \right)^3 + \dots \quad (4.19)$$

The collective mass operator in the diagonal approximation is defined to be:

$$\Sigma(\omega, E) = \langle E | \Delta K(\omega) | E \rangle \quad (4.20)$$

The geometrical series above can be summed up resulting for the propagator of eq. (4.14):

$$\pi_{\alpha\beta\gamma\delta}(\omega) = \int dE \left[\frac{\chi_{\alpha\beta}(E) \chi_{\delta\gamma}^*(E)}{\omega-E-\Sigma(E, \omega)} - \frac{\chi_{\delta\gamma}(E) \chi_{\alpha\beta}^*(E)}{\omega+E+\Sigma(E, \omega)} \right] \quad (4.21)$$

We got therefore a compact equation for the particle hole propagator in terms of the shell model wave functions χ and of the collective mass operator Σ .

Σ is a complex function and in general includes p-h unlinked and p-h linked diagrams shown in fig. 36, where the first two terms represent the selfenergy insertions on a particle (hole) line, and the third one plays the role of a correction to the residual interaction, an "induced interaction".

The explicit equations for $\Sigma(\omega, E)$ are shown in the appendix, section 17, in Fourier-Bessel formalism. Actually the whole set of equations developed here leading to the final propagator expression could have been written down from the very beginning in Fourier-Bessel form, as it is easy to be seen from section 2.

Our model for Σ is that the dominant mechanism is the coupling of (1p-1h) RPA states to 2p-2h intermediate states, whereby the particle (or hole) undergoes a scattering with a phonon. Since the latter is also described by a p-h superposition we express, ultimately, ΔK in terms of the RPA-residual interaction V .

For those states which are discrete one must make an analytical continuation of the propagator to the complex energy plane by adding a quantity $i \Delta/2$ which is to be taken small in comparison to the width Γ of the resonance to be studied.

The idea here is to remember the large pole density that we must deal with. The pole density grows very rapidly, being of the order of 10^3 2p-2h states/MeV in ^{208}Pb at giant resonance energies. Of course, the energy averaging parameter should be kept larger than the average pole spacing $\Delta\epsilon$. Therefore we get the condition $\Delta\epsilon \ll \Delta \ll \Gamma$. We shall return to this point later.

According to eq. (4.21) we are now able to calculate the full particle hole propagator π . However, in order to have the nuclear response to some external perturbation, it suffices to know the imaginary part of π , as we know for instance from the inelastic electron scattering, cf. (FW71).

From eq. (4.21) remembering that $\Sigma(E, \omega)$ is complex we can take the imaginary part of the energy denominator and define

$$P(\omega, E) = \frac{1}{2\pi} \frac{\text{Im}[\Sigma(\omega, E)]}{(\omega - E - \text{Re}[\Sigma(\omega, E)])^2 + \frac{1}{4} \text{Im}[\Sigma(\omega, E)]^2} \quad (4.22)$$

$P(\omega, E)$ is the general distribution function depending on the excitation energy ω and on the variable E describing the (1p-1h) energy distribution of the unperturbed phonon.

It is normalized as

$$\int_{-\infty}^{\infty} d\omega \int_{-\infty}^{+\infty} dE P(\omega, E) = 1 \quad (4.23)$$

It is clear that one must also consider the contribution coming from the second term in eq. (4.21) with the fully positive energy denominator. However, it will play a secondary role in the determination of the phonon distribution function, since the value of this denominator shall be much larger than that of the first one, where one has partial cancellation. Moreover, it will be always connected to the ground state correlation terms, which are significantly smaller than the forward going graphs.

We shall furthermore define

$$\Gamma(\omega, E) = 2 \text{Im} \Sigma(\omega, E) \quad (4.24)$$

and

$$\Delta E(\omega, E) = \text{Re} \Sigma(\omega, E) \quad (4.25)$$

We can interpret these two quantities to be the (energy dependent) spreading width and energy shift respectively.

The three last quantities defined above ($P, \Gamma, \Delta E$) still have the E -dependence. For analyzing and comparing with experimental values we shall deal with the quantities integrated over E_ρ , i.e. keeping only the ω -dependence.

In this case $P(\omega)$, for instance, shall determine the probability of finding a specific phonon at the energy ω .

Knowing $\text{Im } \pi(\omega, E)$, we can immediately write down the structure function for a certain operator

$$\hat{F}(q) = \sum_{i=1}^A F(q\vec{r}_i) \delta(\vec{r} - \vec{r}_i) \quad (4.26)$$

to be (FW71, WF82):

$$S_F(q, \omega) = \int dE \langle J(E) | \hat{F} | 0 \rangle \cdot P(\omega, E) dE \quad (4.27)$$

where $J(E)$ denotes any excited state of momentum angular J .

Of course, for each $\hat{F}(q)$ one can determine a (q -dependent) sum rule for the strength $S_F(q, \omega)$.

$S_F(q, \omega)$, also called structure factor, contains the full information about the nuclear response to an external operator $\hat{F}(q)$ and is therefore directly related to the differential cross section for the process.

5. The Residual Interaction

When theoretically studying giant resonances we are interested in a first step in determining their basic properties, namely energy and transition probability. Moreover one must be able to describe the isoscalar ($T=0$) and the isovector ($T=1$) branches for a given multipolarity.

A reasonable theory should not only describe one special resonance in a specific nucleus but must be able to reproduce its mean energy and width as a function of the mass number A . Experimentally, these quantities show a characteristic dependence such as $E \cdot A^{1/3} \sim \text{constant}$.

Therefore, one should have a universal force in the sense that as few parameters as possible should be present and that those parameters should not change with A or with the multipolarity of the state to be studied. This universality should be present at least over a reasonable range of energies, multipolarities and values of the mass number A .

Those conditions are for sure not fulfilled for forces of multipole multipole type. On the other hand, forces based on a realistic description of the nucleon nucleon interaction (e.g. G-matrix calculations) have not been developed enough in order to permit a general application.

Descriptions based on Skyrme interactions used in a selfconsistent calculation should lead to reliable results if one could be able to find an unambiguous description of the ground state properties (KK77). Unfortunately, this is not the case in the actual stage of selfconsistent calculations.

The closest fulfillment of the main requirements above is achieved for the forces derived in the frame of the theory of finite Fermi systems (Mi65, Sp77).

Here one deals with an effective particle-hole interaction, which is non-local and has a finite range. If one forgets about non-central contributions (like the tensor forces), then the particle-hole effective force can be written in a general form as

$$F^{ph}(\vec{r}_1, \vec{r}_2) = (f_0 + f'_0 \vec{\tau}_1 \vec{\tau}_2 + g_0 \vec{\sigma}_1 \vec{\sigma}_2 + g'_0 \vec{\sigma}_1 \vec{\sigma}_2 \vec{\tau}_1 \vec{\tau}_2) \cdot F(\vec{r}_1, \vec{r}_2) \quad (5.1)$$

The parameters f_0 , g_0 , f'_0 , g'_0 are density dependent (only in an infinite medium they would turn out to be pure constants) and the density dependence is taken to be a Fermi function:

$$F(\rho) = F^{ext} + (F^{int} - F^{ext})\rho(r) \quad (5.2)$$

being F any element of the set $\{f_0, g_0, f'_0, g'_0\}$ and

$$\rho(r) = [1 + \exp(r-R/a)]^{-1} \quad (5.3)$$

R and a must be approximately given by the nuclear radius and diffuseness and in this sense A -dependent. This is also expected since the density dependence of the Landau parameters represents the fact that the effective interaction must depend on the nuclear density at the point where the interaction takes place.

The relation between the parameters above and those directly connected to p-p, n-n and p-n interactions is given by

$$f_{ext(int)}^{pp} = f_{ext(int)}^{nn} = f_{ext(int)} + f'_{ext(int)} \quad (5.4)$$

$$g_{ext(int)}^{pp} = g_{ext(int)}^{nn} = g_{ext(int)} + g'_{ext(int)} \quad (5.5)$$

$$f_{ext(int)}^{pn} = f_{ext(int)}^{np} = f_{ext(int)} - f'_{ext(int)} \quad (5.6)$$

$$g_{ext(int)}^{pn} = g_{ext(int)}^{np} = g_{ext(int)} - g'_{ext(int)} \quad (5.7)$$

This set of parameters has been for the first time successfully adjusted to the excitation energies and electromagnetic transition probabilities in ^{208}Pb as well as isotopic shifts, momenta and transition probabilities in the neighbouring nuclei already in the early seventies (RB73, RS73, RS78, cf. also the review SW77).

A large number of calculations attests that the force above leads to a very good description not only of the low-lying states but also of the giant resonance region (cf. BK77, SW77, SW81, SG82 and references therein).

In this work we use the set of parameters adjusted by (RS78) to investigate the properties of ^{208}Pb and of ^{16}O . The set of force parameters used can be found in table 4.

Since our main interest in this work concerned the giant resonances of natural parity, finite range terms of the force play almost no role and we worked with the short range interaction of ref. (RS78).

6. Transition Probabilities

For any physical probe used in the nuclear study one can associate an operator, e.g. to photons an electromagnetic operator. Having such an operator one can obtain the nuclear response to an excitation for a certain energy and momentum.

For the photon as a nuclear probe things become simple since we have the momentum which is fixed ($k = E/\hbar c$) and we are then able to calculate the transition probability for a certain energy.

For an inelastic electron scattering one is able for each energy to vary the transferred momentum, since

$$q_{\mu}^2 \geq 0 \quad \text{or} \quad k^2 - \frac{\omega^2}{c^2} \geq 0 \quad (6.1)$$

is the only condition we have.

In studying photo-absorption, one can start from basic principles (interaction of a radiation with a nucleus) (BW62) and derive the form of the multipole operator.

Usually this is done in the so-called long wave approximation (LWA), where basically during the derivation one approximates the spherical Bessel functions which appear by the first term of the ascending series (AS65) and perform the resting derivation obtaining for the static part in the quantum mechanical case:

$$Q_J = \sum_i \frac{e}{2} (1-\tau_3^i) r_1^J Y_{JM}(\Omega_i) \quad (6.2)$$

If one performs the derivation without using the LWA it yields

$$\hat{K}_J = \sum_i \frac{e}{2} (1-\tau_3^i) K(k_i r_i) Y_{JM}(\Omega_i) \quad (6.3)$$

with

$$K_J(kr) = (J+1)j_J(kr) - kr j_{J+1}(kr) \quad (6.4)$$

Both functions can be expressed in terms of their ascending series and we have

$$K_J(kr) = \frac{k^J r^J}{(2J+1)!!} \left[(J+1) - \frac{(kr)^2}{2J+3} \left(\frac{J+1}{2} + 1 \right) + \frac{k^4 r^4}{2(2J+3)(2J+5)} \left(\frac{J+1}{4} + 1 \right) + \dots \right]$$

or

$$K_J(kr) = \sum_{n=0}^{\infty} \frac{(-1)^n [J+2n+1]}{n! 2^n (2J+2n+1)!!} (kr)^{2n+J} \quad (6.5)$$

It is clear that taking only the first term of this expansion we retrieve, except for the factor $\frac{k^J (J+1)}{(2J+1)!!}$ the usual r^J -dependence of the LWA. The higher the multipolarity the better the approximation turns out to be in a reasonable energy (k) range.

In case of having a monopole ($J=0$) state, then the operator $K_J(kr)$ reduces to

$$K_J(kr) = \cos kr \quad (6.6)$$

If in this case one again expands it in an ascending series getting:

$$K_J(kr) = 1 - \frac{1}{2!} (kr)^2 + \frac{1}{4!} (kr)^4 - \dots \quad (6.7)$$

The corresponding LWA-operator is $Q_{J=0}^{(1)}(r) = 1$.

Now, since one believes that for a monopole state (which is a compression mode) the radial integral of its transition density must vanish, the transition probability turns out to be

$$B \sim \left| \int \rho_{0+}(r) \cdot 1 \cdot r^2 dr \right|^2 \sim 0 \quad (6.8)$$

The second term of the expression for $K_J(kr)$ is then taken, namely

$$Q_{J=0}^{(2)}(r) = - \frac{(kr)^2}{2} \quad (6.9)$$

Actually one usually lets the $k^2/2$ factor be treated later and simply takes

$$Q_{J=0}(r) = r^2 \quad (6.10)$$

as the monopole operator.

It is clear that $k^2/2$ times this operator shall give the same nuclear response as the $K_J(kr) = \cos kr$ if

$$\begin{aligned} \text{a) } \frac{(kr)^4}{4!} &<< \frac{(kr)^2}{2!} \quad \text{and} \\ \text{b) } \left| \int \rho_{0+}(r) r^2 dr \right| &= 0 \end{aligned} \quad (6.11)$$

The first condition is well fulfilled at the energy region we are interested in. The second, however, is only valid for an extreme collective state like the GMR. For all other states (of the same multipolarity), condition (b) can be very far from being fulfilled and the absolute value of the transition probability for those non-strongly collective states can be wrong (in the LWA) by a factor of 3 or 4.

For the monopole, the only reliable operator is the full one:

$$K_0(kr) = \cos kr \quad (6.12)$$

The physical reason is that (according to eq. (6.8)) the monopole cannot be excited in photonuclear reactions; one needs electron or hadron scattering to do it. In both cases the Bessel function operator $K_J(kr)$ gives either a good (hadron) or an exact description (electron scattering).

If one is interested in the full structure function of the nuclear system one must also run over the q -dependence of the excitation operator to get

$$S(q,k) = \sum_n |\langle n(k)_J | K_J(q) | 0 \rangle|^2 \quad (6.13)$$

This accounts for the full information about the response of the nuclear system to an operator $K_J(q)$. It is, for instance, the case of the results of high resolution inelastic electron scattering.

It is also clear that depending on the spatial form and momentum range of the operator one will be able to probe a different aspect of the nuclear structure.

The well known " r^J " form for the multipole operator is suitable for studying the collective surface modes, but its functional form must be altered in order to be able to study compression modes or even to compare the different amounts of surface and compressional strength "seen" by a physical probe (e.g. hadronic scattering).

The reduced transition probability for a process induced by an operator M_J for a given multipolarity J from the ground state $|0\rangle$ to an excited state n is given by

$$B(J,E) = \sum_c |\langle n_J^c(E) | \hat{M}_J | 0 \rangle|^2 \quad (6.14)$$

Here c denotes all the open channels at a given energy E .

Knowing the solutions of the RPA for any energy one can write down $B(J)$ in terms of the RPA amplitudes or directly in terms of the nuclear transition density (or its Fourier transform, the form factor).

For an operator

$$\hat{M}_J = \sum_i M(r_i) Y_{JM}(\Omega_i) \quad (6.15)$$

we can write the transition probability in terms of the wave functions directly obtained by solving the RPA equations with the choice (3.5) as:

$$B_J(M) = \langle \tilde{n} | \hat{M} | 0 \rangle \left[\langle \tilde{n} | \hat{M} | 0 \rangle \right]^* \quad (6.16)$$

$$B_J(M) = \sum_{c, c', c''} \langle \tilde{n}^{c'} | \hat{M} | 0 \rangle U_{c', c}^+ U_{c c''} \langle 0 | \hat{M} | \tilde{n}^{c''} \rangle \quad (6.17)$$

with

$$U_{\tilde{n}} = \left[(1 - i\pi \tilde{X})^{-1} \right] \cdot \Delta^- \quad (6.18)$$

and Δ^- defined by eq. (3.15).

Using the results of section 15 and definition (2.27) we write eq. (6.17) as

$$B_J(M) = \sum_{a, b, c} \sum_{\mu, \eta=1}^N F_{\mu}^J \tilde{A}_{\mu}^a U_{ac}^+ U_{cb} \tilde{A}_{\eta}^b F_{\eta}^J \quad (6.19)$$

with

$$F_{\mu}^J = \int_0^{\infty} M_J(r) j_J(q r) r^2 dr \quad (6.20)$$

In eq. (6.19) the summations over a, b and c are to be understood over open channels only.

In the special case of photoabsorption

$$M_J(r) = M_J(kr) = (J+1)j_J(kr) - (kr)j_{J+1}(kr) \quad (6.21)$$

and we can write down the photoabsorption cross section for a given multipolarity J to be:

$$\sigma_J^{\text{phot}}(E) = \frac{8\pi^3}{k} \frac{e^2}{J(J+1)} B_J(EJ, 0 \rightarrow J) \quad (6.22)$$

where $B_J(EJ, 0 \rightarrow J)$ is the transition probability for the operator

$$M_J(kr) = (J+1)j_J(kr) - kr j_{J+1}(kr) \quad (6.23)$$

and $k = \frac{E}{\hbar c}$.

In the long wave limit, $\sigma_J^{ph}(E)$ reduces to its more familiar form:

$$\sigma_{ph}^{LWA}(E, J) = 8\pi^3 \frac{e^2}{[(2J+1)!!]^2} \frac{J+1}{J} k^{2J-1} \left| \int r^{J+2} \rho_J(r) dr \right|^2 \quad (6.24)$$

or in terms of the Fourier-Bessel formalism:

$$\sigma_{phot}^{LWA}(E, J) = 8\pi^3 \frac{J+1}{J} \frac{e^2}{[(2J+1)!!]^2} k^{2J-1} \cdot \quad (6.25)$$

$$\cdot \sum_{\mu, \nu=1}^N \sum_{\text{open channels}} P_{\mu}^J(k) \hat{A}_{\mu}^{a+} \hat{A}_{ac}^U \hat{A}_{cb}^U \hat{A}_{\eta}^b P_{\eta}^J(k)$$

with

$$P_{\mu}^J(k) = \int r^{J+2} j_J(q_{\mu} r) dr \quad (6.26)$$

In the case of the dipole we project out the spurious state in the usual way:

$$\hat{A}_{\sim}^{eff} = \frac{eN}{N+Z} \hat{A}_{\sim\mu}^{\pi} - \frac{eZ}{N+Z} \hat{A}_{\sim\mu}^{\nu} \quad (6.27)$$

7. Numerical Considerations

In connection with the Fourier-Bessel expansion method the first question concerns the number of terms needed in the expansion to get the physics of the problem well described (FL72).

In another context, namely, in the analysis of experimental data on inelastic (e,e')-scattering, first estimates have been made by Borysowiak and Hetherington (BH73) and Friar and Negele (FN67) who found that the number of expansion terms M should be of the order of 10.

We note that the optimal value of M is directly connected with the value of the cutoff radius R and the maximum momentum transfer $q_{\max}$ (FN67). A value of R which is too small implies a loss of physical information. A too large value implies a larger expansion basis, just in order to fit a function which is equal to zero (the vanishing tail of the transition density).

In tables 1 and 2 we show convergence tests performed for the lowest 3^- , and for the 5_2^- and 5_3^- states in ^{208}Pb within the framework of the discretized Fourier-Bessel RPA. For the collective 3_1^- state, as well as for almost pure one-particle states (the $5_{2,3}^-$), we can see the fast convergence of the method. With 8 to 10 coefficients we already obtain a reasonable precision. With about 12 coefficients final convergence is reached for the energies, as well as for the more sensitive BE-values.

The same features are encountered in the tests for the continuum Fourier-Bessel RPA. For instance, we show in fig. 1 the E1-photoabsorption cross section in ^{16}O plotted for various numbers of expansion coefficients. With 6 coefficients the essential features are roughly reproduced, and from 6 to 8 coefficients an extremely fast convergence takes place. All curves corresponding

to $8 < M < 15$ lie in between $M=8$ and $M=14$, which are shown explicitly. After $M=10$, the curves become undistinguishable, showing that final convergence has been obtained.

In order to attain the best description of the sharp GMR peaks in ^{208}Pb , the Fourier-Bessel RPA code was run with double precision.

Great care has also been taken in the precise description of the peak with a fine energy mesh in order to get the best possible final values for the multipole strength in comparison with the classical sum rules.

The whole important giant resonance region has been scanned in steps of 10 keV or even finer (under the monopole and quadrupole peaks in ^{208}Pb we used a 2 keV resolution).

For ^{208}Pb two different single particle model spaces have been used, both generated by a Saxon-Woods potential, the parameters of which have been adjusted by Rinker and Speth (RS78). These are given in table 5. The first one (space I) is similar to the model space of (RS78). It includes 73 single particle states which means that all $3h\omega$ ph-configurations are taken into account, leaving out the $1s_{1/2}^{\pi,\nu}$, $2s_{1/2}^{\nu}$, $1p_{1/2}^{\pi,\nu}$, and $2p_{1/2}^{\pi,\nu}$ holes, however. All bound particle states, as well as 34 continuum states up to the $k_{15/2}^{\pi}$ and $k_{17/2}^{\nu}$ waves are taken into account. The (proton) waves with first resonance peaks sharper than 8 keV were considered as discrete states. The Landau-Migdal interaction of (RS78) was adjusted in this space; its parameters are given in table 4.

The second model space (II) was constructed so as to exhaust all coupling possibilities up to the 8^+ state. In other words, we work now in a complete particle-hole space and our model space spans the complete single particle basis for these multipolarities. It is important to note that with a discretized continuum

it is impossible to have a complete p-h model space. Only taking into account the full continuum one is able to achieve it.

The smaller space (I) contains already almost all important particle-hole components, yet does not completely exhaust the sum rule strength. Therefore we performed all calculations in space II and occasionally discuss which qualitative features are missing in space I.

For ^{16}O we choose the Woods-Saxon potential of (KB74) which reproduces fairly well the known experimental single particle energies around the Fermi surface. The parameters of this potential well are given in table 6.

8. Transition Densities

A number of calculations have already been made aiming to describe the nuclear transition densities. For that phenomenological models (Ta56, BT75, WU66, SJ50), sum rule methods (KF70, No71, DF73) and microscopic calculations, mainly in the framework of RPA (TW73, RS74, SW77, RS78, BT75) have been employed.

From the simple phenomenological (hydrodynamical) models one gets a transition density whose radial dependence is essentially given by $j_J(kr)$ (Steinwedel-Jensen model (SJ50)) or by $r^{J-1} \partial \rho_0 / \partial r$ (Tassie model (Ta56)), where J is the multipolarity of the vibration and ρ_0 is the unperturbed (spherical) nuclear density.

Using the expression for the classical sum rule for an operator $\hat{F}$ and the assumption that the whole strength for a given multipolarity J is exhausted by only one (collective) state, one is able to derive an expression for the transition density $\rho_J(\hat{F})$. These forms are exactly identical to the hydrodynamical models referred to above (for the monopole instead of the Tassie model one must take a variation of it, the Werntz-Überall model (WU66)).

This means in all these cases one gets a transition density which (except for $J=0$) is surface peaked and presents no structure in the internal region. However, the experimental measurements of the transition densities, especially through inelastic electron scattering, which have been highly improved in the last years, show in many cases a wiggly behaviour in the interior.

Even the most recent microscopic calculations (e.g. (RS78)), although giving a very good description of the overall behaviour, are not able to fully reproduce the experimental results (He80, PH82, LH79, PH78, PL80).

This discrepancy between experiment and the best existing theories probably has to do with a failure of mean field theories, which do not introduce enough correlations into the nuclear ground state. Phenomenologically, a depletion of the proton $3s_{1/2}$ level removes the discrepancy in the case of the 3_1^- state at 2.6 MeV in ^{208}Pb .

In the present context, we may investigate to which extent the limited model spaces used in several calculations are responsible. One can then study the effect of extending the model space on the transition density. Here, the continuum RPA is an excellent tool since, if we work in a complete p-h space, where all the possible particle-hole coupling possibilities are taken into account, then this corresponds in the usual RPA with discretized continuum to taking all the different $\kappa\omega$ -contributions from $0\hbar\omega$ to infinity. One could then expect to be able to improve the description of the inner behaviour of the transition density.

Fig. 2 compares the transition density obtained in the model space II with the experimental data. Although the high-lying continuum waves do influence the shape of the transition density in the interior, due to their large- q components, our calculation clearly illustrates that one has to go beyond the conventional RPA to fully understand the collective 3_1^- state in ^{208}Pb .

Among the effects that could account for the remaining discrepancies, one should quote: inclusion of higher (n-particles n-holes) configurations, meson exchange currents, relativistic effects and core polarization (PL80). Heisenberg (He80) pointed out that a strong depletion of the proton $3s_{1/2}$ orbit could account for a part of the discrepancy, since this particle state gives rise to the enhancement of the innermost lobe.

In figs. 3 and 4 we show the transition densities for the lowest lying 2^+ and 4^+ states, respectively. In the 2^+ -case, where experimental data exist, we obtained fair agreement. As a general trend we note that the theoretical transition densities are shifted to slightly larger r -values compared to the experimental ones. This is very likely due to the force used which has been adjusted originally in a discretized single particle space. It is obvious that the radial shape of these single particle wave functions is different from an exact continuum wave. These differences are especially large for the high lying waves. If now a full continuum calculation is made, the continuum-continuum contributions are overestimated, especially in the surface region. A new adjustment of the whole set of force parameters would eliminate this shift. Such an analysis, however, is not the aim of the present work. In addition one has to bear in mind that a zero range force is a too great simplification at least as far as the spin-isospin channel is concerned (SK80). Use of a finite range force would bring effectively a cutoff for the higher momenta in the particle-hole propagator affecting the influence of the high-lying continuum waves at the surface region.

The states analyzed above are typical examples of extremely collective surface vibrations, the so-called low-lying phonons or capillarity waves. Their surface feature is obviously shown by their sharp peak near the nuclear surface.

However, a large amount of information can be obtained from high-lying states and very especially from those states whose transition density is innerly peaked and reasonably collective. Such states correspond to volume oscillations, local compressions of the nuclear matter. From these compression modes direct information about the compressibility constant of nuclei can be obtained. The microscopic analysis of such states is made in section 11.

We want here only to remind that from the very idea that such modes represent a (local) compression of the nucleus, one must infer also the existence of a region where the density is rarefied. Therefore the density variation (transition density microscopically) must present a node. Since the mode is volume peaked, the node must be near the surface.

In fig. 5 we show the transition density for the monopole giant resonance (breathing mode) which is the lowest lying compression mode. It is an isoscalar volume oscillation, as expected with one node to the surface. The isovector admixture in this state can be seen as the dotted line and is negligible. Once more the main difference to the hydrodynamical models is given by the wiggles in the interior which are essentially caused by the shell structure (compare for instance with (WU66)).

The transition density for the first isoscalar dipole compression mode at 21.8 MeV is shown in fig. 6 (squeezing mode). It confirms that the state is a collective volume oscillation of the density with one node at the nuclear surface at 6.5 fm.

We show also in fig. 7 the transition density for the first isoscalar quadrupole compression mode (at 30 MeV), which is very similar to the transition densities for the well established (first) monopole and dipole isoscalar compression modes.

Even from a very simple liquid drop model one sees that for each multipolarity one could have (in principle) many modes, characterized by its energy. This means, in terms of standing waves, characterized by the number of nodes in the transition density. We show in fig. 8 the transition density for the second isoscalar monopole compression mode at 28 MeV. The transition density as expected has two nodes.

CHAPTER II - ANALYSIS OF GIANT RESONANCES

9. Qualitative Considerations

One of the most interesting and current questions in the physics of the giant resonances concerns the possibility of finding concentrated strength of multipolarities other than the dipole.

In the particle hole descriptions of the giant resonances one recognizes that the collectivity of the state is to be provided by the particle hole residual interaction. It depends, therefore, on which specific particle hole states one has to compose the collective one. On the other hand, the question of concentration of the collective state depends crucially on the concentration of the component single particle states. In other words, it would be improbable to have giant resonant states with a small escape width if the single particle states which compose the GR are very broad.

If one looks at the natural width of single particle states, one notes that because of the large Coulomb barrier the widths of the proton single particle states in heavier nuclei are narrower than those of lighter nuclei. In addition, the centrifugal barrier gives also rise to relatively narrow neutron resonances for heavy nuclei, if those states are not too high up in the continuum.

One can then generally infer that in heavier mass nuclei it is more likely to find concentrated isoscalar strength of higher multipolarity ($L > 2$) and relatively narrow isovector quadrupole and monopole resonances.

In fig. 9 some examples of single particle scattering cross sections are shown. In the case of ^{208}Pb we found that even for very high partial proton waves (like the m- and n-wave) the width is still quite narrow

(~ 0.5 MeV) which will give rise to narrow collective states. On the other hand, the width of the corresponding neutron states is already extremely wide.

In our microscopic model a collective state is a coherent superposition of proton and neutron particle hole states. In many cases this leads to the mixing of very broad neutron resonances with narrow proton states.

Test calculations with the continuum code showed that the continuum-continuum coupling, mediated by the propagators and by the solution of the RPA equations themselves, introduces a strong mixing between the various particle-hole channels. One obtains e.g. a widening of a sharp peak even by the introduction of one single broad wave.

Already from this qualitative picture one may have only little hope to find concentrated strength for higher multipoles, even in heavy mass nuclei.

Let us now proceed in studying the natural parity giant resonances with our formalism. We shall firstly analyze the surface modes and then the volume ones.

10. Surface (Vibrational) Modes

Short time after the first indication of the giant dipole resonance (BK47, BK48) one was able to describe it as an (isovector) density fluctuation (GT48, SJ50). Moreover this extremely collective state, as well as the later observed GQR (PW71, PW72, BG72, FT72, NT72) were very soon recognized to be a surface vibration of the nuclear matter, and correspondingly (roughly) described by macroscopic models.

However, if one wants to go beyond an approximate description of the gross features of the GMR, one must work within a sound microscopic model.

In the following we present the analysis of the most important multi-polarities for $^{16}_0$ (1^- , 2^+ , 3^-) and ^{208}Pb (1^- , ..., 7^-) for the surface modes paying special attention to the question quoted before about the existence of concentrated strength in a 1p-1h picture.

10.1 The $^{16}_8\text{O}$

$^{16}_0$ is one of the best studied nuclei, from the experimental as well as from the theoretical point of view. Since there exist already 1p-1h continuum calculations for this nucleus (BH67, RM67, MS67, MS69, MS81, SB75, Bi71, Bi72, KB74, KS74, BT75), we used it as a first test of our code.

However, there are several interesting questions that are worth-while to be followed in $^{16}_0$, since we have a very accurate calculation, for instance, the question about the width of the octupole giant resonance.

In addition, we also investigated the modification which arose from a rather extended integration over the single particle continuum. The most

ambitious calculations so far included it only up to 30 MeV, a limit which we surpassed by a factor of 10.

Former continuum calculations usually fitted the giant dipole resonance (BH67, BT75) in order to reproduce reasonably well the location of one of the main peaks but not the splitting. Krewald et al. (KB74, KS74) were able to reproduce quite well the location of the dipole, but again a splitting could only be achieved with very strong spin force parameters.

In the present calculations we take the same particle-hole interaction as used for the ^{208}Pb , changing only slightly the overall force parameter and the nuclear radius and diffuseness in order to reproduce the position of the lowest 3^- bound state without any fit to the GDR or use of absorptive potentials.

10.1.a The dipole

Experimentally one finds the GDR to be split in two main peaks at 22.3 and 24.0 MeV as well as some secondary peaks dispersed from 22 to 27 MeV. From our calculations we get the GDR split also in two main peaks at 24.9 and 26.2 MeV (cf. fig. 10) exhausting from 24 to 28 MeV roughly 60 % of the isovector EWSR (BM75, OC73, cf. also section 16). Over the whole energy range (12.5 to 35.0 MeV) we obtain 108 % of the classical isovector sum rule in good agreement with the experimental data.

The somewhat higher location of the E1-strength is connected with the too strong f'_0 parameter. The present force parameters have been adjusted by Rinker and Speth (RS78) without taking into account the effects of the dynamical theory of Brown, Dehesa and Speth (BD79).

For each multipolarity we present the strength distribution (BEJ/MeV) and the percentage exhaustion of the energy weighted sum rule. For the multipole operator

$$\hat{Q}_J = \frac{e}{2} \sum_{i=1}^A (1-\tau_3^i) r_i^J Y_{JM}(\Omega_i) \quad (10.1)$$

with $\tau_3 = -1$ for protons and $+1$ for neutrons we find the classical energy weighted sum rule to be (cf. section 16)

$$S_J^T = \frac{1}{2} \langle 0 | [Q_J^+, [H, Q_J]] | 0 \rangle = \frac{J(2J+1)^2}{4\pi} \frac{(\hbar c)^2}{2mc^2} Z \langle r^{2J-2} \rangle_\pi \quad (10.2)$$

S_J^T corresponds to a sum (integration) over the whole bound (continuum) spectrum for the multipolarity J. This summation we can perform explicitly from the RPA solutions for the bound states

$$S_J^B = \sum_{n=\{\text{bound states}\}} E_n(\text{BEJ})_n \quad (10.3)$$

and over the continuum

$$S_J^C(E_0, E) = \int_{E_0}^E E' \text{BEJ}(E') dE' \quad (10.4)$$

between any two energies E_0 and E .

The percentage exhaustion of the energy weighted sum rule as a function of the upper integration energy is given by

$$\frac{100 S_J^C(E)}{S_J^T} \quad (10.5)$$

and the integration shall be performed from the continuum threshold.

To this quantity we shall usually add the percentage exhaustion of the bound states:

$$\frac{100 S_J^B}{S_J^T} \quad (10.6)$$

The isospin decomposition of the BEJ in its isoscalar and isovector parts and the exhaustion of the corresponding T=0 and T=1 EWSR is also given. The isoscalar multipole operator is defined to be

$$\hat{Q}_J^{T=0} = \frac{e}{2} \sum_{i=1}^A (1 - \langle \tau_3^i \rangle) r_i^J Y_{JM}(\Omega_i) \quad (10.7)$$

and the isovector

$$\hat{Q}_J^{T=1} = \frac{e}{2} \sum_{i=1}^A (\langle \tau_3^i \rangle - \frac{1}{2}) r_i^J Y_{JM}(\Omega_i) \quad (10.8)$$

with

$$\langle \tau_3^i \rangle = \frac{N-Z}{A} \quad (10.9)$$

In fig. 11 we show this percentage exhaustion for the isovector part of the (proton) energy weighted sum rule starting from the energy of the first open channel up to a certain excitation energy E:

$$\frac{100 S_J^C(E, T=1)}{S_J^T(T=1)} \quad (10.10)$$

10.1.b The quadrupole

In fig. 12 the result of our continuum calculation of the giant quadrupole resonance (GQR) is shown. The present result is essentially the same as in an earlier calculation of Krewald et al. (KB74, KS74). The theoretical width of the GQR is too small compared to the experimental one (KW75). The larger width is well understood theoretically (DK77) within a 2p-2h approach. The 2p-2h

configurations mix strongly with the $1p-1h$ configurations which build up the GQR and this gives rise to a fragmentation of the 2^+ strength. From fig. 13 one notices that more than 20 % of the total EWSR (isoscalar plus isovector) lies beyond 36 MeV. Since the narrow resonance is of dominant isoscalar character, one may conclude that an essential part of the isovector strength (~ 40 %) lies above 36 MeV.

10.1.c The octupole

The E3-resonance (fig. 14) is a good example for the qualitative picture presented in section 9. We find in our microscopic calculation that the $g_{9/2}$ state is the main particle component in the $3\hbar\omega$ -excitations. This wave, however, is extremely broad. Therefore, the strength of the giant octupole resonance obtained in our approach is completely smeared out. Between 12.5 and 39 MeV we find only 31 % of the EWSR, cf. fig. 15. This shows that the major fraction is shifted to very high excitation energies, where it gives rise only to the background, thus eliminating any hope of detecting a giant octupole resonance in light nuclei.

10.2 The ^{208}Pb

In light nuclei like ^{16}O it seems impossible, at least in our model, to find concentrated strength for multipolarities higher than E2. Even for the quadrupole, the experimental width is rather large (SW81, BK77). In the following we shall discuss the situation in ^{208}Pb . We shall mainly concentrate on the question whether or not higher multipole resonance ($J>2$) can exist in heavy mass nuclei.

A summary of our results for the surface modes is given in table 3 where the exhausted fraction of the EWSR is shown for bound states as well as for the continuum region. All electric multipole states are included up to E5. In the following subsection we will discuss these results in more detail.

10.2.a The dipole

The giant E1 (isovector branch) is found in our calculation as to be fragmented into three comparable peaks, fig. 16 (bottom), at 10.8, 11.6 and 12.1 MeV. The centroid energy is at 12.1 MeV, which is slightly lower than the experimental value of 13.5 MeV (Ha79). The experimental width is 4 MeV.

In order to analyze those structures in terms of their isospin, we made the explicit projection into an isoscalar ($T=0$) and isovector part ($T=1$), for all states in ^{208}Pb . The dipole plays in this case a special role, since the surface isoscalar vibration corresponds to a motion of the nucleus as a whole, being therefore completely spurious. In fig. 16 (upper) one sees the proton response (BE1-value) to the operator $\hat{Q} \cdot \mathbf{r}^J$ quoted before (which will be used for all the surface giant resonances). Here are the isovector and the (spurious) isoscalar mixed and this is the reason why one cannot (and should not!) obtain 100 % exhaustion of the proton EWSR. This can be seen from fig. 17, where we show the percentage exhaustion of the EWSR, for proton (upper), isoscalar (center) and isovector components (bottom).

In the outer picture of fig. 16 we show the isoscalar response to the dipole operator. As quoted above, since it is spurious and in the effective RPA we get rid of the spurious state just by forcing it to have a negative energy, one should have in the central part of fig. 17 a zero exhaustion of

the isoscalar dipole EWSR. We get indeed only 4 % over the whole energy range (8 to 60 MeV). This tells us that the error in our projection and in the numerics is less than 4 %.

The bottom part of fig. 17 shows the exhaustion of the isovector EWSR as a function of the energy. It should be noted here that such a kind of plot of the data is extremely useful in order to recognize where concentrated strength is to be found.

For instance, the central picture of fig. 17 tells us that no isoscalar concentrated strength is to be found (the curve is completely flat over the whole region). On the other hand, the bottom picture shows an abrupt growth of the percentage exhaustion of the EWSR, telling us that a large amount of strength is to be found around 12 MeV. In other words, a (nearly) zero-slope region indicates absence of strength and the larger the slope the more concentrated is the strength. The absolute height of the curve tells us the fraction of the sum rule exhausted to this point.

From the ratio of momenta of the strength distribution one can learn about the width of the distribution. The variance σ_k of a distribution, defined to be

$$\sigma_k = \left[\frac{m_{k+1}}{m_{k-1}} - \left(\frac{m_k}{m_{k-1}} \right)^2 \right] \quad (10.11)$$

is connected to the width. The various momenta are defined by:

$$m_k(E_1, E_2) = \int_{E_1}^{E_2} \omega^k BE(J, \omega) d\omega \quad (10.12)$$

where E_1 and E_2 are any two energies and $BE(J, \omega)$ is the usual transition probability for a multipolarity J and energy ω .

If the distribution were Gaussian, the width would be given by

$$\Gamma = 2.35 \sigma_k \quad (10.13)$$

However, the formula can be used, for suitable distributions, at least as a guideline.

In the case of the isovector dipole giant resonance one obtains a width of 1.2 MeV (calculating the different moments between 11 and 14 MeV) and the centroid at 12.1 MeV.

The width we calculate in a 1p-1h RPA is only the escape width. To this value one must add the (spreading) width coming from the contribution of higher configurations (mainly 2p-2h). Therefore, one should not expect to describe the experimental width including only one particle one hole configurations.

The centroid of the distribution, as referred to above, lies about one MeV below the experimental value (Ha79). However, the dynamical theory of Brown, Dehesa and Speth (BD79) points out that the peak energy of the giant dipole resonance can only be reproduced, if all second order diagrams (with respect to the effective interaction) are taken into account. A random-phase calculation based on an interaction derived from a G-matrix should seriously underestimate the energy of the giant dipole resonance. Since we are mainly interested in the decay widths of the giant resonances in the present context, we used the effective interaction of Rinker and Speth (RS78), which was fitted to reproduce the giant dipole resonance in a discretized space. In the energy region from 11 to 15 MeV, the present calculation exhausts 70 % of the isovector sum rule (see fig. 17 bottom), the rest of the strength is distributed up to 30 MeV excitation energy.

10.2.b The quadrupole

The quadrupole excitation spectrum of ^{208}Pb is shown in fig. 18, decomposed into proton, isoscalar and isovector excitations. The giant quadrupole resonance (GQR), found experimentally at 10.9 MeV (ST77, YM76, LB72, Be76, HB77, YR77, BS79, BB79, SW81, HH79, LB79, YK81, CC80) with a width of 2.6 MeV, comes in the present model at 10.2 MeV as a single narrow peak. The isospin decomposition clearly shows that this state is predominantly an isoscalar state, exhausting 70 % of the isoscalar EWSR (see fig. 19). The isoscalar strength is already completely exhausted below 20 MeV. The isoscalar part of the bound states contributes 15 % to the EWSR, being in fig. 19, the intercept of the curve at 8 MeV. The inclusion of 2p-2h states bring significant contributions concerning the total width and shall be discussed in sections 12 and 13. However, its effect concerning the sum rule exhaustion is only a smoothening of the steep growth around 10.3 MeV and does not change the total exhaustion over say 4 MeV around the main peak. This question shall be discussed in more detail in section 13.2.

The isovector GQR is found at 22 MeV with a decay width of 5 MeV in excellent agreement with recent results of Grenoble and Darmstadt which find this state at 21.5 and 24.0 MeV respectively. It exhausts 65 % of the isovector sum rule between 19 and 24 MeV (see fig. 19). A comparison with discretized calculations (RS78, DK78) shows that now about 20 % more strength is found below the isovector peak. This difference is easily understandable, since we now work in a complete ph-space. As expected, the continuum contribution should be especially significant for highly excited states.

10.2.c The octupole

Fig. 20 shows the BE3 strength of ^{208}Pb , obtained in a complete ph-space. In contrast to the quadrupole strength, the octupole strength is seen to be very much fragmented. Therefore the sum rule plot (fig. 21) is of great help to assess where concentrated sum rule strength may be found. The dominant isoscalar octupole state above the particle threshold is located at 17.9 MeV. It exhausts 21 % of the isoscalar EWSR between 17.4 and 18.4 MeV. The many fragments between 17 and 21 MeV add up to 63 % of the isoscalar EWSR in this energy region. Beyond 23 MeV less than 10 % of the isoscalar EWSR is found. The width of this distribution, calculated from the momenta under Gaussian assumption between 17.5 and 22.0 MeV is 2.4 MeV. Therefore the suggestion of Bonin et al. (Bo81) that L=3 strength might dominantly contribute to the state seen at 24.8 MeV using 120 MeV/nucleon α -particles has to be questioned.

The nuclear continuum in the energy range from 8 to 60 MeV contributes 72 % of the isoscalar EWSR, to which 24 % from the low-lying bound states have to be added.

Three experimental groups have recently looked for high-lying octupole strength. Using 800 MeV protons, Carey et al. (CC80) found an isoscalar L=3 state at 19.7 MeV with a width of approximately 5.3 MeV. This state, however, contributed only 20 % to the EWSR. Morsch et al. (MR80) found a state at 17.5 MeV exhausting 60 % of the sum rule using 172 MeV α -particles, while Yamagata et al. (YK81) found a state at 20.5 MeV exhausting 78 ± 15 % of the EWSR with a 110 MeV ^3He beam. Given the large widths of the high-lying octupole strength, the peak energies seem to be in reasonable agreement with the present calculation. The value of (CC80) for the sum rule exhaustion remains to be understood. The different isospin character of the proton and the α -particle can only partially

be held responsible for this discrepancy, as is illustrated in fig. 20 (upper) and 20 (center). Recently, Zhalov (Zh81) proposed background subtraction to be responsible for this discrepancy. Simulating the background subtraction of (CC80), in his theoretical cross sections derived from Skyrme RPA calculations, he claims that about 50 % of the strength has been erroneously subtracted. This would explain the discrepancy between (CC80) and other experimental results.

The isovector giant octupole state is centered at 28 MeV and has a width of 4 MeV. It exhausts 60 % of the isovector EWSR between 24 and 32 MeV. Since one has to add the spreading width to the decay width obtained in the present calculation, it is probably fair to say that a direct experimental observation of this state is very unlikely. Nevertheless, its contribution to the general background in this energy region has certainly to be taken into account.

10.2.d The hexadecapole

For the hexadecapole (see fig. 22) the use of space I (73 single particle states) or space II (a complete p-h state) brings some significative changes. (The figures presented here correspond to the complete space II.) For space I one gets a total exhaustion of the proton EWSR of 70 % from 8 to 60 MeV, with no significative structures, except a tiny state at 10.7 MeV.

Calculations with the complete p-h space show a sharp peak at 10.7 MeV exhausting 16 % of the isoscalar EWSR. Beyond 40 MeV less than 10 % of the isoscalar E4-strength is to be found.

This result is consistent with other theoretical calculations (KS74, RS78, DK77) and confirms former fits to experimental data for (α, α') (MS80, HH79, JM80), (p, p') (W079) and (d, d') (MS80). The best fit was obtained by including

20 % E4 strength in the GQR region. This hexadecapole contribution must be included in any analysis of data in this region.

Approximately 40 % of the isoscalar EWSR are exhausted in the energy region between 24 and 32 MeV (fig. 23). This contribution is mainly built from $4\hbar\omega$ excitations and consequently a considerable spreading width due to e.g. 2p-2h admixtures in the wave function has to be expected. Therefore this concentration of strength should not be called a resonance, yet it is concentrated enough to give a substantial contribution to the background.

The isovector strength in the energy region between 8 and 30 MeV is distributed over a large number of peaks. No concentrated strength is found in the whole spectrum. Even the strength centered around 35 MeV which carries 70 % of the isovector sum rule has a decay width of more than 14 MeV, which makes an experimental detection impossible at the actual stage of experiments.

10.2.e The higher multipoles

The E5 state, whose BE value is shown in fig. 24, shows the typical behaviour already announced: the strength is distributed over several peaks leading to a smooth exhaustion of the EWSR as seen from fig. 25, with no pronounced structures. We note that not only for this curve but also for the corresponding curves for the sum rule exhaustion for even higher multipoles we have roughly a linear behaviour. This clearly demonstrates that no concentrated E5 strength may be expected.

A recent theoretical prediction (BP79) of a significant fraction of E5 strength centered at 20 MeV, after initial experimental evidence (DH79) has not been confirmed by the 255 MeV ^{14}N inelastic scattering experiment by Garg et al. (GB80). The 6^+ strength shown in fig. 26 shows some concentrated strength in

the 10-11 MeV region, where the giant quadrupole (and hexadecapole) is found. Since this strength contributes less than 10 % of the 6^+ EWSR, it cannot be called a resonance but will contribute to the background of the giant quadrupole resonance.

The 7^- strength completely fragments (see fig. 27) and therefore may only be expected to contribute to the direct reaction background.

11. Compression (Volume) Modes

11.1 Qualitative considerations

From the very early macroscopic (liquid drop) (SJ50, GT48, WU66) models one recognized that besides the possibility of exciting surface oscillations in a nucleus, one can also have volume oscillations (BM75). Such modes involve a compression of the local density.

Solving the wave equation for such a state one immediately sees that the excitation energies of the different modes are connected to the nuclear matter incompressibility (TK81, JJ80, B180, EH81, BL79). This fact lets these states play a special role among all nuclear excitations, since they allow us to infer information about nuclear matter properties.

Under reasonable assumptions one is able, in the frame of macroscopic models, to estimate the energy of the compression modes and from that obtain information about K , the incompressibility modulus.

However, in such calculations one is always bound to (a) the assumption that one has a pure volume oscillation, what can hardly be the case in a real nucleus; (b) the use of suitable forces from which one is able to perform the needed derivation of final relations and of the sum rules.

One is then faced to the need of a sound calculation, with a good force, just as in the case of surface modes, in order to be able to get (as far as possible) a model independent prediction of those states and consequently of the incompressibility modulus.

Moreover, since volume and surfaces modes may mix, one is tempted to search for a less model-dependent definition of a compression mode.

In the following, a nuclear excitation will be called a compression mode, if it does not contribute to the photonuclear sum rule, but might be seen via different excitation mechanisms, such as e.g. inelastic α -scattering. In this sense, a volume oscillation is a compression mode, since its transition density has a node which strongly reduces the electromagnetic transition strength.

If we think in terms of standing waves in a fluid, then the excitation energy of a compression mode is expected to be much larger than the excitation energy of a giant surface mode of the same multipolarity. Therefore one has to expect a much stronger coupling of these states to the continuum which will introduce a large escape width. In addition, a large spreading width due to the decay into np-nh configurations has to be considered. As a consequence the chance of finding concentrated compression mode strength is the largest in heavy nuclei, where the shell spacing is small enough, even though the number of low-lying phonons which contribute to the spreading width is larger.

Indeed, it was in ^{208}Pb that the first compression mode was experimentally found by Marty et al. (M175, FT76, ST77, HB77, YR77), the so-called breathing mode, and also recently the first non-monopole compression mode by Morsch et al. (MR80, DM81).

Using 172 MeV α -particles as a probe they were able to identify an isoscalar dipole state at 21.5 MeV excitation energy, the so-called "squeezing mode", thus confirming microscopic predictions based on the random-phase approximation (RPA) (WK78, GS81).

In this work we intend not only to analyze the well established monopole and dipole compression mode with a basis in the random phase approximation, which is a reliable microscopic formalism to obtain positions, widths and also the transition rates for these states. We also want to present a systematic

way of localizing compression mode strength, in order to find out whether concentrated compression strength of larger multipolarities might be found experimentally. Because of the relatively large excitation energies of compression modes, the coupling to the continuum has to be treated very carefully. In this aspect, the Fourier-Bessel RPA is a very suitable tool, since the formalism allows us to take into account the full one particle continuum (in complete particle hole spaces).

In section 13.1 we shall treat again the giant monopole resonance taking into account the effects of two particles two holes.

11.2 Method

A necessary condition that an excited state $|J^\pi, E\rangle$ may be called a compression mode is that its electromagnetic transition strength

$$B(EJ) = \langle J^\pi, E | \hat{Q}_J | 0 \rangle \quad (11.1)$$

with

$$\hat{Q}_J = \sum_i \frac{e}{2} (1 - \tau_3^i) r_i^J Y_{JM}(\hat{r}_i) \quad (11.2)$$

should vanish. In terms of the radial transition density $\rho_J(r)$ (see eq.(2.14) and (2.15)) this means

$$\int \rho_J^E(r) r^{J+2} dr = 0 \quad (11.3)$$

On the other hand, strength should be seen by different excitation mechanisms, such as inelastic α - and electron scattering. Transition operators representing

specific projectiles are discussed in detail e.g. in refs. (Be69, OW79) and could be embodied in our method (Br82), however, this is not the aim of this work. Since we do not want to concentrate on the specific shape of such an operator, we choose the following set of operators to use:

$$\hat{C}_{J,k} = \sum_i \frac{e}{2} (1-\tau_3^i) r_i^{J+2k} Y_{JM}(\hat{r}_i) \quad k = 0,1,2,\dots \quad (11.4)$$

The idea is that characteristic deviations from the photonuclear response may already be detected by analyzing the strength function

$$B_{J,k}(E) = \left| \int \rho_J^E(r) r^{J+2k+2} dr \right|^2 \quad (11.5)$$

for some few values of k .

In order to analyze the different nuclear responses to the different operators $\hat{C}_{J,k}$ and, at the same time, have an information about the localization of the strength, we study the sum rules corresponding to the operators of eq. (11.4). For each probe $\hat{C}_{J,k}$ we calculate the nuclear strength function $B_{J,k}(E)$ for any excitation energy E , multipolarity J^π and for $k=0,1,2,\dots$. For each $\hat{C}_{J,k}$ one can write a classical EWSR of the form (UT74, No71):

$$\pi S^T(k) = \frac{(\hbar c)^2}{4\pi} \frac{Z}{2mc^2} (2J+1) \left[J(J+2k)^2 + J(J+1) \right] \langle r^{2J+4k-2} \rangle_\pi \quad (11.6)$$

Here $\langle \rangle_\pi$ denotes the ground state expectation value over protons only. One can also write down, for each case, linearly energy weighted sum rule calculated directly from the RPA, as a function of the upper integration limit

$$S_J^C(E,k) = \int_0^E \omega \cdot B_{J,k}(\omega) d\omega \quad (11.7)$$

which is readily the generalization of eq. (10.4).

For ω less than the continuum threshold the integration is to be understood as a sum over bound states.

Now we can construct the function describing the relative exhaustion of the classical EWSR for each upper limit E as

$$S_J(E, k) = \frac{100 S_J^C(E, k)}{S_J^T(k)} \quad (11.8)$$

An increase of the curve $S_J(E, k)$ indicates the presence of strength and its slope tells how concentrated it is. In fig. 28 we demonstrate this effect: e.g., in the upper picture $S_J^{T=0}(E, 1)$ and $S_J^{T=0}(E, 2)$ are compared and in the center and bottom pictures $S_J^{T=0}(E, 0)$ and $S_J^{T=0}(E, 1)$ are compared.

For any interval ΔE , we can directly read from the ordinates the corresponding exhaustion of the classical EWSR in this interval.

In general it is more appropriate to have the isoscalar and isovector strengths displayed separately. In this case we must correspondingly change eq.(11.6), i.e. sum the expectation value of $r^{2J+4k-2}$ over both proton and neutron states and simultaneously change the factor Z into Z^2/A for the isoscalar state and into NZ/A for the isovector state, respectively (see section 16).

In order to recognize location (and width) of possible compression modes we only need to compare $S_J(E, k \neq 0)$ with $S_J(E, k=0)$, see fig. 28. Compression modes should appear as an increase of $S_J(E, k)$ in an energy region where $S_J(E, k=0)$ shows no appreciable raising, or where $S_J(E, k=0)$ is already saturated.

One decisive feature of this calculation is the fact that one is able to take into account the full continuum and, moreover, to exhaust every possible particle hole coupling to a given multipolarity. If one works in a limited

ph-space, one cannot even approximately exhaust the sum rules for the higher compression modes.

For the sake of simplicity we display only the responses to $\hat{C}_{J,0}$ and $\hat{C}_{J,1}$ (for the monopole $\hat{C}_{J,1}$ and $\hat{C}_{J,2}$ respectively). It is however clear that in some cases, especially for the higher multipoles, the higher operators $\hat{C}_{J,k>1}$ can give a dominant contribution. However calculations made with the higher operators show negligible changes on the centroid value. Only the relative amount of exhausted sum rule between $\hat{C}_{J,0}$ and $\hat{C}_{J,k}$ is changed, as expected, since we scan different r-regions of the transition densities. Using inelastic electron scattering as a probe has the disadvantage that isoscalar and isovector modes are excited simultaneously. We feel therefore that α -particles or possibly heavy ions are a more adequate probe to map out isoscalar compression modes. Therefore we concentrated our analysis on the behaviour of the $\hat{C}_{J,k}$ operators (eq. (11.4)).

In the following we shall analyze the most important multipolarities in detail, paying attention here to the question of concentrability of the strength and possibility of experimental observation.

11.3.a The monopole

Since the theoretical possibility of a (monopole) volume oscillation had been suggested, an intense study with a number of probes has been carried out. However, it was only in 1975 that the first sound experimental evidence for the monopole giant resonance has been found. Marty et al. (MM75) suggested that differences in their inelastic deuteron results could be due to a giant monopole resonance located slightly above the giant quadrupole resonance. The

following years (until nowadays) brought steadily new indications and clear confirmations about the location and strength of the giant monopole (ST77, HB77, YR77, BB79, B180, CC80, YK81, MR80, BB79, LB79). In photoabsorption reactions it is impossible to observe a giant monopole resonance, due to the fact that photons have spin one. If one tries to use, theoretically, the multipole operator with $J=0$, to study the monopole, one will get only a negligible result due to the orthogonality between the ground state and the excited states.

By our very definition

$$\hat{C}_{J,k=0} \equiv \hat{Q}_J$$

that means when $k=0$, $\hat{C}_{J,k}$ reduces exactly to the usual multipole operator. However, as discussed above, it will not excite the monopole state. Therefore, we compare, in fig. 28, the percentage exhaustion of the corresponding isoscalar energy weighted sum rule for the operator $\hat{C}_{J=0,k=1}$ and $\hat{C}_{J=0,k=2}$.

The nuclear response to $\hat{C}_{J=0,k=1}$, due to its importance, is also shown separately in fig. 29. Here, like in the case of the surface mode, for a better analysis of nature and concentration of strength, we decompose the different multipolarities in its isospin components.

The upper curve of fig. 29 shows the proton nuclear response to $\hat{C}_{J=0,k=1}$, the center picture the isoscalar, and the bottom picture the isovector response to the same operator.

The upper picture (proton response) has the three most important peaks at 13.6, 13.9, 14.2 MeV and a smaller one at 16 MeV. In addition there is a very broad bump at much higher energies.

Looking at the center picture (isoscalar part, one realizes that the structures around 14 MeV are almost pure isoscalar resonances. This is confirmed by fig. 5 where the transition density for the strongest monopole state is shown. Here we decomposed in its isospin components, and the isoscalar is not only much larger but has also the typical volume structure, as discussed in section 8. The isovector contribution oscillates around zero, giving a negligible contribution when calculating the BE-value.

The ratio of first to zeroth momentum of this distribution defined by eq. (10.12) in one MeV around the main peak gives us a centroid of 13.9 MeV, which is in very good agreement with the recent experimental values of Jülich, $E_{0+} = 13.8 \pm 0.3$ MeV (MR79, MS80, Mo81); Orsay, $E_{0+} = 13.9$ MeV (WM80); Texas A&M, $E_{0+} = 13.7 \pm 0.4$ MeV (YB81) and Grenoble, $E_{0+} = 13.2 \pm 0.3$ MeV (BB79). The three main peaks exhaust 72 % of the classical isoscalar EWSR (cf. section 16) (compare with fig. 30, which shows the exhausted fractions of the corresponding EWSR). This value is to be compared with the experimental values for the percent exhaustion of the isoscalar sum rule: Jülich: $S_{0+} = 85$ %; Orsay: $S_{0+} = 100 \pm 20$ %; Texas A&M: $S_{0+} = 90 \pm 20$ % and Grenoble: $S_{0+} = 92$ %. The references are the same as above.

For the isoscalar response, the classical sum rule value is exhausted at 30 MeV, where also saturation occurs.

The width of each peak is very small compared with the experimental one. However, it must be remembered that we are until now treating only 1p-1h contributions. Therefore, only the escape width is obtained. However, one expects that for the monopole the main contribution comes from the spreading width (DK77). This part represents the coupling of the single particle states to phonons and shall be treated in section 13. We would like to call into question, however, a

recent theoretical attempt to explain the experimental width mainly by the decay width (VS81). Van Giai et al. produce with a selfconsistent Skyrme calculation very large decay widths. This is mainly due to the unperturbed HF single particle states which are too high in the continuum and therefore have an unrealistically high width. The problem with Skyrme forces is that because of the non-uniqueness (KK77) of the parametrization in principle any width can be reproduced.

We note also that the contribution of higher correlations quoted above shall influence strongly the theoretical width found, but will not change appreciably either the position (centroid of the state) or the fraction of the sum rule exhausted by the resonance.

The whole analysis done until now for the monopole corresponds to the state identified through the steep increase of the percentage exhaustion of the EWSR, that means, the full curve in the upper picture of fig. 28. To the dotted curve we shall come later.

In fig. 31 we have the equivalent of fig. 28 for the isovector component. Here is shown, for the monopole, in the upper picture, the response to the isovector part of $\hat{C}_{J,k=1}$ and $\hat{C}_{J,k=2}$. Again, we single out the so-called giant monopole (isovector branch now) in figs. 29 and 30.

In the bottom picture of fig. 29 we have the isovector response to $\hat{C}_{J=0,k=1}$. The isovector giant monopole resonance is about 9 MeV wide and located around 27 MeV.

From the bottom picture of fig. 30 we can see that this resonance exhausts 70 % of the isovector EWSR from 23 to 35 MeV.

There are attempts to identify this state experimentally through a pion-charge-exchange reaction (BJ81) exciting directly the isobar analog of this state which lies much lower (Coulomb barrier) in energy in the neighbour nucleus. An open question is whether 2p-2h (and higher) correlations would smear out that state. Qualitatively, one can think that in the case of high-lying states the escape width might be dominant. We shall come back to this point in the analysis of the dipole compression mode and later.

11.3.b The dipole

The clearest case to be analyzed in detail to see the method at work is that of the isoscalar dipole. The operator $\hat{C}_{1,0} \sim r$ gives rise to no intrinsic isoscalar excitation, since it describes only the movement of the center of mass (protons and neutrons moving together in the same direction). Therefore, its whole contribution is called spurious. The calculated percentage exhaustion of the isoscalar dipole linearly weighted sum rule $S_{J=1}^{T=0}(E, k=0)$ for the operator $\hat{C}_{1,0}$ is only a flat curve reaching no more than 4 % over the whole energy region, as seen in fig.28 (center), full line. Since this result should be exactly zero, we know that the error in the projection of the spurious state added to those of the numerics do not surpass 4 %. On the other hand, for the operator $\hat{C}_{1,1}$ the function $S_{J=1}^{T=0}(E, k=1)$ shows a clear and steep growth around 23.0 MeV, proving the presence of concentrated isoscalar strength not seen with the usual multipole operator (fig.28, center, dotted line). Between 20.0 and 26.0 MeV, the isoscalar dipole exhausts 62 % of the isoscalar linear EWSR. Full saturation is obtained around 50 MeV. We note that for such a highly excited state the contribution of the continuum waves is crucial, because only if we treat the continuum correctly, we can have a complete ph-space. Calculations made in a large discretized ph-space including all the $3\hbar\omega$ and many of the $4\hbar\omega$

contributions show that one can only obtain about 50 % of the EWSR. The reason is to be found in the strong contribution of the high-lying continuum waves, which cannot be correctly taken into account in a discretized calculation.

The centroid of the distribution can be calculated by the ratios of the various momenta defined by eq. (10.12). The many possible centroid energies as well as the various variances are shown in table 7. Just like in the case of the monopole, we shall be referring to the ratio m_1/m_0 when we talk about centroid energy, although any of the other ratios showed in table 7 can be understood as such.

The main reason to show the whole set of ratios m_k/m_{k-1} and $\sqrt{m_k/m_{k-2}}$ is that if the distribution were infinitely concentrated (a δ -function), all those ratios would coincide. Therefore, the closer the different centroid lies, the more concentrated is the distribution in energy; see (BL79).

This additional information shall be very important for us, since many of the states discussed here are not concentrated in one bump but are composed of a series of peaks, like it is the case of the isoscalar dipole compression mode, even though we are now able to define a width for the distribution, by eq. (10.13).

The isoscalar dipole resonance, also called squeezing mode, is seen in fig. 32 as the bump around 23.0 MeV (upper picture). Here the nuclear response to the isoscalar (upper picture) and isovector (bottom one) parts of the operator $\hat{C}_{J=1,k=1}$ is shown. The strength is fragmented over 8 major components, which already individually have a significative decay width due to the coupling to the continuum.

The centroid of the isoscalar dipole strength, summed between 20 and 26 MeV, is 23 MeV. Although fragmented in many peaks, the distribution following the criterion explained above is fairly concentrated, since the values of from σ_{+2} to σ_{-2} are practically constant.

The agreement between our result for the centroid of the resonance and the experimental value of Morsch et al. (MR80) is good, if one takes into account the large experimental width $\Gamma^{\text{exp}} = 5.9 \pm 0.8$ MeV. The same is valid for the results of Djalali et al. (DM82), who claim to see the isoscalar dipole compression mode at (21.5 ± 0.2) MeV with a width of $\Gamma^{\text{exp}} = (5.7 \pm 0.2)$ MeV. If the distribution were Gaussian, its full width at half maximum (FWHM) would be given by $\Gamma = 2.35 \sigma_1$.

If one uses this relation for the isoscalar dipole, one gets from table 7 $\Gamma^G = 3.4$ MeV. This is within 3 % also the result that one could read, directly from fig. 32 for the FWHM, namely 3.5 MeV. The result is not surprising and only shows that the relation above, between Γ and σ_1 , works reasonably good for a concentrated distribution, and we can take it, at least as a guide rule.

Since the values of σ in table CM are constant, within 10 %, for all states discussed here, we shall always compare the width obtained as if the distribution were Gaussian, Γ^G and that directly read from the strength distribution.

That the state discussed above is indeed a compression mode can be seen also from the form of its transition density shown in fig. 6 and discussed in section 8.

Obviously the results of this section cannot yield the full experimental width, since they are based entirely on $1p-1h$ configurations (escape width). There exists a strong admixture of higher configurations ($np-nh$) which give rise to the spreading width. In the present case of the isoscalar dipole, however, one realizes that the escape width is responsible for 60 % of the experimentally found total width of this resonance. This indicates that the isoscalar dipole resonance decay is to a large extent a direct process. This means, the cross section for particle decay may be fairly large. Therefore coincidence experiments (Wa81) could be used for a further investigation of these resonances.

We can now proceed similarly for the other states: from the curve for the percentage exhaustion of the corresponding EWSR we locate the possible concentrated strength in energy and directly read its contribution to the sum rule and its decay width.

From the central picture of fig. 31 we can identify the isovector dipole compression mode to be located around 36.0 MeV which is also the centroid of the distribution, calculated from 28.0 to 48.0 MeV (see table 7). The same information yields fig. 32. There in the bottom picture one sees the isovector response to the operator $\hat{C}_{J=1,k=1}$.

Directly from this picture one can read the FWHM which yields a width $\Gamma = 15.3$ MeV for the isovector dipole compression mode. The variance $\sigma_1 = 5.0$ MeV results in a width $\Gamma^G = 11.8$ MeV.

Within the half-maxima of the distribution one finds 15 % of the isovector EWSR, as can be seen from fig. 31 (center).

Again for this state one may assume that the escape width shall give the largest contribution to the total width of the state.

11.3.c The quadrupole

From the bottom picture of fig. 28, the change in the slope of the dotted curve around 30.0 MeV, where the usual BE2-strength is already saturated (full curve), indicates the isoscalar quadrupole compression mode. The centroid of the distribution is at 30.3 MeV and within an energy range of 5 MeV 10 % of the isoscalar EWSR are found for the operator $\hat{C}_{J=2,k=1}$.

Fig. 33 (upper picture) shows the isoscalar nuclear response to the operator $\hat{C}_{2,1}$ and the first isoscalar quadrupole compression mode is seen to be the bump around 30 MeV. From this picture, or from the bottom picture of fig. 28, one can directly read the FWHM to be $\Gamma = 5.3$ MeV. From table 7 one gets a variance $\sigma_1 = 2.0$ MeV which yields $\Gamma^G = 4.8$ MeV. Both results are consistent, revealing some concentrated strength.

The transition density for the isoscalar quadrupole compression mode is shown in fig. 7 and presents the typical behaviour of a volume mode (see section 8). We remark here that the percentage of strength exhausted by the state refers to the operator $\hat{C}_{2,1}$. For other operators $\hat{C}_{2,k}$ and their corresponding sum rule this percentage may change, reflecting the fact that the set $\hat{C}_{J,k}$ is a good tool in localizing the compression modes, but need not to be the optimal operator for each state. This (optimal) operator must be determined in each case (Be69, Br82).

From fig. 31, bottom picture, the isovector quadrupole compression mode is seen through the change in slope of the response to the operator $\hat{C}_{2,1}$ around 43.0 MeV (dotted curve). In this energy region, the response to the $\hat{C}_{2,0}$ operator (usual BE2-value) is already saturated, as seen from the flat full curve.

The state is seen in fig. 33 (bottom picture) as the broad bump around 43 MeV. The centroid of the distribution is found to be at 43.4 MeV with a variance of 5.2 MeV, leading to $\Gamma^G = 12.3$ MeV. From fig. 33 one gets a FWHM $\Gamma = 17.5$ MeV. Over 17 MeV, we get 14 % of the isovector EWSR for the operator $\hat{C}_{2,1}$.

11.3.d Higher modes and the second monopole

We performed this study also for the 3^- and 4^+ states in ^{208}Pb . Both isoscalar (fig. 34) and isovector strength (fig. 35) of these high multipolarities are already too much spread (more than 20 MeV) and located beyond 50 MeV. In addition to the isoscalar monopole resonance at $E = 13.9$ MeV and the isovector monopole resonance at $E = 27$ MeV, already discussed in section 11.2.a, one may expect even further monopole compression strength.

As discussed before, from a naive macroscopic model one may expect to have many compression modes of one determined multipolarity. It would correspond to different numbers of nodes for the standing waves in the liquid drop. However, microscopically there is little or no hope to have such higher modes of a given multipolarity, since they would lie even higher in energy and must be extremely spread out.

On the other hand, the (first) monopole compression mode (the usual giant monopole) lies still relatively low in energy, and we can think of theoretically looking for its higher companion.

From the upper picture of fig. 28 one sees that the second isoscalar monopole is rather spread. It is indicated by the varying slope of the percentage exhaustion of the nuclear response to operator $\hat{C}_{0,2}$ (dotted curve). In this region, the full curve which corresponds to the $\hat{C}_{0,1}$ operator is already saturated.

That it is indeed a second compression mode can be verified from its transition density, shown in fig. 8. As expected, the isoscalar component (full line) presents two nodes (compare with fig. 5, which corresponds to the giant monopole, the first monopole compression mode).

The centroid of this distribution, as seen from table 7, is at 29.9 MeV. The FWHM taken from fig. 28 is $\Gamma = 11.8$ MeV. The variance calculated between 24.0 and 38 MeV is $\sigma_1 = 3.7$ MeV, resulting in a width $\Gamma^G = 8.5$ MeV.

A second isovector monopole is indicated by the raising dotted curve beyond 45 MeV; however, it exhausts no more than 6 % of the isovector EWSR beyond 45 MeV for the operator $\hat{C}_{0,2}$.

Second compression modes of other multipolarities (if any exist) will lie at energies higher than 60 MeV and must be completely spread out.

CHAPTER III - ANALYSIS OF THE CONTRIBUTION OF TWO PARTICLES TWO HOLES WITH CONTINUUM TO THE GIANT RESONANCES

12. General Considerations

As discussed in the preceding paragraphs, the 1p-1h RPA with continuum gives highly satisfactory results for the location and strength of the giant resonances.

However, considering only 1p-1h contributions only the escape width is described. For the surface modes and the first monopole compression mode, this yields only a fraction of the total (experimental) width, the largest contribution coming from the spreading width (2p-2h correlations and higher).

One believes that the higher correlations must, in most of the cases, play a minor role. Therefore, a sound calculation taking into account the 1p-1h + 2p-2h contribution should be able to describe the experimental data very closely. However, concerning the width, this was not the case for the most well known giant resonances in the discretized 2p-2h calculations recently performed (WM82, WF82, BB82). Here a width ≈ 1.0 MeV was found in ^{208}Pb for the monopole and also for the quadrupole. Bertsch, Bortignon and Broglia (BB82) try to explain this small width obtained theoretically through the cancellation effect between particle, hole and induced graphs. For the case of monopole, calculations of Van Giai and Sagawa (GS81) are quoted, since the referred authors obtain an escape width for the giant monopole of about 2.0 MeV. This width would sum up to the spreading width obtained in (BB82) to give the experimental width of about 3.0 MeV (MR79, MS80, WM80, YB81, BB79).

However, the results of (GS81) are very different from all other continuum calculations for the monopole in ^{208}Pb (KS74, HK82, BT75) and may be due to the single particle spectrum used (which lies high in energy) and to the force needed to obtain a good value for the location of the giant monopole (HK82).

In the following we relate to a calculation made with the 2p-2h formalism of section 4 for the monopole and quadrupole giant resonances in ^{208}Pb .

Since we had already solved the RPA problem (solutions of the continuum FB-RPA), we are able to calculate the coupling matrix elements $\gamma_{\alpha\alpha'}^{J,2}(E)$ and the mass operator $\Sigma(E, \omega)$. Here a $2\hbar\omega$ Woods-Saxon basis was used as model space. Again, as in the case discussed in section 7, some of the sharpest single particle states were taken to be discrete, treating the ϵ -dependence as a δ -function. On the other hand, the inner phonons, contributing to the mass operators (cf. fig. 36), were also taken from a discretized calculation.

Clearly the large continuum effect came from the outer RPA-phonons (or, in other words, from the outer p-h pairs) which were taken with their full continuum dependence. For discrete states and also for those taken to be discrete, one used an averaging energy parameter $\Delta = 0.05$ MeV. This value is small enough to permit us to see any oscillations that are physically significant in the quantities of interest.

The space of RPA-phonons was taken to cover all the range of multipolarities from $J=0$ to $J=9$ for the electrical excitations of natural parity. For each multipolarity the whole spectrum was treated. For discrete states, all p-h excitations were taken into account, since in a limited model space no truncation is needed in order to avoid divergences, even when dealing with a short range interaction (BG81). The space taken in these calculations provide a large phase space, due to the high density of states.

13. Analysis of Results - 2p-2h with Continuum

13.1 The monopole

As quoted in section 12.2.a, the spreading of the monopole giant resonance can be observed from fig. 37, where we show the distribution function $P_{0+}(\omega)$ (once integrated over the E-dependence). The distribution function describes the probability of finding the GMR at an energy ω , being normalized to 1. From this distribution one can calculate the various momenta (according to eq. and also the variance σ (cf. eq. (10.11)). The first momentum gives us the centroid of the distribution

$$E_{0+} = 13.7 \text{ MeV}$$

showing that the shift with respect to the 1p-1h solution is rather small.

The variance of the distribution results in a width

$$\Gamma_{0+} = 2.6 \text{ MeV}$$

which is to be compared with the experimental value (MS80, WM80, YB81, DB79)

$$\Gamma_{0+}^{\text{exp}} = 2.7 \pm 0.3 \text{ MeV}$$

showing a very good agreement.

It is important to remember that in other calculations using only discrete states (WM82, WF82, BB82), only a fraction of the total experimental width was obtained. Even in a (discrete) calculation under conditions as similar as possible to ours (same space, same discrete levels, force, etc.) done by Wambach (Wa82) a width

$$\Gamma_{0+} = 1.1 \text{ MeV}$$

was found, as it can be seen from fig. 38. This shows that the continuum-yields an essential contribution in the case of the giant monopole. This discards the

suggestion of Broglia et al. (BB82) that the lacking width (other than the spreading width) should be found in the escape width for the monopole. It also rules out the possibility of strong thermalization (large contribution of higher excitations as 3p-3h, 4p-4h, etc.).

An interesting information which can be obtained from our results concerns the structure function defined by eq. (4.27). We calculated the structure function for the operator $M(qr)$ given by eq. (6.4) for the monopole the 1p-1h approximation, as can be seen in fig. 39. Here, $S(q, \omega)$, the structure function, is plotted as a function of the transferred momentum q and excitation energy ω . One sees that the structure function presents definite features as a function of the transferred momentum. As a function of the energy, three isolated sharp peaks dominate. We note the scale, which runs over 1 MeV, showing the isoscalar giant monopole in the 1p-1h approximation to be very sharp ($\Gamma \sim 0.1$ MeV).

Now, we calculate the same structure function $S(q, \omega)$ for the same operator $M(qr)$ but in the 2p-2h level, taking into account all the contributions given in fig. 38. This result is shown in fig. 40.

Now, we still have a similar structure as a function of the transferred momentum, having a maximum around 0.85 fm^{-1} . The energy dependence, however, shows now a complicated structure, spread over some MeV, with many (8) comparable peaks between 13.3 and 14.7 MeV.

It is well known that the monopole cannot be excited by photoabsorption. However, what one notes from fig. 40, is that also inelastic electron scattering at very low energies (near the photon print, this means, $q \sim E/\hbar c$) is unable to excited the monopole. We see that (although not exactly equal to zero) the nuclear response at very low momentum transfer values is negligible.

13.2 The quadrupole

In the case of the quadrupole, an outstanding and current problem concerns the fraction of the sum rule exhausted in the giant resonance region, say, between 9 and 13 MeV.

As already quoted in section 1, there is a controversy, essentially due to the results obtained in inelastic electron scattering (Ri80, KM81, Ri81), which can find only 50 % of the values encountered in hadron scattering (MR80, CC80, YK81, DM82).

These measurements, made at low energy, with high resolution ($\Delta E = 38$ keV) find (29_{-8}^{+11}) % of the proton EWSR in the region 8-12 MeV.

Some theoretical calculations tried to justify this effect through the coupling of 2p-2h excitations to the giant quadrupole resonance (KM81, BB80, BB82). According to these authors a large fraction of the quadrupole strength should be shifted to higher energies ($E > 12$ MeV) and strongly fragmented in many sharp peaks. However, one should remember that one would need to explain why hadron experiments obtain results in agreement with the usual theoretical calculations with 1p-1h as well as with 2p-2h RPA's (RS76, RS78, DK77, HK82).

Furthermore, none of the calculations quoted above was able to reproduce the width of the quadrupole giant resonance (DK77, BB82), resulting always in a width which is too small.

Similarly to the case of the GMR, one can argue whether the problem does not lie in the treatment of the continuum employed until now. In our calculation we take into account the continuum as described in sections 4, 12.1 and 13.1, aiming to shed some light on the problem.

The distribution function for the GQR is shown in fig. 42. Here two particle-two hole contributions were taken accounting for the single particle continuum. One should compare this result with a similar one performed with a discretized continuum (Wa82) shown in fig. 41. One assigns a significant spreading of the distribution function. A large spreading compared to 1p-1h results (fig. 29) is to be noted. Here, calculations of the first moment result in a centroid at

$$E_{2+} = 11.0 \text{ MeV}$$

The width obtained from the variance of the distribution function is

$$\Gamma_{2+} = 3.1 \text{ MeV}$$

This value is to be compared with the experimental one (MR80, YB81, WM80, DM82)

$$\Gamma_2^{\text{exp}} = 2.7 \pm 0.3 \text{ MeV}$$

showing a very good agreement. Therefore, consideration of 2p-2h contributions with a correct treatment of the single particle continuum alone is able to explain fairly the experimental location and width of the GQR.

Just as in the case of the monopole, thermalization can be neglected. We have a much larger amount of information at disposition in the structure function and we shall use it in the following analysis.

The full 1p-1h+2p-2h structure function for the giant quadrupole is shown in fig. 44. Comparison with fig. 43 showing the 1p-1h structure function for the giant quadrupole makes clear the importance of the 2p-2h contribution in this case.

Remembering now that the (e.g.) Darmstadt experiments (KM81) are made (in the best case) at an energy of 50 MeV and 165° scattering angle, one can easily see that the momentum transfer lies between 0.22 and 0.45 fm^{-1} . On the other hand, hadron experiments, e.g. α -scattering of the Jülich group (MR80), have a full range of possible momentum transfers, typically, for the example above, between 0.5 and 3.0 fm^{-1} , peaking around 1.0 fm^{-1} .

If one looks now again at the structure function, one sees that the maximum of the curve lies at 0.95 fm^{-1} ; this value is almost exactly the peak value of the hadron experiments quoted above. The region of momenta where the Darmstadt experiments were made corresponds approximately to the mean height of the first maximum.

Therefore, we suggest that the difference in results between hadron and electron scattering experiments is mainly due to the bad momentum matching of the (e,e') experiments with the GQR.

Furthermore, we suggest that an inelastic electron scattering performed at sufficiently large momentum transfers (around 1 fm^{-1}) should obtain results similar to those of hadron scattering, due to the better momentum matching.

From our calculations, in the region between 9 and 13 MeV we get about 65 % of the isoscalar EWSR, confirming other theoretical calculations (RS78, RS76, DK77, HK82) made in the frame of the 1p-1h and 2p-2h RPA theory.

CHAPTER IV - CONCLUSIONS AND LAST REMARKS

14.1 General plan

The main objective of this work was the microscopic analysis of the giant multipole resonances.

In the first chapter we develop the theory. We first derive a new formalism for the RPA equations - the Fourier-Bessel RPA, in the 1p-1h approximation. The single particle continuum is fully taken into account. Then we extend the existing formalism for the inclusion of 2p-2h contributions, deriving the equations for the full p-h polarization propagator. The propagator is expressed in terms of the mass operator in closed form, still allowing us to take into account the full continuum correctly.

In chapter II we apply the formalism to the analysis of giant resonances. In a first part we study only surface modes. We develop then a method for finding possible concentrated volume strength and use it systematically to study the compression modes.

Finally we analyze two outstanding current problems concerning the giant monopole and the giant quadrupole in chapter III. The contribution of 2p-2h excitations with continuum is fully considered in the theoretical description of these states.

14.2 About the method

The Fourier-Bessel RPA revealed to be a very suitable method for the analysis of heavy nuclei, states of higher multipolarity, where usual methods encounter severe numerical limitations and for the inclusion of the single particle continuum.

The basis of the method is the fact that one can express the RPA equations solely in terms of the transition density matrix. This allows us to get a set of equations which do not depend on the model space we work with (number of p-h pairs). It depends only on the number of coefficients of the Fourier-Bessel expansion of the transition density.

As remarked in section 2, the Fourier-Bessel RPA allows us to treat even finite range interactions in large spaces with a reasonable amount of numerical effort. Nowadays the importance of such forces is well established. Subnuclear degrees of freedom (Δ -hole excitations for instance) definitely require the finite range one-pion and rho-meson exchange potentials. Earlier formulations, like the one of Bertsch et al. (SB75, BT75), are restricted to zero range interactions only.

A further advantage is that this expansion is converging fast. Because it is a fast method we are able to treat the full single particle continuum, without the restrictive approximations made until now, even for the heaviest nuclei. The same reason allows to treat 2p-2h contributions with continuum.

14.3 About the main results

The methods presented in this thesis were applied both to discrete and continuum states.

In the case of the surface modes, the main question was whether concentrated strength of large multipolarity may be found. This is of particular interest for planning new experiments which search for strength beyond the well established monopole and quadrupole resonance.

For resonances of large multipolarity the treatment of the continuum is very important. Since the width of single particle resonances decreases with increasing mass number, heavy nuclei are the best candidates for showing concentrated giant resonance strength of large multipolarity.

In the case of ^{208}Pb , we show that the giant octupole (isoscalar branch) lies around 18 MeV. The state is fragmented among many peaks having a centroid at 18.8 MeV and an escape width of 2.4 MeV. It exhausts about 63 % of the isoscalar EWSR.

These results are in good agreement with existing experimental results (MR80, CC80, YK81).

The isoscalar hexadecapole presents some concentrated strength around 11 MeV (16 % of the isoscalar EWSR). This result confirms other theoretical predictions (KS74, RS78, DK77) and experimental fits to data (MS80, W079, HH79, JM80).

For the higher multipolarities no concentrated strength has been found. For these states, the decay width is expected to be somewhat large, and the fragmentation caused by the coupling of phonons to the single particle states forming the collective state is especially large. The interplay of both factors leads to the typical picture of a series of small peaks that characterize the higher multipoles.

On the compression (or volume) modes one should remark, that, as in the case of the surface modes, the location of the peak calculated with the 1p-1h RPA with continuum is in good agreement with available experimental results.

As an example, the giant monopole is calculated to be at 13.9 MeV. Experimental values lie at 13.8 ± 0.3 MeV. Also concerning the exhausted fraction

of the corresponding sum rules one must assign a good agreement with experiments. Our theoretical results indicate about 72 % of the EWSR to lie in the giant resonance region. Experimental results indicate around 90 %. However, one should note the dependence of these results on the extraction of the background, what leads to large error bars, typically of 20 %.

The giant isoscalar dipole state is found at 23 MeV (centroid value), with an escape width of 3.4 MeV. Experimental indications give a location at 21.3 MeV (MR80) and 21.5 MeV (DM82) and a width of 5.9 ± 0.8 MeV (MR80) and 5.7 ± 0.2 MeV (DM82). Agreement between our results and experimental ones is fair, if one takes into account the large experimental width.

An important point to be made here concerns the escape width of high lying states, especially those of compression modes. Here, as seen in the case of the isoscalar dipole, the escape width is dominant, constituting about 60 % of the total experimental width. This suggests the possibility of experimentally observing such states through their decay properties. Due to the large decay width one may expect a fairly large cross section for particle evaporation, as it is the case in light nuclei (Wa81).

The method we developed to systematically search for compression modes is based on the different exhaustion of the corresponding sum rules for different operators for volume states. The method was practically tested in the examples above, and the good results give us confidence for the analysis of other volume modes. We find theoretical indications for an isoscalar quadrupole, an isovector monopole, an isovector dipole and a second isoscalar monopole compression mode.

All those states have a fairly large escape width, to which a spreading width must be added, possibly leading to a further widening of the state. However, the possibility of observing these states through their decay properties

as suggested above and the possibility of employing other kinds of reactions - e.g. π -scattering for the isovector monopole (BJ81) - may lead to new indications in the future. Even higher volume modes have also been investigated, but they show up to be hopelessly spread out.

However, one should not forget that all these contributions must be taken into account when extracting the background of nuclear reactions, since, with variable strength, they generally shall be excited, contributing to the direct reaction background.

14.4 Effect of 2p-2h contributions with continuum

Two case studies were selected due to the present day interest of the problems.

14.4.a The giant monopole

The isoscalar giant monopole resonance, already studied in the frame of the 1p-1h continuum RPA, was reanalyzed taking into account now the 2p-2h contributions.

As a matter of fact for the monopole and quadrupole giant resonances, the escape width can only account for a small fraction of the total experimental width.

1p-1h RPA results give an escape width $\Gamma^\uparrow$ of 0.1 MeV and the experimental width is found to be about 2.7 ± 0.3 MeV (MS80, WM80, YB81, DL79). However, if one treats now the two particle-two hole contributions, taking into account also the single particle continuum, one obtains a width $\Gamma = \Gamma^\uparrow + \Gamma^\downarrow = 2.6$ MeV.

This answers a question by Bertsch et al. (Be71, BB82) whether the escape width should be dominant in the case of the giant monopole, since large cancellations among different contributions to the mass operator cancel the 2p-2h components of the wave function.

Clearly this is not the case, since we still have a small (pure) escape width and get a result which is fully consistent with experimental values when taking into account 2p-2h contributions with continuum.

It appears that a combined treatment of 2p-2h and continuum is important. The main point is that all former calculations were made with discretized continuum, this means, not taking into account the effect of the escape width on the 2p-2h contributions. We see now that this effect is a major one and cannot be neglected.

Calculations that resulted in a large (~ 2.0 MeV) escape width for the monopole should be reviewed (GS81), the problem probably lying in the single particle spectrum used and consequently in the parameters used in the force in order to obtain a good fit to experiments. The non-uniqueness of the choice of the force parameters for Skyrme-like forces is actually a well known problem (KK77, HK82a).

Furthermore, the structure function for the monopole was calculated, allowing new insight into the q-dependence of the giant resonance.

14.4.b The giant quadrupole

As quoted before, the escape width calculated for the quadrupole is much smaller than the experimental one.

From our calculations we get $\Gamma_{2+}^{\uparrow} = 0.1$ MeV, whereas the experimental value is $\Gamma_{2+}^{\text{exp}} = 2.7 \pm 0.3$ MeV (MR80, YB81, WM80, DM82). Again, discretized calculations trying to describe the width of the quadrupole with higher correlations (2p-2h) were not able to achieve it (BB82, WM82, Wa82).

A very recent calculation where all the phonons of natural parity up to $J=9$ in a fairly large model space ($2\hbar\omega$) could account also only for less than half of the experimental width (Wa82). Now, taking into account the continuum

$$\Gamma_{2+} = \Gamma_{2+}^{\uparrow} + \Gamma_{2+}^{\downarrow} = 3.1 \text{ MeV}$$

This result is in good agreement with experimental data and confirms the fact we discussed in the case of the monopole. The effect of the interplay of the natural width of each single particle state brings an important contribution to the total width, allowing us to reproduce experimental data.

A further problem, for which many solutions have been proposed, was still to be solved. It concerns the amount of the EWSR exhausted by the giant quadrupole in the giant resonance region (~ 9 to 13 MeV).

Results of inelastic electron scattering at low energy (~ 50 MeV) (KM81, Ri81) yield a result which is roughly half of the strength found in any kind of hadron scattering (MR80, YB81, WM80).

To tackle this problem we make use of the full nuclear information contained in the structure function, calculated with continuum in the 2p-2h approximation, as seen in fig. 43. Here we recognize the very definite momentum dependence of the nuclear quadrupole response, which is also seen in hadron scattering (Mo82). It gives us a first (secondary) maximum at about 0.5 fm^{-1} and peaks effectively at about 1.0 fm^{-1} .

If we now return to the Darmstadt experiments, which originated the controversy, we see that they are made in a momentum transfer range of 0.22 to 0.45 fm^{-1} . This means, they probe the outer slope of the first (secondary) maximum. Of course, in this case the peak to noise ratio must be smaller than if one sits at the absolute maximum of the giant resonance (relative to the momentum transfer). On the other hand, the very exhaustion of the (q-dependent) sum rules is different depending on the region one probes.

Hadron scattering, due to the distortion and absorption effects, does not have a definite q-transfer like it is the case in inelastic electron scattering. The range of q-transfer in this case goes typically from 0.5 to 3.0 fm^{-1} , with a peak at about 1.0 fm^{-1} . Therefore, hadron scattering is twofold favoured in probing (e.g. the isoscalar quadrupole) giant resonance, since it covers a larger q-range and also has its maximum q-transfer in the region of the maximum of the giant quadrupole resonance (relative to q).

Therefore we may consider also this problem as solved and suggest that inelastic electron scattering at larger momentum transfer yields results very similar to those of hadronic reactions.

14.5 The very last remarks

Using the formalism developed in this work we could make a contribution to the understanding of some current problems. However, many open questions must still be analyzed.

The application of the formalism to other nuclei must be undertaken and a systematic study of the width and strength should be performed. One should

also generalize the formulation of the Fourier-Bessel RPA to treat also deformed nuclei as well as include explicitly finite range interactions.

The wave functions of the continuum Fourier-Bessel RPA are extremely suitable as an input for the analysis of $(e,e'p)$ reactions. Coincidence experiments yield already new information about the microscopic structure of giant resonances and their decay channels. At the moment some theoretical effort is done in order to carry out such a program (KC82).

The Fourier-Bessel RPA may also be a suitable tool for studying the inclusion of subnuclear degrees of freedom. Here one could also avoid the large matrices involved when using the usual RPA formulation in configuration space.

Appendices

15. The Transition Probability Formulae

We start from the transition probability formula

$$B_J(M_J) = |\langle n | \hat{M}_J | 0 \rangle|^2 \quad (15.1)$$

If $\langle n |$ is an RPA state, then

$$\langle n | \hat{M}_J | 0 \rangle = \sum_{1,2} [X_{12} + (-1)^J Y_{12}] \langle 1 | \hat{M} | 2 \rangle \quad (15.2)$$

where 1(2) is a particle (hole) index. Now

$$\langle 1 | \hat{M} | 2 \rangle = \langle 1 | e M(r) Y_J(\Omega) | 2 \rangle = \langle 1 | \hat{M} | 2 \rangle = e \langle 1 | M(r) | 2 \rangle \langle 1 | Y_J | 2 \rangle \quad (15.3)$$

From (Ed57) we can write down

$$\langle 1 | Y_J | 2 \rangle = \frac{1}{2} \frac{[1+(-1)^{\ell_1+\ell_2+J}]}{\sqrt{4\pi}} (2J+1)^{1/2} (k_1 k_2)^{1/2} (-1)^{j_1-1/2} \begin{pmatrix} j_1 & J & j_2 \\ -1/2 & 0 & 1/2 \end{pmatrix} \quad (15.4)$$

and

$$\begin{aligned} \langle n | \hat{M}_J | 0 \rangle &= \sum_{1,2} [X_{12} + (-1)^J Y_{12}] \cdot \int dr r^2 M(r) \cdot R_1(r) R_2(r) \\ &\cdot \frac{1}{2} [1+(-1)^{\ell_1+\ell_2+J}] (2J+1)^{1/2} (k_1 k_2)^{1/2} (-1)^{j_1-1/2} \begin{pmatrix} j_1 & J & j_2 \\ -1/2 & 0 & +1/2 \end{pmatrix} \end{aligned} \quad (15.5)$$

Recalling now definition (2.17) we can write eq. (15.5) as

$$\langle n | \hat{M}_J | 0 \rangle = \int dr r^2 M(r) \rho_J(r) \quad (15.6)$$

and

$$B_J(M) = \left| \int dr r^2 M(r) \rho_J(r) \right|^2 \quad (15.7)$$

16. Sum Rules

In this work we are essentially concerned with the classical EWSR and with the more general formulation of Noble (No71).

The classical EWSR can be easily derived, in the case of a hermitian single particle operator, since it is defined to be

$$S_k = \sum_n (E_n - E_0)^k |\langle n | F | 0 \rangle|^2 \quad (16.1)$$

A general formula can be derived to express the different momenta in terms of chained commutators with the hamiltonian, see for instance (BL79a). However, we shall write down only the case of the linearly weighted sum rule, where (SF74)

$$\sum_n (E_n - E_0) |\langle n | F_{JM} | 0 \rangle|^2 = \frac{1}{2} \langle 0 | [F_{JM}, H], F_{JM} | 0 \rangle \quad (16.2)$$

Now, defining

$$F_{JM}^\pi = \frac{e}{2} \sum_{i=1}^A (1 - \tau_3^{(i)}) r_i^J Y_{JM}(\Omega_i) \quad (16.3)$$

to be the (proton) multipole operator. From that operator one can extract the isoscalar and isovector branches through:

$$F_{JM}^\pi = F_{JM}^\pi(T=0) + F_{JM}^\pi(T=1)$$

with

$$F_{JM}^\pi(T=0) = \frac{e}{2} \sum_{i=1}^A (1 - \langle \tau_3^{(i)} \rangle) r_i^J Y_{JM}(\Omega_i) \quad (16.4)$$

and

$$F_{JM}^\pi(T=1) = \frac{e}{2} \sum_{i=1}^A (\tau_3 - \langle \tau_3^{(i)} \rangle) r_i^J Y_{JM}(\Omega_i) \quad (16.5)$$

and the expected value $\langle \tau_3^{(i)} \rangle$ is

$$\langle \tau_3^{(i)} \rangle = \frac{N-Z}{A} \quad (16.6)$$

For each of these branches we derive a corresponding sum rule:

$$S^{T=0}(J) = \frac{(\hbar c)^2 e^2}{8\pi (mc^2)} \cdot J(2J+1)^2 \frac{Z^2}{A} \langle r^{2J-2} \rangle_{\text{ground state}} \quad (16.7)$$

and

$$S^{T=1}(J) = \frac{(\hbar c)^2 e^2}{8\pi (mc^2)} \cdot J(2J+1)^2 \frac{NZ}{A} \langle r^{2J-2} \rangle_{\text{ground state}} \quad (16.8)$$

For the protons we have

$$S^\pi(J) = \frac{(\hbar c)^2 e^2}{8\pi (mc^2)} \cdot J(2J+1)^2 Z \langle r^{2J-2} \rangle_{\text{over protons of g.s.}} \quad (16.9)$$

and

$$\langle r^{2J-2} \rangle = \frac{1}{\int \rho(r) d^3r} \cdot \int \rho(r) r^{2J-2} d^3r \quad (16.10)$$

The density in the case is the full ground state density for the isovector and isoscalar branches and the proton ground state density for the proton sum rule. For the analysis of the compression modes we need to know the sum rules to other powers of r . Then, for the general form

$$\hat{C}_{J,k}^\pi = \sum_{i=1}^A (1 - \tau_3^i) r_i^{J+2k} Y_{JM}(\Omega_i) \quad (16.11)$$

we can write down the sum rule to be (UT74):

$$S_J^\pi(k) = \frac{(\hbar c)^2}{4\pi} \frac{Z}{2mc^2} (2J+1) \left[J(2k+J)^2 + J(J+1) \right] \langle r^{2J+4k-2} \rangle_{\text{gs} \cdot \pi} \quad (6.12)$$

As before, the expected value is to be understood over the protons of the ground state. For the isoscalar and isovector parts, the same kind of alterations in the formula is needed.

Here, one must consider, in the case of the dipole (isoscalar branch) the subtraction of the contribution coming from the spurious state. This is found to be (De73, HD81):

$$\Delta S_{J=1}^{T=0}(k=1) = \frac{25(\hbar c)^2}{8\pi(mc^2)} Z^2 \langle r^2 \rangle_{g.s.}^2 \quad (16.13)$$

The correct value of the sum rule for the isoscalar dipole being given by

$$S_{J=1}^{\pi(ef.)}(k=1) = S_{J=1}^{\pi}(k=1) - \Delta S_{J=1}^{T=0}(k=1) \quad (16.14)$$

17. Derivation of the Mass Operator

As seen in section 4, the mass operator $\Sigma(\omega, E)$ shall have three different terms indicated in fig. 36.

We shall develop now the Fourier-Bessel equations for each of these terms. We start with the so-called induced interaction term. We can write (for one of the possible choices of χ):

$$\begin{aligned} \Sigma(E, \omega) = -2 \sum_{\substack{\alpha, \beta, \alpha', \beta' \\ C_\alpha, C_\alpha', J', r}} \int d\varepsilon_\alpha d\varepsilon_{\alpha'} X_{\alpha\beta}^{C_\alpha, J, \tau}(E, \varepsilon_\alpha) X_{\alpha'\beta'}^{C_{\alpha'}, J, \tau}(E, \varepsilon_{\alpha'}) \\ \cdot \int dE' \frac{\gamma_{\alpha\alpha'}^{J', \tau}(\varepsilon_\alpha, \varepsilon_{\alpha'}, E') \gamma_{\beta\beta'}^{J', \tau}(E') U(j_\beta J J' j_{\alpha'}; j_\alpha j_{\beta'})}{\omega - E' - \varepsilon_{\alpha'} - \varepsilon_\beta} \end{aligned} \quad (17.1)$$

where

$$U(j_\beta J J' j_{\alpha'}; j_\alpha j_{\beta'}) = \sqrt{2j_\alpha + 1} \sqrt{2j_{\beta'} + 1} (-1)^{j_\beta + J + J' + j_{\alpha'}} \begin{pmatrix} j_\beta & J & j_{\alpha'} \\ j_{\alpha'} & J' & j_{\beta'} \end{pmatrix} \quad (17.2)$$

is just a recoupling coefficient, and the γ -functions are defined by:

$$\begin{aligned} \gamma_{\alpha\alpha'}^{J', \tau}(\varepsilon_\alpha, \varepsilon_{\alpha'}, E') = \sum_{\substack{\beta\beta' \\ C_\beta}} \int d\varepsilon_\beta \left[\langle (j_\alpha j_{\alpha'})_T^{J'} | F^{ph} | (j_\beta j_{\beta'})_T^{J'} \rangle \right. \\ \cdot \left. X_{\beta\beta'}^{C_\beta, J', \tau}(E', \varepsilon_\beta) + \langle (j_{\alpha\alpha'})_T^{J'} | F^{ph} | (j_\beta j_{\beta'})_T^{J'} \rangle \gamma_{\beta\beta'}^{J', \tau}(E', \varepsilon_\beta) \right] \end{aligned} \quad (17.3)$$

and

$$\begin{aligned} \gamma_{\beta\beta'}^{J', \tau}(E') = \int d\varepsilon_\gamma \sum_{\substack{\gamma, \gamma' \\ r' C_\gamma}} \left[\langle (j_\beta j_{\beta'})_T^J | F^{ph} | (j_\gamma j_{\gamma'})_T^{J'} \rangle X_{\beta\beta'}^{J', \tau}(E', \varepsilon_\gamma) \right. \\ \left. + \langle (j_\beta j_{\beta'})_T^J | F^{ph} | (j_\gamma j_{\gamma'})_T^{J'} \rangle \gamma_{\beta\beta'}^{J', \tau}(E', \varepsilon_\gamma) \right] \end{aligned} \quad (17.4)$$

The C-indices run over open channels, the ϵ -integrations over particle energies and the E-dependence over the RPA eigenvalues, which may be discrete (under the continuum threshold).

We remember that the p-h matrix element of the residual interaction is given by:

$$\begin{aligned} \langle (j_p j_h)_{\tau}^{J'} | F^{ph} | (j_p j_h)_{\tau}^{J'} \rangle &= \frac{C_0}{4\pi} \int dr r^2 R_p^{\tau}(\epsilon_p, r) \cdot R_h^{\tau}(r) \\ &\cdot R_p^{\tau}(\epsilon_p, r) R_h^{\tau}(r) \cdot (-1)^{k_p + k_{p'}} \hat{k}_p \hat{k}_h \hat{k}_{p'} \hat{k}_{h'} \\ &\cdot \left(\begin{matrix} j_p & j_h & J' \\ 1/2 & -1/2 & 0 \end{matrix} \right) \left(\begin{matrix} j_{p'} & j_{h'} & J' \\ 1/2 & -1/2 & 0 \end{matrix} \right) h(r) + (-1)^{\ell_p + \ell_{p'} + j_h + j_{h'} + 1} h'(r) \\ &\cdot \left(\begin{matrix} j_p & j_h & J' \\ 1/2 & 1/2 & -1 \end{matrix} \right) \left(\begin{matrix} j_{p'} & j_{h'} & J' \\ 1/2 & 1/2 & -1 \end{matrix} \right) \end{aligned} \quad (17.5)$$

Definitions and notations are identical to those of section 2. Defining also a transition density $\bar{\rho}$, summed over all channels, we get

$$\begin{aligned} \gamma_{\alpha\alpha'}^{J', \tau}(\epsilon_{\alpha}, \epsilon_{\alpha'}, E') &= C_0 \sum_{\tau'} \int dr r^2 h_{\tau\tau'}(r) C_{\tau}^{\alpha\alpha'}(\epsilon_{\alpha}, \epsilon_{\alpha'}, r) \rho_{J', \tau}^{\tau'}(E', r) \\ &+ \sum_{\tau'} \int dr r^2 h'_{\tau\tau'}(r) F_{\tau}^{\alpha\alpha'}(\epsilon_{\alpha}, \epsilon_{\alpha'}, r) \sigma_{J', \tau}^{\tau'}(E', r) \end{aligned} \quad (17.6)$$

and

$$\begin{aligned} \gamma_{\beta\beta'}^{J', \tau}(E') &= C_0 \sum_{\tau'} \int dr r^2 h_{\tau\tau'}(r) C_{\tau}^{\beta\beta'}(r) \rho_{J', \tau}^{\tau'}(E', r) \\ &+ C_0 \sum_{\tau'} \int dr r^2 h'_{\tau\tau'}(r) F_{\tau}^{\beta\beta'}(r) \sigma_{J', \tau}^{\tau'}(E', r) \end{aligned} \quad (17.7)$$

For numerical purposes it is convenient to write

$$C_{\tau}^{\alpha\alpha'}(\epsilon_{\alpha}, \epsilon_{\alpha'}, r) = R_{\alpha}^{\tau}(\epsilon_{\alpha}, r) R_{\alpha'}^{\tau}(\epsilon_{\alpha'}, r) W_{\alpha\alpha'} \quad (17.8)$$

and

$$F_{\tau}^{\alpha\alpha'}(\varepsilon_{\alpha}, \varepsilon_{\alpha'}, r) = R_{\alpha}^{\tau}(\varepsilon_{\alpha}, r) R_{\alpha'}^{\tau}(\varepsilon_{\alpha'}, r) K_{\alpha\alpha'} \quad (17.9)$$

with

$$W_{\alpha\alpha'} = \frac{(-1)}{\sqrt{4\pi}} \sqrt{k_{\alpha} k_{\alpha'}} \begin{pmatrix} j_{\alpha} & j_{\alpha'} & J' \\ 1/2 & -1/2 & 0 \end{pmatrix} \quad (17.10)$$

and

$$K_{\alpha\alpha'} = \frac{(-1)^{k_{\alpha}+k_{\alpha'}+\ell_{\alpha}+J'}}{\sqrt{4\pi}} \cdot \sqrt{k_{\alpha} k_{\alpha'}} \begin{pmatrix} j_{\alpha} & j_{\alpha'} & J' \\ +1/2 & +1/2 & -1 \end{pmatrix} \quad (17.11)$$

We can Fourier-Bessel expand eq. (17.6) to get

$$\begin{aligned} \gamma_{\alpha\alpha'}^{J\tau}(\varepsilon_{\alpha}, \varepsilon_{\alpha'}, E') &= \sum_{\tau'} \sum_{\mu\nu\eta} R_{\mu}^{\alpha\tau}(\varepsilon_{\alpha}) R_{\nu}^{\alpha'\tau'}(\varepsilon_{\alpha'}) \\ &\cdot \left[W_{\alpha\alpha'} A_{\mu}^{J\tau'}(E') T_{\mu\nu\eta}^{\tau\tau'} + \bar{B}_{\mu}^{\tau\tau'}(E') S_{\mu\nu\eta}^{\tau\tau'} K_{\alpha\alpha'} \right] \end{aligned} \quad (17.12)$$

where R_{μ} is the FB expansion coefficient of $R(r)$ and

$$T_{\mu\nu\eta}^{\tau\tau'} = C_0 \int dr r^2 h_{\tau\tau'}(r) j_{J'}(q_{\mu}r) j_{J'}(q_{\nu}r) j_{J'}(q_{\eta}r) \quad (17.13)$$

and

$$S_{\mu\nu\eta}^{\tau\tau'} = C_0 \int dr r^2 h'_{\tau\tau'}(r) j_{J'}(q_{\mu}r) j_{J'}(q_{\nu}r) j_{J'}(q_{\eta}r) \quad (17.14)$$

From eq. (17.7) we similarly get

$$\gamma_{\beta\beta'}^{J'\tau'}(E') = \sum_{\tau\mu\nu} \tilde{C}_{\mu}^{\beta\beta'\tau} \left[W_{\mu\nu}^{\beta\beta'\tau\tau'} A_{\nu}^{J'\tau'}(E') + K^{\beta\beta'} \cdot S_{\mu\nu}^{\tau\tau'} A_{\nu}^{J'\tau'}(E') \right] \quad (17.15)$$

With eqs. (17.12) and (17.15) we are now able to calculate directly in FB formalism the mass operator (eq. (17.1)).

Of course, one can make an entirely analogous derivation for the particle and for the hole terms contributing to $\Sigma^{\text{coll}}(E, \omega)$. On the other hand, mere interchange of X and Y's shall yield the other contributions to each term.

We note also that for numerical applications, due to the large number of integrations needed in the calculation of $\Sigma(E, \omega)$, further parametrizations of the single particle energy dependences are recommended.

The set of basis functions giving the parametrization should be suitably chosen so as to permit us analytical integration of the largest number of terms, leaving us finally only with summations over FB indices.

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Table Captions

Table 1: The convergence of energy (column 3) and BE3 (column 4) for the low-lying 3^- state in ^{208}Pb for increasing number of expansion coefficients M (column 1). The spatial resolution achieved in each case is in column 2.

Table 2: The convergence of energy (columns 2 and 4) and BE5 (columns 3 and 5) for the second and third 5^- state in ^{208}Pb - non-collective states - for increasing number of expansion coefficients M (column 1). Just as in table 1, no fitting to the experimental values was intended.

Table 3: For each multipolarity J and parity π (column 1) the value of the classical EWSR S_J^T (in $e^2 b^J$) is shown in column 2 for the surface modes

$$S_J^T = \frac{J(2J+1)^2}{4\pi} \frac{\hbar^2 c^2}{2mc^2} Z \langle r^{2J-2} \rangle_{\pi\text{-g.s.}}$$

In column 3 the percentage exhaustion of the EWSR due to the bound states is shown.

$$S_J^B = \sum_{n=\{\text{bound states}\}} E_n(\text{BEJ})_n$$

In column 4 the percentage exhaustion of the EWSR in the continuum region between the energies E_0 (column 5) and E (column 6) is shown:

$$S_J^C = \int_{E_0}^E \omega \text{BEJ}(\omega) d\omega$$

The asterisk reminds that the dipole data refer only to the isovector part. For all the other multipolarities the proton-BEJ is shown.

The table demonstrates that it is essential to work in a complete particle-hole space in order to exhaust the sum rule.

Table 4: The force parameters for the Landau-Migdal interaction discussed in section 6 are shown, as well as the complementary parameters (force constant, interpolation radii and diffuseness) which must be adjusted to each nucleus.

Table 5: Woods-Saxon parameters for ^{208}Pb . The Woods-Saxon interaction is defined by

$$V(r) = V_0 \left[1 + \exp\{(r-R_0)/\alpha_0\} \right]^{-1} + V_c(r) - \lambda \frac{\hbar^2}{m_c^2 c^2} \frac{\ell \cdot s}{2r} \frac{d}{dr} \left[1 + \exp\{(r-R_\lambda)/\alpha_\lambda\} \right]^{-1}$$

where

$$V_c(r) = \begin{cases} \frac{Ze^2}{2R_c} \left[3 - (r/R_c)^2 \right] & r \leq R_c \\ \frac{Ze^2}{r} & r > R_c \end{cases}$$

Table 6: For the same Woods-Saxon interaction as above the parameters for ^{16}O are shown.

Table 7: For the most important compression modes the ratio of moments, m_{i+1}/m_i and $\sqrt{m_{i+2}/m_i}$, as well as the variances σ_k , defined by eq. (10.11) are shown. For each multipolarity the isospin branches are shown separately (isoscalar and isovector branches). The integration range in each case is shown in the second row.

Table 1

M	$(r/R)=P/q_1^3$	$E(3_1^-)$ [MeV]	$BE(3_1^-)$ [$e^2 b^3$]
15	0.061	2.84418	0.65971
14	0.064	2.84417	0.65966
13	0.069	2.84415	0.65965
12	0.074	2.84412	0.65938
11	0.080	2.84431	0.65865
10	0.087	2.84289	0.66146
9	0.096	2.84159	0.66704
8	0.106	2.76638	0.67088
7	0.119	2.70478	0.65292
6	0.135	1.81893	1.54604
5	0.156	2.68486	0.77555
4	0.186	1.21470	0.92254

Table 2

M	$E(5_2^-)$ [MeV]	$BE(5_2^-)$ [$e^2 10^2 b$]	$E(5_3^-)$ [MeV]	$BE(5_3^-)$ [$e^2 10^4 b$]
15	3.86075	4.54147	4.84402	9.22531
14	3.86075	4.54064	4.84402	9.22345
13	3.86075	4.53988	4.84402	9.22201
12	3.86075	4.53976	4.84402	9.22203
11	3.86075	4.53745	4.84402	9.20203
10	3.86079	4.51885	4.84400	9.20121
9	3.86046	4.57258	4.84400	9.42388
8	3.86032	4.65225	4.84400	9.21839
7	3.84464	4.17611	4.84320	7.91769
6	3.39268	3.07058	3.83153	332.24701
5	3.14102	20.07831	3.65055	562.872201
4	3.81409	3.32134	4.17941	44.45361

Table 3

J^π	$S_J^T [e^{2b^J}]$	$\frac{100 S_J^B}{S_J^T}$	$\frac{100 S_J^C(E_o, E)}{S_J^T}$	$\frac{100(S_J^B + S_J^C(E_o, E))}{S_J^T}$	$E_o \text{ [MeV]}$	$E \text{ [MeV]}$
1^-	12.187 *	2.3 *	94.9 *	97.2 *	8.0	60.0
2^+	19.849	9.1	92.5	101.6	8.0	60.0
3^-	21.693	11.5	85.7	97.2	8.0	60.0
4^+	20.632	6.9	90.8	97.7	8.0	60.0
5^-	18.685	3.7	94.9	98.6	8.0	60.0

Table 4

Force Parameters					
f_{in}	f_{ext}	f_{in}	f_{ext}	g	g'
0.20	-2.45	1.50	1.50	0.55	0.70

Parameters for the Continuum Calculations				
nucleus	overall force constant C (MeV fm ³)	interp. radius R (fm)	r_o (fm)	diffuseness α (fm)
¹⁶ O	418.852	3.07	1.200	0.60
²⁰⁸ Pb	318.261	7.22	1.219	0.60

Table 5

	V_o (MeV)	R_o (fm)	α_o (fm)	λ	R_λ (fm)	α_λ (fm)	R_c (fm)	$\langle r^2 \rangle^{1/2}$ (fm)
proton	-60.40	7.46	0.79	20.254	7.20	0.59	7.4062	5.440
neutron	-44.32	7.46	0.66	24.866	6.96	0.64		5.704

Table 6

nucleus	charge	V_o [MeV]	r [fm]	a_o [fm]	V_{LS} [MeV]	r_{LS} [fm]	a_{LS} [fm]	b [fm]
$^{16}_O$	p	-52.88	3.0828	0.53	5.533	3.0828	0.53	1.639
	n	-53.14	3.0828	0.53	5.186	3.0828	0.53	1.639

Table 7

$J^\pi(T)$	$0^+ (T=0)$ (first)	$0^+ (T=0)$ (second)	$1^- (T=0)$ (first)	$2^+ (T=0)$ (first)	$0^+ (T=1)$ (first)	$0^+ (T=1)$ (second)	$1^- (T=1)$ (first)	$2^+ (T=1)$ (first)
integr. range	13.5-14.3	24.0-38.0	20.0-26.0	26.0-34.0	20.0-37.0	44.0-60.0	28.0-49.0	34.0-54.0
m_3/m_2	13.875	30.789	23.226	30.634	28.787	51.230	37.774	44.676
m_2/m_1	13.870	30.324	23.136	30.504	28.228	50.950	37.069	44.058
m_1/m_0	13.865	29.869	23.046	30.370	27.673	50.675	36.373	43.427
m_0/m_{-1}	13.860	29.412	22.954	30.232	27.131	50.407	35.698	42.792
m_{-1}/m_{-2}	13.855	28.977	22.863	30.092	26.608	50.148	35.052	42.165
m_{-2}/m_{-3}	13.850	28.566	27.770	29.950	26.112	49.896	34.446	41.554
$\sqrt{m_3/m_1}$	13.872	30.556	23.181	30.569	28.506	51.090	37.420	44.366
$\sqrt{m_2/m_0}$	13.867	30.093	23.091	30.437	27.950	50.812	36.720	43.741
$\sqrt{m_1/m_{-1}}$	13.862	29.636	23.000	30.301	27.401	50.541	36.034	43.108
$\sqrt{m_0/m_{-2}}$	13.857	29.194	22.908	30.162	26.868	50.277	35.374	42.478
$\sqrt{m_{-1}/m_{-3}}$	13.852	28.771	22.816	30.021	26.359	50.022	34.748	41.859
σ_2	0.261	3.752	1.440	1.995	3.972	3.777	5.112	5.217
σ_1	0.263	3.712	1.445	2.018	3.918	3.730	5.032	5.235
σ_0	0.258	3.643	1.448	2.038	3.837	3.675	4.911	5.210
σ_{-1}	0.270	3.548	1.448	2.052	3.728	3.608	4.756	5.142
σ_{-2}	0.259	3.426	1.447	2.063	3.601	3.545	4.572	5.039

Figure Captions

- Fig. 1: In order to demonstrate the convergence of the Fourier-Bessel method, the photoabsorption cross section of the dipole strength in ^{16}O is shown as a function of the excitation energy for different numbers M of Fourier-Bessel expansion coefficients. Beyond $M=10$, the curves are virtually indistinguishable.
- Fig. 2: The charge transition density of the 3^- state at $E_x = 2.615$ MeV in ^{208}Pb including the effects of the full continuum is displayed as a function of the radial coordinate (solid). The dotted curves are the experimental error bars taken from ref. (He80).
- Fig. 3: The charge transition density of the 2^+ state at 4.44 MeV in ^{208}Pb as a function of the radial coordinate. The solid line refers to the continuum calculation, the dotted lines are the experimental error envelopes from ref. (He80).
- Fig. 4: The charge transition density of the 4^+ state at 4.73 MeV in ^{208}Pb as a function of the radial coordinate.
- Fig. 5: Isoscalar (full curve) and isovector (dotted curve) transition densities for the first monopole compression mode (at 13.5 MeV).
- Fig. 6: Isoscalar (full curve) and isovector (dotted curve) transition densities for the first isoscalar dipole compression mode (at 21.9 MeV).
- Fig. 7: Isoscalar (full curve) and isovector (dotted curve) transition densities for the first isoscalar quadrupole compression mode (at 27.9 MeV).

- Fig. 8: Isoscalar (full curve) and isovector (dotted curve) transition densities for the second isoscalar monopole compression mode (at 46.7 MeV). As expected, the curve presents two nodes and is volume peaked.
- Fig. 9: Cross sections of several partial waves in ^{16}O and ^{208}Pb , obtained by scattering from a Saxon-Woods potential, as a function of the incident energy. Since in a light nucleus like ^{16}O only the $f_{7/2}$ partial wave has a resonant behaviour, concentrated multipole strength beyond $L=2$ is highly unlikely. In ^{208}Pb , the widths of the proton partial waves are extremely small so that in principle resonances of large multipolarity may be expected. Note, however, that the neutron partial waves contribute with significant decay widths.
- Fig. 10: Isovector $B(E,1)$ strength in ^{16}O as a function of the excitation energy.
- Fig. 11: Exhaustion of the isovector energy weighted sum rule for dipole excitations in ^{16}O as a function of the upper integration limit E (see eqs. (10.1) to (10.10)).
- Fig. 12: $B(E,2)$ strength in ^{16}O as a function of the excitation energy.
- Fig. 13: Exhaustion of the energy weighted sum rule for quadrupole strength (isoscalar + isovector) in ^{16}O as a function of the upper integration limit.
- Fig. 14: $B(E,3)$ strength in ^{16}O as a function of the excitation energy.

Fig. 15: Percentage exhaustion of the linearly weighted sum rule (isovector + isoscalar) for the giant octupole resonance as a function of the upper integration energy E in MeV for ^{16}O .

Fig. 16: Isovector component of the BE1-value in unities of $e^2\text{fm}^2/\text{MeV}$ against the excitation energy in MeV for ^{208}Pb .

Fig. 17: Percentage exhaustion of the linearly weighted sum rule (isovector part) for the giant dipole resonance as a function of the upper integration energy for ^{208}Pb .

Fig. 18: BE2-value in unities of $e^2\text{fm}^4/\text{MeV}$ against the excitation energy in MeV for ^{208}Pb .

top - isoscalar plus isovector

middle - isoscalar

bottom - isovector

Fig. 19: Percentage exhaustion of the linearly weighted sum rule for the giant quadrupole resonance as a function of the upper integration energy for ^{208}Pb , decomposed into protons (top), isoscalar (middle) and isovector (bottom) contribution.

Fig. 20: BE3-value in unities of $e^2\text{fm}^6/\text{MeV}$ against the excitation energy in MeV for ^{208}Pb .

top - isovector plus isoscalar components

middle - only isoscalar ($T=0$) component

bottom - only isovector ($T=1$) component

- Fig. 21: Percentage exhaustion of the linearly weighted sum rule for the giant octupole resonance as a function of the upper integration energy for ^{208}Pb .
top - isovector plus isoscalar components
middle - only isoscalar (T=0) component
bottom - only isovector (T=1) component
- Fig. 22: BE4-value in unities of $e^2\text{fm}^8/\text{MeV}$ against the excitation energy in MeV for ^{208}Pb , decomposed into proton (top), isoscalar (middle) and isovector (bottom) contribution.
- Fig. 23: Percentage exhaustion of the linearly weighted sum rule for the E4 state as a function of the upper integration energy E in MeV for ^{208}Pb .
top - isoscalar plus isovector
middle - isoscalar
bottom - isovector
- Fig. 24: BE5-value in unities of $e^2\text{fm}^{10}/\text{MeV}$ against the excitation energy in MeV for ^{208}Pb , decomposed into proton (top), isoscalar (middle) and isovector (bottom) contribution.
- Fig. 25: Percentage exhaustion of the linearly weighted sum rule for the E5 state as a function of the upper integration energy E in MeV for ^{208}Pb .
top - isoscalar plus isovector
middle - isoscalar
bottom - isovector
- Fig. 26: BE6-value in unities of $e^2\text{fm}^{12}/\text{MeV}$ against the excitation energy in MeV for ^{208}Pb .

Fig. 27: BE7-value in unities of $e^2 \text{fm}^{14} / \text{MeV}$ against the excitation energy in MeV for ^{208}Pb .

Fig. 28: The percentage exhaustion of the isoscalar part of the 0^+ , 1^- and 2^+ states is shown as a function of the upper integration energy (see eq. (11.8)). For the monopole the full curve corresponds to the $\hat{C}_{J,1}$ operator and the dotted one to the $\hat{C}_{J,2}$ operator. For the 1^- and 2^+ the full curve corresponds to the $\hat{C}_{J,0}$ operator and the dotted one to the $\hat{C}_{J,1}$. In each case for the lowest k-operator full saturation of the sum rule (11.6) is reached. For the higher k-operator not all the strength is found below 60 MeV and must be found beyond that value.

Fig. 29: BE0-value against the excitation energy in MeV for ^{208}Pb .
top - isoscalar plus isovector
middle - only isoscalar (T=0) component
bottom - only isovector (T=1) component
All three BE0-values are given in unities of $e^2 \text{fm}^4 / \text{MeV}$.

Fig. 30: Percentage exhaustion of the linearly weighted sum rule for the giant monopole resonance as a function of the upper integration energy E in MeV in ^{208}Pb .
top - isoscalar plus isovector
middle - only isoscalar (T=0) component
bottom - only isovector (T=1) component

Fig. 31: Percentage exhaustion of the isovector compression mode strength for $J^\pi = 0^+, 1^-$ and 2^+ , as a function of the excitation energy. The full and dotted curves correspond to the same operators described in fig. 28.

- Fig. 32: The nuclear response to the operator $\hat{C}_{1,1}$ is shown as a function of the excitation energy. The isoscalar dipole compression mode is to be recognized in the upper picture as the sequence of peaks around 23 MeV. The isovector dipole compression mode (lower picture) comes out to be a large bump peaked at 33 MeV.
- Fig. 33: The nuclear response to the operator $\hat{C}_{2,1}$ is shown. The isoscalar quadrupole compression mode (upper picture) peaks at 31 MeV. The isovector one (lower picture) is the strength fragmented around 44 MeV.
- Fig. 34: The percentage exhaustion of the isoscalar part of the 3^- and 4^+ states is shown as a function of the upper integration energy (see eq. (11.7)). For both states the full curve corresponds to the operator $\hat{C}_{J,0}$ and the dotted one to the $\hat{C}_{J,1}$. A good fraction of the strength is to be found beyond 60 MeV.
- Fig. 35: The same as in fig. 34, for the isovector branch. Most part of the strength is to be found beyond 60 MeV.
- Fig. 36: The structure of the mass operator as used in this work. The first and second terms are the particle and hole selfenergy graphs, the third one is the induced interaction.
- Fig. 37: The distribution function for the giant monopole resonance in ^{208}Pb as a function of the energy. Physically it shows the probability of finding the (originally) unperturbed phonon at a certain energy. Here the two particle-two hole contributions were calculated with consideration of the single particle continuum.

- Fig. 38: The distribution function for the giant monopole resonance in ^{208}Pb calculated with a discretized continuum (Wa82).
- Fig. 39: The structure function - nuclear response to an operator $F(q,r)$ - as a function of energy and transferred momentum for the GMR in ^{208}Pb . Here we consider only 1p-1h excitations.
- Fig. 40: The structure function for the giant monopole resonance in ^{208}Pb with consideration of 2p-2h contributions with continuum.
- Fig. 41: The distribution function for the giant quadrupole resonance in ^{208}Pb , calculated with a discretized continuum (Wa82).
- Fig. 42: The distribution function for the GQR in ^{208}Pb . Here the 2p-2h contributions were calculated taking account of the single particle continuum.
- Fig. 43: The structure function for the GQR in ^{208}Pb , considering only 1p-1h excitations.
- Fig. 44: The structure function for the GQR in ^{208}Pb considering also 2p-2h contributions and accounting for the single particle continuum.

M = NUMBER OF EXPANSION COEFFICIENTS

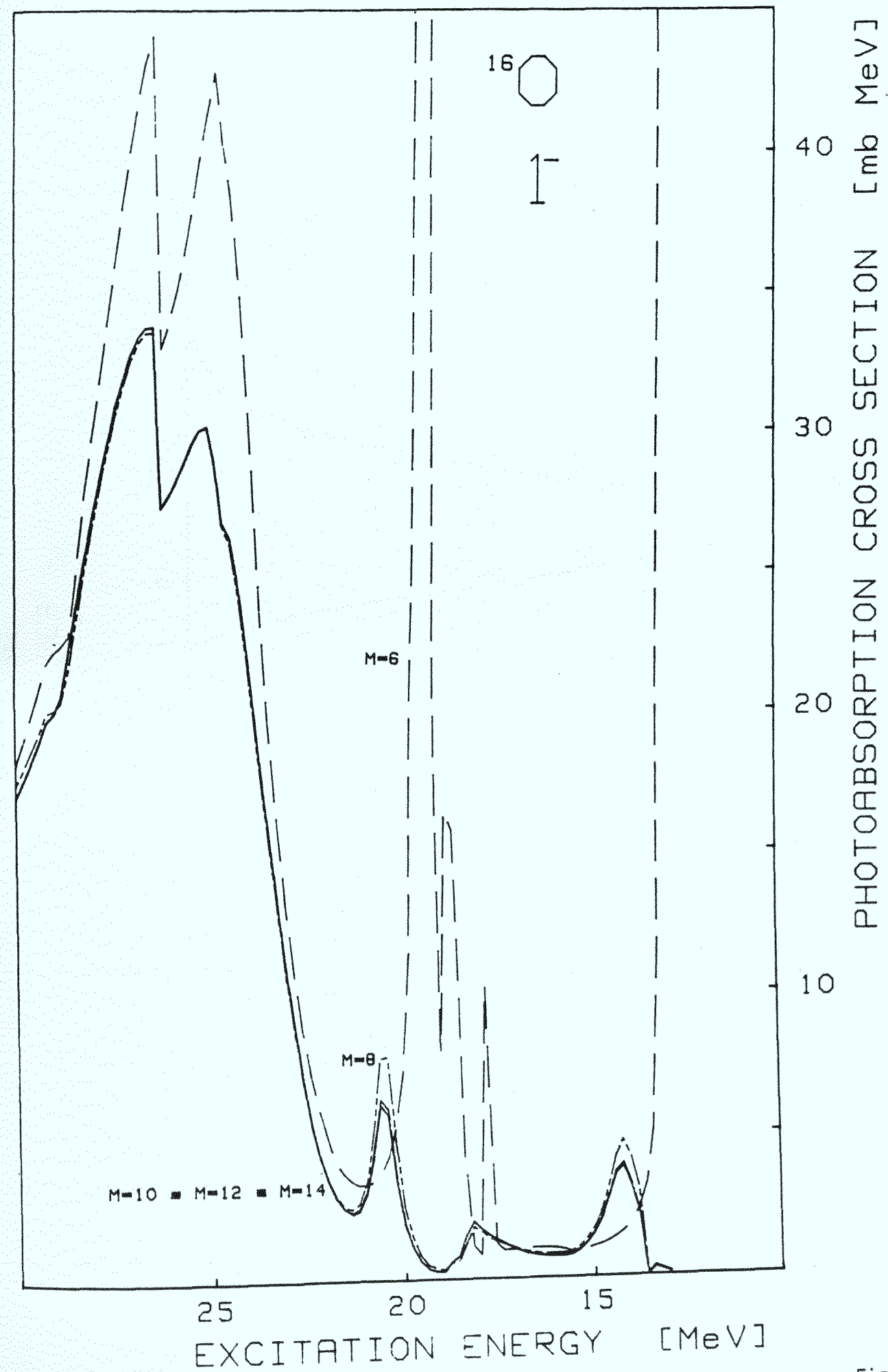
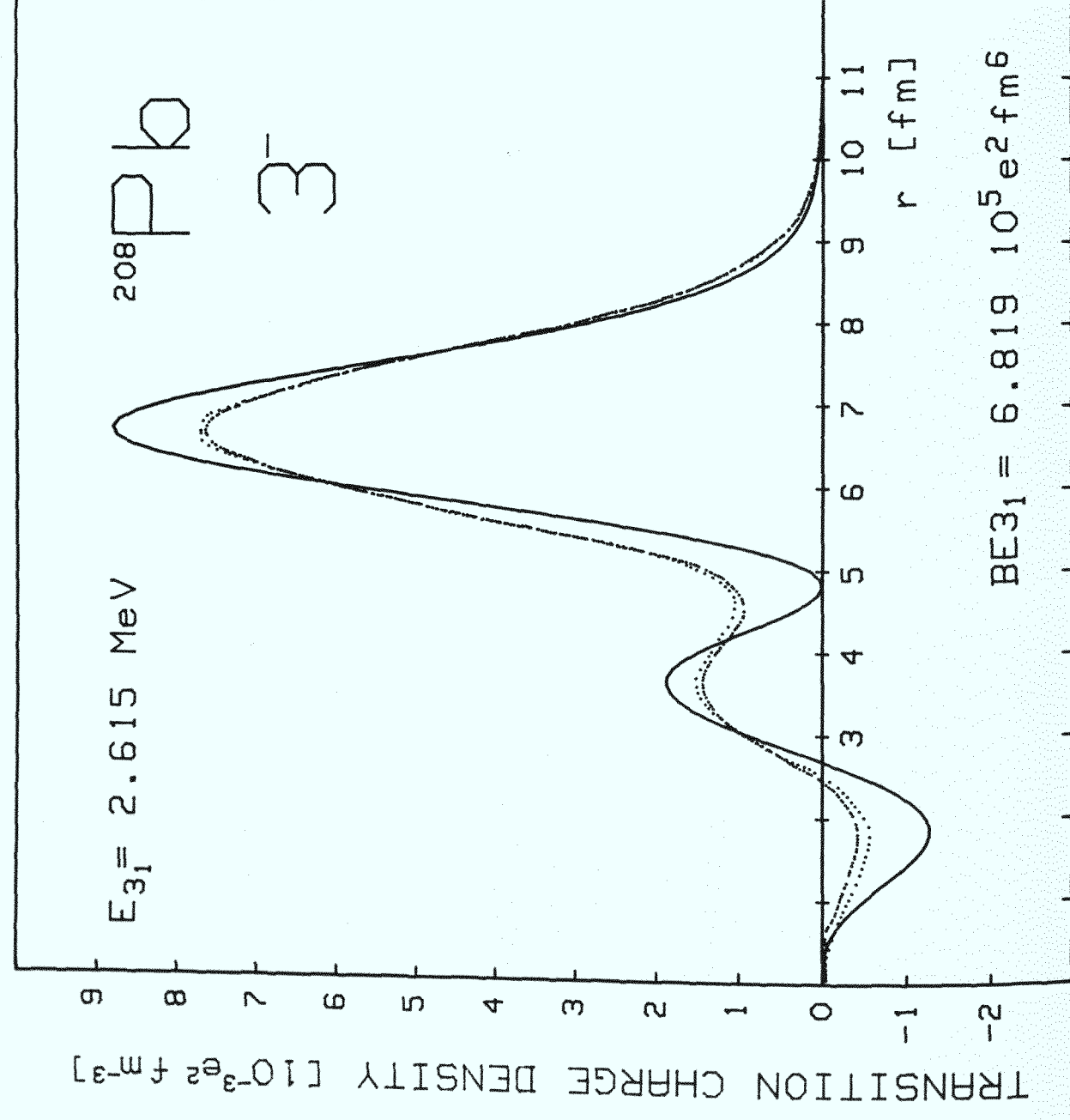


Fig. 1



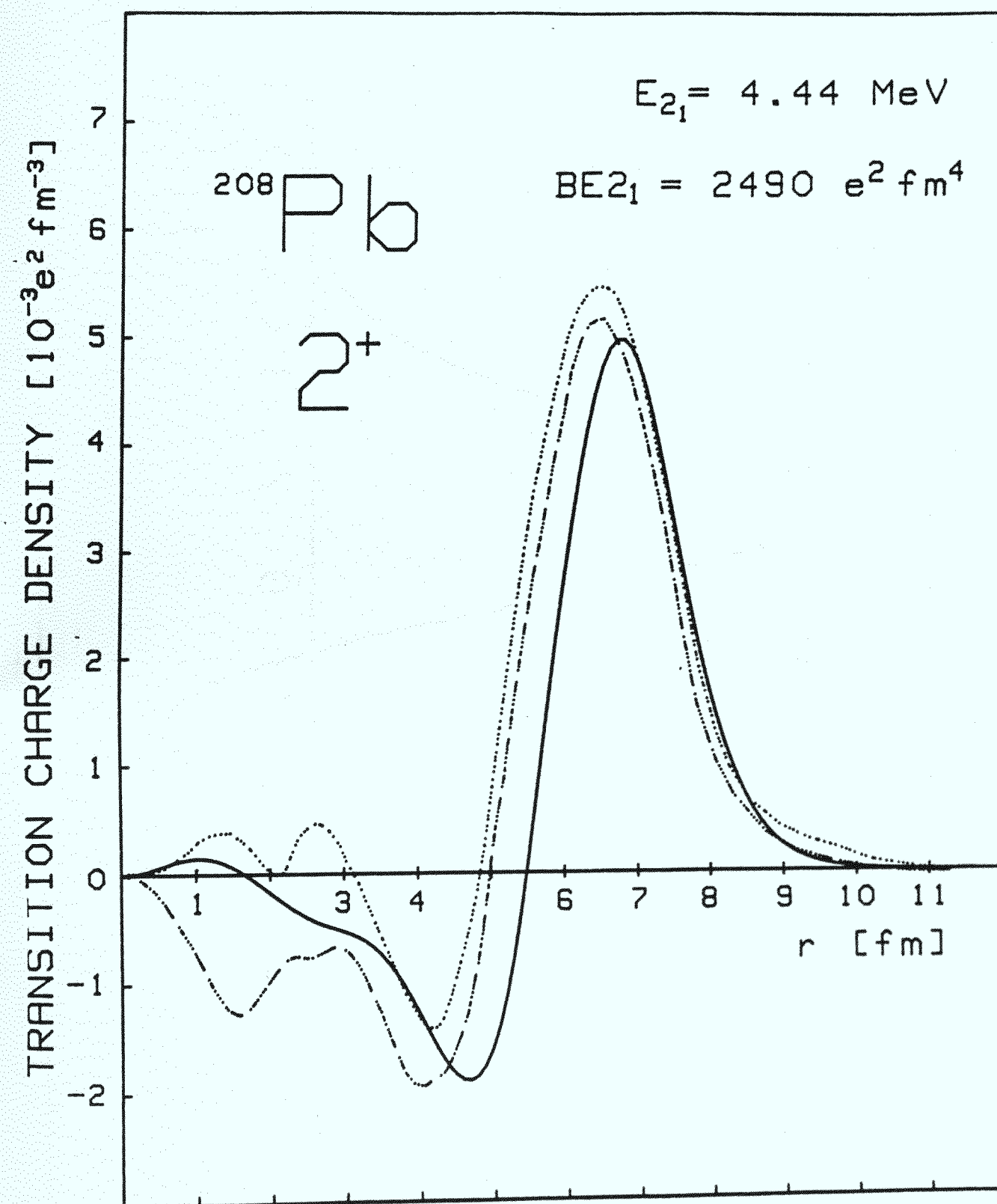


Fig. 3

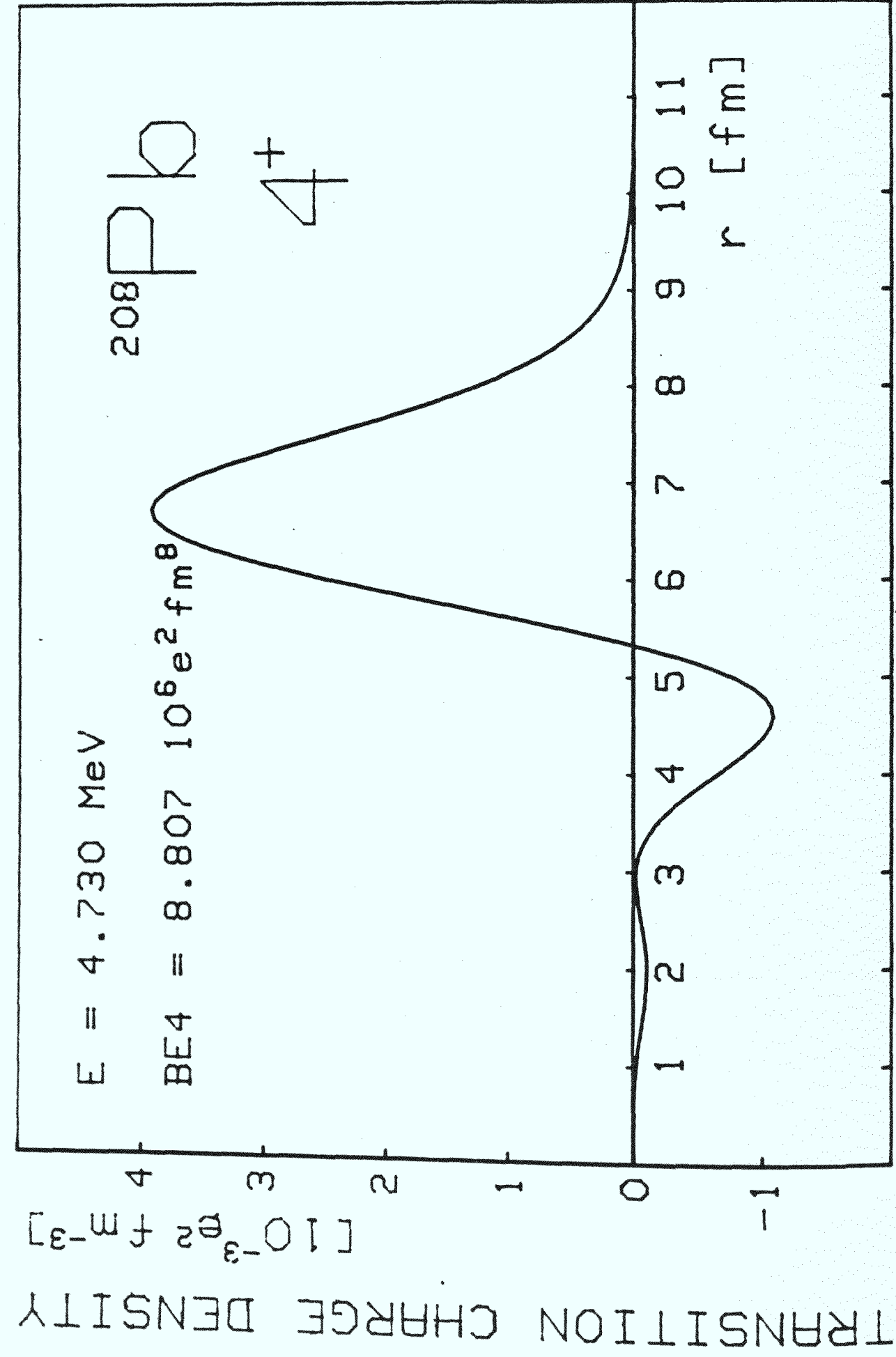


Fig. 4

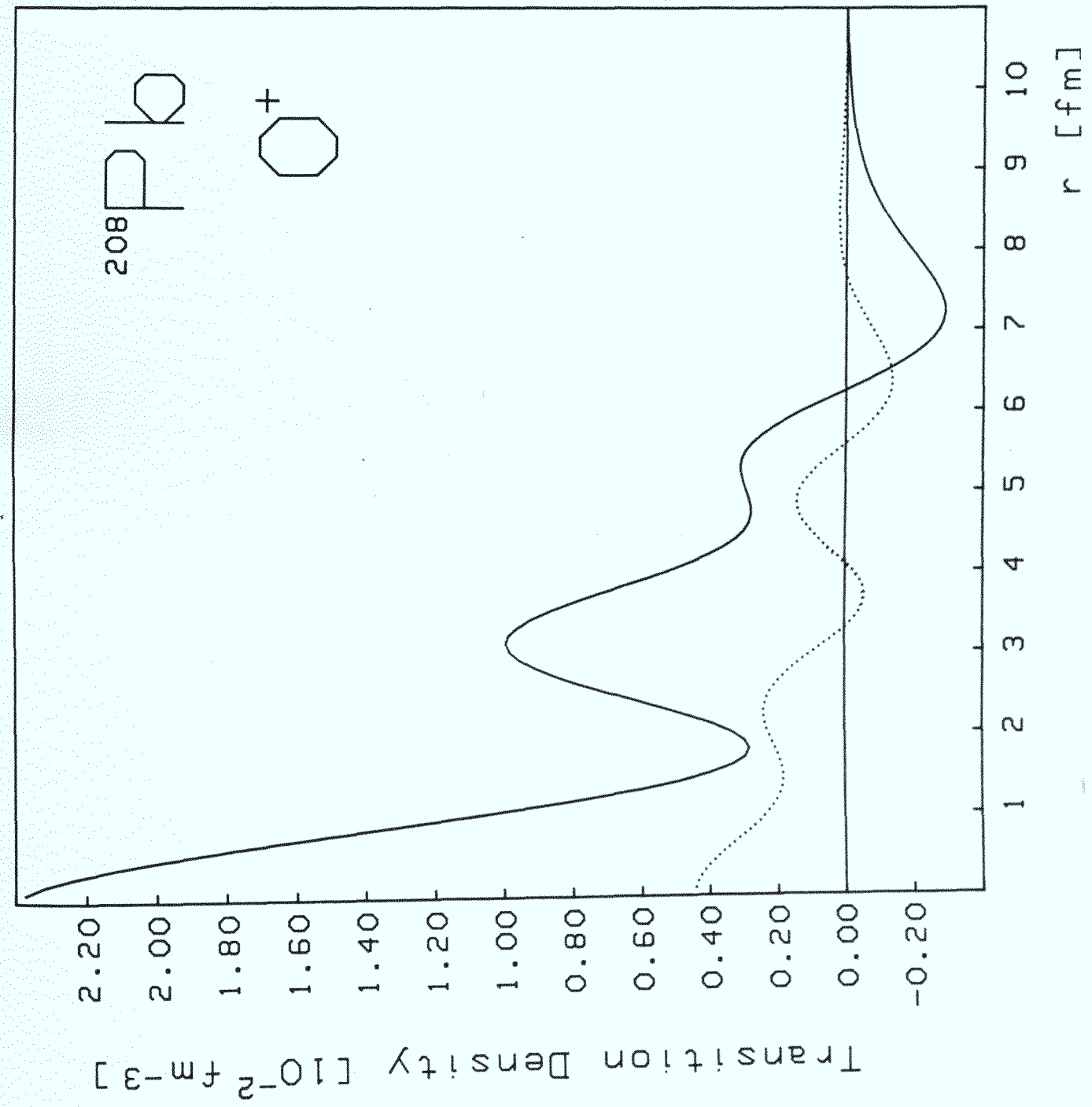


Fig. 5

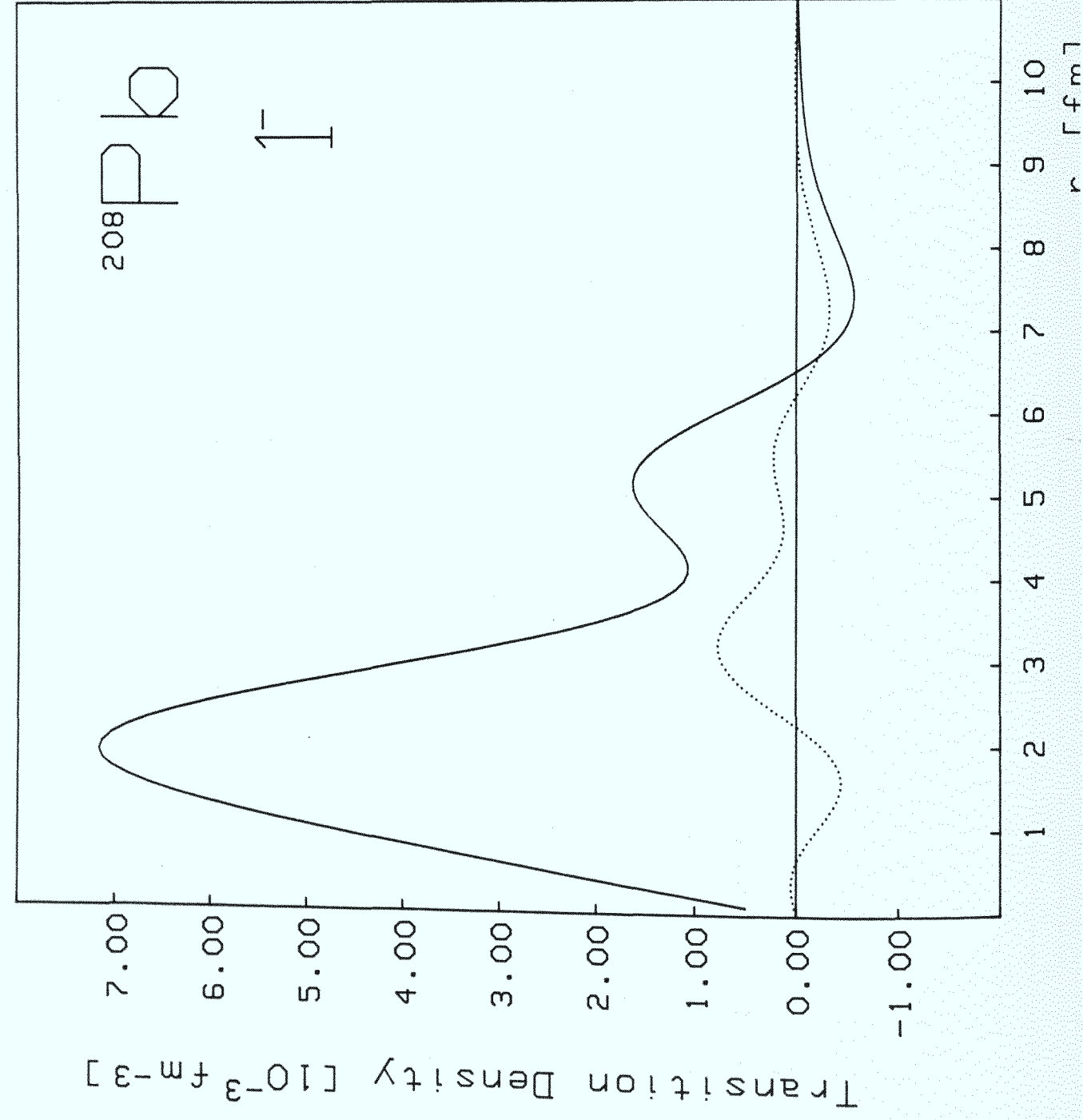


Fig. 6

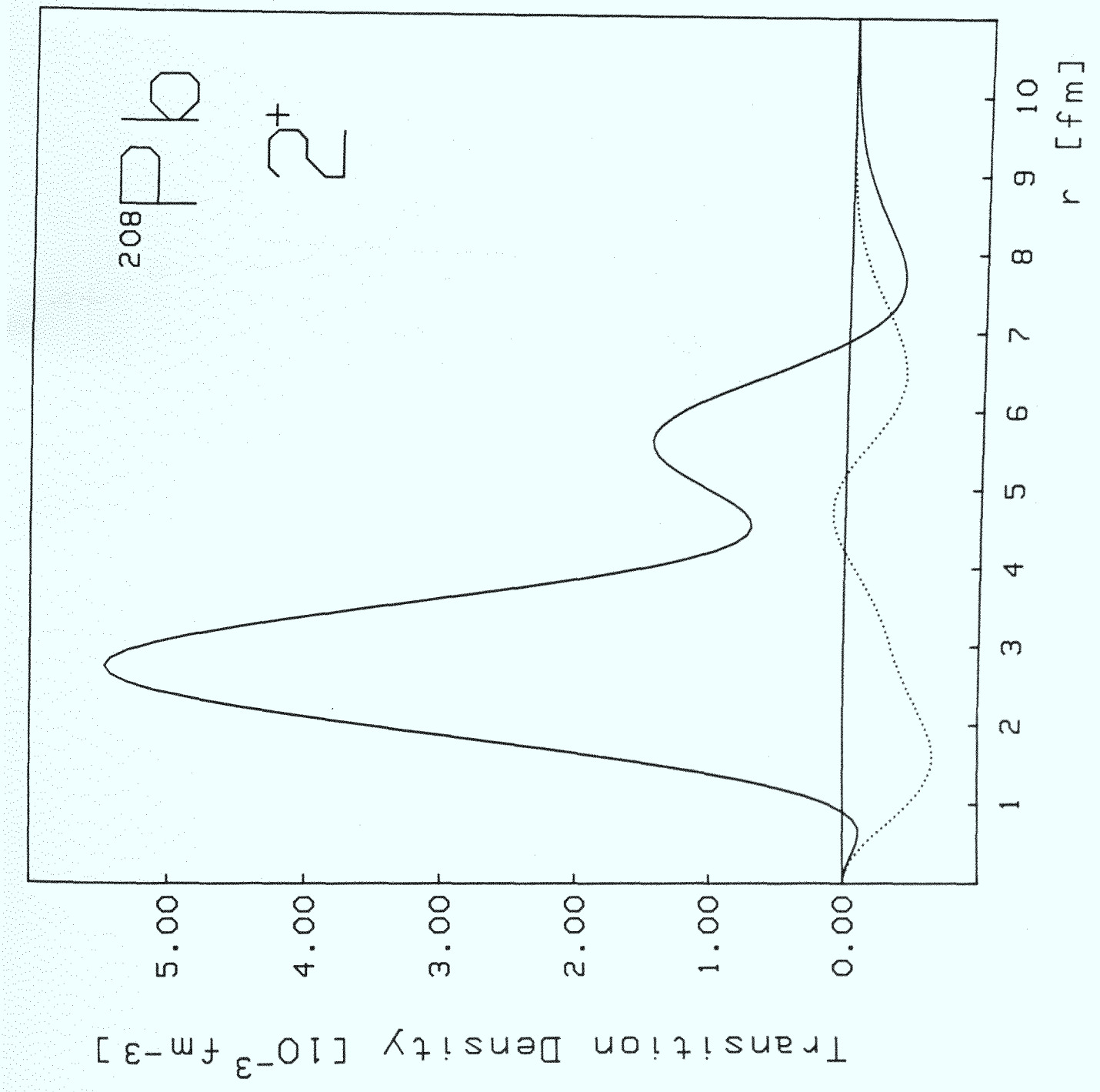
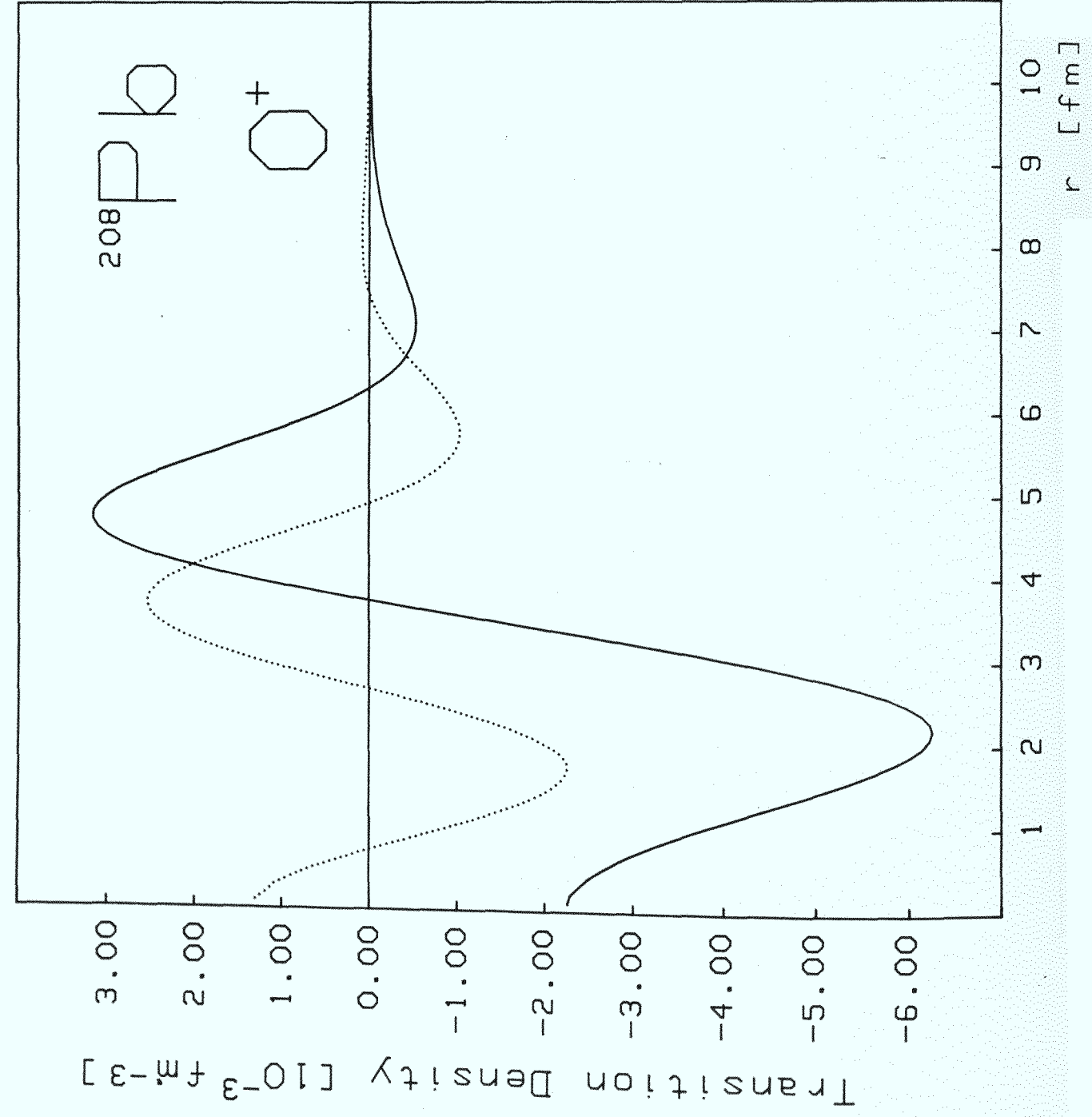


Fig. 7



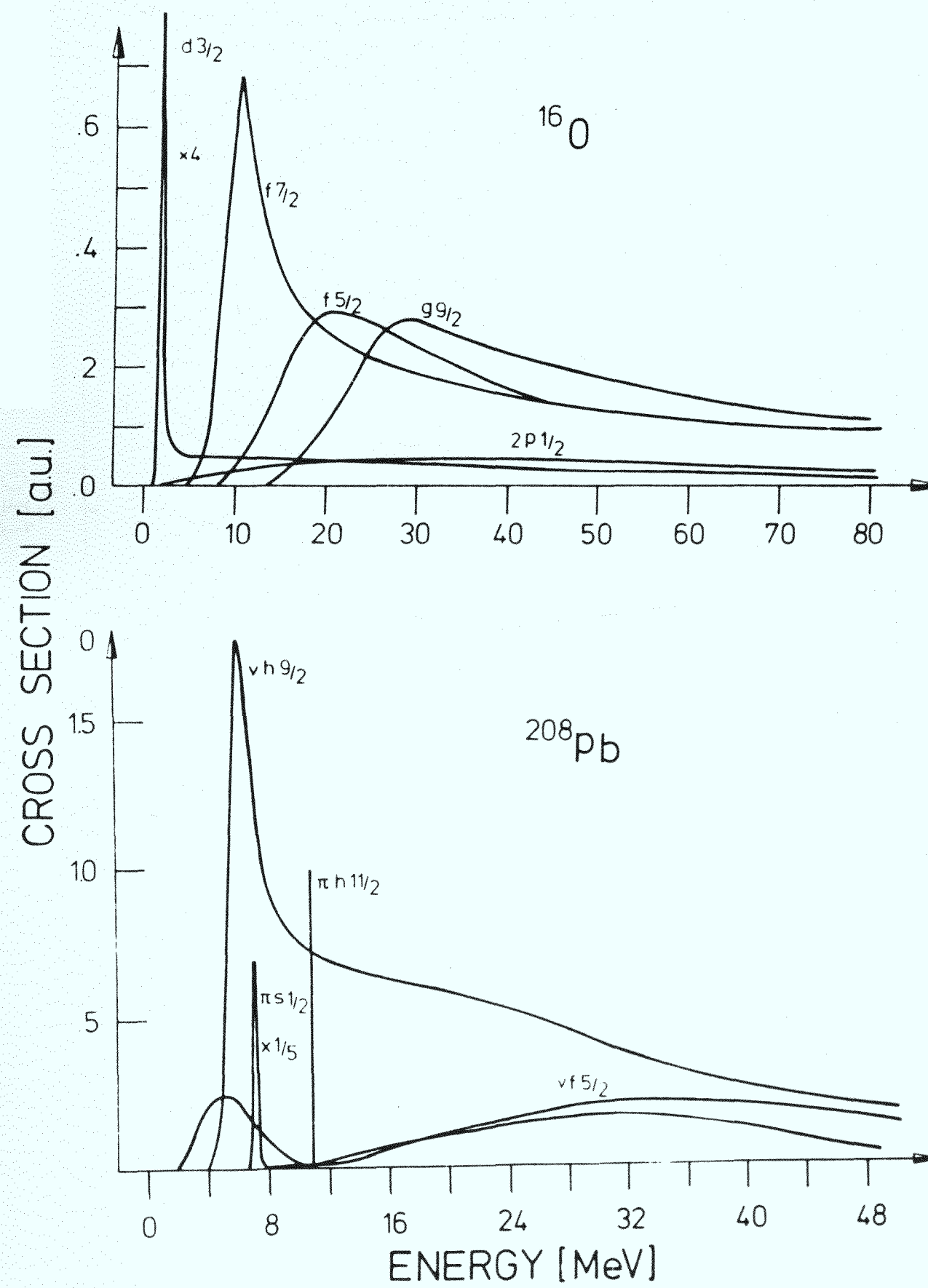
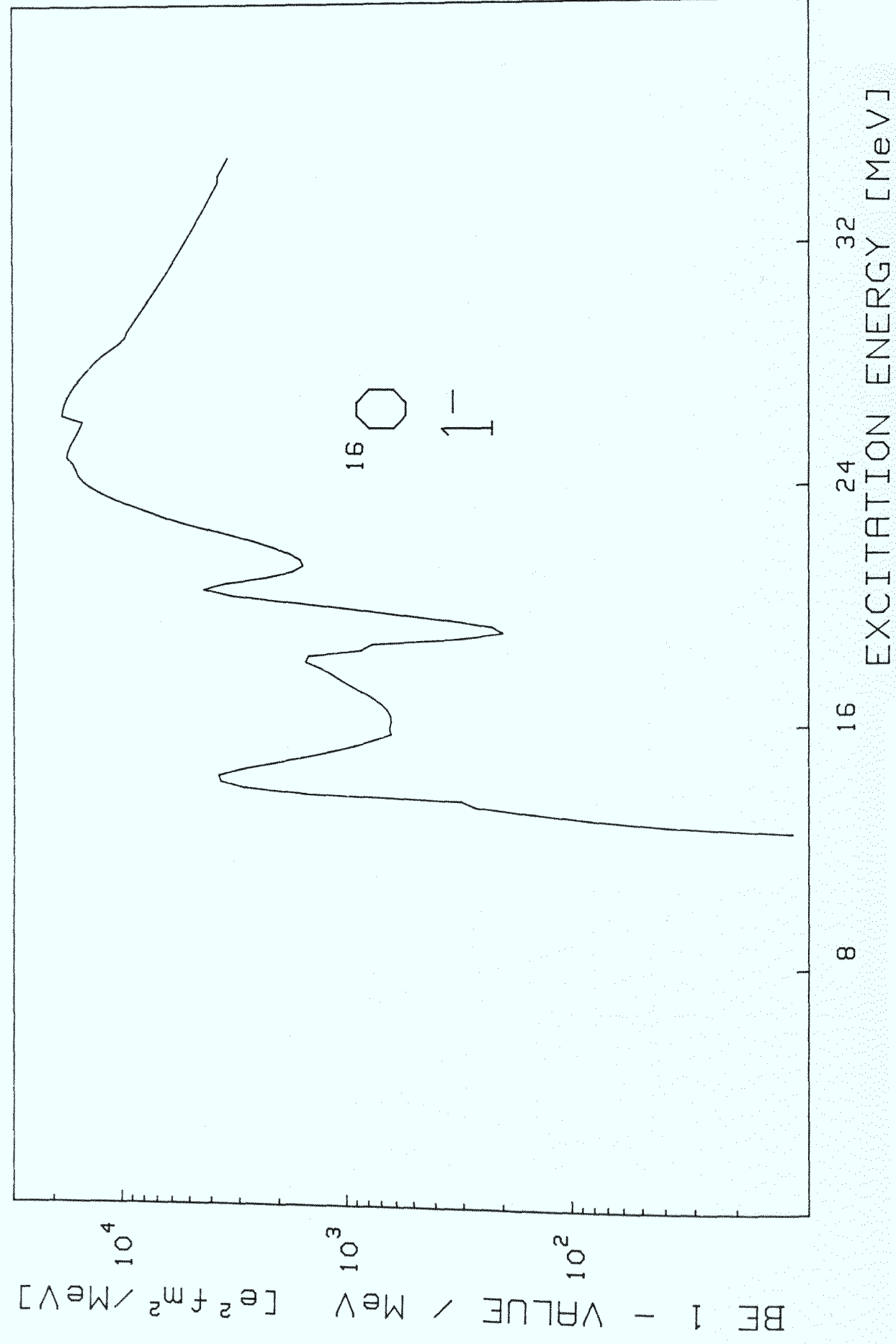


Fig. 9



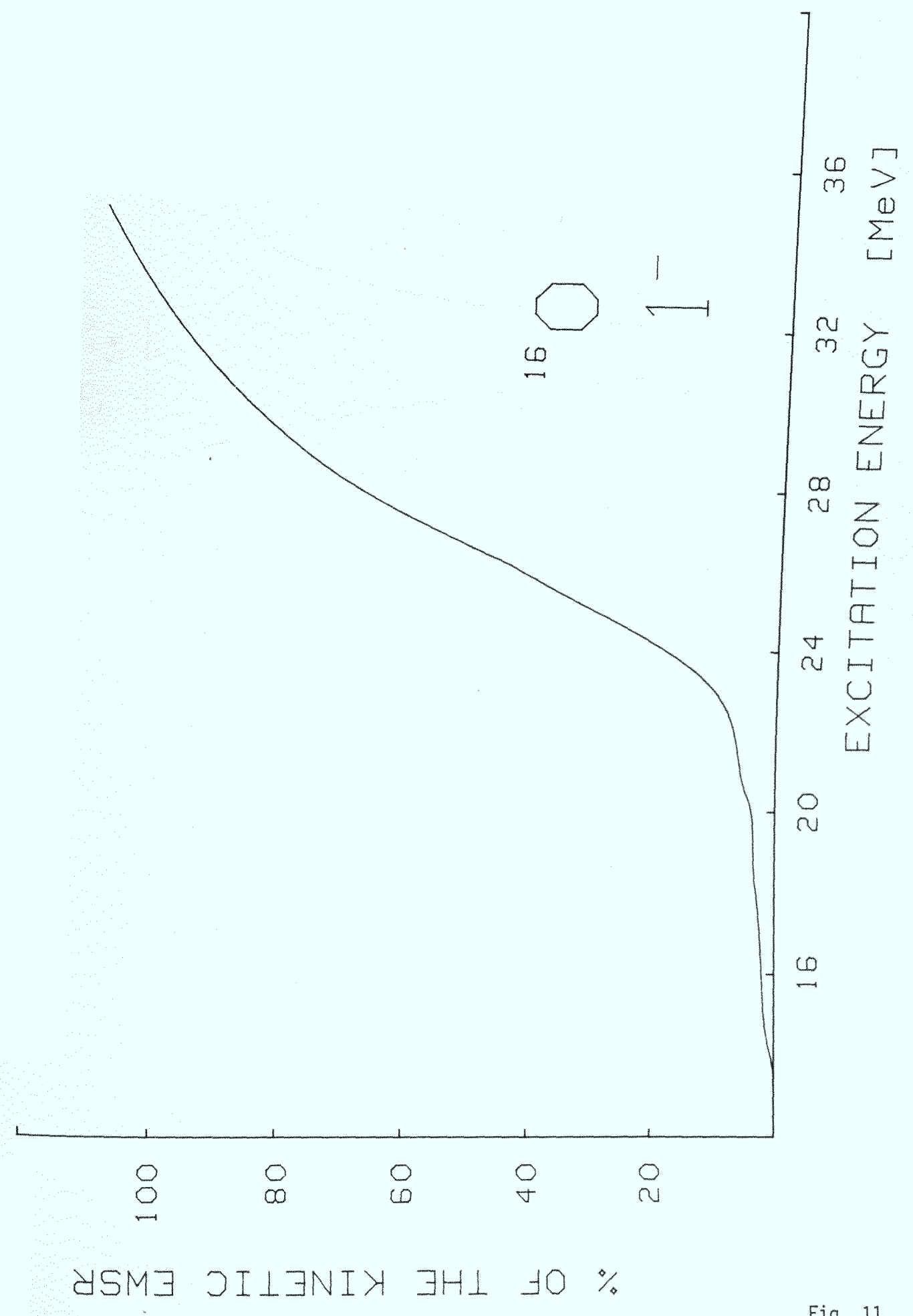


Fig. 11

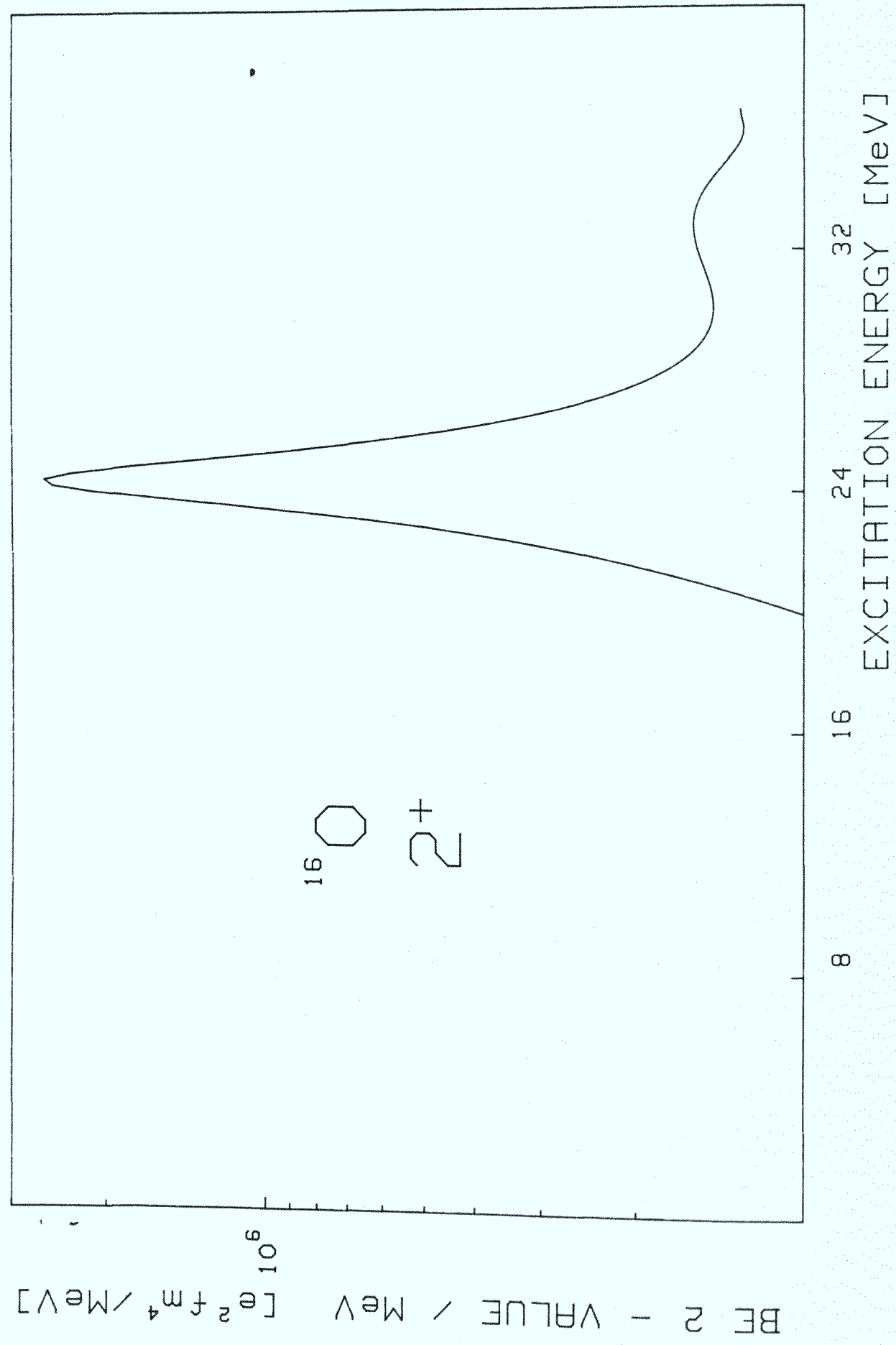


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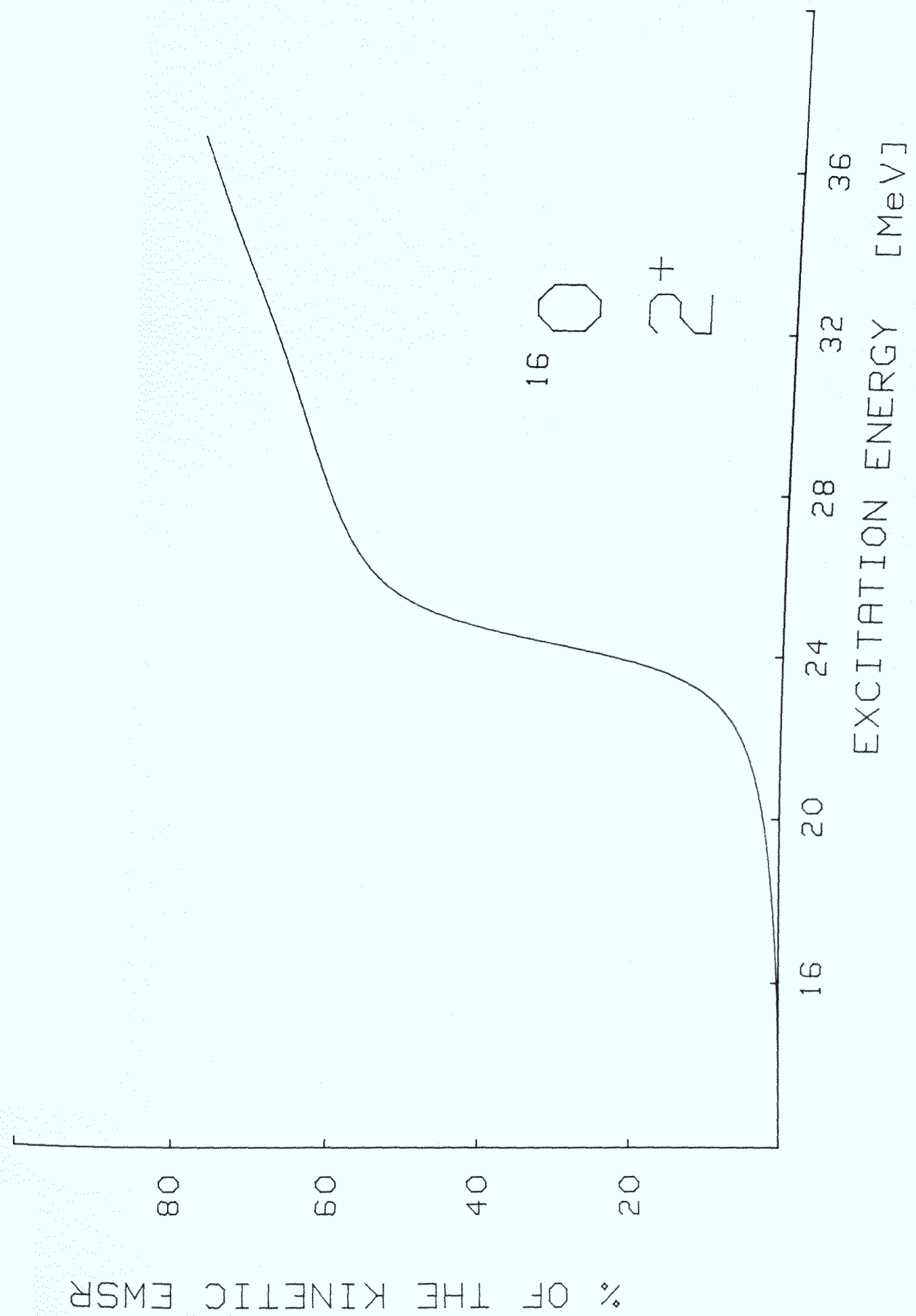


Fig. 13

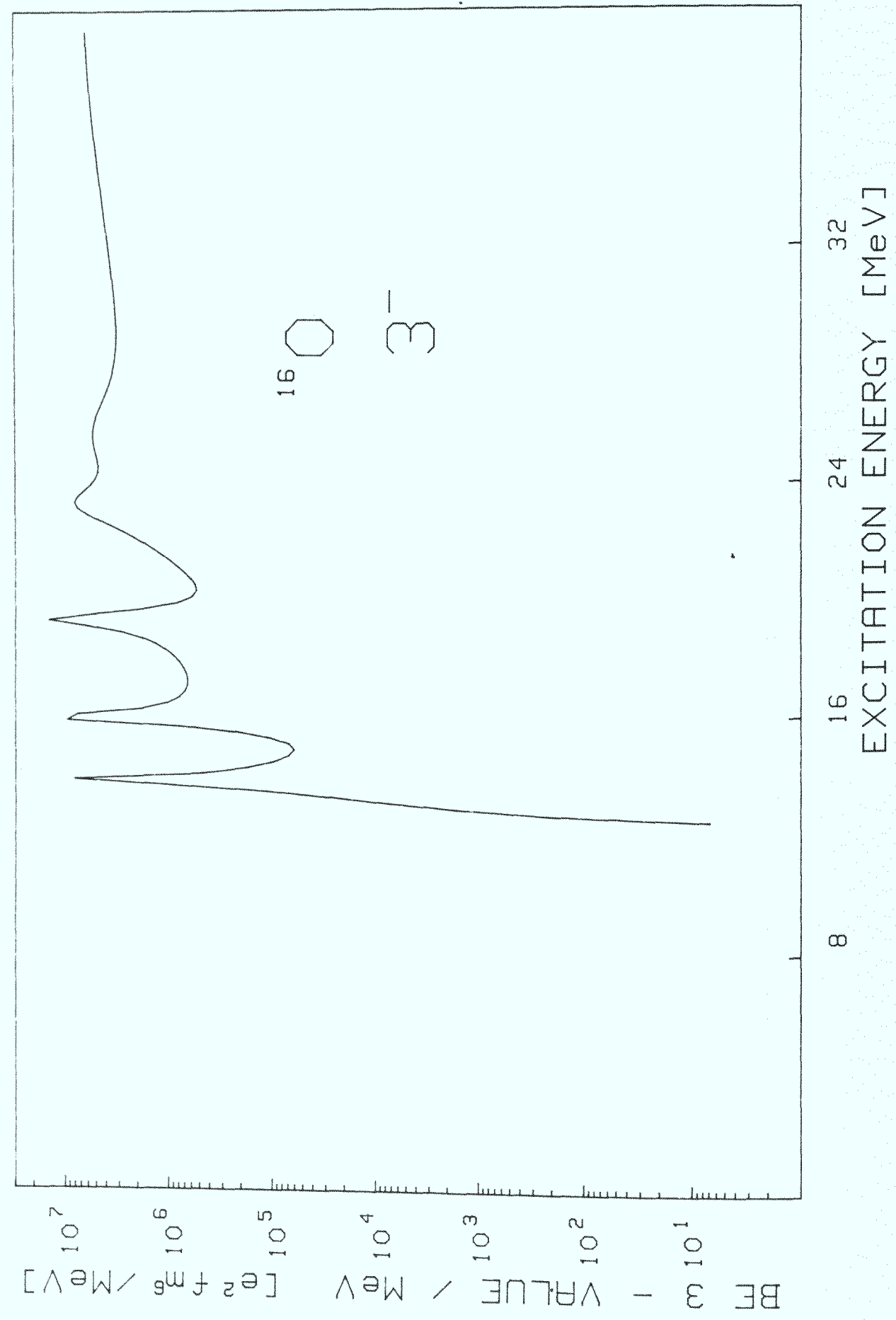


Fig. 14

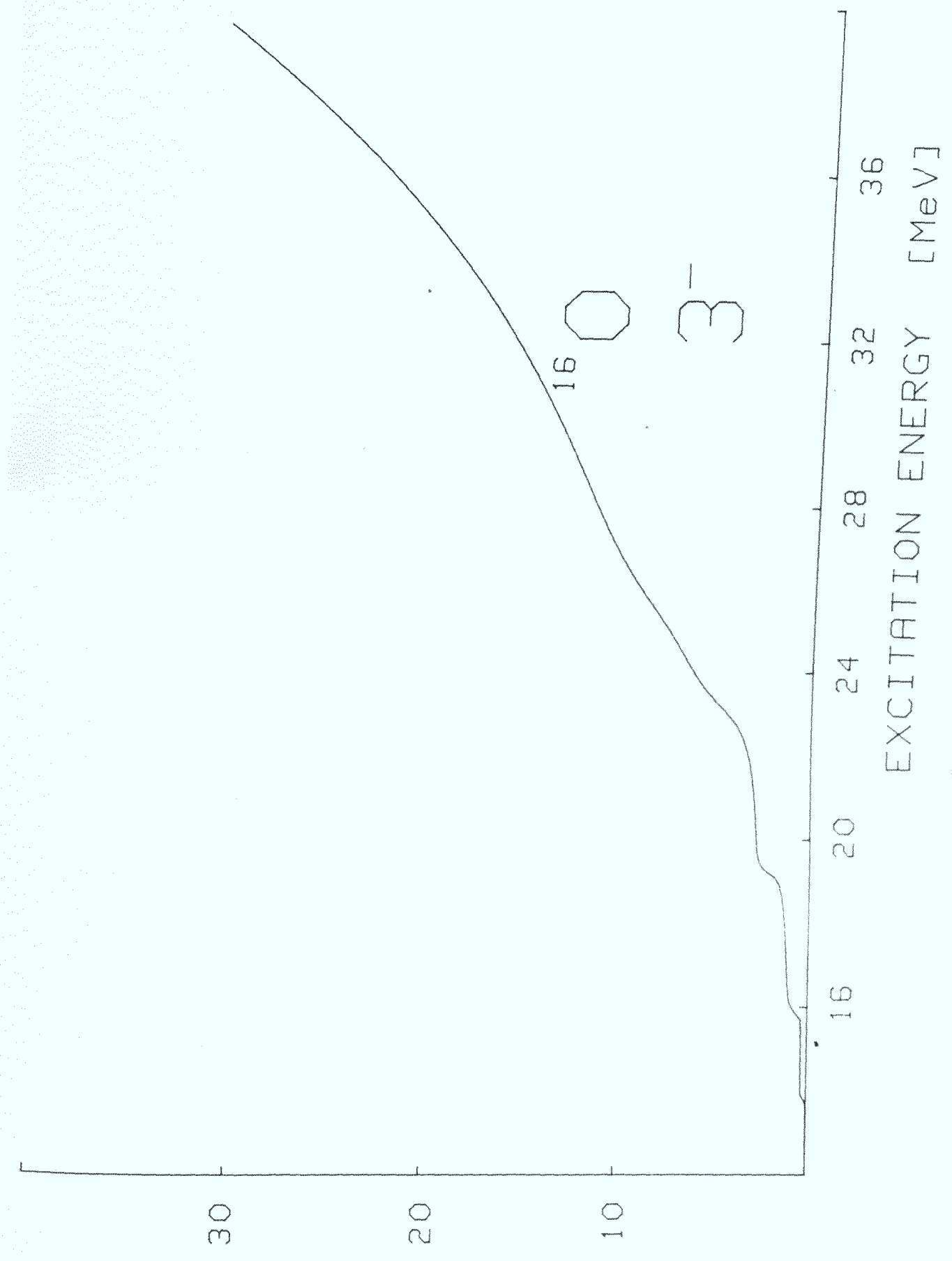


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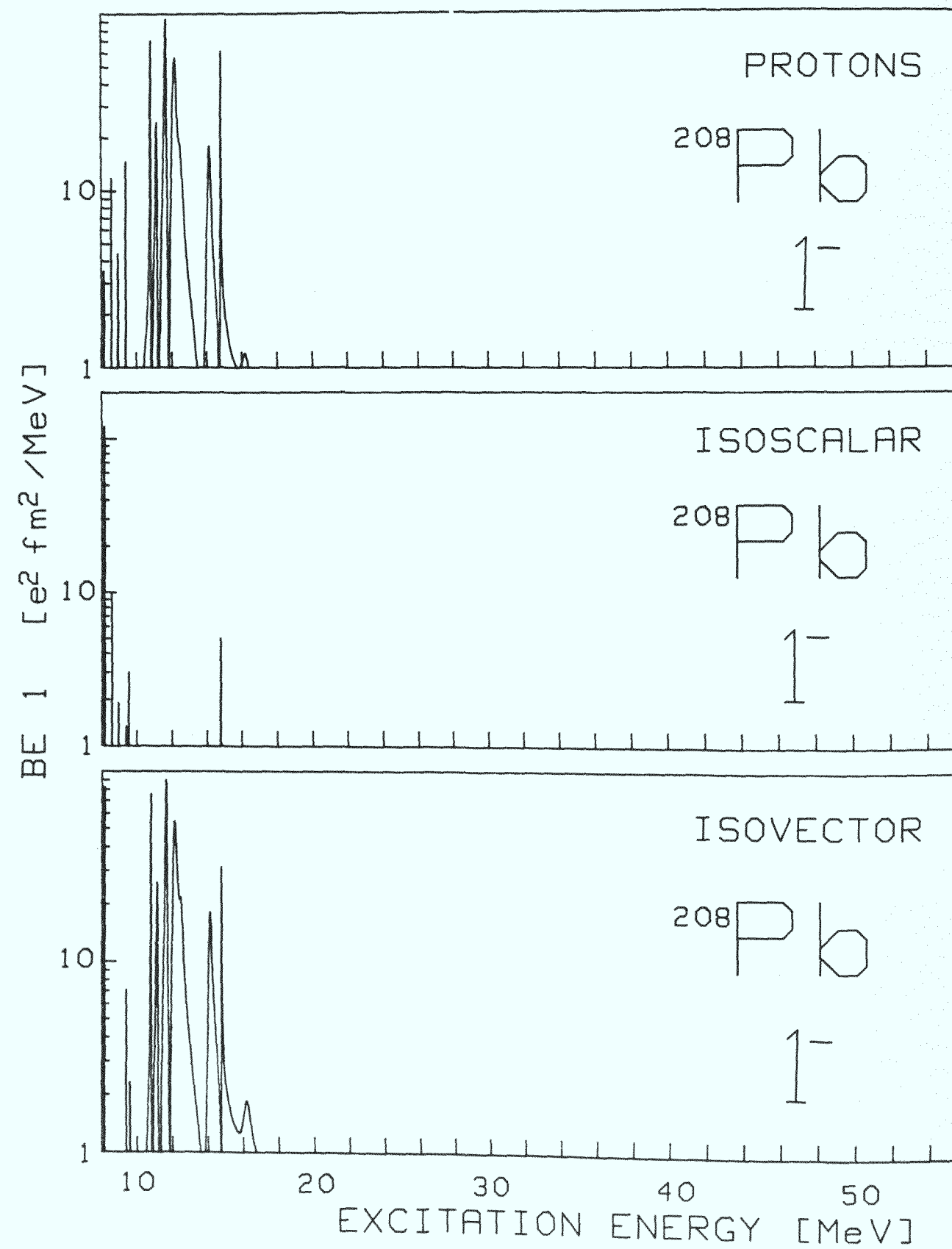


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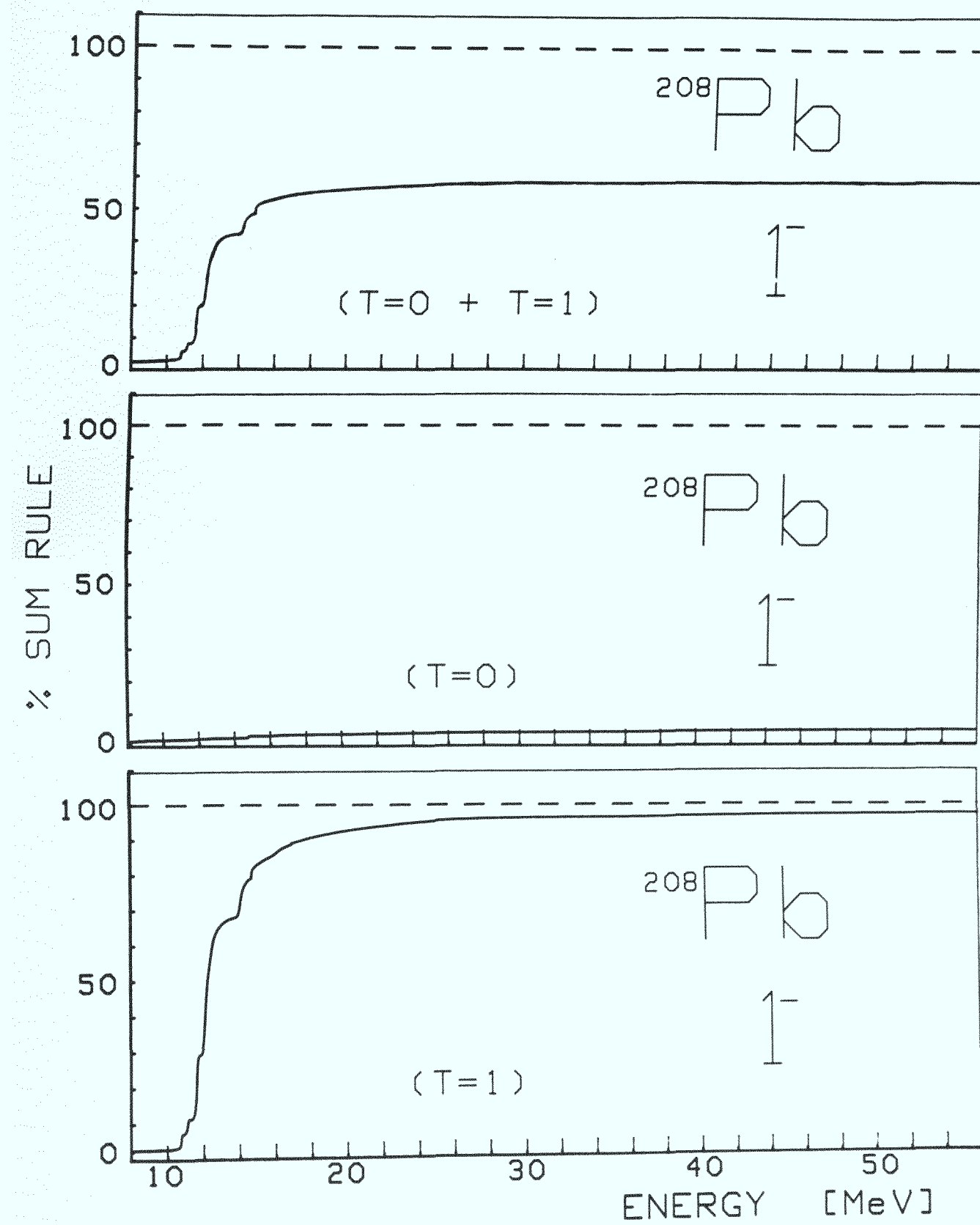


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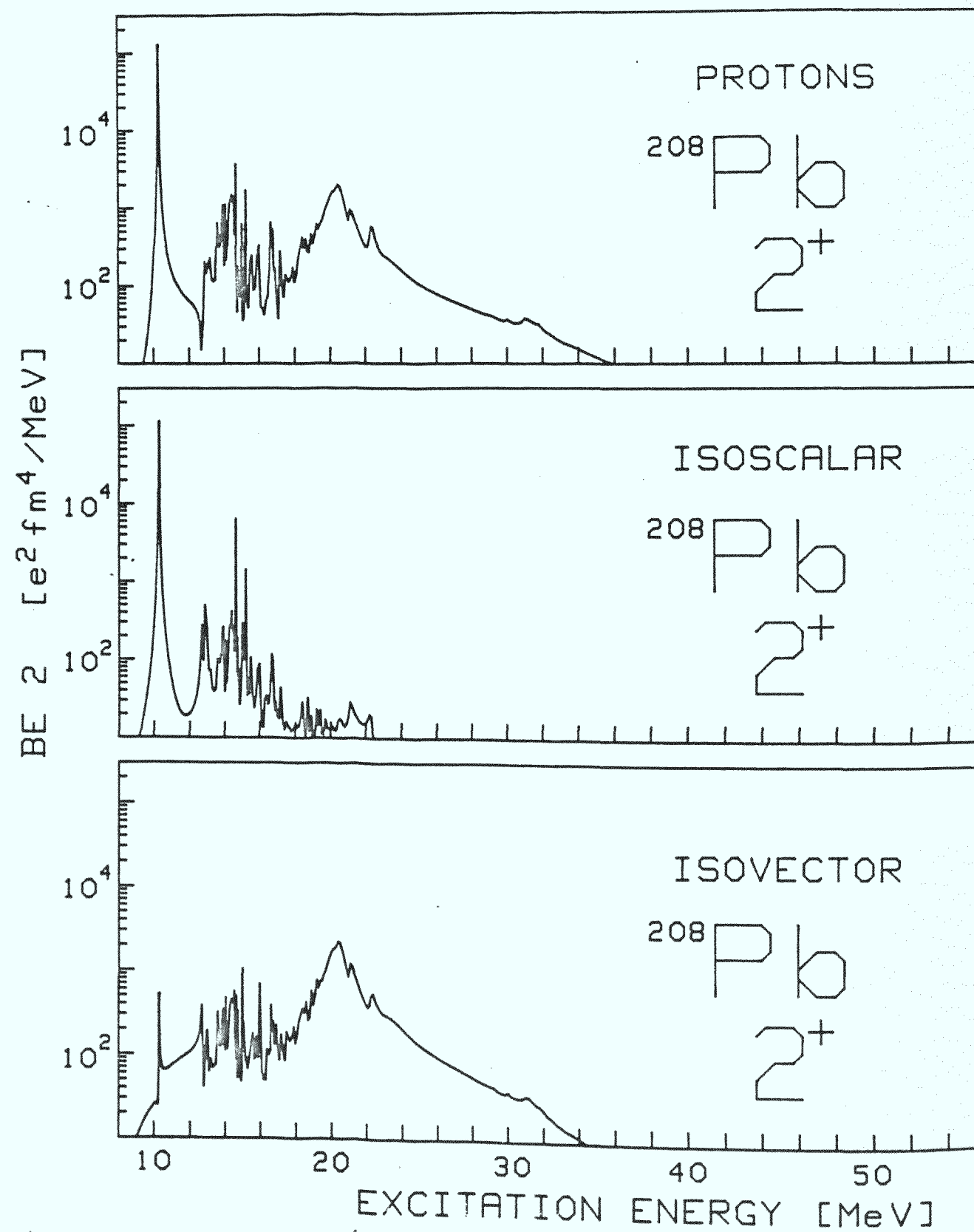


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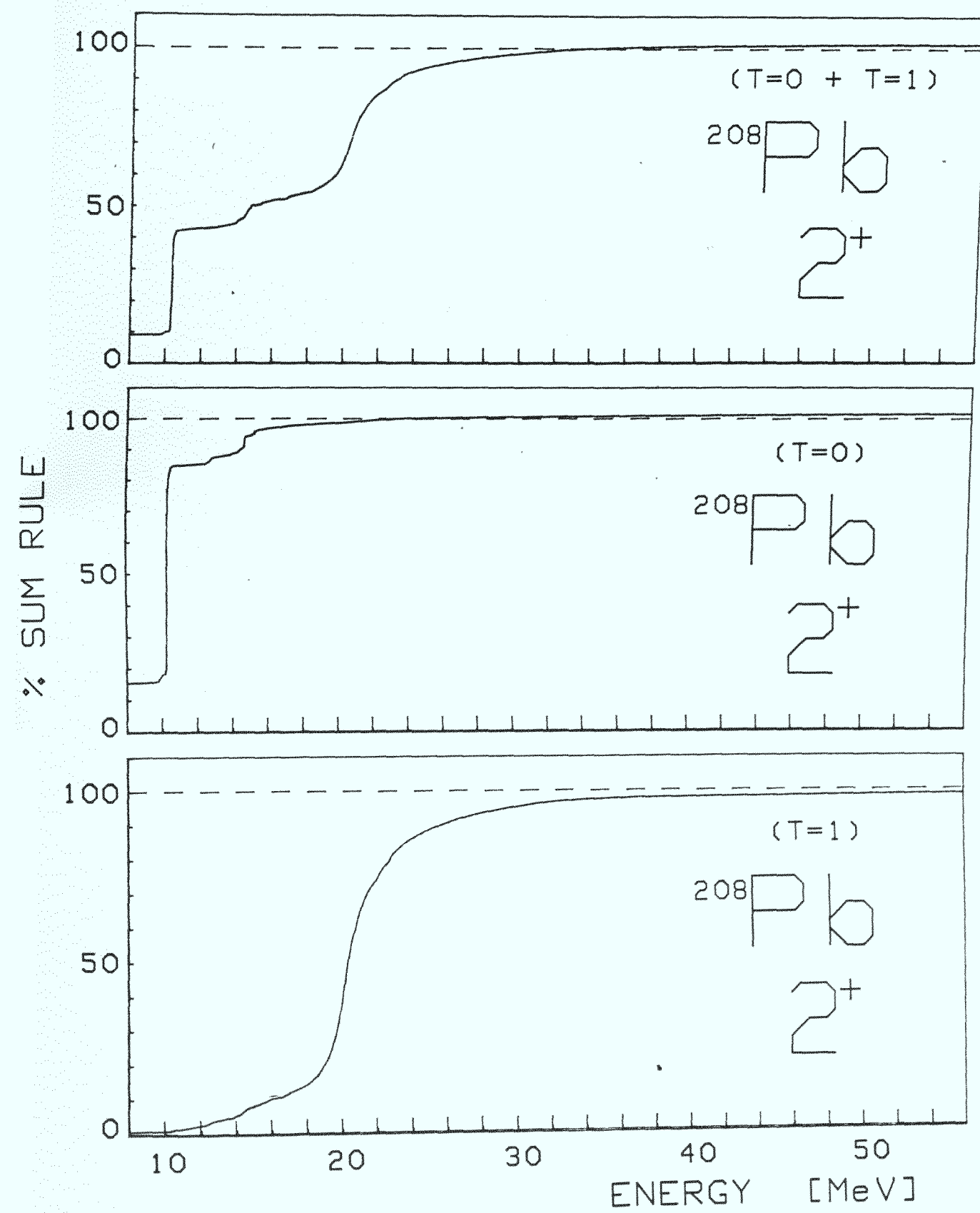


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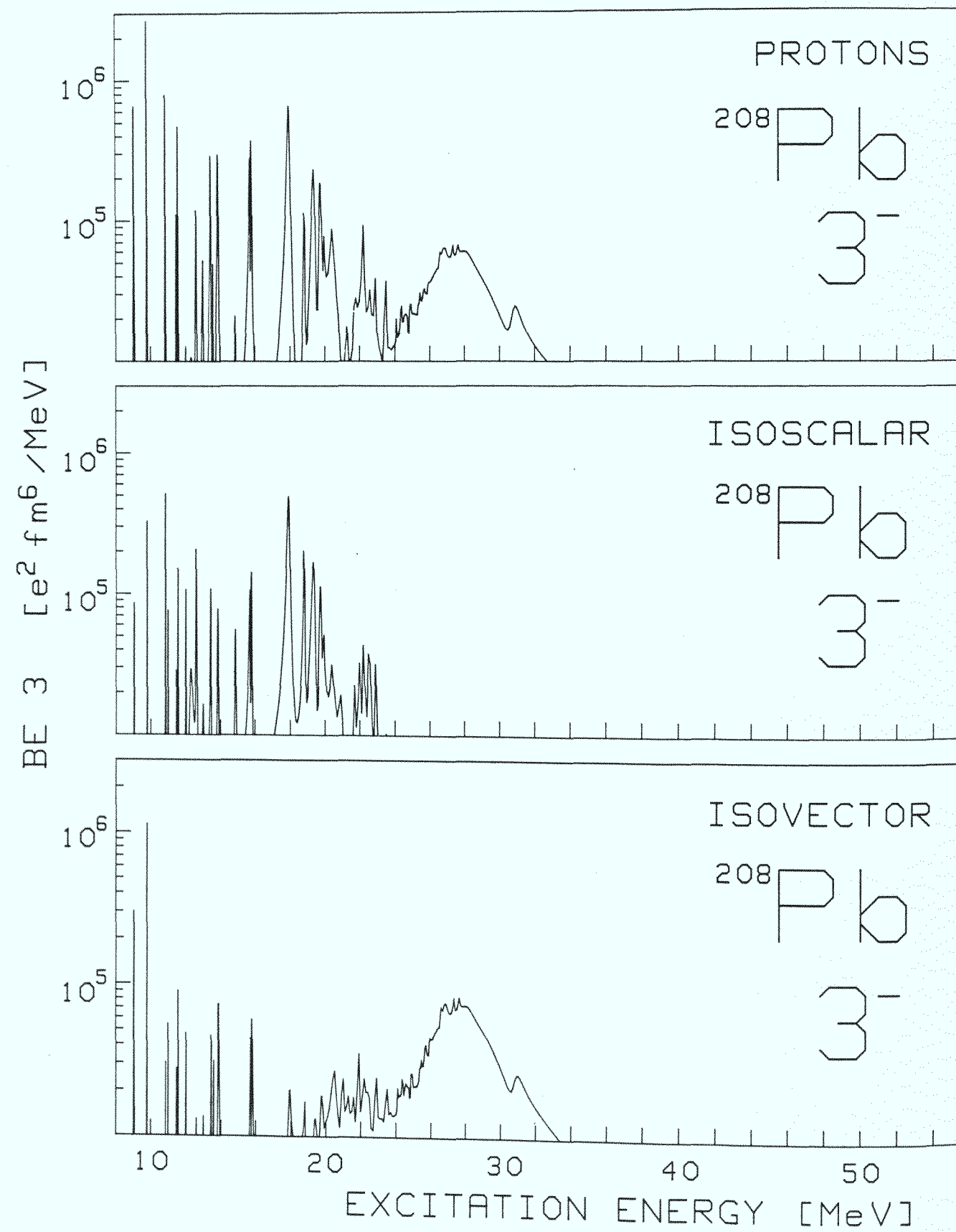


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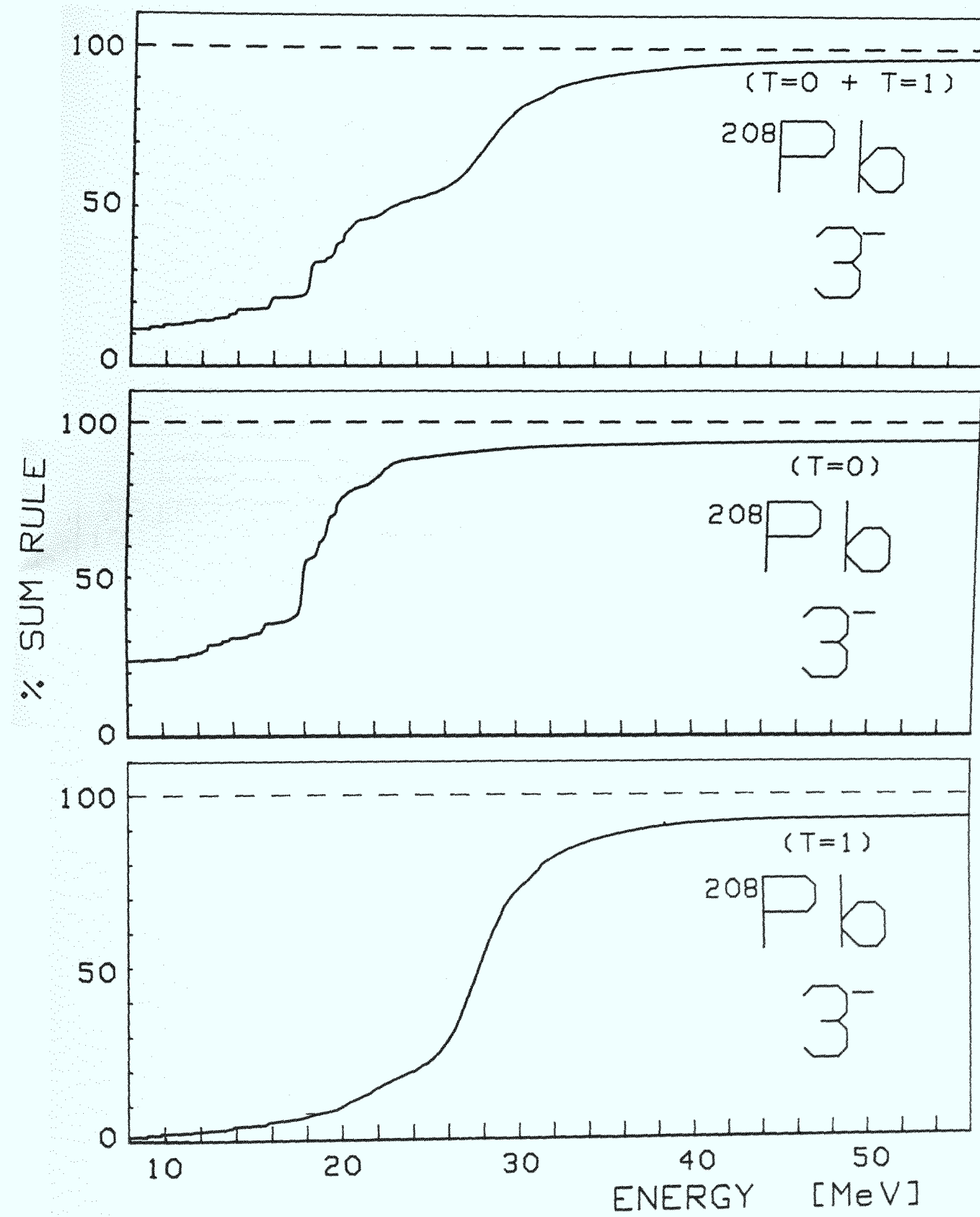


Fig. 21

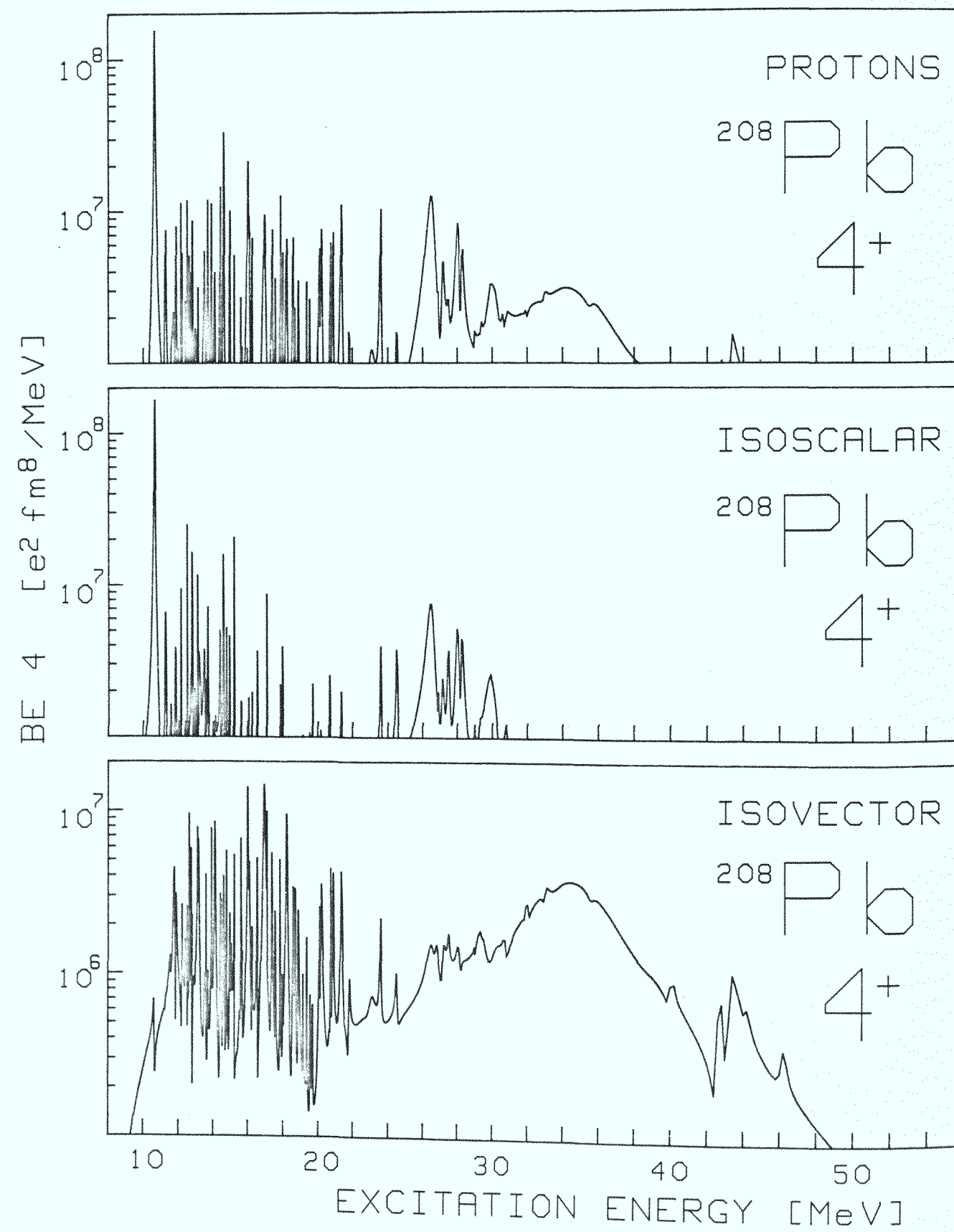


Fig. 2

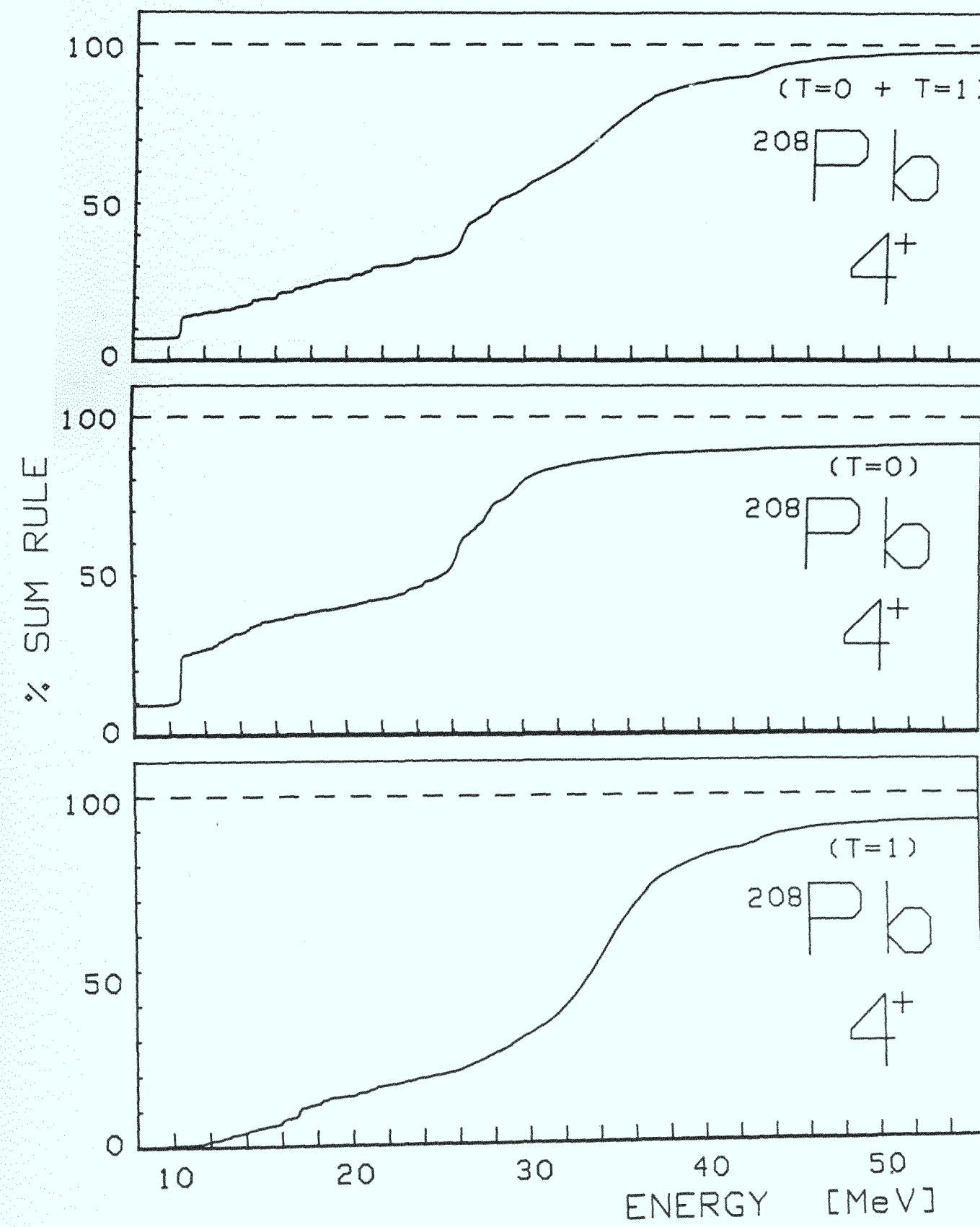


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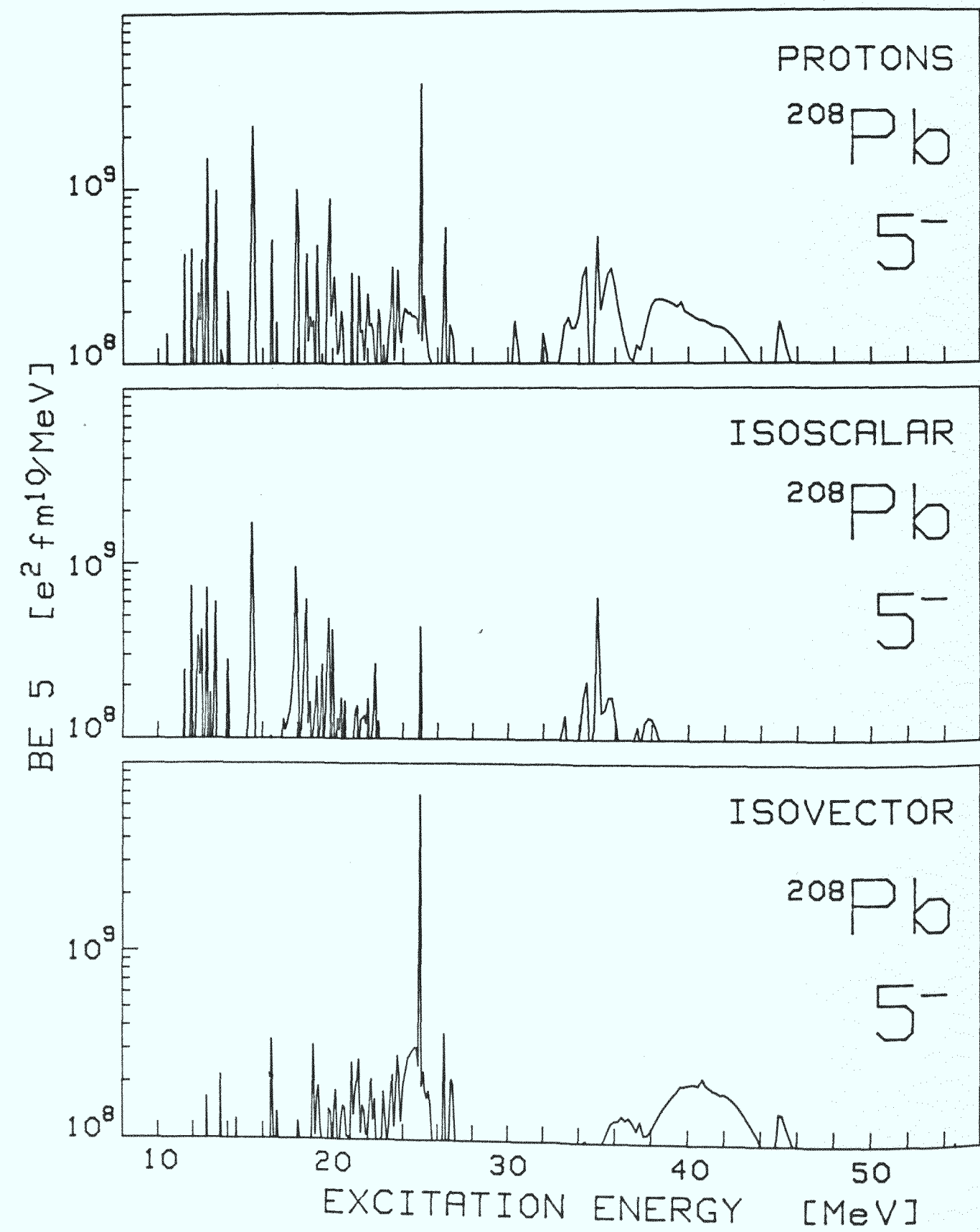


Fig.

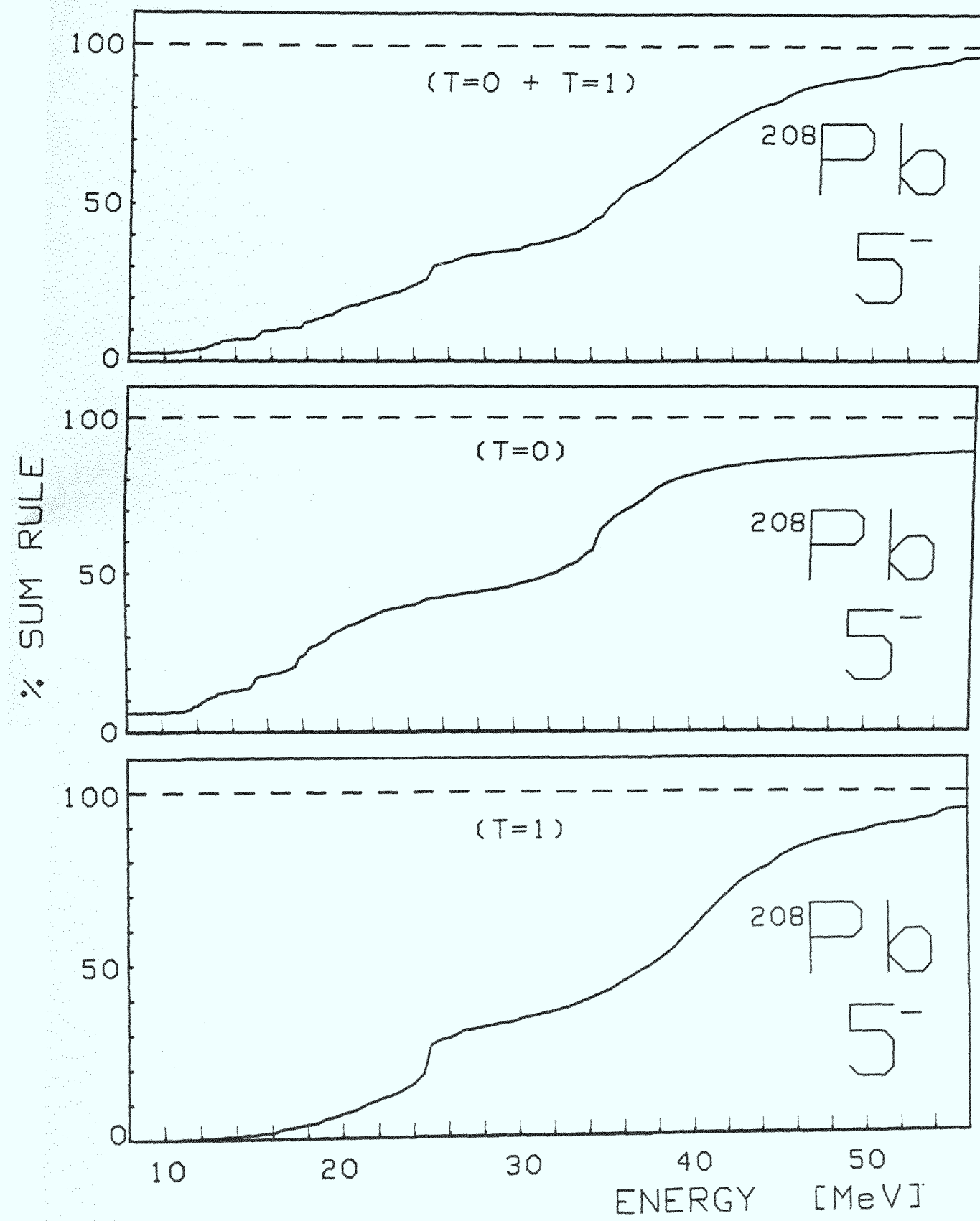


Fig. 25

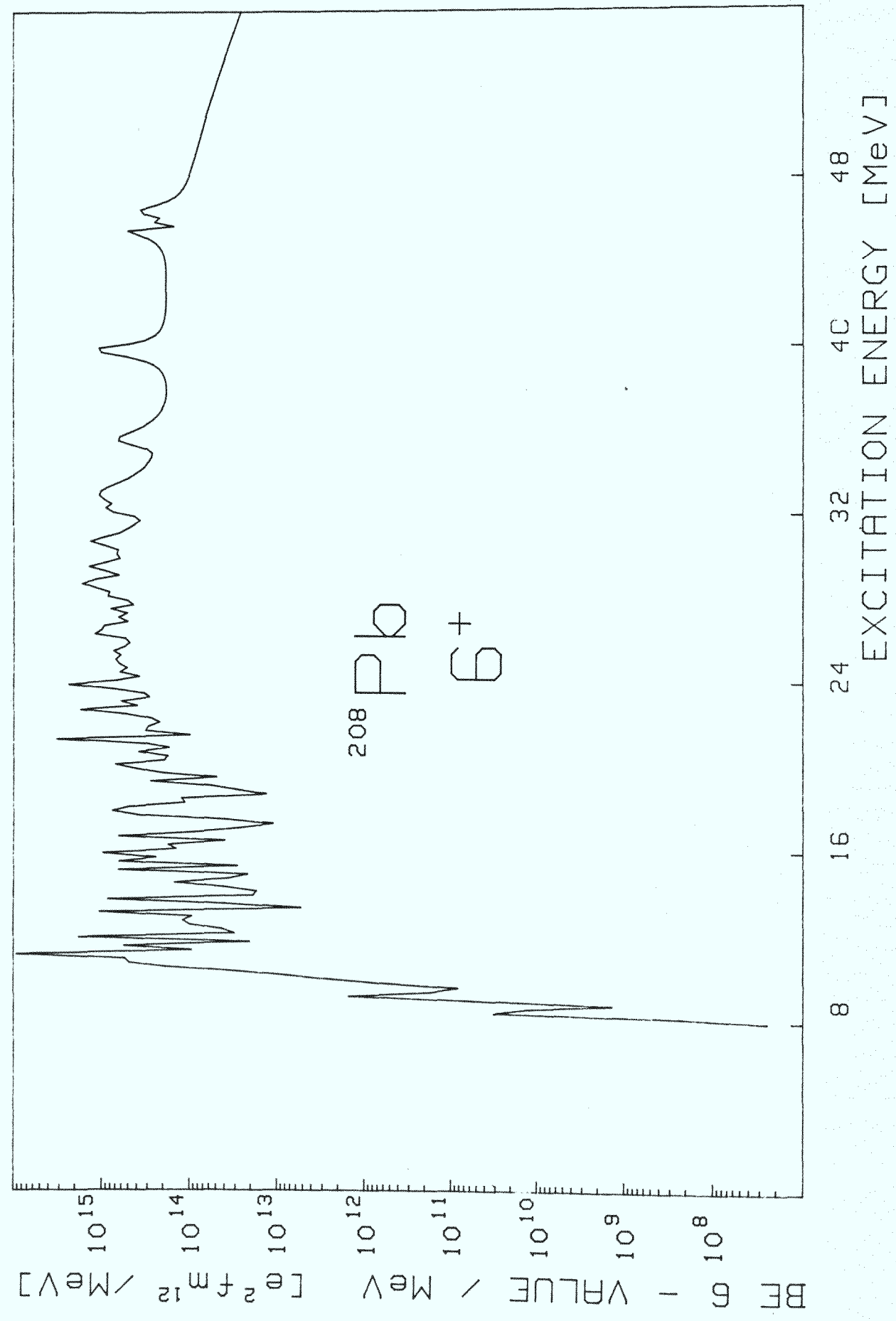


Fig. 26

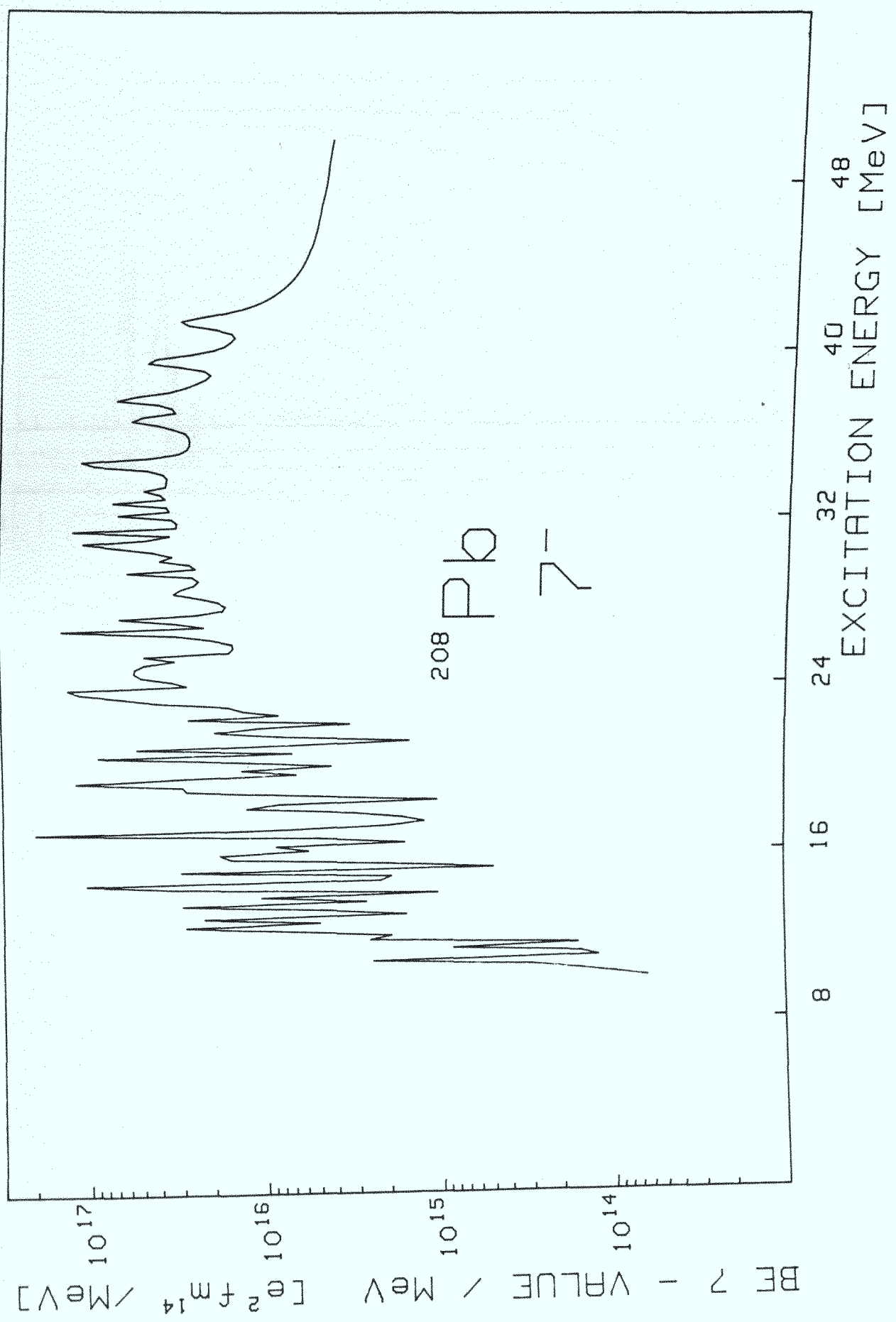


Fig. 27

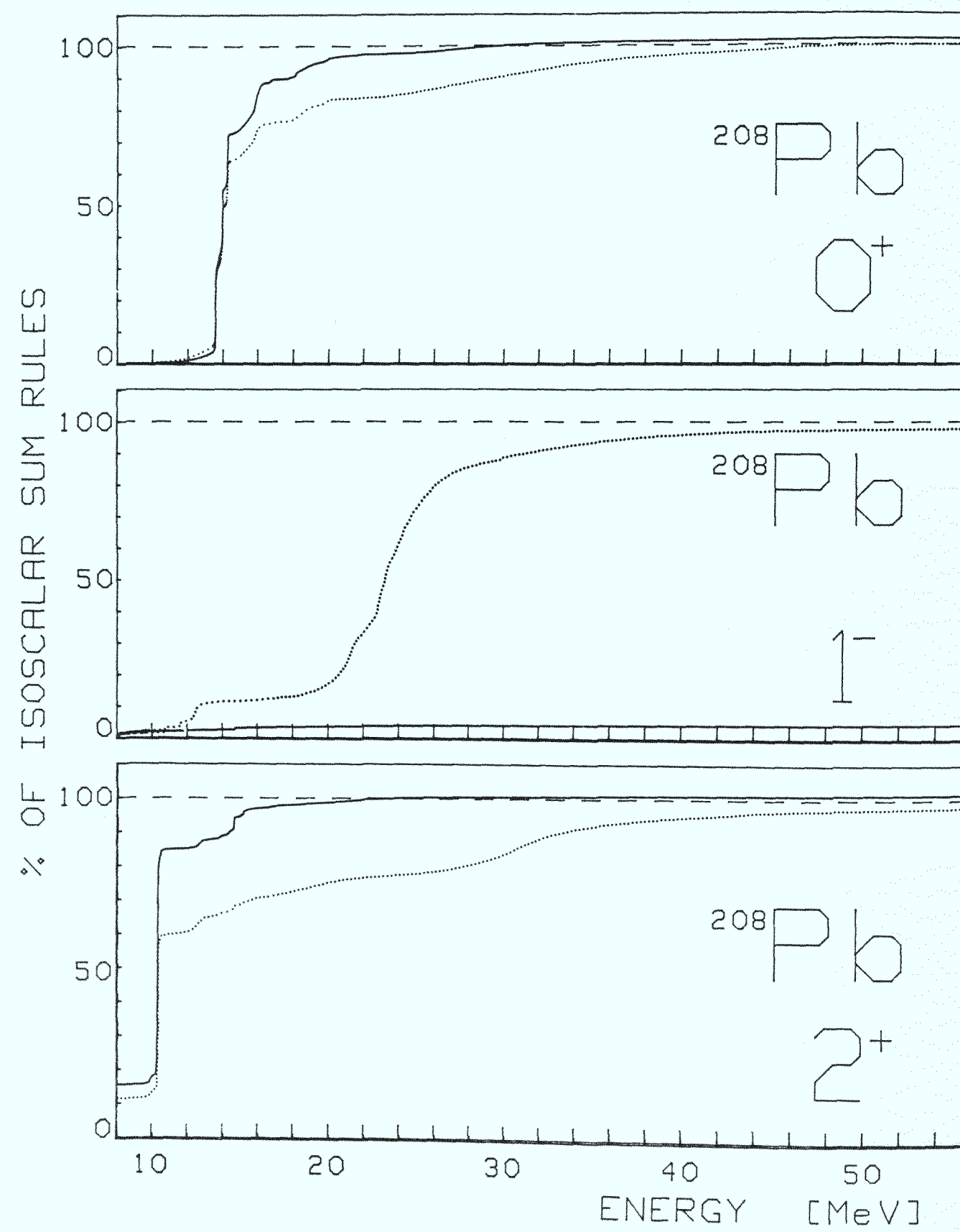


Fig. 28

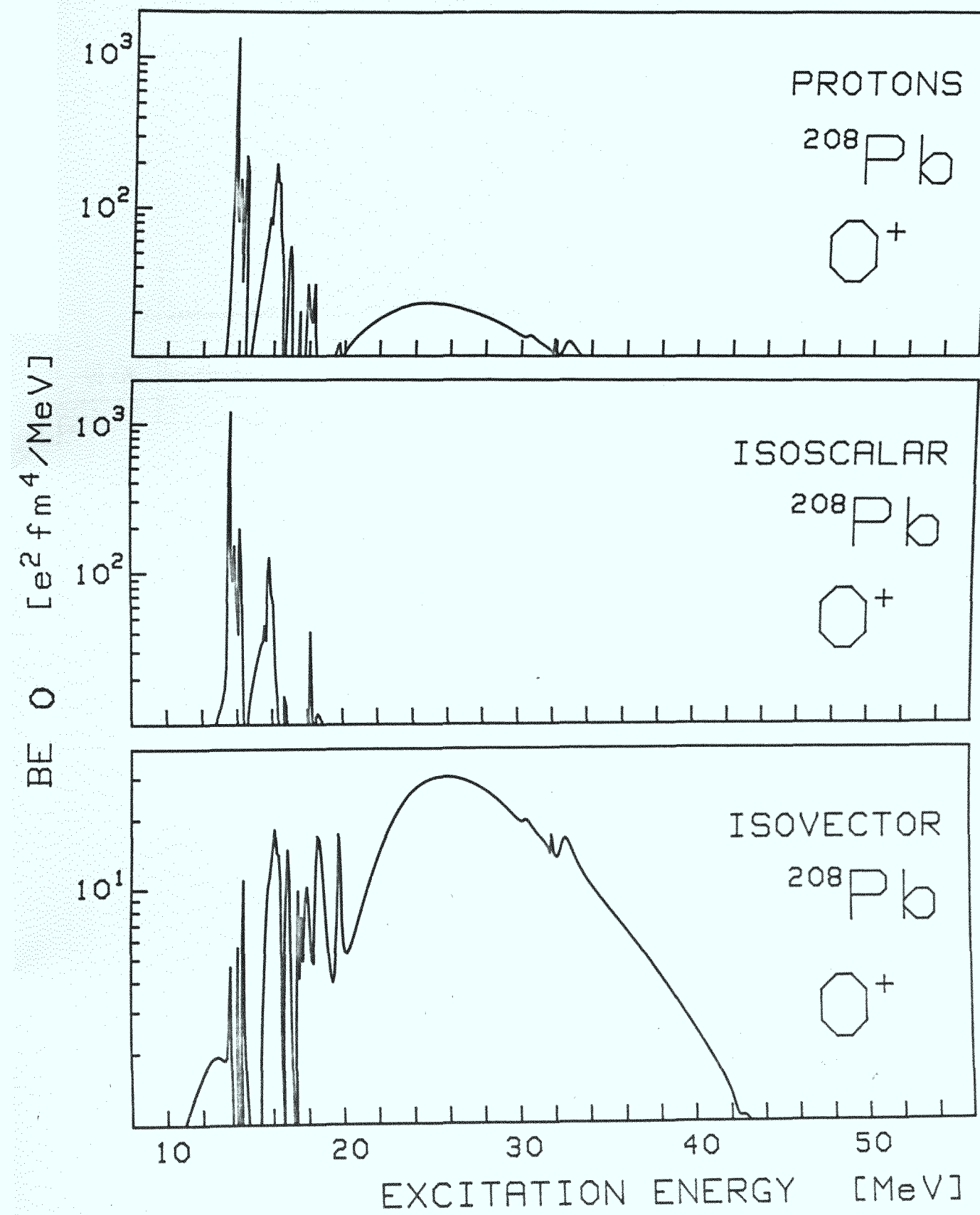


Fig. 29

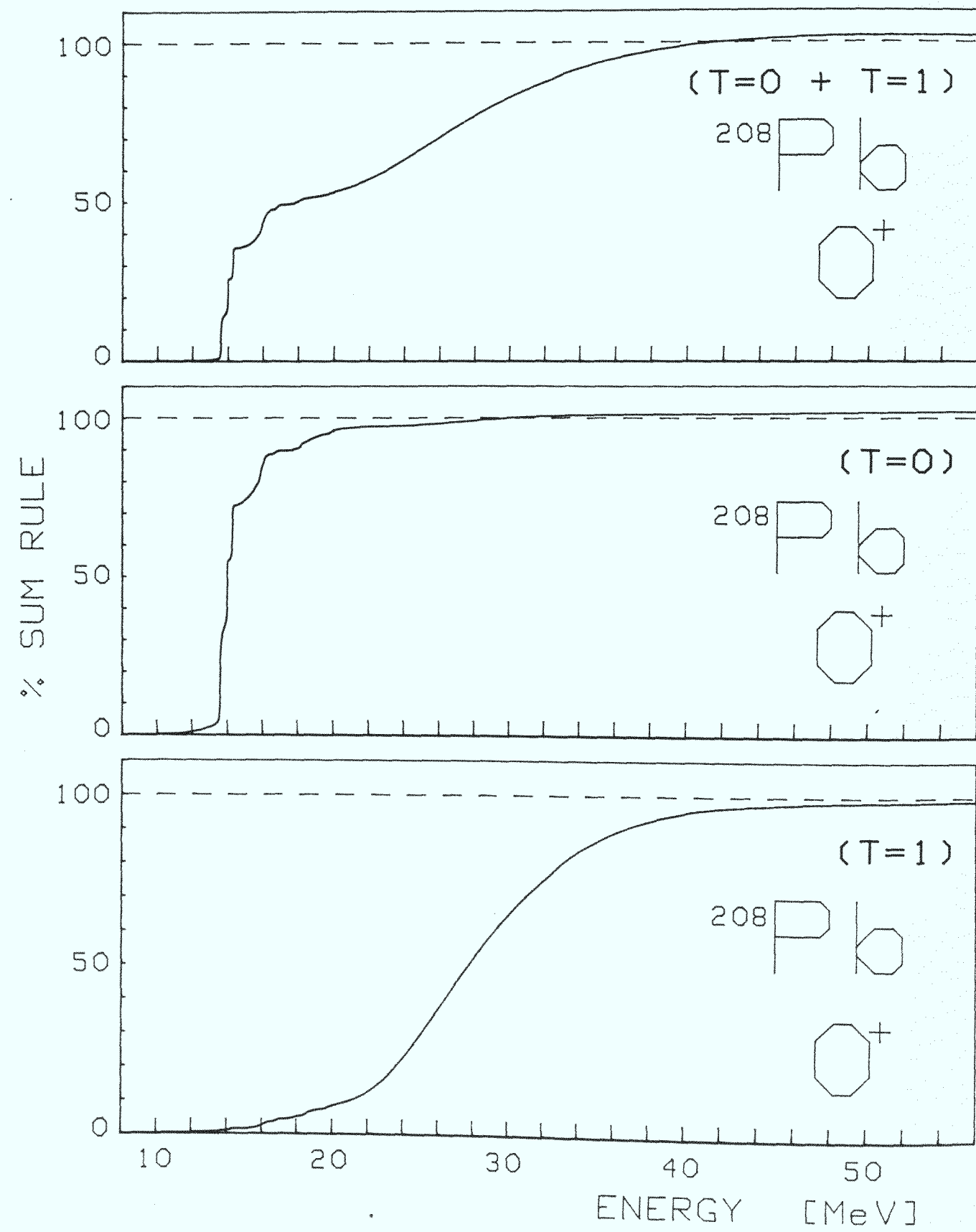


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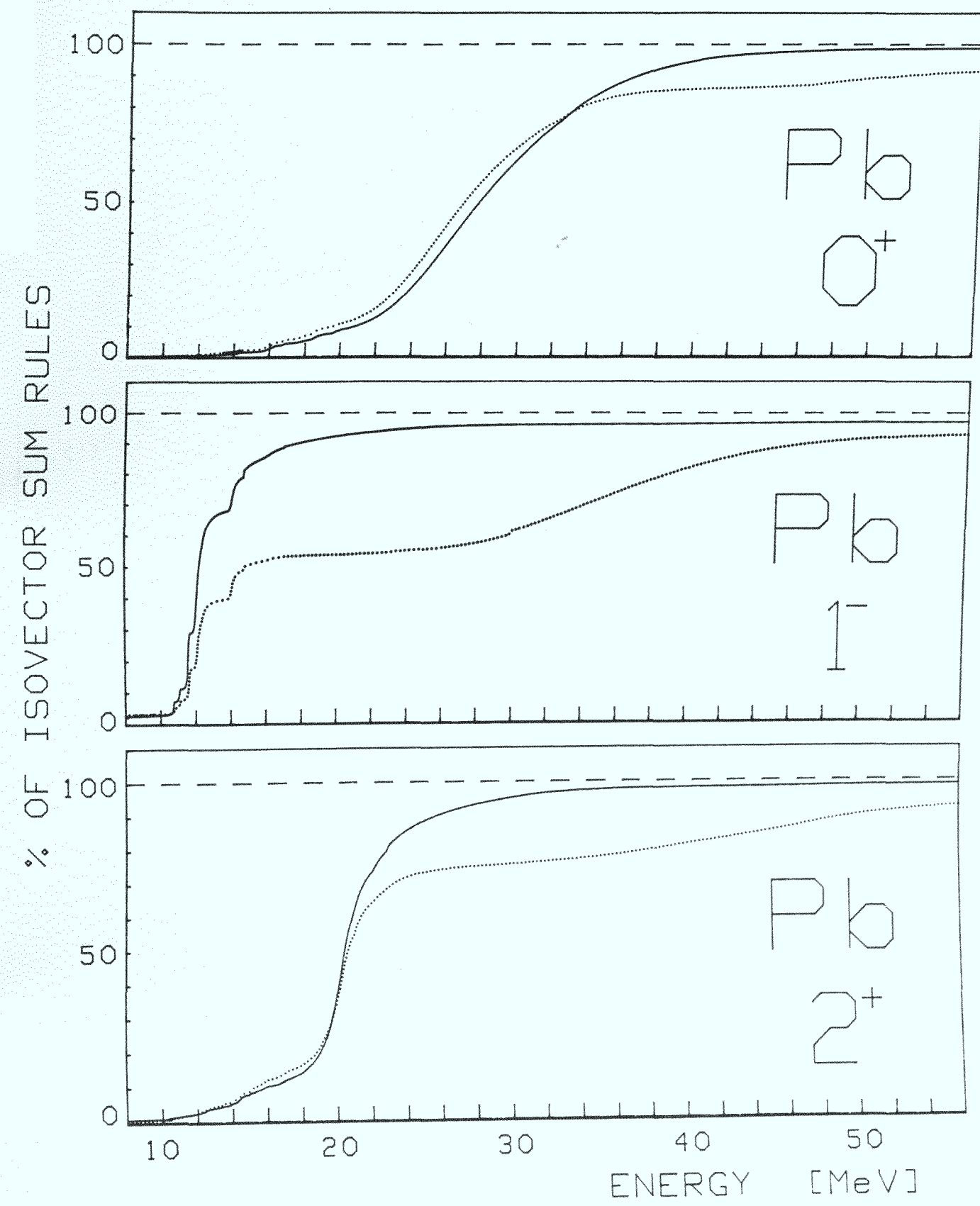


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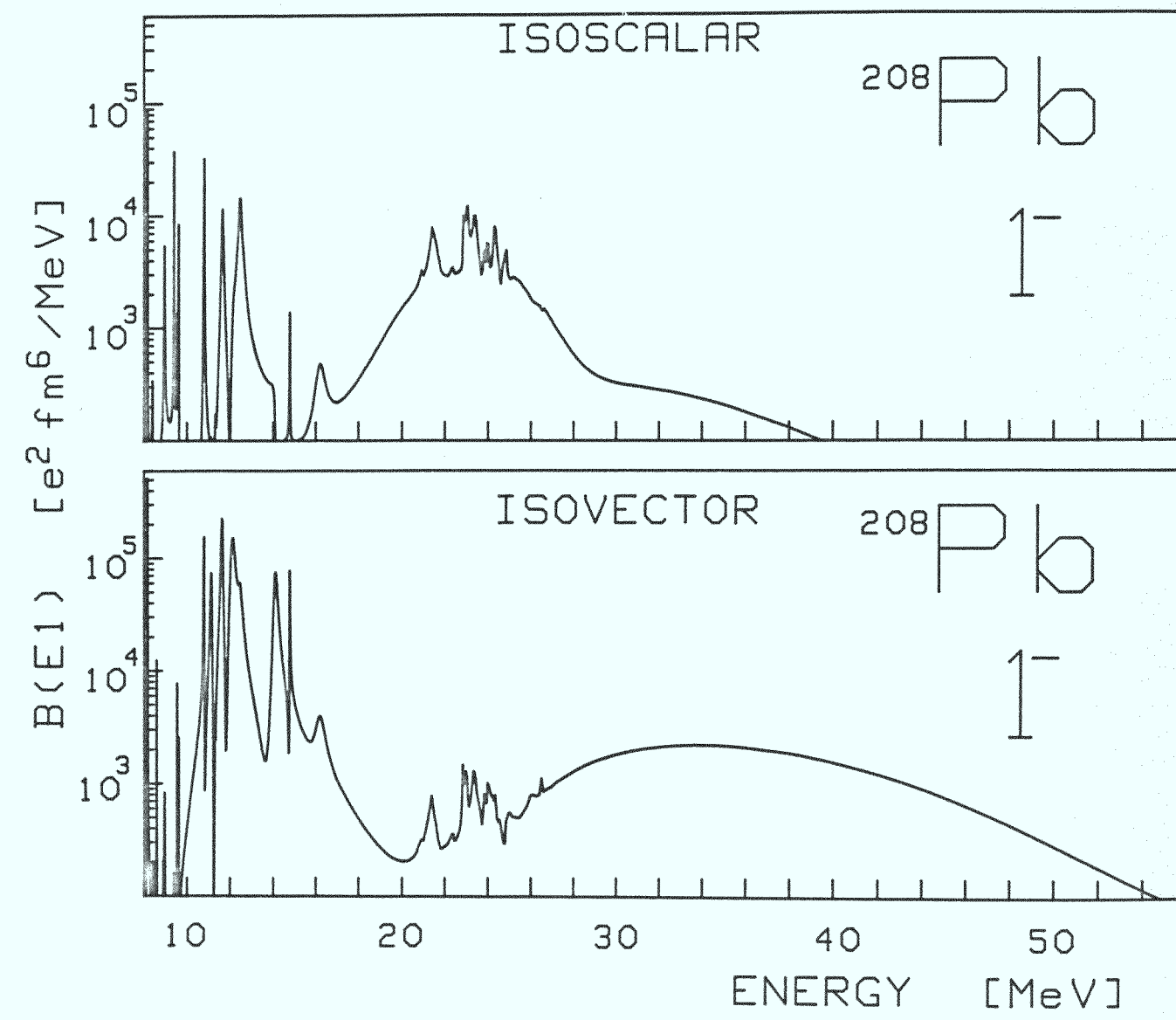


Fig. 32

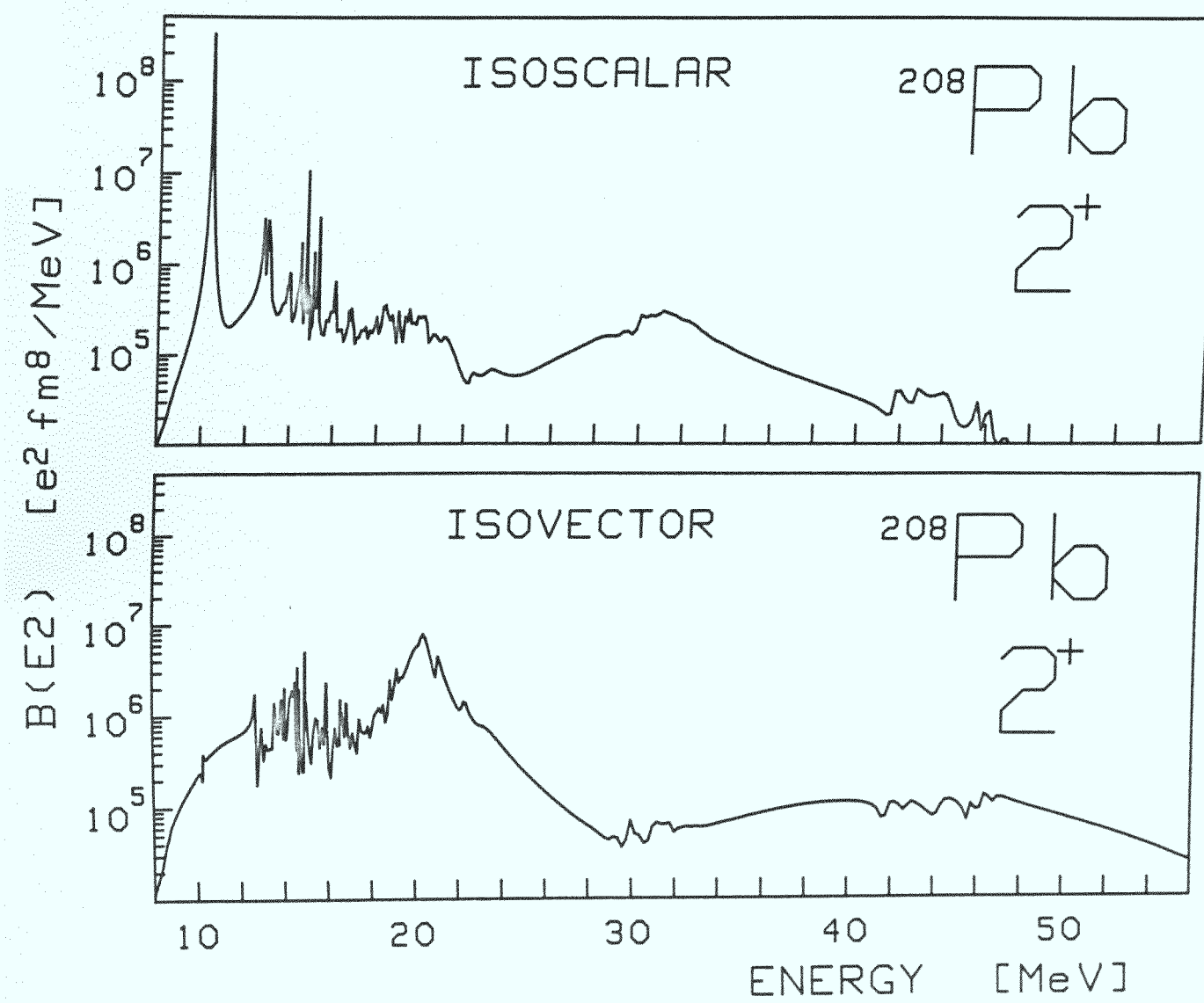


Fig. 33

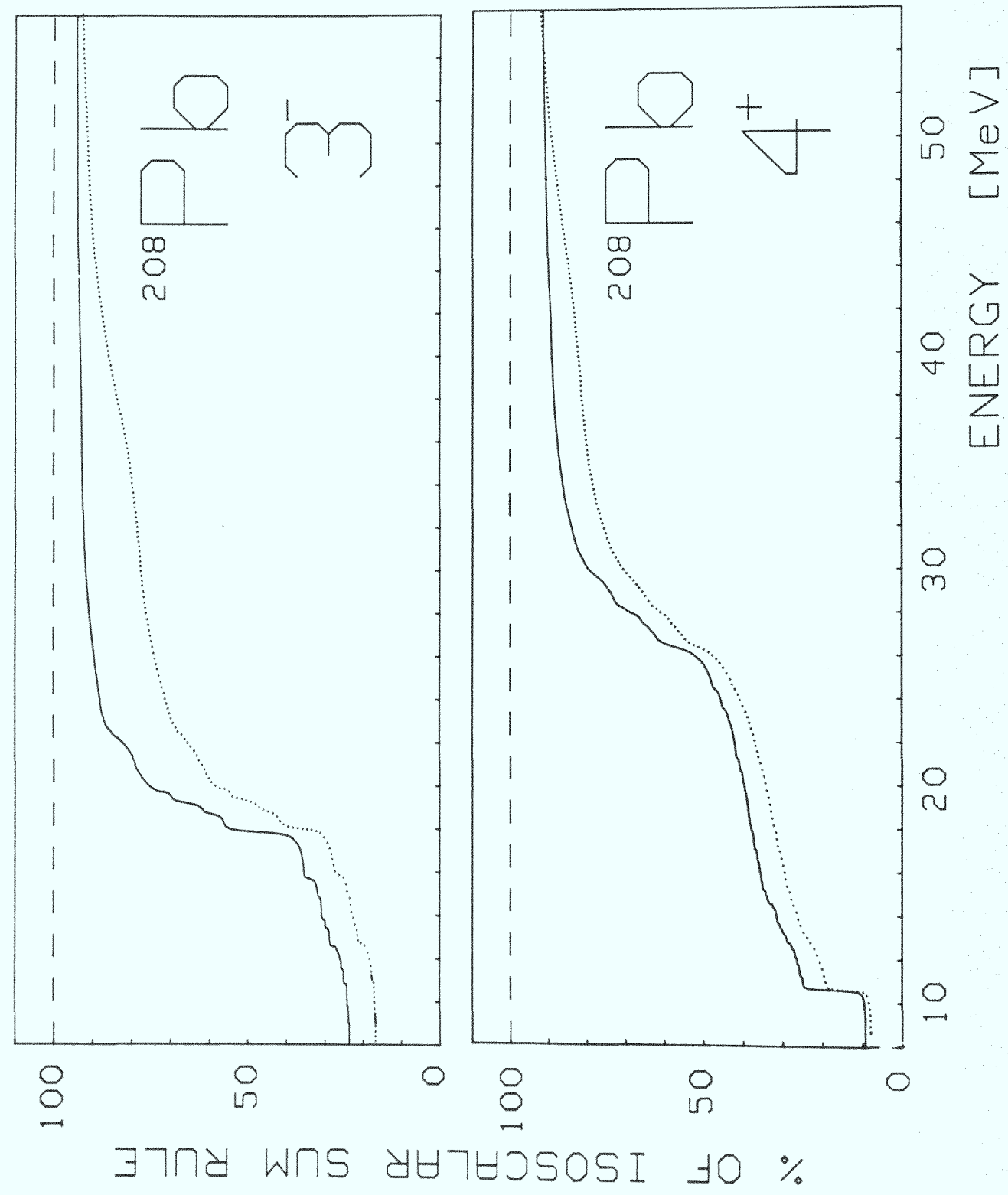


Fig. 34

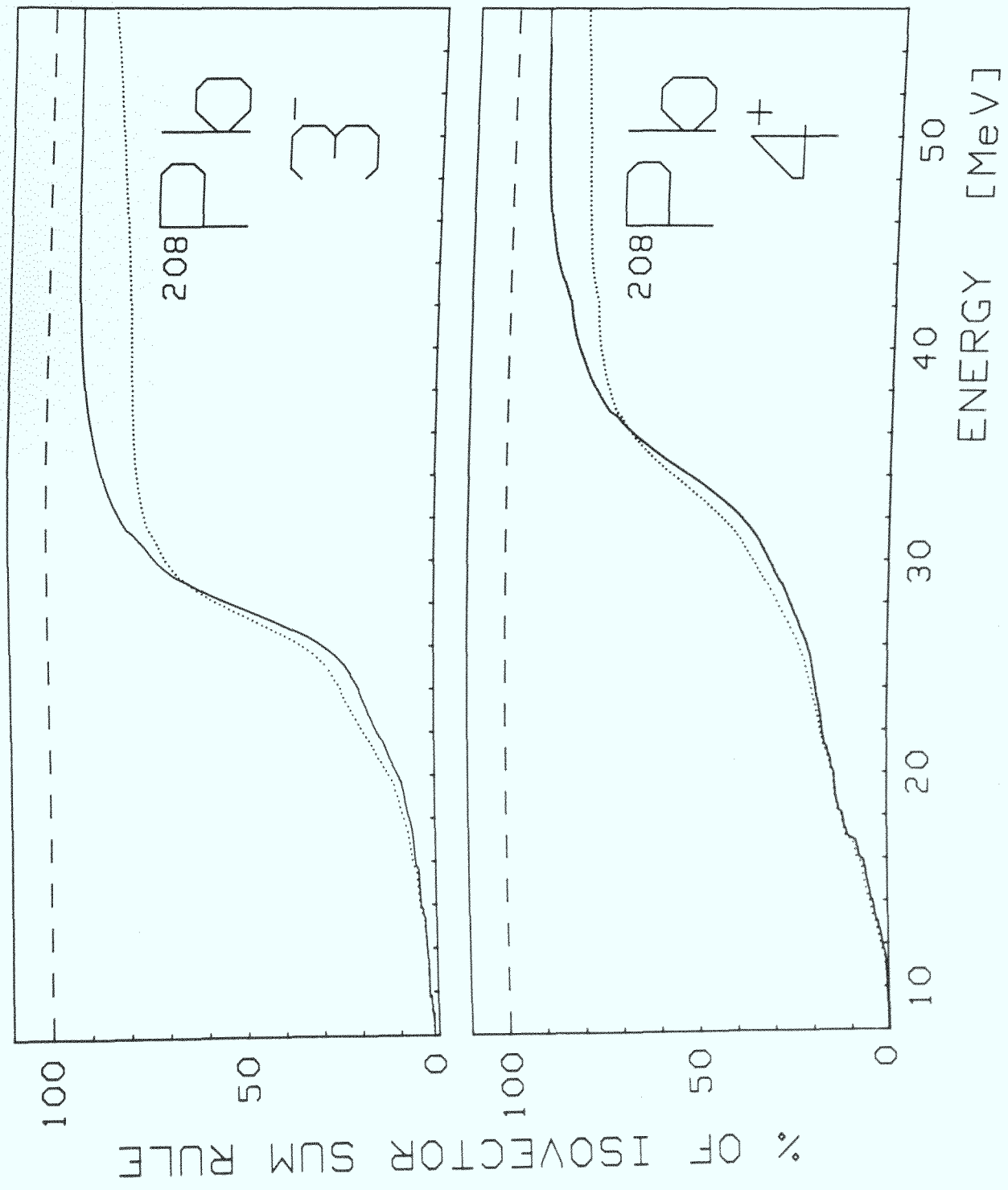


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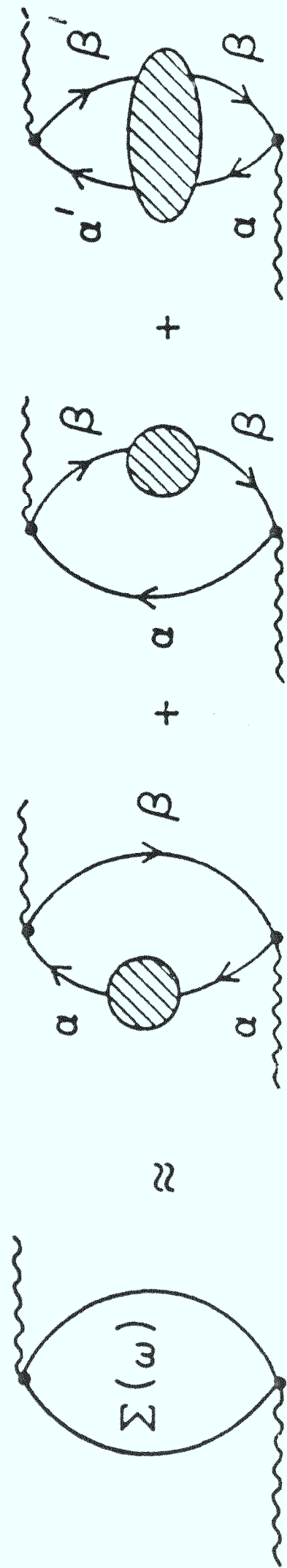


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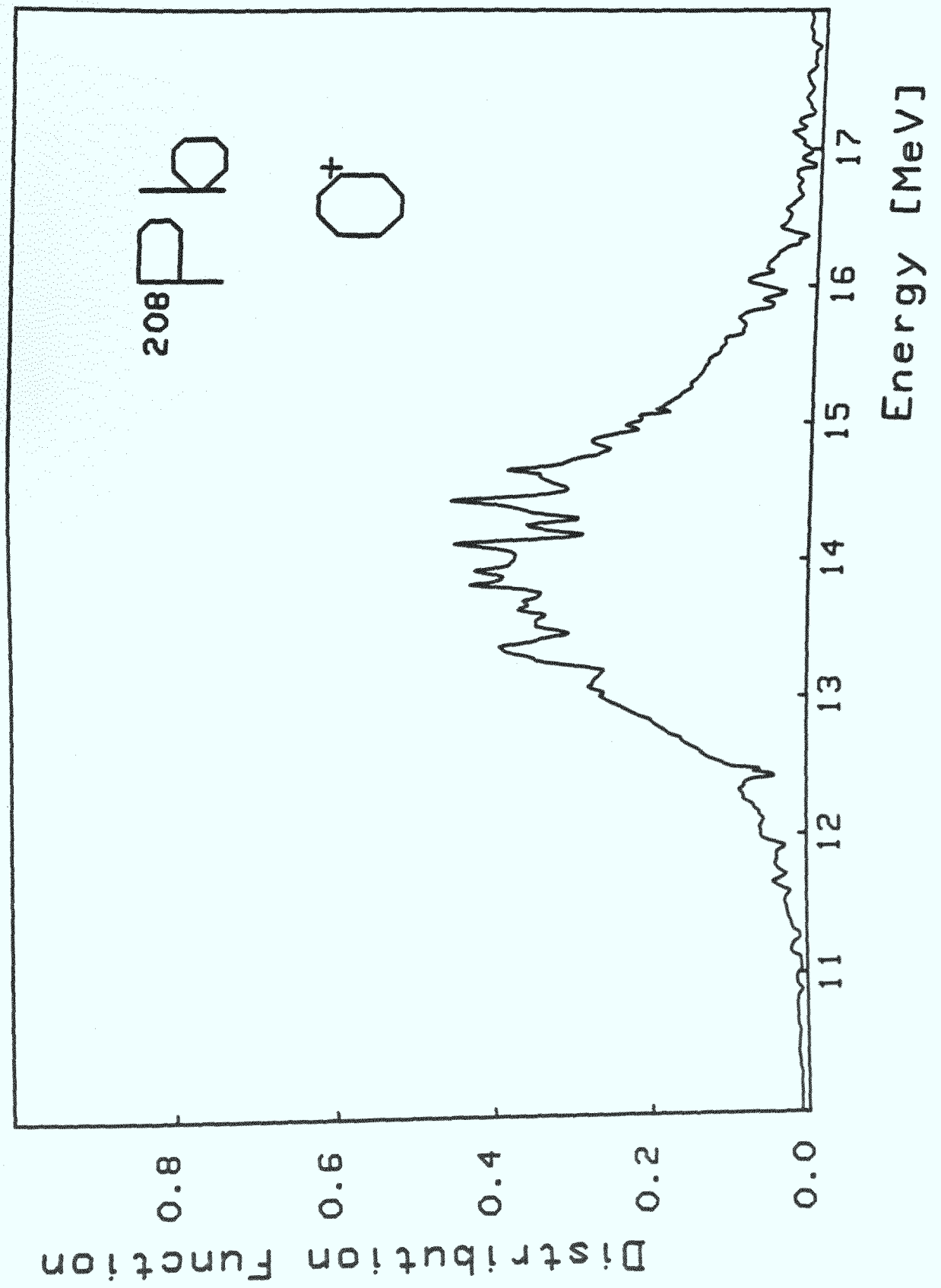


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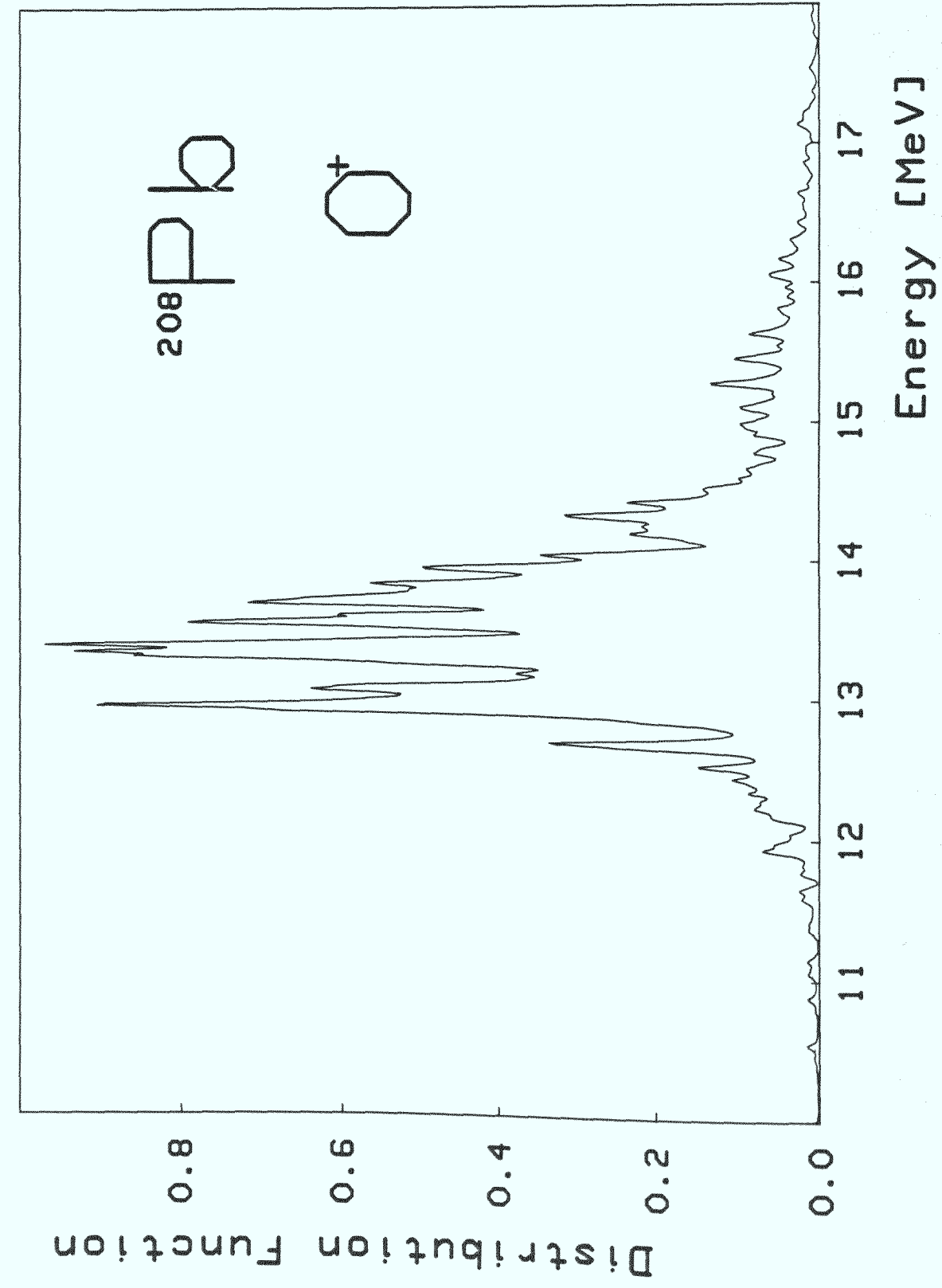


Fig. 38

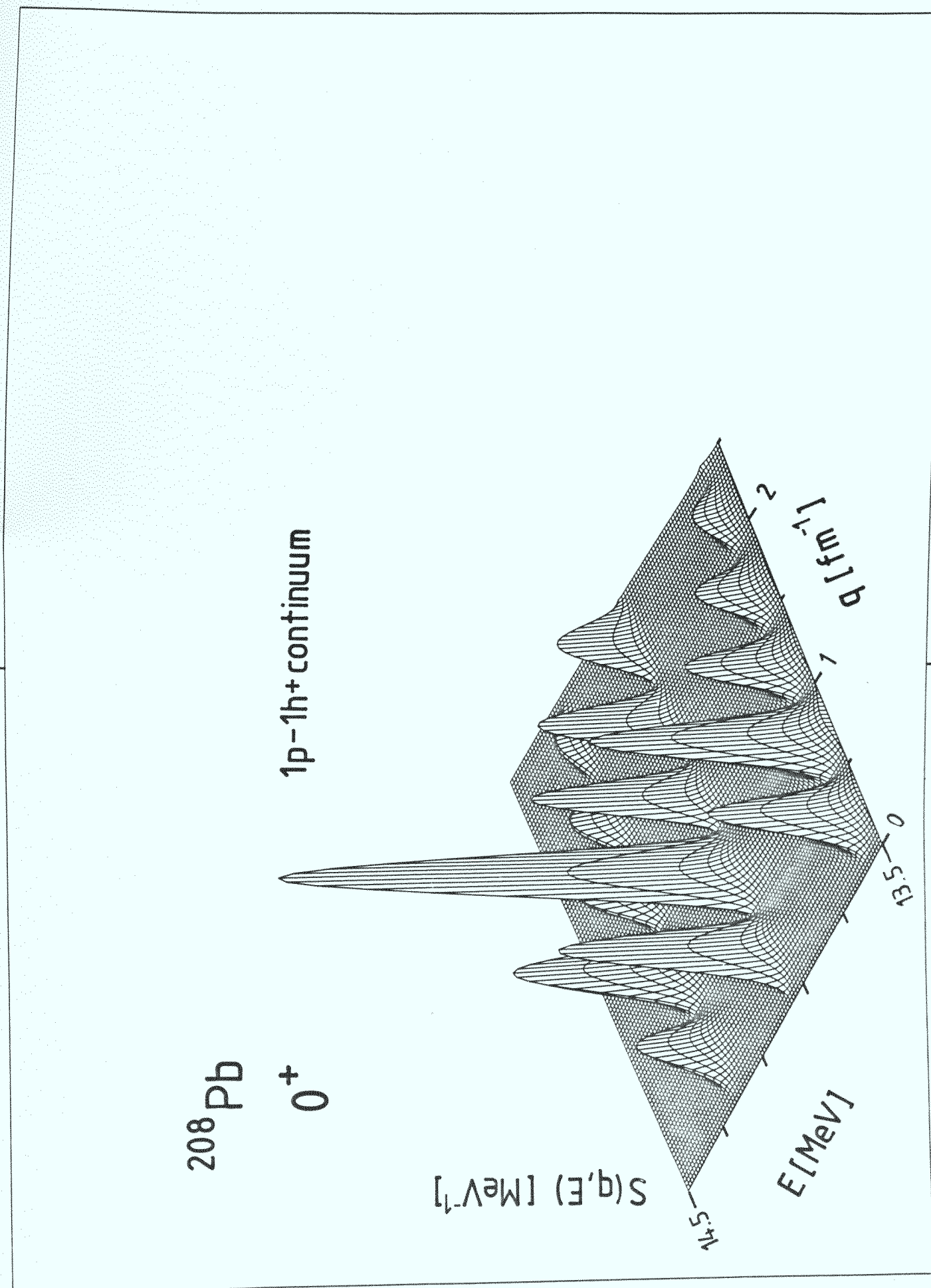


Fig. 39

^{208}Pb
 0^+

$1p-1h+2p-2h$
+continuum

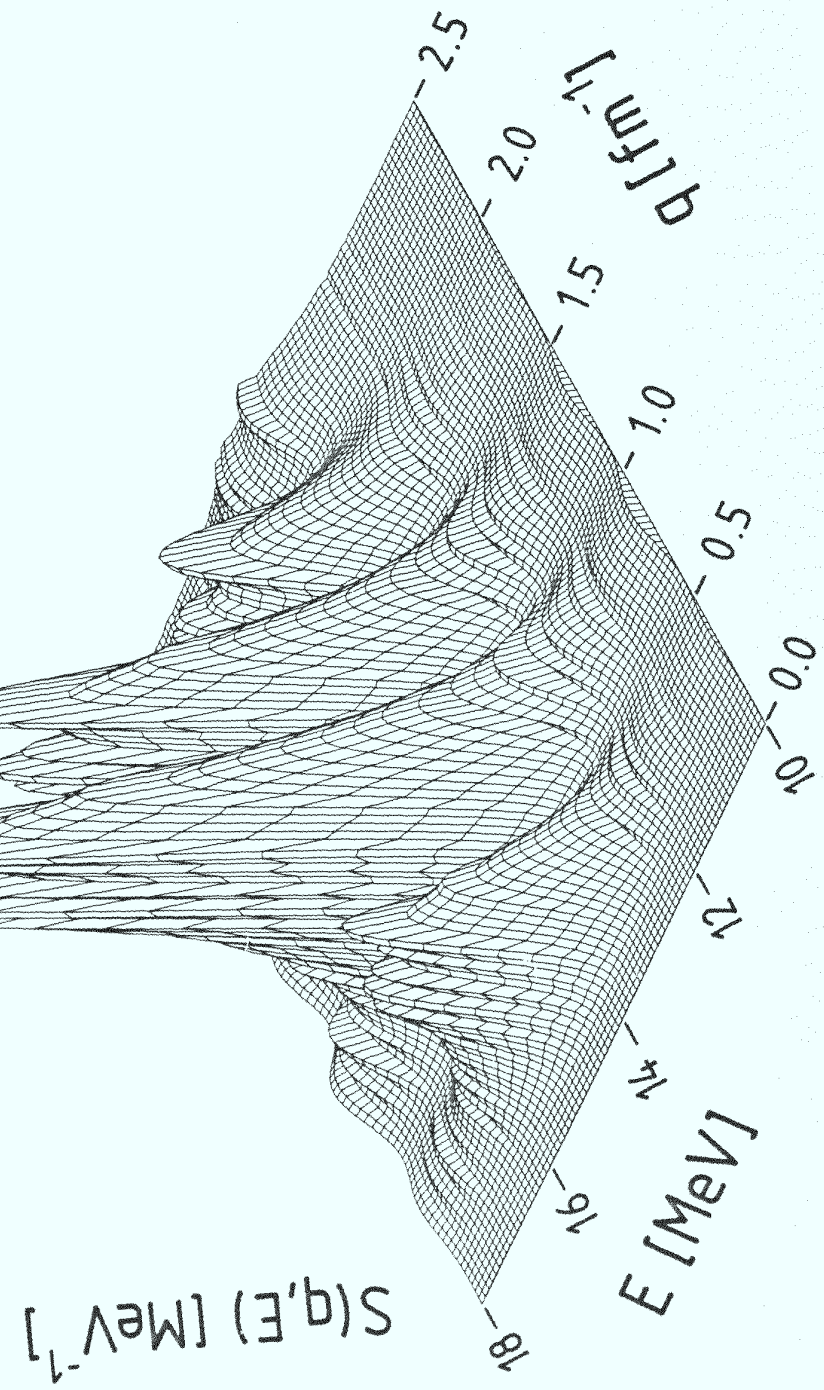


Fig. 40

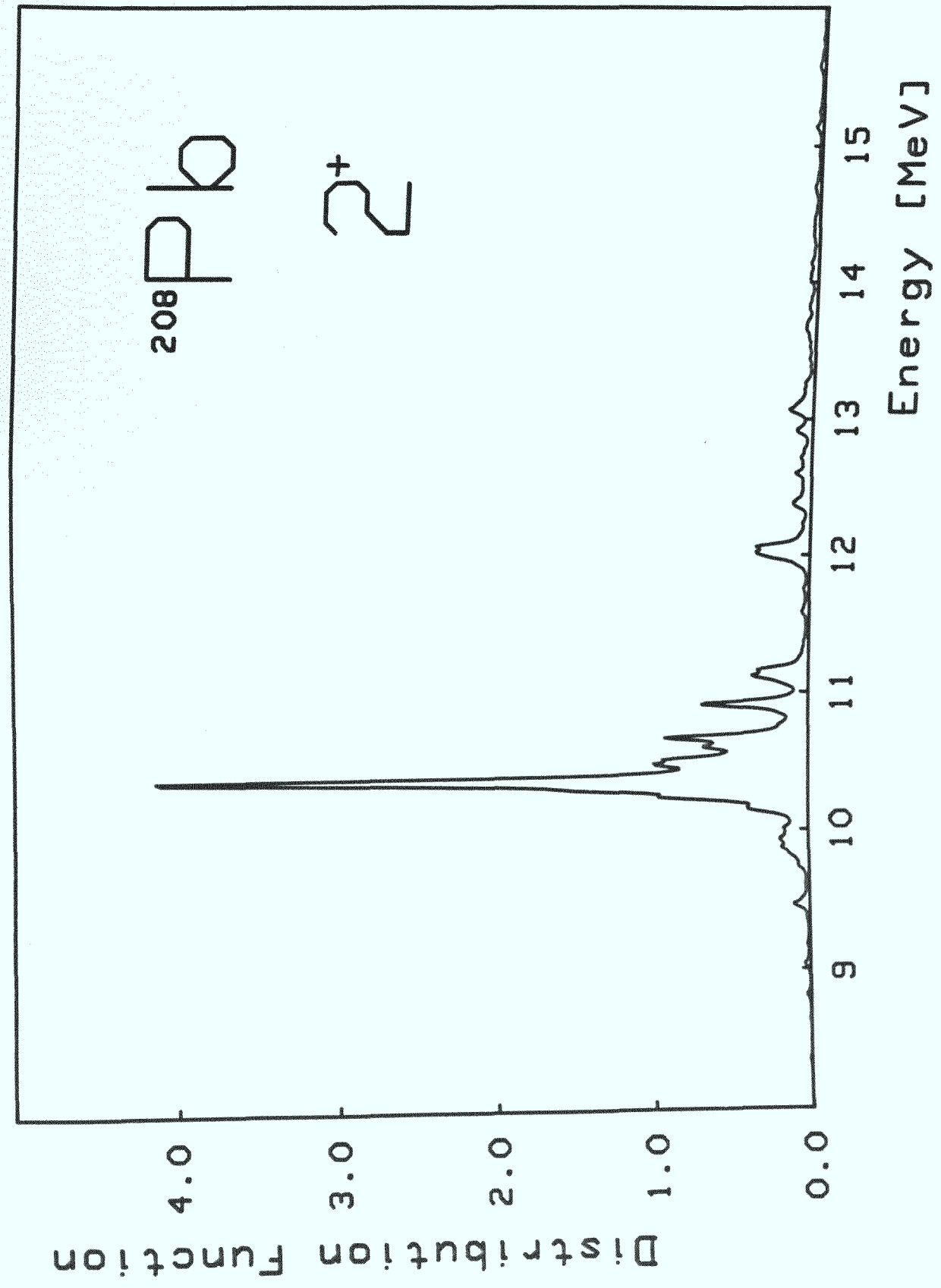


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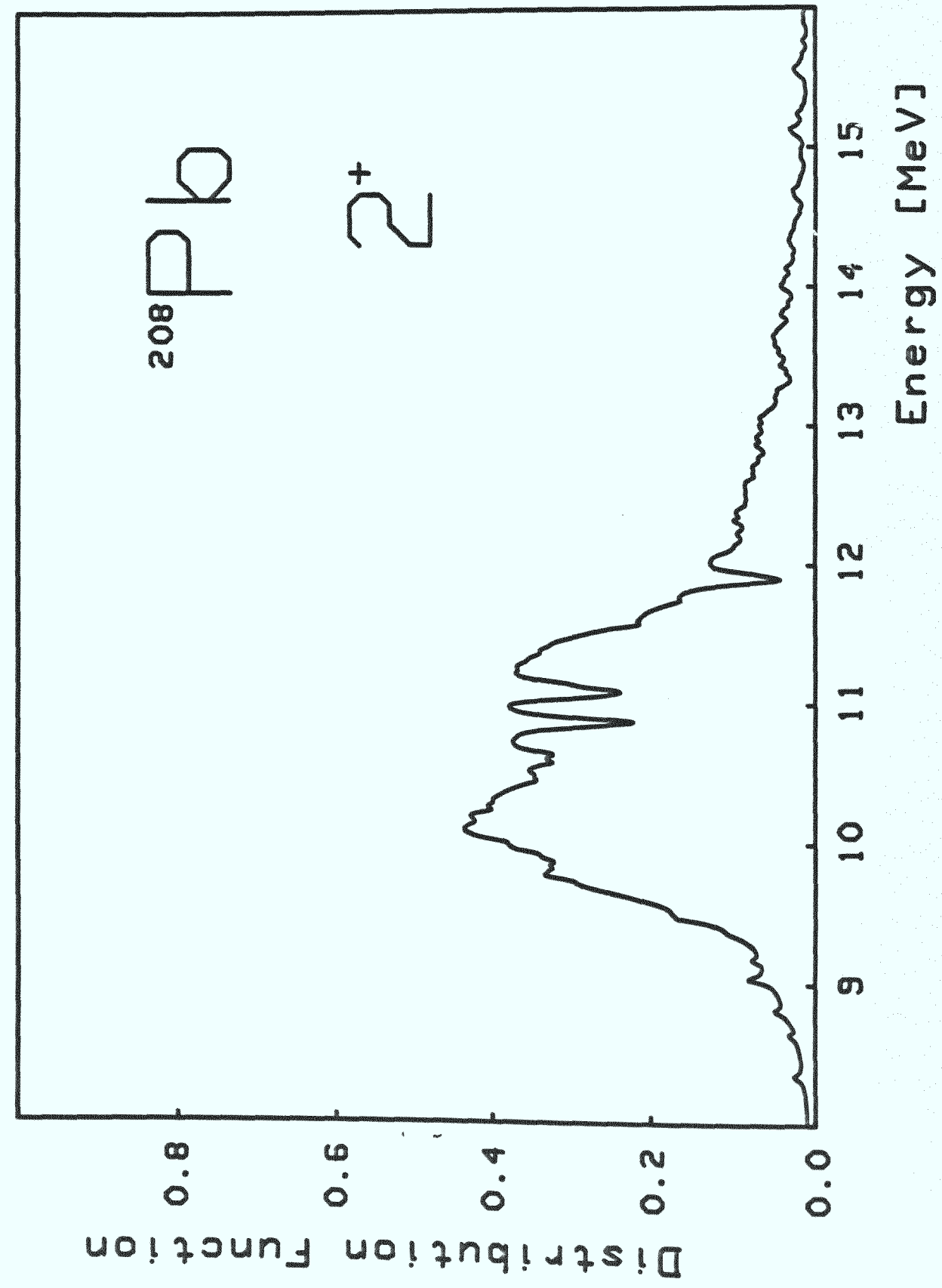


Fig. 42

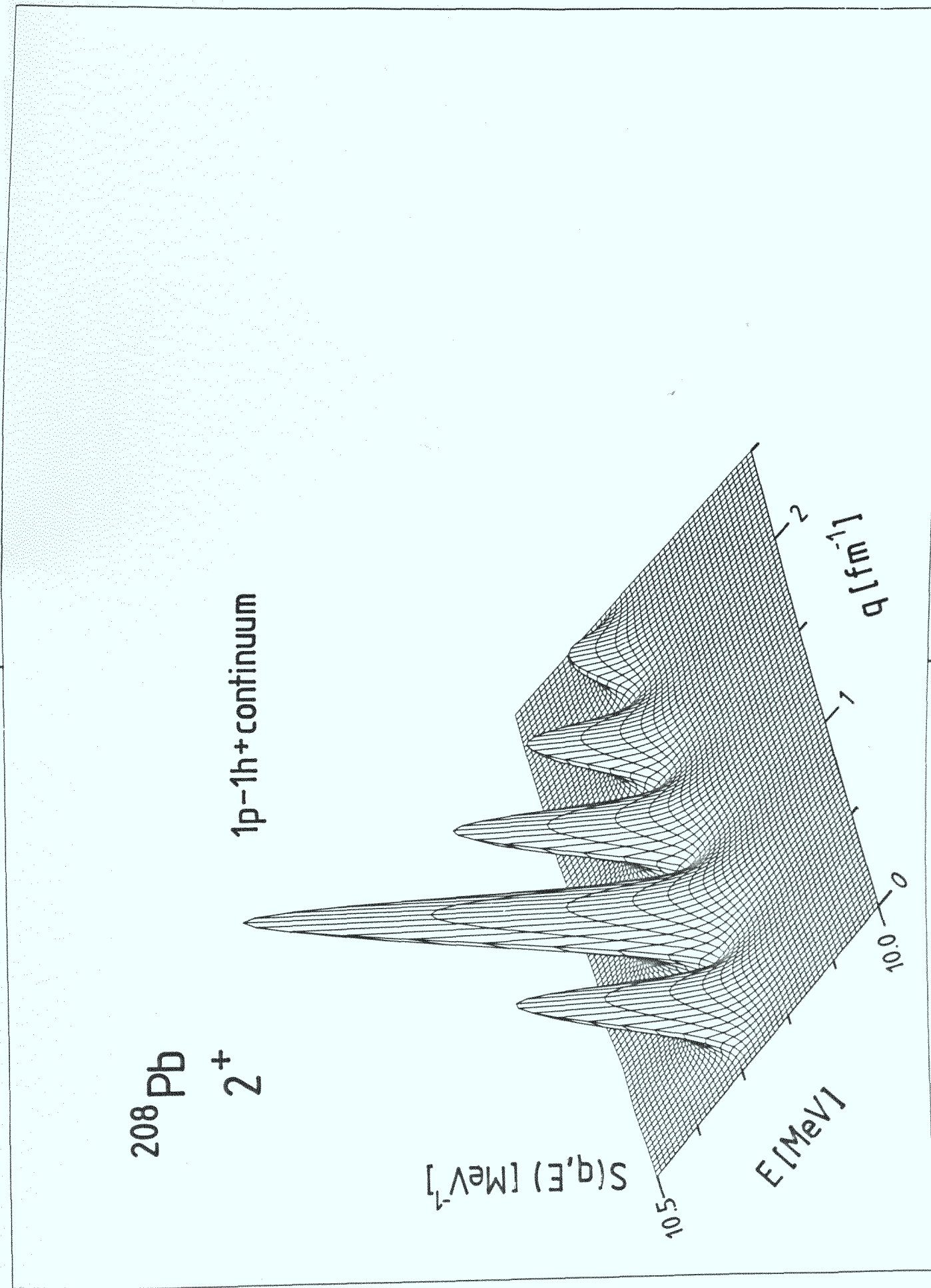


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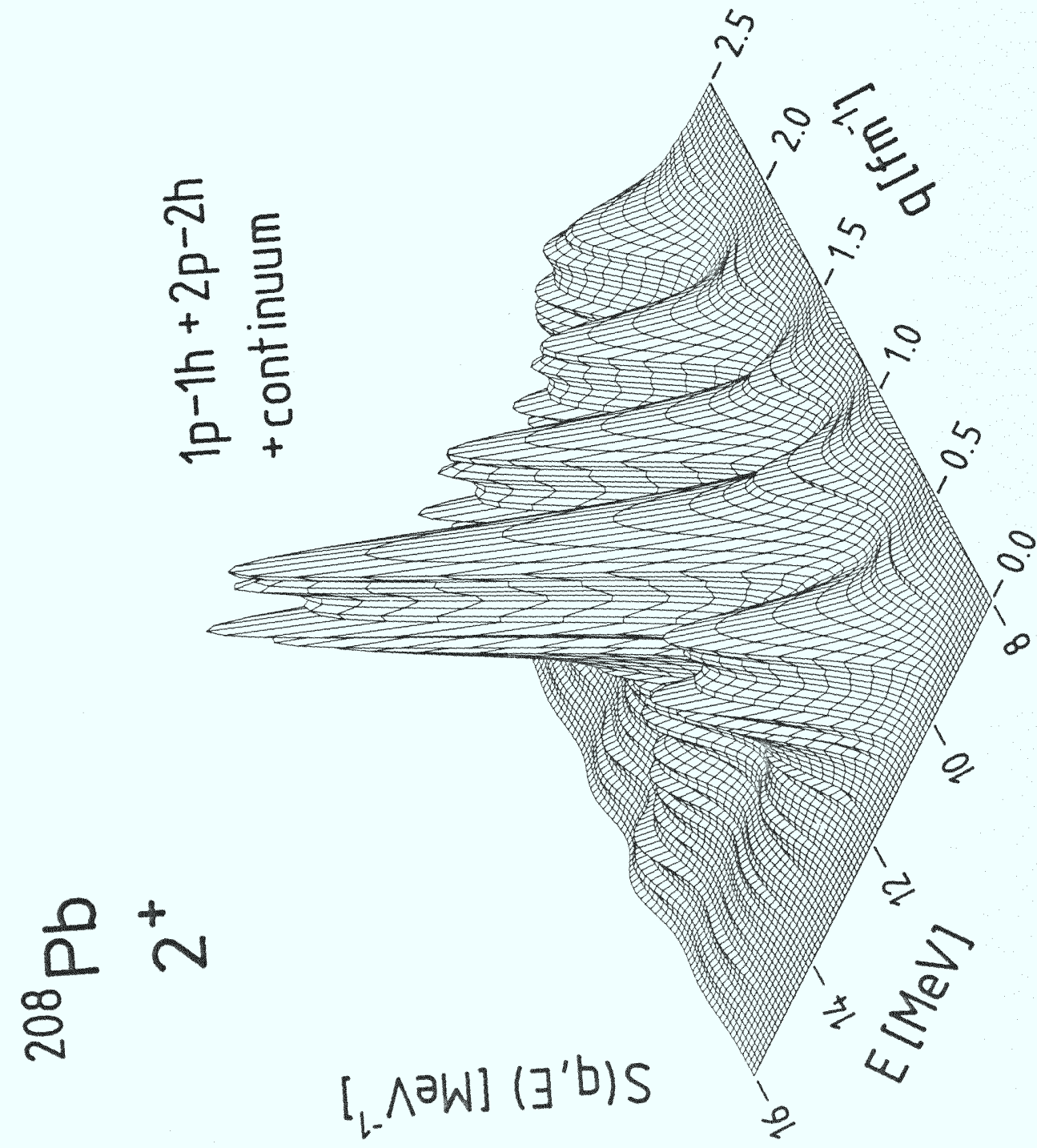


Fig. 44