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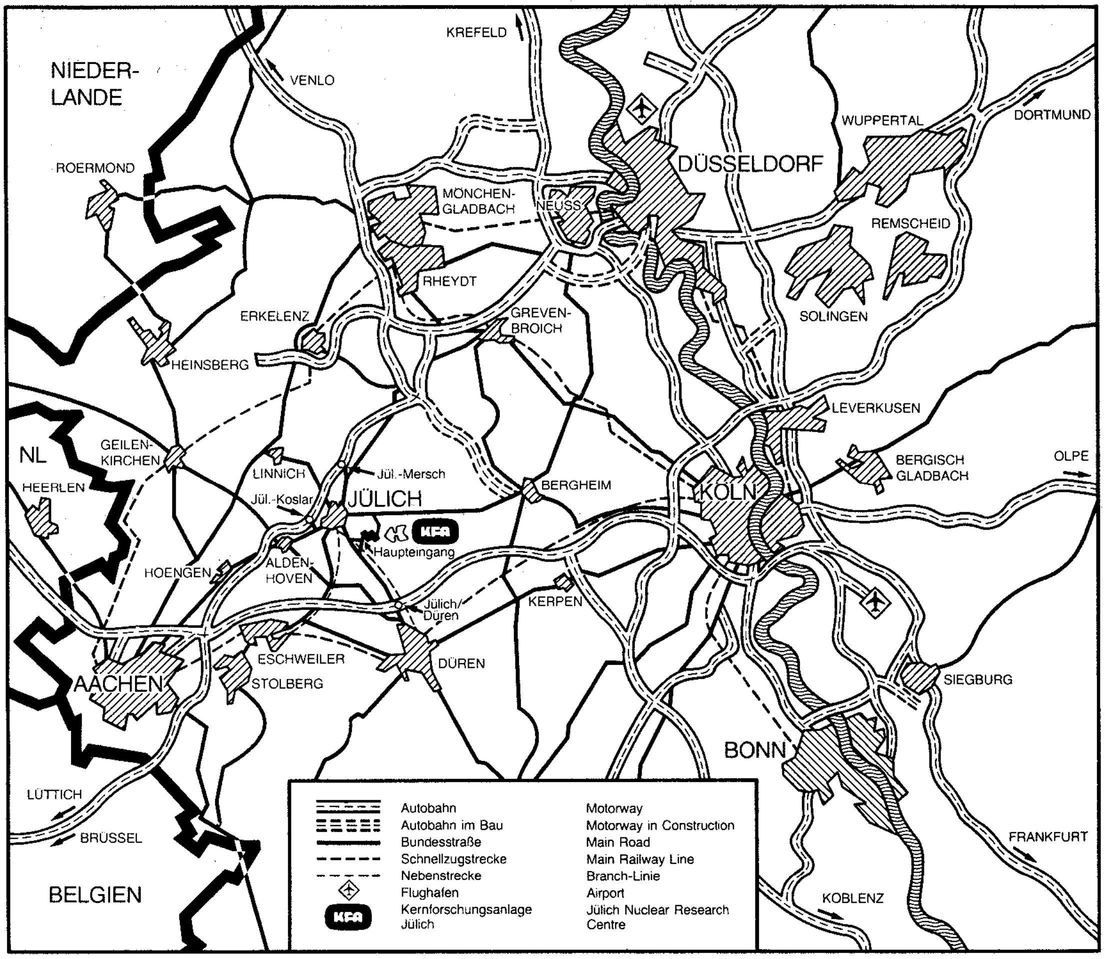

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ABSTRACT. - It is shown that the electron and ion diamagnetic drifts stabilize the rippling mode in the straight cylindrical tokamak model. Parallel electron heat conduction is further stabilizing if the parameter $\eta_e = d\ln T_e/d\ln N$ is positive. This has as consequence that the mode does not survive at temperatures exceeding, typically, 50 eV for standard values of magnetic field and density. The collisional drift wave is found to be always stable even when the effect of the tokamak current is included in the calculation.

1. INTRODUCTION

Plasma transport in fusion devices is observed to be largely non-classical. It has been conjectured that the anomalous losses arise through scattering by plasma instabilities. There is indeed a wealth of evidence from laser (Surko and Slusher, 1980 a and b; Semet et al., 1980; Meyer and Mahn, 1981; Evans et al., 1983; Brower et al., 1983), microwave (Mazucato, 1982; Equipe TFR, 1983) and Langmuir probes (Oktsuka et al., 1978; Zweben and Taylor, 1981; Zweben and Gould, 1983) diagnostic experiments that microturbulence is a common feature of high temperature plasmas. The observed frequency ω is of the order of the electron diamagnetic frequency ω_e^* and the fluctuation level n/N usually increases from less than 1 % in the center up to 10 - 100 % at the plasma edge. We believe that drift instabilities and particularly the trapped electron mode explain satisfactorily the observed turbulence and transport in the temperature and density gradient layer which separates the plasma core from the edge region (Hasselberg and Rogister, 1983). This mode is, however, stable in the cold plasma periphery and it is necessary to look for other instabilities to explain the transport there. Practically, this problem is important, e.g. for the design of divertors.

The rippling instability, which is driven by the resistivity gradient, has been proposed as a candidate in previous papers (Carreras et al., 1982; Callen et al., 1983) but these calculations, as well as former ones (Kadomtsev and Nedospasov, 1960; Kadomtsev, 1961; Furth, Killeen and Rosenbluth, 1963; Hoh, 1964; Rutherford, 1979) have largely neglected the role of electron and ion diamagnetism. This approximation fails in the edge plasma of a tokamak as recognized by Carreras et al. (1982). It is our aim to remedy this situation.

The most striking consequence of the introduction of electron and ion diamagnetic drifts in the theory is that the rippling mode is stabilized at temperatures exceeding, typically, 50 eV assuming usual magnetic field strengths and densities for tokamaks. This wave frequency is approximately $\omega = \omega_e^*(1+1.71\ \eta_e)$. Parallel electron heat conduction is also stabilizing if $\eta_e>0$ but can be destabilizing if $\eta_e<0$.

We have extended the theoretical development to include the collisional drift mode. These calculations confirm the previous results (Rogister and Hasselberg, 1979; Cordey et al., 1979; Hasselberg and Rogister, 1980), using however a different approach (kinetic versus macroscopic), that these modes are stable within the cylindrical model. They show further that the tokamak current is not destabilizing.

Our work thus implies that neither the rippling mode nor the collisional drift mode can explain the observed turbulence in the whole plasma layer where the trapped electron instability is absent (roughly for $T_e<$ a few hundreds eV).

The paper is organized as follows. The plasma model and the expansion technique are outlined in Section 2. The radial eigenvalue equation is derived in Section 3 and solved in Section 4. The conclusions are presented in Section 5.

2. THE MODEL

In order to reduce the basic equations to a tractable, yet consistent form, it is useful to order the typical parameters of the plasma and of the instability in powers of an appropriate expansion parameter. We choose here

$$\frac{\nu_{ei}}{\Omega_i} \sim \frac{a_i}{L} \sim \frac{m_e}{m_i} \sim \lambda. \tag{1}$$

To illustrate this scaling, consider the plasma edge of a standard tokamak with B = 4 Tesla (toroidal magnetic field), a = 20 cm (minor radius), $T_e = T_i = 10^2$ eV (temperature) and $N = 10^{13}$ cm^{-3} (density). We find $\tau_e = 2 \times 10^{-6}$ sec. (Braginskii's collision time, Braginskii 1965), $\Omega_i = 4 \times 10^8$ assuming Hydrogen (ion gyrofrequency), $c_s = 10^7$ cm sec^{-1} (ion sound speed = $(T_e/m_i)^{1/2}$), $a_i = 2.5 \times 10^{-2}$ cm (ion Larmor radius); hence $\nu_{ei}/\Omega_i = a_i/a = 1/800$ (We do not differentiate yet between the various equilibrium length-scales: a, R (major radius), L_s (shear length), neither between the magnitude of the poloidal and toroidal fields B_θ and B since q $\sim$ 1 (safety factor)). The mass scaling is merely introduced to easily differentiate between the electron and ion behaviour across and along the magnetic field.

We also order the components of the equilibrium electron and ion flow velocities with respect to the sound speed. The perpendicular momentum equations show that $\hat{n} \times \vec{U}_J^{(o)} = 0$ where j is the particle species (i ≡ ions; e ≡ electrons), and

$$\hat{n} \times \vec{U}_J^{(1)} = -\frac{c}{B}\frac{\vec{\nabla}P_J}{q_J N} \tag{2}$$

where $\hat{n} \equiv \vec{B}/B$. We assume the velocity along the field to be of order of the sound speed. Hence we write

$$\hat{n}\cdot\ (\vec{U}_e^{(o)}-\vec{U}_i^{(o)}) = 2\ \tau_e \frac{q_e}{m_e}\vec{E}\cdot\hat{n}. \tag{3}$$

In connection with (1), Equ.(3) implies that

$$\frac{e\ E_\varphi L}{T_e} \sim \lambda \tag{4}$$

E_φ is the applied toroidal field, q_J is the electric charge ($q_i = -q_e = e$), and P_J is the kinetic pressure. The gradients are here of course in the radial direction $\hat{p}$.

We anticipate that the modes

$$\Phi\ (\vec{r},t) = \Phi\ (x)\ \exp\ (-i\omega t + im\theta - il\varphi) \tag{5}$$

are localized around the rational surfaces defined by $lq(r) + m = 0$ and have a radial width of order $a_s = c_s/\Omega_i$. The derivatives in the directions of the magnetic field and the binormal $\hat{b} = \hat{n} \times \hat{p}$ are thus

$$\hat{n}\cdot\vec{\nabla} = ik_{||} = ik_\theta x/L_s, \tag{6a}$$

where $L_s = -(B/B_\theta)\ (d\ln q/dr)^{-1}$ is the shear length and $x \sim a_s$ is the distance from the rational surface, and

$$\hat{b}\cdot\vec{\nabla} = i\ k_\theta\ B/B_\varphi \tag{6b}$$

where $k_\theta = m/r$. To allow parallel electron heat conduction to play a role, we require

$$\omega \sim (\chi_{||,e} / N) \; (\hat{n}\cdot\vec{\nabla})^2 \tag{7}$$

where $\chi_{||,e} = 3.16 \; NT_e\tau_e/m_e$. To allow electron diamagnetism to have an influence on the dispersion relation we also impose

$$\omega \sim \omega^* \tag{8}$$

where $\omega_J^* = k_\theta \; c \; T_J \; / \; q_s B_\phi L_N$ is the diamagnetic frequency ($L_N^{-1} = d\ln N/dr$). The scaling relations (1), (7), and (8), imply that we take

$$k_\theta L \sim 1. \tag{9}$$

There follows that $\omega \sim \lambda\nu_{ei}$ and $k_{||}L \sim \lambda$. We note also that $k_{||}\lambda_{m.f.p.} \sim \lambda^{1/2}$ which justifies the use of macroscopic equations to describe the instability ($\lambda_{m.f.p.}$ is the mean free path).

It remains to assess the role of magnetic fluctuations. They usually appear through the parallel component of the vector potential and are to be taken into account when $\omega a_{||}/c \sim k_{||}\Phi$. Normalizing the perturbed potential to T_e/e (i.e. $e\Phi^{(o)}/T_e \sim \mu$, μ = linearization parameter; similarly $n^{(o)} \sim \mu N$, $t^{(o)} \sim \mu T$ and $\vec{u}^{(o)} \sim \mu\vec{c}_s$) and noting that the perturbed velocity along the field is $u_{||}^{(o)} \sim \tau_e k_{||} e\Phi/m_e \sim c_s e\Phi^{(o)}/T_e$, Ampere's equation implies that this condition is satisfied when $\beta \sim 1$ where β is the ratio of kinetic and magnetic pressures. This is orders of magnitude larger than the actual β at the plasma edge. We therefore neglect the role of magnetic fluctuations in deriving the eigenvalue equation.

We now take the curl of the perpendicular momentum equation and form the scalar product of the result with the unit vector $\hat{n}$ in order to draw $\vec{\nabla}\cdot(N\vec{u}_\perp)$ for later insertion in the continuity equation. We obtain to lowest significant order:

$$\vec{\nabla}\cdot(N\vec{u}_J+n\vec{U}_J) = -\frac{c}{B}\,\hat{n}\cdot(\vec{\nabla}N\times\nabla\Phi - \vec{\nabla}n\times\vec{E}) - \frac{c}{q_J B}\,\text{sign}\,q_J\,\frac{\partial}{\partial x}\,\hat{b}\cdot\vec{r}$$

$$-\,\Omega_J^{-1}\,N(\frac{\partial}{\partial t} + \vec{U}_J\cdot\vec{\nabla})\,\frac{\partial}{\partial x}\,\hat{b}\cdot\vec{u}_J \qquad (10)$$

where $\vec{r}$ is the sum of the perturbed friction and thermal forces:

$$\hat{b}\cdot\vec{r} = -\frac{m_e N}{\tau_e}\,\hat{b}\cdot(\vec{u}_e-\vec{u}_i) + \frac{3}{2}\,\frac{N}{\Omega_e\tau_e}\,\frac{\partial t_e}{\partial x}$$

In these two equations, $\hat{b}\cdot\vec{u}$, n, Φ, and t_e stand for $\hat{b}\cdot\vec{u}^{(o)}$, $n^{(o)}$, $\Phi^{(o)}$, and $t_e^{(o)}$. We note that $\hat{p}\cdot\vec{u}_J^{(o)} = 0$,

$$\hat{b}\cdot\vec{u}_J^{(o)} = \frac{c}{B}\,(\hat{p}\cdot\vec{\nabla}\Phi^{(o)} + \frac{\hat{p}\cdot\vec{\nabla}p_j^{(o)}}{q_j N}), \qquad (11a)$$

and

$$\hat{p}\cdot\vec{u}_J^{(1)} = -\frac{c}{B}\,(\hat{b}\cdot\vec{\nabla}\Phi^{(o)} + \frac{\hat{b}\cdot\vec{\nabla}p_J^{(o)}}{q_J N}) \qquad (11b)$$

The electron inertia term in Eq. (10) is of course negligible.

The parallel component of the electron momentum equation leads to

$$N(u_e-u_i)_{||} + n\,(U_e-U_i) = -\,2\,\frac{\tau_e}{m_e}\,\hat{n}\,\vec{\nabla}\,(q_e N\Phi + p_e + 0.71\,N\,t_e)$$

$$+\,\frac{3}{2}\,\frac{t_e}{T_e}\,N\,(U_e-U_i)_{||} \qquad (12)$$

where the last term arises from the perturbation of the plasma resistivity. Summing up the parallel electron and ion momentum equations, one obtains also

$$m_i N\,(\frac{\partial}{\partial t} + \vec{U}_i\cdot\nabla)\,u_{i||} = -\,\hat{n}\cdot\vec{\nabla}\,(p_e+p_i). \qquad (13)$$

Equations (10) to (13) have to be combined with the continuity and the heat equations. Charge neutrality implies

$$\Omega_i^{-1}\left(\frac{\partial}{\partial t}+\vec{U}_i\cdot\vec{\nabla}\right)\frac{\partial^2}{\partial x^2}\left(\Phi+\frac{p_i}{q_iN}\right)=2\,\tau_e\Omega_e\,(\hat{n}\cdot\vec{\nabla})^2\left(\Phi+\frac{p_e}{q_eN}+0.71\,\frac{t_e}{q_e}\right)$$

$$-\frac{3}{2}\,(U_e-U_i)_{||}\,\frac{B}{c\,T_e}\,\hat{n}\cdot\vec{\nabla}t_e. \tag{14}$$

whereas the ion continuity equation becomes

$$\frac{\partial n}{\partial t}=\frac{c}{B}\Big[\hat{n}\cdot(\vec{\nabla}N\times\vec{\nabla}\Phi-\vec{\nabla}n\times\vec{E})+\frac{1}{q_i}\frac{\partial}{\partial x}\,\hat{b}\cdot\vec{r}$$

$$+\Omega_i^{-1}N\left(\frac{\partial}{\partial t}+\vec{U}_i\cdot\vec{\nabla}\right)\frac{\partial^2}{\partial x^2}\left(\Phi+\frac{p_i}{q_iN}\right)+\Omega_i\left(\frac{\partial}{\partial t}+\vec{U}_i\cdot\vec{\nabla}\right)^{-1}$$

$$(\hat{n}\cdot\vec{\nabla})^2\,\frac{p_e+p_i}{q_i}\Big] \tag{15}$$

It remains to consider the heat equations. We obtain to lowest significant order:

$$\left(\frac{\partial}{\partial t}+\vec{U}_e\cdot\vec{\nabla}-\frac{c\hat{p}\cdot\vec{\nabla}P_e}{q_eNB}\,b\cdot\vec{\nabla}\right)\left(\frac{3}{2}\,Nt_e-nT_e\right)+\frac{c}{b}\left(T_e\hat{p}\cdot\nabla N-\frac{3}{2}\,Np\cdot\nabla T_e\right)\hat{b}\cdot\vec{\nabla}\Phi$$

$$=-\vec{\nabla}\cdot\vec{q}_e+\tilde{Q}_e, \tag{16}$$

where we define the electron heat flux as

$$\vec{q}_e=0.71\,\Big[P_e\,(u_e-u_i)_{||}+p_e\,(U_e-U_i)_{||}\Big]\hat{n}-\frac{3}{2}\,\frac{P_e}{\Omega_e\tau_e}\,\hat{n}\times(\vec{u}_e-\vec{u}_i)-\chi_{\|,e}\,\hat{n}\cdot\vec{\nabla}t_e\,\hat{n}$$

$$-\chi_{\perp,e}\,\vec{\nabla}_{\perp}t_e. \tag{17a}$$

Other terms appearing in the definition of Braginskii have been taken care of in the left-hand side of Eq. (16). We note also that $\tilde{Q}_e$ is the perturbed electron heat source. A similar equation holds for the ions with

$$q_i = - \chi^i \hat{n}\cdot\vec{\nabla} t_i \hat{n} - \chi_\perp^i \vec{\nabla}_\perp t_i . \tag{17b}$$

(these terms actually yield contributions formally of order $\lambda^{1/2}$ and $\lambda^{-1/2}$ to the equations).

Although Eqs. (11) to (17) are simpler than the original fluid equations, further reductions are necessary to arrive at concrete results. These will be considered in Section 3.

3. DERIVATION OF THE EIGENMODE EQUATION

In order to proceed efficiently, we need to introduce further restrictions on the scaling. We shall introduce two assumptions each of which has a broad range of validity under the conditions prevailing at the edge of a Tokamak plasma.

$$\text{(a)} \qquad \frac{m_e \nu_{ei}}{m_i \omega} \ll 1, \qquad \frac{L_N^2}{L_s^2} \ll 1, \tag{18a}$$

and

$$\frac{m_e \nu_{ei}}{m_i \omega} \frac{L_s^2}{L_N^2} \sim 1 \tag{18b}$$

where ω is the complex wave frequency. We note that ω is either larger or of the order of the diamagnetic frequency as shown by the calculation.

$$\text{(b)} \qquad \hat{p}\cdot\vec{U}_J \; \hat{p}\cdot\vec{\nabla}, \; \hat{n}\cdot\vec{U}_J \; \hat{n}\cdot\vec{\nabla} \ll \hat{b}\cdot\vec{U}_J \; \hat{b}\cdot\vec{\nabla} \tag{19}$$

so that $\vec{U}\cdot\vec{\nabla} = i\, \omega_J^* (1+\eta_J)$, where $\eta_J = (d\ln T_J/dr)(d\ln N/dr)^{-1}$, as can be seen from Eq. (2) and the definition of ω_J^*.

These relations permit considerable simplifications listed below.

(i) Comparing the ratios of the $(\hat{n}\cdot\nabla)^2$ terms and of the $\partial^2/\partial x^2$ terms in Eqs. (14) and (15), we obtain after reduction:

$$\frac{(\hat{n}\cdot\nabla)^2}{\partial^2/\partial x^2}: \quad 1 \text{ (Eq. 14)}; \qquad \frac{m_e \nu_{ei}}{m_i \omega} \text{ (Eq. 15)} \tag{20}$$

which implies that one could neglect either the $(\hat{n}\cdot\vec{\nabla})^2$ term in Eq. (15) or the $\partial^2/\partial x^2$ term in Eq. (14), the latter approximation leading to the drift branch, the former to the rippling branch. The ordering (20) should however be reexamined in the eventuality that the dispersion relation of the rippling mode yields approximately $q_e N\Phi + p_e + 0.71\ Nt_e \cong 0$, which would tend to reduce the disparity between the ratios in (20) and hence between the absolute values of the radial turning points. As this may be the case (depending however of the plasma parameters and of the mode number) we shall retain the two branches simultaneously in the following calculation. By balancing the $\partial^2/\partial x^2$ term and the $(\hat{n}\cdot\vec{\nabla})^2$ term for both the rippling and the drift branches we obtain that the absolute value of the linear turning point is bounded from above and below according to

$$\frac{m_e \nu_{ei}}{m_i \omega} \frac{L_s^2}{L_N^2} \left(\frac{\omega}{\omega^*}\right)^2 \lesssim \left(\frac{x_t}{a_s}\right)^4 \lesssim \frac{L_s^2}{L_N^2} \left(\frac{\omega}{\omega^*}\right)^2 . \tag{21}$$

(ii) We can now estimate the effect of the parallel electron heat conduction in Eq. (16) by comparing $\chi_{\parallel,e}(\hat{n}\cdot\nabla)^2 t_e$ and $\omega N t_e$. We find

$$\left(\frac{m_i \omega}{m_e \nu_{ei}}\right)^{1/2} \frac{L_N}{L_S} \frac{\omega^*}{\omega} \lesssim \frac{\chi_{\parallel,e}(\hat{n}\cdot\vec{\nabla})^2}{\omega N} \lesssim \frac{m_i \omega}{m_e \nu_{ei}} \frac{L_N}{L_S} \frac{\omega^*}{\omega} \tag{22}$$

Note that for the rippling branch, $\chi_{\parallel,e}(\hat{n}\cdot\vec{\nabla})^2/\omega N$ is strictly of the order of $(a_s/x_t)^2$ if $q_e N\Phi >> p_e + 0.71\ Nt_e$, i.e. if $\omega >> \omega^*$. Therefore, in this limit and in the framwork of the fluid model $(x_t^2/a_s^2 >> 1)$, parallel electron heat conduction cannot play a significant stabilizing role. Previous estimations of this effect using resistive magnetohydrodynamics were thus inconsistent. Since we allow the upper bound in (22) to be larger than unity, it is necessary to retain here the $\chi_{\parallel,e}$ term.

(iii) The effect of perpendicular electron heat conduction can be estimated as

$$\frac{m_e \nu_{ei}}{m_i \omega} \frac{L_N}{L_S} \frac{\omega^*}{\omega} \leq \frac{\chi_{\perp,e} \, \partial^2/\partial x^2}{\omega N} \lesssim \left(\frac{m_e \nu_{ei}}{m_i \omega}\right)^{1/2} \frac{L_N}{L_S} \frac{\omega^*}{\omega} \tag{23}$$

and is always negligible.

(iv) We estimate also that (see Eq. 16)

$$\frac{\tilde{Q}_e}{\omega^* N q_e \Phi} \sim \frac{k_{||} U_{||,e}}{\omega^*} << 1 \tag{24}$$

(assumption b). Hence the electron heat source is negligible.

(v) The role of parallel ion heat conduction is negligible. The effect of perpendicular conduction is bounded from above by

$$\frac{\chi_{\perp,i} \, \partial^2/\partial x^2}{\omega N} \lesssim \left(\frac{m_i}{m_e}\right)^{1/2} \left(\frac{m_e \nu_{ei}}{m_i \omega}\right)^{1/2} \frac{L_N}{L_S} \frac{\omega^*}{\omega} \tag{25}$$

and will be neglected althoug this implies strictly a further hypothesis.

(vi) In Eq. (15) the term $c\hat{n} \cdot (\vec{\nabla} x \vec{E})/B$ is negligible. Indeed

$$\frac{\hat{n} \cdot (\vec{\nabla} x \vec{E})}{\Omega_i^{-1} N \omega \partial^2 \Phi/\partial x^2} \sim \frac{B_\theta L_S}{B \, L_N} \frac{m_e \nu_{ei}}{m_i \omega} \frac{\omega^*}{\omega} \frac{k_{||} U_{||,e}}{\omega} << 1 \tag{26}$$

since $B_\theta L_S / B L_N \sim 1$.

(vii) The ratio of the term $\partial \hat{b} \cdot \vec{r}/\partial x$ to the finite Larmor radius term in Eq. (15) is

$$\frac{q_i^{-1} \, \partial \hat{b} \cdot \vec{r}/\partial x}{\Omega^{-1} N \omega \partial^2 \Phi/\partial x^2} \sim \frac{m_e \nu_{ei}}{m_i \omega} << 1 \ . \tag{27}$$

(viii) We note finally that the ratio $k_{||} U_{||,e}/\omega$ is bounded from above as follows:

$$\frac{k_{||}U_{||,e}}{\omega^*} \lesssim \frac{U_{||,e}}{c_s} \left(\frac{\omega}{\omega^*}\right)^{1/2} \left(\frac{L_N}{L_S}\right)^{1/2} \tag{20}$$

The right-hand side of this inequality is always smaller than unity in the plasma edge layer.

We can now rewrite Eqs. (14), (15), and (16) as follows:

$$A^2 \frac{\partial^2}{\partial\xi^2} - ib\xi^2 \left(\varphi - \tilde{n} - 1{,}71\ \tilde{t}_e\right) - a\xi\tilde{t}_e = 0 \tag{29}$$

$$\tilde{n} = \frac{\omega_e^*}{\omega}\varphi + A^2 \frac{\partial^2\varphi}{\partial\xi^2} + A^{-1}\ c\xi^2\left\{ \frac{\omega_e^*}{\omega} \left[1+\tau\ (1+\eta_i)\right] \varphi + \tilde{t}_e\right\} \tag{30}$$

and

$$(1 + 1.86\ ib\xi^2)\tilde{t}_e = \eta_e \frac{\omega_e^*}{\omega}\varphi + \frac{2}{3}\left(\tilde{n} - \frac{\omega_e^*}{\omega}\varphi\right) - 0.47\ ib\xi^2\ (\tilde{n}-\varphi) \tag{31}$$

where we have defined the following dimensionless parameters:

$$a = \frac{3}{2} k_{||}' a_s\ (U_e - U_i)_{||}\ \omega^{-1}, \tag{32a}$$

$$b = 2\ \tau_e\ k_{||}'^2\ a_s^2 c_e^2 \omega^{-1}, \tag{32b}$$

$$c = k_{||}'^2\ a_s^2\ c_s^2\ \omega^{-2}. \tag{32c}$$

and $A = 1 - \omega_i^*(1+\eta_i)/\omega$, $\tau = T_i/T_e$, $\tilde{t}_e = t_e/T_e$, $\tilde{n} = n/N$, $\varphi = e\Phi/T_e$, $\xi = x/a_s$. In Eq. (30) we have treated the finite Larmor radius (f.l.r.) term and the sound term by expansion, i.e. we have approximated $\tilde{n}$ by $\omega_e^*\varphi/\omega$.

Eqs. (29-31) will be solved in the next section in successive degress of complexity.

4. SOLUTION OF THE EIGENVALUE EQUATION

4.1 The Rippling Mode

We first neglect the role of parallel electron heat conduction ($b \to 0$) in Eq. (31) and neglect the f.l.r. and sound terms in Eq. (30). We obtain $\tilde{n} = \omega_e^* \varphi/\omega$ and $\tilde{t}_e = \eta_e \tilde{n}$. Eq. (29) yields

$$A^2 \frac{\partial^2 \varphi}{\partial \xi^2} - ib\xi^2 \left[1 - (1+1.71\eta_e)\, \omega_e^*/\omega\right]\varphi - a\eta_e\, (\omega_e^*/\omega)\xi\, \varphi = 0 \qquad (33)$$

The solution is of the form

$$\varphi = \varphi(0) \exp\left[-\Theta^{1/2}\, (\xi+\delta)^2/2\right] \qquad (34)$$

where $\Theta^{1/2} = \delta^{-2} = (ib^\dagger\rho)^{1/2}$ and $\delta = -ia^\dagger\eta_e\omega_e^*/2b^\dagger\rho\omega$; here $a^\dagger = a/A^2$, $b^\dagger = b/A^2$, and $\rho = 1 - (1+1.71\,\eta_e)\,\omega_e^*/\omega$. The dispersion relation is

$$(i\, b^\dagger\, \rho)^{3/2} = (a^\dagger\, \eta_e\omega_e^*/2\omega)^2 \qquad (35)$$

with the constraint $-\pi/2 < \arg \Theta^{1/2} = \arg(-\rho^2\omega^2) < \pi/2$ which states that the eigenfunctions are spatially bounded. There are three limiting cases to consider.

(a) In the limit $\omega \gg \omega_J^*$, Eq. (35) yields

$$\Gamma^{5/2} = \frac{|k_\theta| a_s c_s |L_s|\, (U_e - U_i)_{||}^2/c_s^2}{4\, L_\eta^2\, (2\, \tau_e\, m_i/m_e)^{3/2}}$$

$$= \tau_R^{-5/2}\, S\, |m|\, \frac{|L_s|}{r}\, \left(\frac{cE_{||}/B}{2L_\eta/\tau_R}\right)^2 \qquad (36)$$

where we have defined the resistivity length scale $L_\eta = -2\, L_N/3\, \eta_e$, and $\Gamma = -i\omega$. The parameter $S = \tau_R/\tau_H$ has its usual meaning with $\tau_R = 4\pi r^2/\eta_{||}$ as the resistive diffusion time scale and $\tau_H = r/v_A$ as the hydromagnetic time scale. $v_A = B/(4\pi N m_i)^{1/2}$ is the Alfvèn speed. Eq. (36) is the standard dispersion relation for tearing modes (Furth, Killeen and Rosenbluth, 1963; Callen et al., 1983).

To quantify the assumption that $\Gamma >> \omega_J^*$, we divide both sides of the equality by $\omega_e^{*5/2}$ and obtain the condition

$$\frac{9}{16}\frac{\eta_e^2}{}\left(\frac{m_e \nu_{ei}}{2m_i \omega_e^*}\right)^{3/2} \frac{L_S}{L_N} \frac{(U_e - U_i)_{||}^2}{c_s^2} >> 1, \tag{37a}$$

which upon insertion of the parameters of Sec. 2 and the assumptions that L_S = 200 cms, L_N = 5 cms, V/Z_{eff} = 1 volt (loop voltage/effective charge) from which there follows that $(U_e - U_i)_{||}/c_s = T\, A_i^{1/2}/N$, leads to

$$0.35\ \eta_e^2\ m^{-3/2}\ A_i^{-1/2}\ N^{-1/2}\ T^{-7/4}\ B^{3/2} >> 1. \tag{37b}$$

The units are (T) = 100 eV, (N) = 10^{13} cm^{-3} and (B) = 40 kG. When the inequality (37b) is reserved, we conclude from Eq. (35) that either $\rho \to 0$ or $A \to 0$, i.e. either

$$\omega = \omega_e^*\ (1+1.71\ \eta_e) \tag{38a}$$

or $\quad \omega = \omega_i^*\ (1+\eta_i)$.

We now analyse these two possibilities.

b) $\omega = \omega_i^*\ (1+\eta_i) + i\Gamma$, $\Gamma << \omega$. The solution is still given by Eq. (35) where $\delta = -\, ia\eta_e\omega_e^*/2b\rho\omega_i^*\ (1+\eta_i)$ and $\rho = 1+(1+1.71\eta_e)/\tau(1+\eta_i)$. Hence δ^2 and therefore $\Theta^{1/2}$ are negative and there are no bounded solutions.

c) $\omega = \omega_e^*\ (1+1.71\eta_e) + i\Gamma$, $\Gamma << \omega$. Here $\delta = -a\eta_e\omega_e^*/2b\Gamma$ and $(\Gamma/\omega)^3 = -\, b^\dagger(a\eta_e\omega_e^*/2b\omega)^4$ (here $\omega = \omega_e^*\ (1+1.71\ \eta_e)$). Hence the roots $\Gamma = [\exp\,(\pm i\pi/3), -1] \times (b^\dagger\omega^3)^{1/3}\ (a\eta_e\omega_e^*/2b\omega)^{4/3}$ of which only the last one - stable - corresponds to a bounded solution.

We conclude that in the absence of parallel electron heat conduction and finite ion Larmor radius effects, the rippling instability disappears if the inequalities (37a) or (37b) are reversed, i.e. if diamagnetic effects are important. Typically, this occurs when $T_e \gtrsim 50$ eV. We now investigate the role of finite $\chi_{\|,e}$ and f.l.r. in case (c) above.

Eq. (38a) implies that the second term in Eq. (33) can be balanced only if $\rho \propto \varphi - \tilde{n} - 1.71\, \tilde{t}_e \to 0$. The effect of parallel electron heat conduction and f.l.r. will thus enter first through this term when $b\xi^2 \sim \Gamma/\omega \ll 1$ [cf. Eq. (31)], respectively when either $\partial^2/\partial\xi^2$ or $c\xi^2 \sim \Gamma/\omega \ll 1$ [cf. Eq. (30)]. The normalized electron temperature fluctuation then becomes, up to first order in Γ/ω:

$$\tilde{t}_e = \eta_e \frac{\omega_e^*}{\omega} \varphi - 1.05\, i\eta_e \frac{\omega_e^*}{\omega} b\xi^2 + \frac{2}{3} (\tilde{n} - \frac{\omega_e^*}{\omega} \varphi) \tag{39}$$

(We note that $1.05\, b\xi^2 = (2/3)\, k_{\|}^2\, \chi_{\|,e}/N$). The eigenvalue equation (29) thus yields

$$A^2 \frac{\partial^2 \varphi}{\partial \xi^2} + b\xi^2 \left[(\frac{\Gamma}{\omega} + 1.80\, \eta_e \frac{\omega_e^*}{\omega} b\xi^2) \varphi + 2.14\, i\, (\tilde{n} - \frac{\omega_e^*}{\omega} \varphi) \right] - a\, \xi\, \eta_e \frac{\omega_e^*}{\omega} \varphi = 0 \tag{40}$$

It is clear from this equation that parallel electron heat cnduction is further stabilizing (if $\eta_e > 0$). It can further be shown that the term $(\tilde{n} - \omega_e^* \varphi/\omega)$ is negligible as long as $b\langle\xi^2\rangle \ll 1$ as assumed in the derivation of Eq. (39) and (40) [$(\langle\xi^2\rangle)^{1/2}$ being the characteristic width of the wave function]. Indeed the second term in the right-hand side of Eq. (30) yields, in (40), a contribution of order $b\langle\xi^2\rangle$ when compared with the first term; whilst the last term of Eq. (30) yields a contribution of order

$m_e \nu_{ei}/m_i \nu$ when compared, in Eq. (40), with $1.80\ \eta_e (\omega_e^*/\omega) b\xi^2 \varphi$. When $b\langle\xi^2\rangle$ is larger than unity, Eq. (40) is no longer valid and one obtaims the drift branch.

4.2 The collisional drift branch

If $b\langle\xi^2\rangle \gg 1$, the temperature fluctuation is, in lowest order,

$$t_e^{(o)} = -0.47\ i \frac{b\xi^2}{1+1.86ib\xi^2} \left(\frac{\omega_e^*}{\omega} - 1\right) \varphi \tag{40a}$$

Eq. (29), where we write $\tilde{n} = \omega_e^* \varphi/\omega + (\tilde{n} - \omega_e^* \varphi/\omega)$ and treat the second term by expansion, yields in lowest order:

$$\omega = \omega_e^* . \tag{41}$$

Let $\omega = \omega_e^* + i\Gamma$, $\Gamma \ll \omega_e^*$. Then, from Eq. (40a), $t_e^{(o)} = 0$, and from Eq. (31):

$$t_e^{(1)} = \frac{(\eta_e - 0.47 b\xi^2 \Gamma/\omega_e^*)\varphi - 0.47ib\xi^2(\tilde{n} - \omega_e^* \varphi/\omega)}{1 + 1.86\ i\ b\ \xi^2} ; \tag{40b}$$

in the denominator, the first term is retained to insure proper behaviour as $\xi \to 0$. Introducing in Eq. (29) yields:

$$A^2 \frac{\partial^2 \varphi}{\partial \xi^2} + c\ \xi^2 \varphi - i \frac{\Gamma}{\omega_e^*} \varphi + \frac{1.71 + i(a/b)\xi^{-1}}{1.06ib\xi^2 + 2.86} \eta_e \varphi = 0. \tag{42}$$

In the denominator, we have neglected two terms: $1/ib\xi^2$ and $0.47\ ia\xi$; indeed here $\langle\xi^2\rangle \sim c^{-1/2} \sim L_S/L_N$, and $a(\langle\xi^2\rangle)^{1/2} \sim \left[(U_e - U_i)_{||}/c_s\right](L_N/L_S)^{1/2} \ll 1$. In the limit $a \to 0$, this equation reduces to the drift wave eigenvalue equation derived in (Rogister and Hasselberg, 1979; Cordey et al., 1979; Hasselberg and Rogister, 1980) and admits stable roots only. We shall briefly review this case and give a hint that the term arising from the plasma current $[\eta_e(a/b)\xi^{-1}]$ can be treated by perturbation methods; we then proceed with the perturbed eigenvalue problem.

a) Effect of the electron thermal conductivity.

We define

$$\xi_1 \sim \frac{a}{b} \sim \frac{m_e\nu_{ei}}{m_i\omega_e^*} \frac{L_S}{L_N} \frac{(U_e-U_i)_{||}}{c_s}, \tag{45a}$$

$$\xi_2 \sim \frac{1}{b^{1/2}} \sim \left(\frac{m_e\nu_{ei}}{m_i\ \omega_e^*}\right)^{1/2} \frac{L_S}{L_N}, \tag{45b}$$

$$\xi_3 \sim \frac{1}{c^{1/4}} \sim \left(\frac{L_S}{L_N}\right)^{1/2}. \tag{45c}$$

Solution of Eq. (42) by matched asymptotic expansion is possible if $\xi_1<<\xi_2<<\xi_3$, i.e. if $(m_e\nu_{ei}/m_i\omega_e^*)^{1/2}\ (U_e-U_i)_{||}/c_s<<1$ and $(m_e\nu_{ei}/m_i\omega_e^*)^{1/2}$ $(L_S/L_N)^{1/2}<<1$. We assume also $\xi_1<<1$ and of course $\xi_3>>1$.

In the range $0 \leq \xi<<\xi_2$, Eq. (42) reduces to

$$2.86\ A^2 \frac{\partial^2 \varphi}{\partial\xi^2} + [1.71 + i(a/b)\xi^{-1}]\ \eta_e \varphi = 0. \tag{44}$$

This equation is amenable to confluent hypergeometric differential equation, but a series expansion suffices since $\xi_1<<1$. We have

$$\varphi = [T_2(\xi) + g\ \xi\ T_1(\xi)\ \ln|\xi|] + \theta_1\ \xi\ T_1(\xi) \tag{45a}$$

$$T_1 = 1 + (g/2)\ \xi - 0.10\ (\eta_e/A^2)\ \xi^2 + \ldots \tag{45b}$$

$$T_2 = 1 + (g/3)\ \xi - 0.30\ (\eta_e/A^2)\ \xi^2 + \ldots \tag{45c}$$

$$g = -i\ (\eta_e/2.86\ A^2)\ (a/b). \tag{45d}$$

up to order ξ_1^2. In the limit $\xi_1<<\xi<<\xi_2$, where matching is sought with the solution in the domain $\xi_1 \ll \xi \ll \xi_3$, the g terms can be neglected which shows that the effect of the current on the dispersion relation can be treated by perturbation: the essential reason for this is the localization of the corresponding term.

In the range $0 \leq \xi \ll \xi_3$, Eq. (42) yields (neglecting the small role of the current):

$$A^2 \frac{\partial^2 \varphi}{\partial \xi^2} + \frac{0.60\ \eta_e}{1+0.37 i b \xi^2} \varphi = 0 \tag{46a}$$

which can be converted into a hypergeometric equation

$$z_1(1-z_1) \frac{\partial^2 \varphi}{\partial z_1^2} + \frac{1}{2}(1-z_1) \frac{\partial \varphi}{\partial z_1} + \frac{1}{4} s(s-1) \varphi = 0 \tag{46b}$$

where $z_1 = -0.37\ ib\xi^2$ and

$$s(s-1) = 1.62\ i\eta_e/A^2 b. \tag{46c}$$

The solution of Eq. (46b) is of the form

$$\varphi = F\left(-\frac{s}{2}, \frac{s-1}{2}; \frac{1}{2}; z_1\right) + \theta_2\, z^{1/2}\, F\left(\frac{1-s}{2}, \frac{s}{2}; \frac{3}{2}; z_1\right) \tag{47}$$

where the F's are hypergeometric functions (Abramowitz and Stegun, 1965).

Finally in the range $\xi_2 \ll \xi \leq \infty$, Eq. (42) is approximated by

$$A^2 \frac{\partial^2 \varphi}{\partial \xi^2} + c\ \xi^2 \varphi - i \frac{\Gamma}{\omega_e^*} \varphi - 1.62\ i\ (\eta_e/b)\xi^{-2} \varphi = 0 \tag{48a}$$

which can be transformed into the confluent hypergeometric equation

$$z_2 \frac{\partial^2 Q}{\partial z_2^2} + \left[(s+\frac{1}{2}) - z_2\right] \frac{\partial Q}{\partial z_2} - \frac{1}{4}\left[(2s+1) + \frac{\Gamma}{A\sqrt{c}\,\omega_e^*}\right] Q = 0 \tag{48b}$$

where $z_2 = i(c^{1/2}/A)\xi^2$, and $\varphi = \xi^s \exp(-i\ c^{1/2}\xi^2/2A)Q$. We note that $c^{1/2}$ =

$|k_\|| a_s c_s \omega^{-1}|$ $(1-i\Gamma/2\omega_e^*)$. For $\Gamma>0$ (by analogy with the definition of the Landau contour (Landau, 1946), the solution is evanescent at large x if $\omega_e^*>0$ (if $\omega_e^*<0$, one has to replace $c^{1/2}$ by $-c^{1/2}$). The solution of Eq. (48b) is

$$Q = U\left[\frac{1}{4}\left(2s+1+\frac{\Gamma}{A\sqrt{c\omega_e^*}}\right),\ s+\frac{1}{2};\ z_2\right] + \theta_3 M\left[\frac{1}{4}\left(2s+1+\frac{\Gamma}{A\sqrt{c\omega_e^*}}\right),\ s+\frac{1}{2};\ z_2\right] \tag{49}$$

where M and U are the two Kummer's functions (Abramowitz and Stegun, 1965).

As $\xi \to \pm\infty$, Re $z_2 \to +\infty$ and $M(a,b;z_2) = \Gamma(b)\Gamma^{-1}(a)\, z_2^{a-b}\, e^{z_2}$ which

diverges. Hence $\theta_3 = 0$. In the same limit $U(a,b;z_2) = z_2^{-a}$. As $\xi<<\xi_3$, i.e. as $z_2 \to 0$, on the other hand, one has

$$\frac{\partial \ln\varphi}{\partial\xi} = \frac{s}{\xi} + 2\left(i\,\frac{c^{1/2}}{A}\right)^{1/2-s} \frac{\Gamma\left[\frac{1}{2}-s+\frac{1}{4}\left(2s+1+\Gamma/Ac^{1/2}\omega_e^*\right)\right]\Gamma(s+\frac{1}{2})}{\Gamma\left[\frac{1}{4}\left(2s+1+\Gamma/Ac^{1/2}\omega_e^*\right)\right]\Gamma(\frac{1}{2}-s)}\,\xi^{-2s}\ldots$$

where $s-1/2 = -\left[(1/4) + i\ 1.62\ \eta_e/A^2b\right]^{1/2}$ and the $\Gamma(x)$ are gamma functions. Considering now the asymptotic behaviour of the hypergeometric functions F for $\xi>>\xi_2$, i.e. $z_1 \to \infty$, we have

$$\frac{\partial \ln\varphi}{\partial\xi} = \frac{s}{\xi} + 2\ (0.37\ ib)^{1/2-s}\left(\frac{1}{2}-s\right)\frac{\Gamma(\frac{s-1}{2})\ \Gamma(\frac{s+1}{2})\ \Gamma(\frac{1}{2}-s)}{\Gamma(s-\frac{1}{2})\ \Gamma(-\frac{s}{2})\ \Gamma(1-\frac{s}{2})}\,\xi^{-2s}\ \ldots$$

for the symmetric mode ($\theta_2 = 0$) and

$$\frac{\partial \ln\varphi}{\partial\xi} = \frac{s}{\xi} + 2\ (0.37\ ib)^{1/2-s}\left(\frac{1}{2}-s\right)\frac{\Gamma(\frac{s}{2})\ \Gamma(1+\frac{s}{2})\ \Gamma(\frac{1}{2}-s)}{\Gamma(s-\frac{1}{2})\ \Gamma(\frac{1-s}{2})\ \Gamma(\frac{3}{2}-\frac{s}{2})}\,\xi^{-2s}\ \ldots$$

for the antisymmetric mode ($\theta_2 = \infty$). Hence the dispersion relations:

$$\frac{\Gamma\left[\frac{1}{2} - s + \frac{1}{4}\,(2s+1+\Gamma/Ac^{1/2}\omega_e^*)\right]}{\Gamma\left[\frac{1}{4}\,(2s+1+\Gamma/Ac^{1/2}\omega_e^*)\right]} = (0.37\ Ab/c^{1/2})^{1/2-s}$$

$$\left[\frac{\Gamma\,(\frac{s-1}{2})\ \Gamma(\frac{1}{2} - s)}{\Gamma\,(s - \frac{1}{2})\ \Gamma(-\,\frac{s}{2})}\right]^2 \frac{s-1}{s}\ , \tag{50a}$$

respectively

$$= (0.37\ Ab/c^{1/2})^{1/2-s} \left[\frac{\Gamma(\frac{s}{2})\ \Gamma(\frac{1}{2} - s)}{\Gamma(s - \frac{1}{2})\ \Gamma(\frac{1-s}{2})}\right]^2 \frac{s}{s-1} \tag{50b}$$

Two limiting cases can occur:

i) Strong shear or weak collisionality limit.

If $(0.37\ Ab/c^{1/2})^{1/2}\ s \ll 1$, which implies $s \ll 1$ since $b/c^{1/2} \sim (\xi_3/\xi_2)^2 \gg 1$, and $s = -1.62\ i\eta_e/A^2b = +0.81\ i\eta_e\ (m_e\nu_{ei}/m_i\omega_e^*)\ (L_S^2/L_N^2)$, the solution of the dispersion relation is approximately

$$\Gamma/\omega_e^* = -\ Ac^{1/2} + i\ 0.99\ \pi^{1/2}\ \eta_e\ (c^{1/2}/Ab)^{1/2} \tag{51}$$

for the symmetric mode. The first term on the right-hand side represents the usual result for shear damping; the second yields a mere frequency shift.

ii) Weak shear or strong collisionality limit.

If $(0.37\ Ab/c^{1/2})^{1/2}\ s \gg 1$, the solution of the dispersion relation is

$$\Gamma/\omega_e^* = (-3 + 2s)\ Ac^{1/2} \tag{52}$$

which shows that parallel electron heat conduction is stabilizing since Re $s<0$. We note that for small s, this result does not simply reduce to the shear induced eigenvalue for the symmetric mode.

b) Effect of the parallel current.

We consider here the simplified equation

$$\left[\frac{\partial^2}{\partial z^2} - 2z\frac{\partial}{\partial z} - \left(1 + \frac{\Gamma}{Ac^{1/2}\omega_e^*}\right) + H'\right] \exp\,(z^2/2)\,\varphi = 0 \tag{53}$$

where $H' = \eta_e\sqrt{i}\,(a/bA^{3/2}c^{1/4})z^{-1}\,\left[2.86 + 1.06\,(bA/c^{1/2})\,z^2\right]^{-1}$ and $z = (ic^{1/2}/A)^{1/2}\,\xi$ in the domain $\sqrt{i}\,(c^{1/2}/A)\,(1-i\varepsilon/2) \times [-\infty,\infty]$. Here $\varepsilon = \Gamma/\omega_e^* \to 0^+$ and ω_e^* is assumed to be positive [see the discussion below Eq. (48b)]. The approximate equation (53) is justified only in the strong shear limit ($s \to 0$) where the eigenfunction $\varphi(z) \cong \varphi(0)\exp\,(-z^2)$ over most of the domain. The unperturbed equation admits the Hermite polynomials $H_n(z)$ as eigenfunctions with the eigenvalues $(1+\Gamma/Ac^{1/2}\omega_e^*) = -2\,n$. Since the perturbation is antisymmetric, one has to proceed to second order to obtain the perturbed eigenvalue. The result is

$$1 + \frac{\Gamma}{Ac^{1/2}\omega_e^*} = \sum_n \left[\int dz\,\exp(-z^2)\,H'H_oH_n\right]^2 / \left[2n\int dz\,\exp(-z^2)\,H_o^2 \int dz\,\exp\,(-z^2)\,H_n^2\right] \tag{54}$$

for the fundamental mode, which shows that the perturbation yields a mere frequency shift. Retaining only the first term ($n = 1$) in the sum, one obtains for example

$$1 + \frac{\Gamma}{Ac^{1/2}\omega_e^*} = i\,\frac{\pi}{3.03}\,\eta_e^2\,\frac{a^2}{b^3A^4}\,.$$

3. CONCLUSIONS

We have shown that introduction of the electron and ion diamagnetic drift in the theory of the rippling mode leads to complete stabilization at temperatures above, typically, 50 eV. Parallel electron heat conduction is further stabilizing if η_e = dlnTe/dlnN >0, but is destabilizing if η_e<0. The collisional trapped electron mode is also stable in the straight cylindrical model, even if the tokamak current is retained in the equations.

REFERENCES

Abramowitz, M. and Stegun, I.A. (1965), Handbook of Mathematical Functions 504, and 556.

Braginskii, S.I (1965) in: Reviews of Plasma Physics, Vol. 1, 205 ff (Consultants Bureau, New York).

Brower, D.L., Bravenec, R., Gentle, K., Kochanski, T., Luhman, N.C.Jr., Peebles, W.A., Phillips, P., Richards, B., and Rowan, W. (1983), Proceedings of the 11th European Conf. on Controlled Fusion and Plasma Physics, Aachen, F.R.G., Vol. I, 73.

Callen, J.D., Carreras, B.A., Diamond, P.H., Benchikh-Lehocine, M.E., Garcia, L. and Hicks, H.R. (1983), Plasma Physics and Controlled Nuclear Fusion Research 1982, Vol. I, 297 (IAEA, Vienna).

Carreras, B.A., Gaffney, P.W., Hicks, H.R., and Callen, J.D. (1982), Phys. Fluids 25, 1231.

Cordey, J.G., Jones, E.M., and Start, D.F.H. (1979), Plasma Phys. 21, 725.

Equipe T.F.R. (1983), Plasma Phys. 25, 641.

Evans, D.E., Doyle, E.J., Frigione, D., von Hellermann, M. and Murdock, A. (1983), Plasma Phys. 25, 617.

Furth, H.P., Killeen, J. and Rosenbluth, M.N. (1963), Phys. Fluids 6, 459.

Hasselberg, G., and Rogister, A. (1980), Plasma Phys. 22, 805.

Hasselberg, G., and Rogister, A. (1983), Nucl. Fus. 23, 1351

Hoh, F.C. (1964), Phys. Fluids 7, 956.

Kadomtsev, B.B., and Nedospasov, A.V. (1960), J. Nucl. Energy Part C 1, 230.

Kadomtsev, B.B. (1961), Zh. Techn. Fiz. 31, 1209 [Sov. Phys-Tech. Phys 6, 882 (1962)].

Mazzucato, E. (1982), Phys. Rev. Lett. 48, 1828.

Meyer, J., and Mahn, C. (1981), Phys. Rev. Lett. 46, 1206.

Ohtsuka, H., Kimura, H., Shimomura, S., Maeda, H., Yamamoto, S., Nagami, M., Ueda, N., Kitsunezaki, A., and Nagashima, T. (1978), Plasma Phys. 20, 749.

Rogister, A. and Hasselberg, G. (1979), Proc. of IAEA Conf. on Plasma Physics and Controlled Thermonuclear Fusion Research, Nucl. Fus. Suppl. 1, 809.

Rutherford, P.H. (1979) in: International School of Plasma Physics: Physics of Plasmas close to Thermonuclear Conditions (Varenna, Italy) Vol. I, 143 (CEC Brussels).

Semet, A., Mase, A., Peebles, W.A., Luhman, N.C.Jr., and Zweben, S. (1980), Phys. REv. Lett. 45, 445.

Slusher, R.E., and Surko, C.M. (1980 a), Phys. Fluids 23, 472.

Surko, C.M., and Slusher, R.E. (19806) Phys. Fluids 23, 2435.

Zweben, S.J., and Gould, R.W (1983), CALTECH REport, Edge Plasma Turbulence in the Caltech Tokamak.

Zweben, S.J., and Taylor, R.J. (1981), Nucl. Fus. 21, 193.