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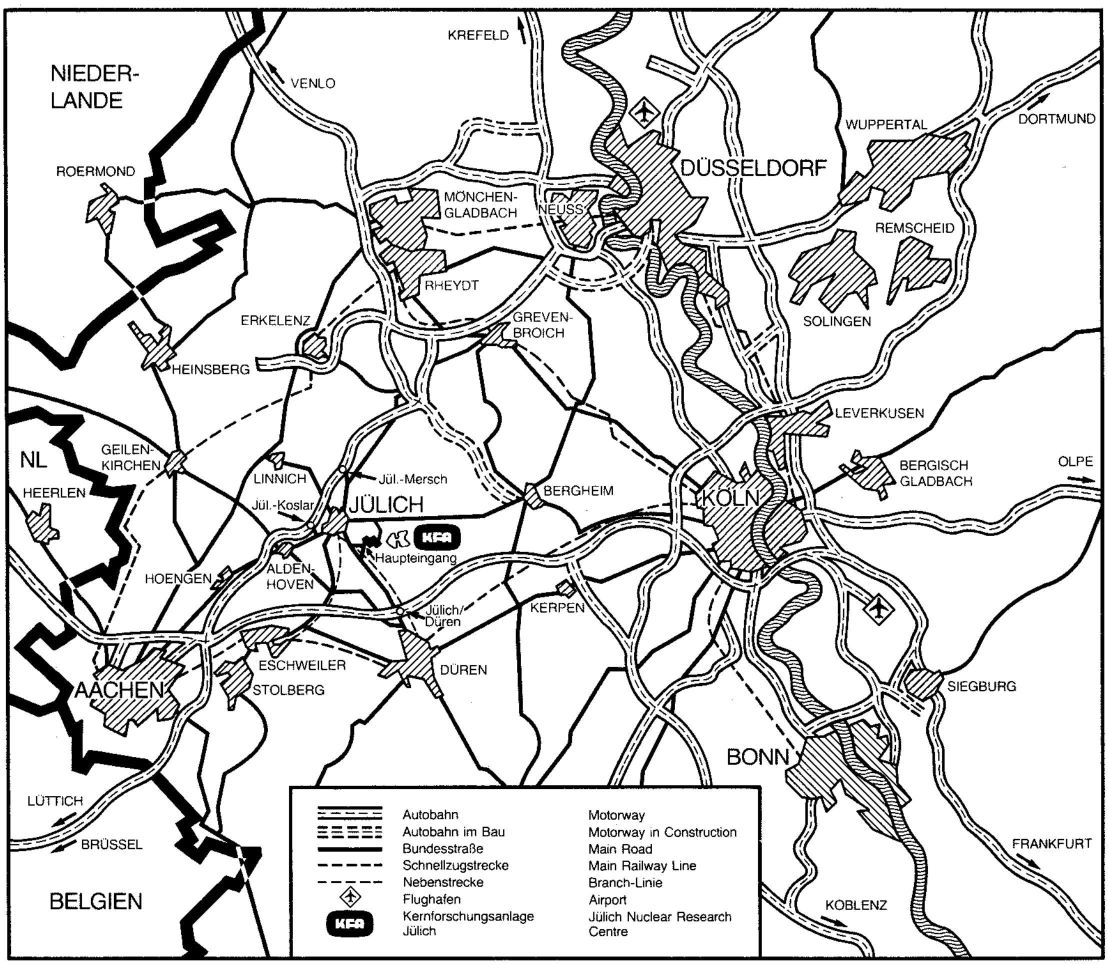

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TRANSPORT PROPERTIES OF DRIFT WAVES

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We present the results and discuss the prospects of a study of plasma transport which is based on a recent theory of drift wave turbulence. The specific questions addressed are the turbulence spectrum, the electron heat conduction, the impurity transport, the evacuation of the power released during sawtooth relaxations, the cleaning action of these sawteeth, the cause of the high density limit and of the gross plasma disruptions.

Anomalous transport in magnetically confined plasmas - be it laminar or disruptive - is probably the most challenging aspect of controlled thermonuclear fusion. Whether the anomalies will ultimately be detrimental or useful may depend on our skill in coping with them and turning them to account. Their understanding passes through three stages: identification of the underlying instabilities, identification of the saturation processes, and evaluation of the resulting transport. Our considerations are restricted here to the plasma layer with steep temperature and density gradients which surrounds the dense, hot core and screens it from the outer zones in contact with walls and limiter. The nonlinear ion Landau damping[1] of the dissipative trapped electron mode provides there a microturbulence spectrum which allows to obtain a laminar electron heat conduction and an enhanced outward impurity transport which are compatible with the observations. The calculation shows furthermore that the trapped electron instability is able to sweep without delay across the turbulent layer the enormous power which is periodically released during the abrupt decay of the sawteeth. The high density limit which corresponds to the onset of gross plasma disruptions is the consequence of the thermal insulation of the core following the improved linear stability of the trapped electron mode.

The dispersion relation of drift waves allows for "distant" interactions to occur in which two modes with almost equal frequencies but different poloidal mode numbers couple to interact resonantly with ions. This form of scattering which we found[1] to play an essential role in the saturation of the instability is also determinant as concerns the transport properties of the turbulence. It gives rise to a large energy transfer from long to short wavelengths; when this is balanced against cascading, which arises from "close interactions", a splitting of the spectrum into two branches occurs. The short wavelength branch which is then maintained filled by the distant interactions accounts for most of the anomalous electron heat flux. It brings, however, only a small contribution to the fluctuation level $\tilde{n}(r)$ and is not observable with present detection systems. This resolves the contradiction which appears when one notes that most plasmas are in a marginally unstable state[2] but that the energy transport estimated from the observed long wavelength branch of the spectrum is plainly insufficient to account for the anomalous losses estimated from power balance[3]. Distant interactions provide also the dominant contribution to the anomalous impurity transport: they give rise to a diffusive flux which is quantitatively consistent with the conclusions from impurity injection experiments[4] whereas the contribution from close interactions is here down in $1/Z$. (The diffusive flux is proportional to the impurity density gradient.)

In order to illustrate and to push further our arguments, we concentrate the discussion on the moderate density discharge in Tokamak Fontenay-aux-Roses (TFR) analysed in Reference 3. For the density fluctuation of the trapped electrons, we adopt the model[5]

$$[\tilde{n}_\ell(x)]_t = i\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \varepsilon_o^{\frac{3}{2}} \frac{\omega_\ell-\omega^*_{e,\ell}}{(1+Z^{-1}_{eff})} \frac{(1+3\eta_e/2)}{\nu_{ei}} \sum_p (-1)^p \phi_\ell(x+p\Delta_\ell)\, S(x;x+p\Delta_\ell) \quad , \tag{1a}$$

in which the function

$$S(x;x+p\Delta_\ell) = \langle\alpha\rangle_\theta \int_o^\pi dz\, z\, \frac{J_1\,[(p+x/\Delta_\ell)z]}{p + x/\Delta_\ell} \frac{J_1\,[(x/\Delta_\ell)z]}{x/\Delta_\ell} \tag{1b}$$

was obtained with the approximation of a pitch angle collision operator and of a parabolic magnetic well. (The factor $\langle\alpha_\theta\rangle = \langle[(1-\cos\theta)/2]^{3/2}\rangle_\theta = 4/3\pi$ corrects for the systematic overestimation linked to the latter). $\nu_{e,i} = 3\sqrt{\pi}/4\tau_e$ where τ_e is the standard collision time of Braginskii; $x = r-r_o$ and $\Delta_\ell = |\ell dq/dr|^{-1}_{r_o}$ are respectively the distances from a reference and between neighbouring rational surfaces of toroidal mode number ℓ; ω^*_e is the electron diamagnetic frequency, $\eta_e = d\ln T_e/d\ln N$, $\varepsilon_o = r_o/R$, and J_1 is the Bessel function of the first kind. Eq. (1) where ω_ℓ is the frequency of the localized eigenmode can easily be corrected for finite values of the collisionality parameter $\nu^* = (1 + Z^{-1}_{eff})\, \nu_{ei} qR/\sqrt{2}\, \varepsilon_o^{3/2}\, c_e$ (q = safety factor; $c_e^2 = T_e/m_e$) and agrees quantitatively for $x = p = 0$ with Eq. (26) of Reference 6. As concerns the normalized potential $\phi = e\Phi/T_e$, a careful analysis of the experimental profiles has substantiated the hypothesis, also favoured by the absence of pronounced ballooning, of radiating eigenmode structures[1], viz. $\phi_\ell \sim \exp[-i(1-i\varepsilon)x^2/2x_t^2]$ where x_t is the Pearlstein Berk turning point; $x_t/\sqrt{\varepsilon} = x_c$ characterises their spatial decay owing to linear ion absorption. The normalized linear growth rate $\tilde{\gamma}_L = \gamma|L_s/\omega L_n|$ that one can now derive from Eq. (1) has the form (L_s it the shear length and L_n the density length scale):

$$\tilde{\gamma} = c\,[1-\exp(-2\Delta/x_t)_\ell\,]\,(1 + \frac{2}{3\eta_e}\frac{y}{1+y})\,\frac{y^{1/2}}{1+y} - 1 \tag{2a}$$

where the factor within the bracket makes the transition between the two asymptotic limits of rational surface overlapping [large or small $(\Delta/x_t)_\ell \equiv (|L_n| (1+y)/qR\hat{\sigma}y)^{1/2}$] and

$$c = \langle\alpha\rangle_\theta \left(\frac{\pi}{2}\right)^{7/2} \left(\frac{m_e}{2\, m_i}\right)^{1/2} \frac{L_s^2}{L_n^2} \frac{3}{2}\, \eta_e\, \hat{\sigma}\nu^{*-1}. \tag{2b}$$

$\hat{\sigma}$ is the shear parameter and $y = k_\theta^2 a_s^2 = (\ell q a_s/r_o)^2$; $a_s^2 = T_e m_i c^2/e^2B^2$. In Eg. (2a), the substracted term arises from shear damping.

The contribution K_e of the drift waves to the electron heat conduction coefficient defined through the radial heat flux $\Gamma = K_e N\partial T_e/\partial r$ is

$$K_e = \frac{5\langle\alpha\rangle_\theta}{3} \frac{(T_i/T_e)^{1/2}}{k_{rc}^5\ a_i^5}\, \nu^{*-1}\, \frac{a_e^2 c_e}{|L_s|}\, G_1 \tag{3a}$$

where $a_i = a_s(T_i/T_e)^{1/2}$;

$$G_1 = \int_0^\infty \frac{dy}{y}\,(1+y)\,\left(1 + \frac{2}{5\eta_e}\frac{y}{1+y}\right)\bar{f} \tag{3b}$$

can be calculated once the spectral function $\bar{f} = 12\,\pi \langle k_r^4 a_i^4\rangle\ |L_s L_n|\ a_s^{-3} y |\phi|_{k_\theta}^2$ is known. The latter obtains from the wave kinetic equation of Reference (1a), viz

$$\{L_1[f(y)] + L_2[f(y^{-1})]\}\, f(y) = -\mathrm{sign}(1-y)\ \ \gamma_L(y)\, \frac{(1-y)^2}{(1+y)^3}\, y^{-3/2}\, f(y) \tag{4}$$

where the operators $L_1 \equiv y\partial/\partial y + (2-y)/(1-y)$ and $L_2 \equiv (2-y)^2\ \partial/\partial y + (2-y)/(1-y)$. Note that the solution of Eq. (4) is subject to the constraint $f \geqslant 0$, hence its nonlinear character. In Eq. (3a) k_{rc} is the cutoff radial mode number set by linear ion Landau damping; we estimate $k_{rc}^2 a_i^2 = a_i^2/\varepsilon x_t^2 \cong 1/4.5$.

Fig. (1) shows the spectrum of the poloidal electric field at the radius $r/a = \rho = 0.45$ where the drift wave transport is largest. The long wavelength branch has a maximum at the point $k_\theta a_s \doteq 0.6$ where the energy transfer in k_θ-space sets in. Experimentally a maximum is observed[7] at $k_\theta a_i \cong 0.5$. Fig. (2a) gives the theoretical values of the contribution K_e of the drift waves to the heat conductivity across the turbulent layer. It is compared with the experimental thermal conductivity χ_e^b inferred from power balance. The heat flux carried by the sawteeth has here been substracted; this explains the deviation from the corresponding curve in Reference 3 at small ρ's. Considering that the trapped electron mode must be progressively replaced by some other transport mechanism beyond the radius where it levels off because of increased collisionality, the agreement between theory and experiment on the heat conduction coefficient is clearly satisfactory. The predicted fluctuation level, however, is somewhat low, e.g. $(\tilde{n})_{th} \sim 1.8 \times 10^{-3}$ compared to $(\tilde{n})_{exp.} \sim 4 \times 10^{-3}$ at $\rho = 0.5$. This is not considered to be a real problem given the experimental uncertainties and the simplifications introduced in the theory.

In a further attempt to check the validity of the model, we have analyzed the transport of high Z impurities. Assuming that $m_I/m_i \ll Z^2$, i.e. $a_I^2/a_i^2 \ll 1$, we find that their diffusion coefficient is of the form

$$D_I \cong \frac{1}{50\pi} \frac{(T_e/T_i)}{k_{rc}^2 a_i^2} \frac{a_s^2 c_s}{L_s} \frac{(-)L_n}{L_s} \int_0^\infty \frac{dy}{y} K(y,y^{-1})\ \bar{f}(y)\ \bar{f}(y^{-1}) \tag{5}$$

where the kernel K is symmetric in y and y^{-1}. Thus only distant interactions contribute to the diffusion in the lowest order of a $1/Z$ expansion and D_I, in agreement with the observed confinement time[4], is independent of both the atomic mass and the charge of the impurity. The ratio $(D_I)_{an}/(D_I)_{n.c.}$ is much larger than unity (= 11.75 on the rational surface $\rho = 0.5$); D_I actually has the order of magnitude ($\sim 1.5\ 10^3\ cm^2/sec$) requested to explain the time history of the line radiation of injected metallic impurities.[4] Fig. 2b shows that the radial profiles of the anomalous diffusion coefficients of the impurities and of the base ions ($D_i = D_e \cong K_e/5$) are remarkably similar.

The ratio of the destabilizing trapped electron term to the instability threshold corresponding to shear damping is less than two across the layer. This property, which seems common to all Tokamaks[2], suggests that the increase of the anomalous heat transport with the linear growth rate is strongly nonlinear and that a relaxation of the profiles has taken place. To check the soundness of this inference we have multiplied the destabilizing trapped electron term by 1.5 in Eq. (2a); the corresponding enhancement factors of K_e and D_I were respectively larger than 10 and than 10^2 at the normalized radius $\rho = 0.5$.

The following picture of a Tokamak discharge then emerges: initially the core temperature rises until the shear stabilization of the dissipative trapped electron mode is defeated in the surrounding transition layer; beyond this marginally stable state, the latter is so leaky that only minor modifications of the profile are required to ensure that the local turbulent transport matches the heat flux originating from the hot core. This scenario implies

- that the physics of drift wave transport cannot be described by scaling laws;
- that the power which is released periodically during the abrupt decay of the sawteeth should be able to create the conditions requested to sustain a swift outflow across the turbulent layer.

As it progresses the heat flow indeed lowers the collisionality along its path which increases the growth rate of the instability and enhances the transport. Since the instability threshold rises with density, both the period and the amplitude of the sawteeth should also increase eventually leading to a major catastrophy: the gross plasma disruption. These predictions are confirmed experimentally[8]. We estimate from the model that the density limit is approximately given by

$$N_o = 7\ 10^{16} \frac{a^{1/2}\ T^2_{e,o}}{R^{3/2}\ Z_{eff}\ A_i^{1/2}}\ F(a/q_aR) \tag{6a}$$

where A_i is the atomic mass. The functional F depends on the density and temperature profiles; F(0.1) = 1 for the parabolic dependences $N \sim (1-\rho^2)$, $T \sim (1-\rho^2)^2$. The two independent combinations of plasma parameters entering in (6a), viz. a/q_aR and $NR^{3/2}\ Z_{eff}\ A_i^{1/2}\ /\ a^{1/2}\ T_e^2$, are those entering the definition of the growth rate, Eq. (2). Eliminating the temperature with the help of Ohm's law, we obtain

$$N_o = 5\ 10^{10} \frac{a^{1/2}\ Z^{1/3}}{R^{1/6}\ A_i^{1/2}} \left(\frac{B_t}{q_oRV}\right)^{4/3} F\left(\frac{a}{q_aR}\right) \tag{6b}$$

which clearly reminds of the Murakami limit[9]. In Eqs. (6a) and (6b), $[T_e]$ = kev, [V] = volt, [B] = Gauss, and [L] = cm.

– According to our numerical results, the sawteeth relaxations should also sweep the impurities at an enhanced rate to the plasma edge; experimentally the cleaning action of the sawteeth is agreed upon but no explanation has yet been provided for the underlying mechanism.

In conclusion we have been able to describe much of the ontology of a Tokamak discharge from first principles. Further work should extend the nonlinear theory to the regime of strong turbulence and attempt to explain the still puzzling line broadening.

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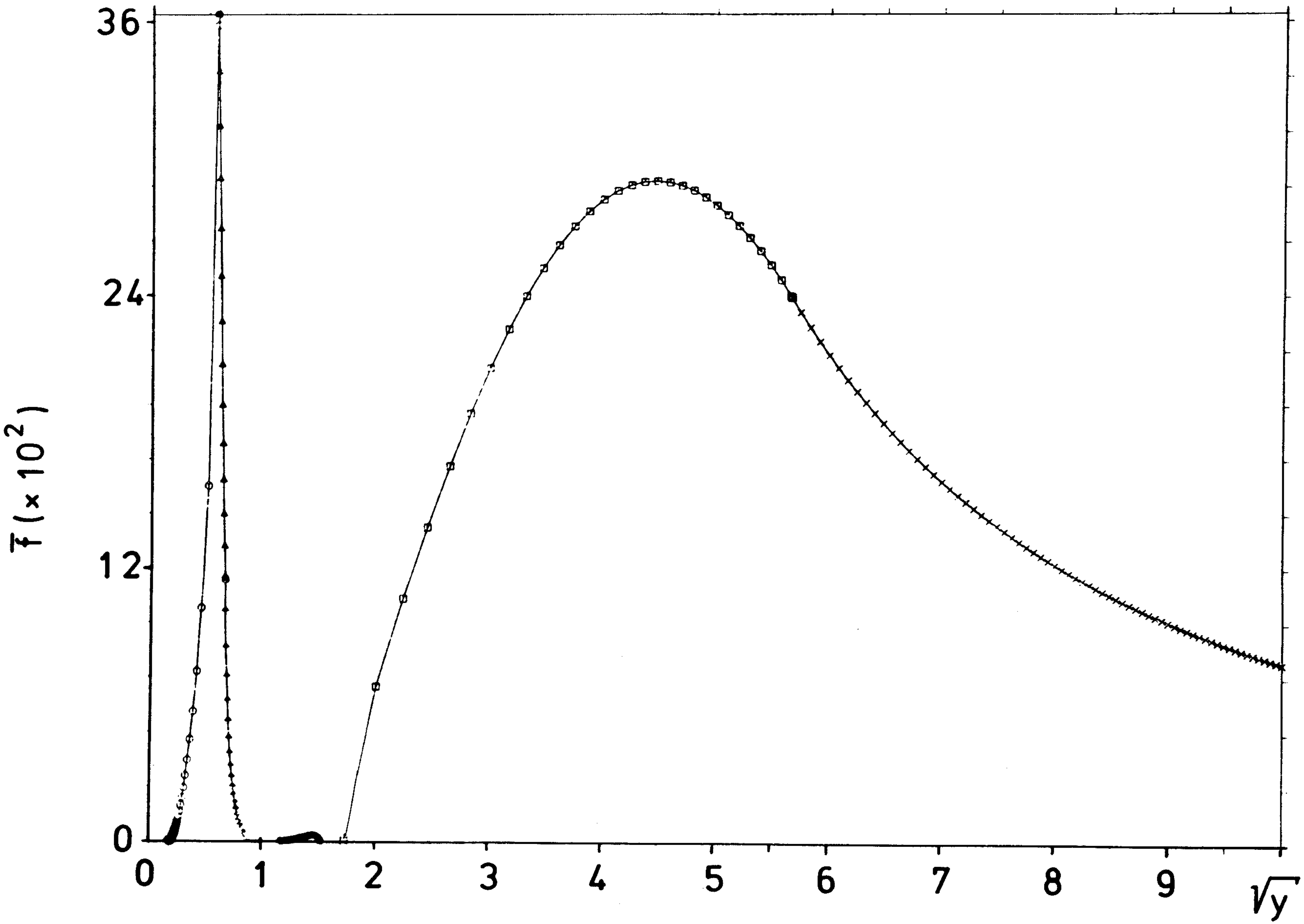
FIGURE CAPTIONS:

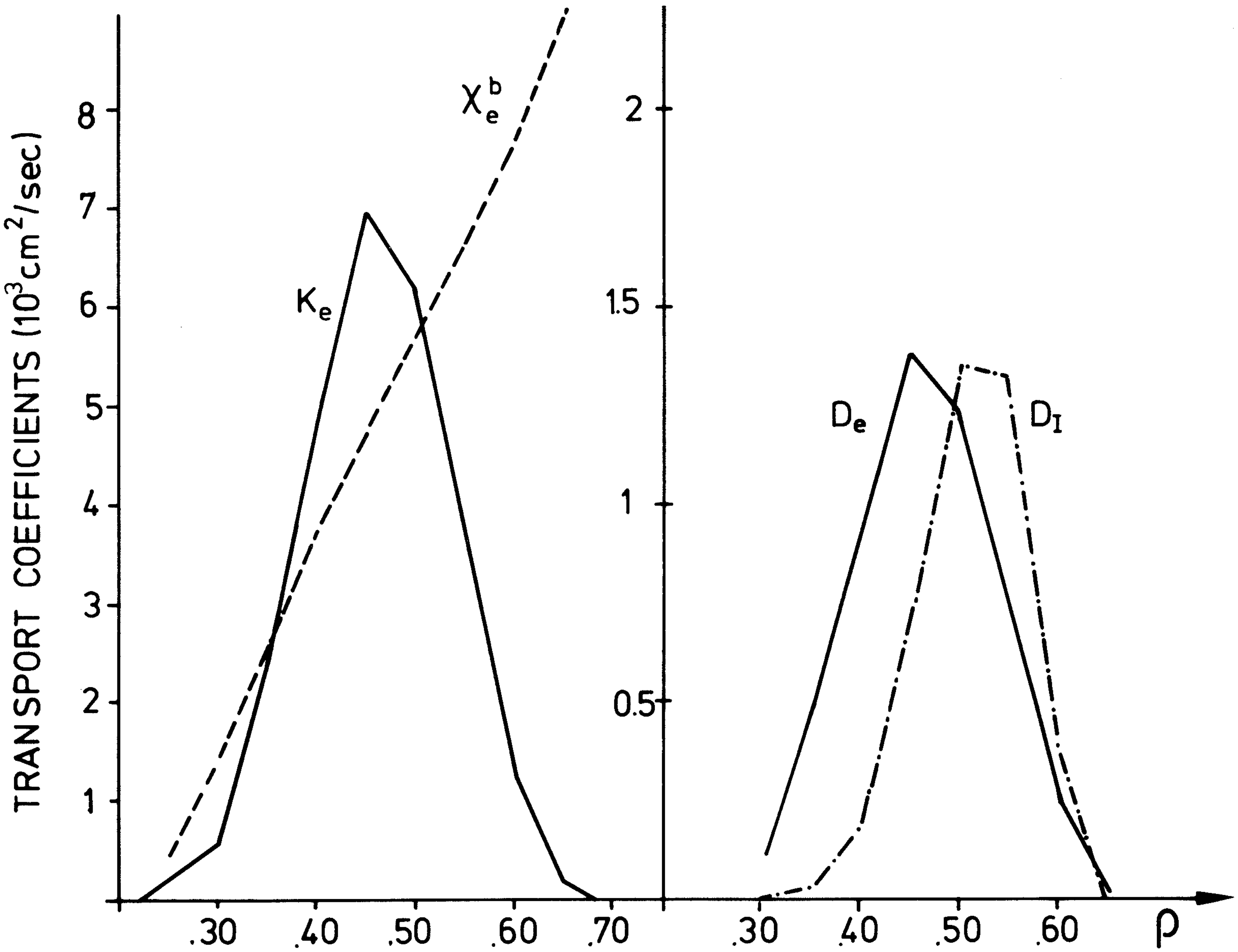
Fig. 1: Electric field energy density $|k_\theta \phi_{k_\theta}|^2 \sim \bar{f}$ versus normalized mode number $\sqrt{y} = k_\theta a_s$.

Fig. 2: (a) Theoretical (K_e) and experimental (χ_e^b) electron heat conduction coefficients;

(b) theoretical diffusion coefficient for impurities (D_I) and electrons (D_e). In abscissa is the normalized plasma radius.



36
24
12
0
f̄ (× 10²)
0
1
2
3
4
5
6
7
8
9
√y



TRANSPORT COEFFICIENTS (10³cm²/sec)
8
7
6
5
4
3
2
1
.30
.40
.50
.60
.70
K_e
χ_e^b
2
1.5
1
0.5
.30
.40
.50
.60
D_e
D_I
ρ