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The influence of cross field diffusion and convection on the impurity ion transport in the limiter shadow region of a tokamak

I. Analytical formulae

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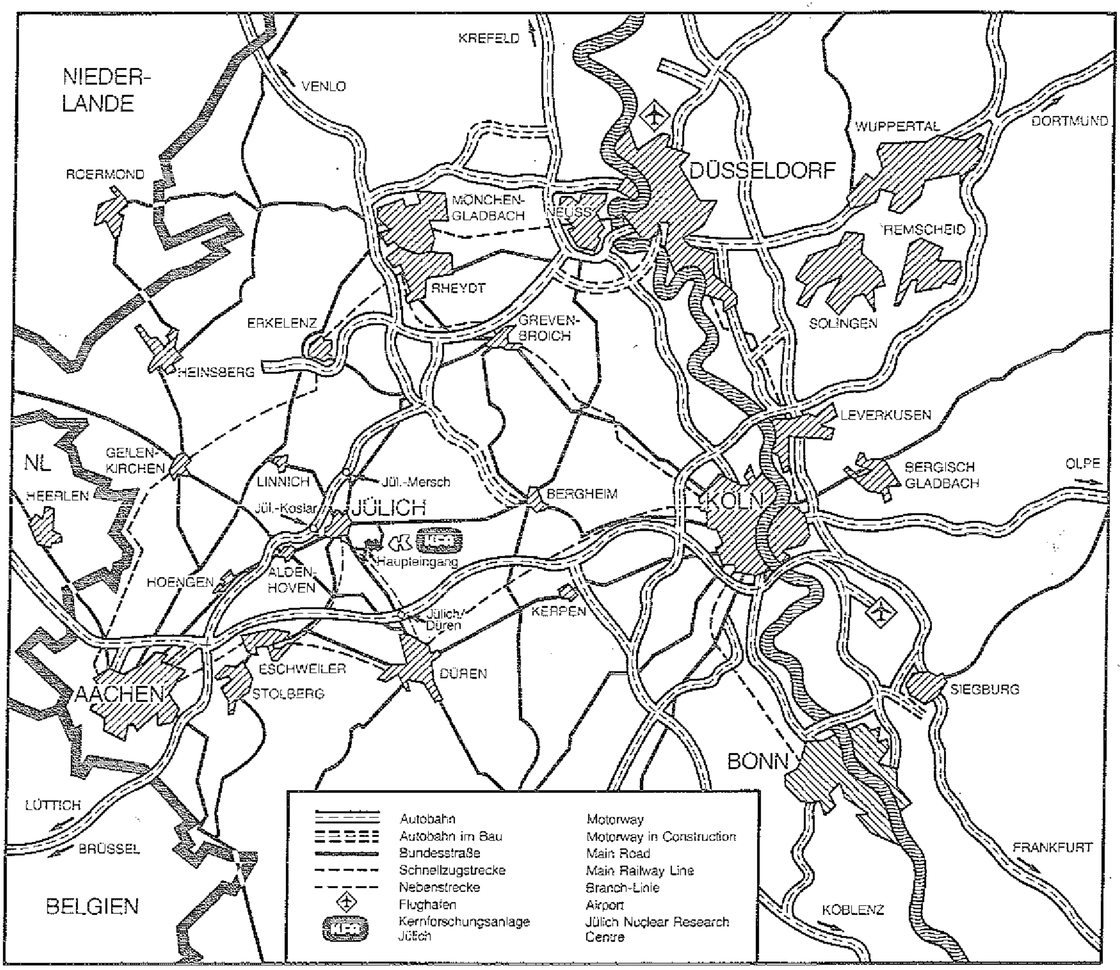
Jül-1971
February 1985
ISSN 0366-0885



Als Manuskript gedruckt

Berichte der Kernforschungsanlage Jülich – Nr. 1971
Institut für Plasmaphysik, Association EURATOM-KFA Jül-1971

Zu beziehen durch: ZENTRALBIBLIOTHEK der Kernforschungsanlage Jülich GmbH
Postfach 1913 · D-5170 Jülich (Bundesrepublik Deutschland)
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Errata: Jül - 1971

p. 8, eq.(14)

$$\left.\frac{\partial(V_\perp a_o)}{\partial r}\right|_{v_\parallel,\zeta} \quad \text{instead of} \quad V_\perp \left.\frac{\partial a_o}{\partial r}\right|_{v_\parallel,\zeta}$$

p.15, last line but one

$n_k \dfrac{\partial n_j}{\partial x_k} = 0$ and using a slab geometry with Cartesian coordinates r,ζ

p.16, formula for the radial impurity influx across the separatrix

$$\Gamma_\perp\left(a,\tau,v_\parallel^\pm(\tau)\right) + D_\perp(\tau)\,\frac{\left(r - a - \displaystyle\int_\tau^\zeta \frac{V_\perp(\xi)}{v_\parallel^\pm(\xi)}\,d\xi\right)}{2\displaystyle\int_\tau^\zeta \frac{D_\perp(\xi)}{v_\parallel^\pm(\xi)}\,d\xi}\; a_o\left(a,\tau,v_\parallel^\pm(\tau)\right)$$

instead of

$$\Gamma_\perp\left(a,\tau,v_\parallel^\pm(\tau)\right)$$

p.16, last line but two

the wall region $r \geq a+\Delta$ we assume $Q_o = 0$ for all ionization states

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Introduction

In reference [1] the transport and redeposition of trace amounts of metal impurities sputtered from a limiter or divertor target by hydrogen ion bombardment were studied without considering diffusion and convection perpendicular to the magnetic field (m.f.). This study had therefore two basic implications:

a) All sputtered impurity atoms, which are ionized in the plasma scrape-off layer (s.o.l.), are redeposited on the target. The ionization probability depends on the plasma parameters and the angular and energy distribution of the sputtered impurity atoms.

b) The charge state population and the penetration depth of the impurities are solely determined by electron impact ionization and deceleration (acceleration) along the m.f.. The driving forces are the presheath electric field and the friction force exerted by the hydrogen ions of the counterstreaming plasma.

In the present report an attempt is made to extend the impurity transport analysis by inclusion of diffusion and convection processes perpendicular to the m.f.. In this more realistic case both the neutral and the ionized impurities may escape the s.o.l. across the magnetic field lines.

Rather than accounting for neoclassical drifts, which as small effects may be treated iteratively within the drift-kinetic formalism of [1] , we consider diffusion and perpendicular convection processes induced by plasma edge turbulence. Because of the lack of a proper theory of plasma turbulence in the s.o.l. we link the time- and gyro-averaged parts of the electric drift and the distribution function of the impurity ions in such a way as to satisfy Bohm's diffusion law and to produce a simple convection term. But even then the resulting drift-kinetic equation for the impurity ions is too complicated to be solved analytically for arbitrary plasma parameters. In the following we therefore assume that the plasma parameters are constant parallel to the target (perpendicular to the m.f.) ignoring an essential feature of the s.o.l. and the most likely mechanism for driving plasma turbulence. In this case there is, however, a rigorous analytical solution to the problem, which may serve as a benchmark for more complicated numerical calculations.

Plasma background and impurity transport model

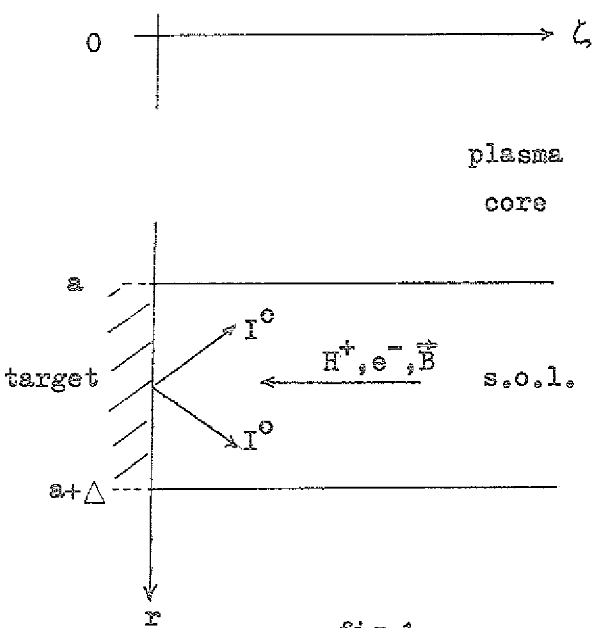


fig.1

In the following we consider the 2-d situation shown in fig.1 :

A hydrogen plasma streams along a m.f. at right angle towards a target of lateral extension Δ . The plasma parameters are constant perpendicular to the m.f. (in radial direction parallel to the target), but vary along the m.f. (in ζ -direction). The velocity distributions of both electrons and hydrogen ions are shifted Maxwellians. Whereas the electron flow velocity is always very small compared to the electron thermal velocity, the flow velocity of the hydrogen ions reaches the ion sound velocity in front of the target. The hydrogen ions impinging on the target sputter impurity atoms, which leave the target surface with a prescribed angular and energy distribution (with a Thompson distribution, for instance). After being ionized the impurity particles undergo Coulomb collisions with the hydrogen ions, a deceleration (acceleration) in the presheath electric field, an electric drift induced by plasma edge turbulence, and further electron impact ionization. Whereas the Coulomb collisions and the presheath electric field cause an impurity ion motion essentially along the m.f., the turbulent electric drift forces the impurity ions to migrate across the m.f..

Since the plasma parameters are constant in radial direction, it is only the source function, which causes the impurity ion distribution to be radially inhomogeneous. The radial inhomogeneity of the source function is, however, a consequence of the combined effect of the angular distribution of the emitted impurity atoms and the lateral bounds of the emitting target surface (due to the finite s.o.l. thickness).

The above model applies to the case of trace impurity amounts, which scarcely interact among themselves and do not markedly influence the plasma properties by impurity-hydrogen ion collisions. It presupposes that the recombination rates are negligibly small for the lower charge states existing in the s.o.l..

Governing equations and their solution

Within the adopted model assumptions the kinetic equations for the various charge states of the impurity particles can be solved step by step beginning with the neutral atomic state. The solution for each charge state yields the source function of the next higher charge state. The plasma properties and the emission data of the target are taken as given quantities.

Basically, we have to solve the kinetic equations for the impurity atoms and ions separately:

a) impurity atoms

Omitting elastic collision processes from consideration we start with the kinetic equation for the velocity distribution G of the impurity atoms

$$- v_{\perp} \cos \varphi \frac{\partial G}{\partial r} + v_{\parallel} \frac{\partial G}{\partial \zeta} = - \nu_o G \qquad (1)$$

where with respect to the drift-kinetic formalism for the impurity ions velocity coordinates $v_{\parallel,\perp}$ parallel and perpendicular to the m.f. and the angle φ between $\vec{v}_{\perp}$ and the negative radial direction have been introduced. $\nu_o = n\, S_o(T_e)$ is the frequency for electron impact ionization of the impurity atoms (n = plasma density, $S_o(T_e)$ = ionization function calculated for a Maxwellian electron distribution function at the temperature T_e). Eq.(1) may be solved by the method of characteristics to yield

$$G(\vec{x}, \vec{v}) = G\left[\vec{x}(0), \vec{v}\right] \exp\left(- \int_0^{\zeta} \frac{\nu_o\left[\vec{x}(\xi)\right]}{v_{\parallel}} d\xi \right) \qquad (2)$$

the integration being performed along the projection of the characteristic on the r, ζ - plane

$$\vec{x}(\xi) = \left(r - \frac{v_{\perp}}{v_{\parallel}} \cos\varphi \; (\xi - \zeta) \; , \; \xi \right) \qquad (3)$$

for a given point $\vec{x} = (r, \zeta)$ under consideration. For radially constant plasma parameters the integral is independent of the angle between the characteristic and the m.f..

The distribution function of the emitted impurity atoms (mass m_z) may be described by the Thompson formula

$$G\left[\vec{x}(0),\vec{v}\right] = \frac{4}{\pi}\,\Gamma_E\left[\vec{x}(0)\right]\,\frac{\gamma^*}{\left(v_\parallel^2 + v_\perp^2 + \gamma^*\right)^3} \qquad \text{for} \quad v_\parallel \geq 0 \tag{4a}$$

where $\gamma^* = 2U_B/m_z$. U_B is the binding energy of the impurity atoms at the target surface. In the low energy regime (some 100's of eV) of the sputtering hydrogen ions, eq.(4a) is a useful approximation to the experimentally observed energy distribution [2] , if U_B is replaced by a lower effective value depending on the mean energy of the incident hydrogen ions.

For numerical purposes it is, however, more convenient to use instead of eq.(4a) a distribution of the form

$$G\left[\vec{x}(0),\vec{v}\right] = \frac{\Gamma_E\left[\vec{x}(0)\right]}{\pi v_{To}^2 V_o}\exp\left(-\frac{v_\parallel^2}{v_{To}^2}\right)\delta\left(v_\perp - V_o\right) \quad , \quad v_{To} = \sqrt{\frac{2kT_o}{m_z}} \tag{4b}$$

which because of the Delta-function in $v_\perp$ reduces the number of multiple integrations in the final result.

Γ_E is the emission flux, which in case of hydrogen ion sputtering reads

$$\Gamma_E\left[\vec{x}(0)\right] = 2\pi\int_{\sqrt{2e\Delta\phi/m_i}}^{\infty} dv_\parallel\, v_\parallel \int_0^{\infty} dv_\perp\, v_\perp\, Y(E')\, F_i\left[\vec{x}(0)\right] \qquad \begin{cases} E' = \dfrac{m_i v_\parallel^2}{2\epsilon_t} \\ \epsilon_t = \text{threshold energy} \end{cases} \tag{5}$$

$\Delta\phi = \phi_p - \phi_w$ = potential drop across the Langmuir sheath at the target (ϕ_w)

$F_i\left[\vec{x}(0)\right]$ is the distribution function of the incident hydrogen ions. In case of a shifted Maxwellian distribution function of the hydrogen ions penetrating the Langmuir sheath with the ion sound velocity c_s

$$F_i\left[\vec{x}(0)\right] \sim \frac{\Gamma_{\parallel i}}{v_{Ti}^4}\exp\left[-\left(\sqrt{\frac{v_\parallel^2}{v_{Ti}^2} - \frac{e\Delta\phi}{kT_i}} - \frac{c_s}{v_{Ti}}\right)^2 - \frac{v_\perp^2}{v_{Ti}^2}\right] \tag{6}$$

with $v_{Ti} = \sqrt{2kT_i/m_i}$ = thermal velocity of the ions accounting for energy $(m_i v_{\parallel}^2/2 + e\phi)$ and flux $(\Gamma_{\parallel i})$ conservation across the collisionless Langmuir sheath.

Combination of eqs.(5) and (6) with the expression for the sputtering yield $Y(E')$ from reference [3]

$$Y(E') = 6.4\times10^{-3} \frac{m_z}{m_H} \gamma^{5/3} (E')^{1/4} (1 - 1/E')^{7/2} \tag{7}$$

with $\gamma = \dfrac{4m_i m_z}{(m_i + m_z)^2}$

shows the coupling between the emission current density Γ_E of the impurity atoms and the incident hydrogen ion flux $\Gamma_{\parallel i}$

$$\Gamma_E \sim \left(\frac{kT_i}{\epsilon_t}\right)^{1/4} \Gamma_{\parallel i} \int\limits_{\max\left(\frac{e\Delta\phi,\ \epsilon_t}{kT_i}\right)}^{\infty} d\chi\, \chi^{1/4} \left(1 - \frac{\epsilon_t}{kT_i \chi}\right)^{7/2} \exp\left[-\left(\sqrt{\chi - \frac{e\Delta\phi}{kT_i}} - \frac{c_s}{v_{Ti}}\right)^2\right] \tag{8}$$

b) <u>impurity ions</u>

The transport of impurity ions is treated within the drift-kinetic formalism of [1] . Inclusion of the nonlinear $(\vec{e}\,\partial f/\partial \vec{v})$ - term due to turbulent fluctuations in both the electric field $\vec{e}$ and the impurity ion distribution function f into the time-averaged drift-kinetic formalism leads under simplifying assumptions (see Appendix) to the equation

$$\frac{1}{2}\left(\frac{\partial}{\partial r}\bigg|_{\zeta, v_\parallel, v_\perp} + \frac{ZeE_\perp}{m_z}\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\bigg|_{\vec{x}, v_\parallel}\right)\left(D_\perp \frac{\partial}{\partial r}\bigg|_{\zeta, v_\parallel, v_\perp} - V_\perp\right)\left(v_\perp \frac{\partial \bar{F}}{\partial v_\perp}\bigg|_{\vec{x}, v_\parallel}\right) +$$

$$+ v_\parallel \frac{\partial \bar{F}}{\partial \zeta}\bigg|_{r, v_\parallel, v_\perp} + \frac{ZeE_\parallel}{m_z}\frac{\partial \bar{F}}{\partial v_\parallel}\bigg|_{\vec{x}, v_\perp} = Q(\vec{x}, v_\parallel, v_\perp) - \nu_z \bar{F} + \nu_c \frac{\partial_s \bar{F}}{\partial \tau}\bigg|_{\vec{x}} \tag{9}$$

for the guiding-centre distribution function $\overline{F}$ of impurity ions in charge state Z. $D_\perp \sim kT_e/eB$ and $V_\perp$ denote anomalous values of the cross diffusion coefficient and the perpendicular convection velocity of the impurity ions. $Q(\vec{x}, v_\parallel, v_\perp)$ is the gyro-averaged source function. For radially constant plasma parameters and constant impurity emission fluxes we obtain from eqs.(2) and (3)

$$Q(\vec{x}, v_\parallel, v_\perp)\Big|_{Z=1} = \begin{cases} \dfrac{\nu_o(\zeta)}{\pi}\, G\left[\vec{x}(0), \vec{v}\right] \dfrac{\varphi_o(\vec{x}, v_\parallel, v_\perp) - \varphi_\Delta(\vec{x}, v_\parallel, v_\perp)}{\exp\left(\displaystyle\int_0^\zeta \frac{\nu_o(\xi)}{v_\parallel}\, d\xi\right)} & \text{for } v_\parallel > 0 \\ 0 \quad \text{for } v_\parallel \leq 0 \end{cases} \tag{10a}$$

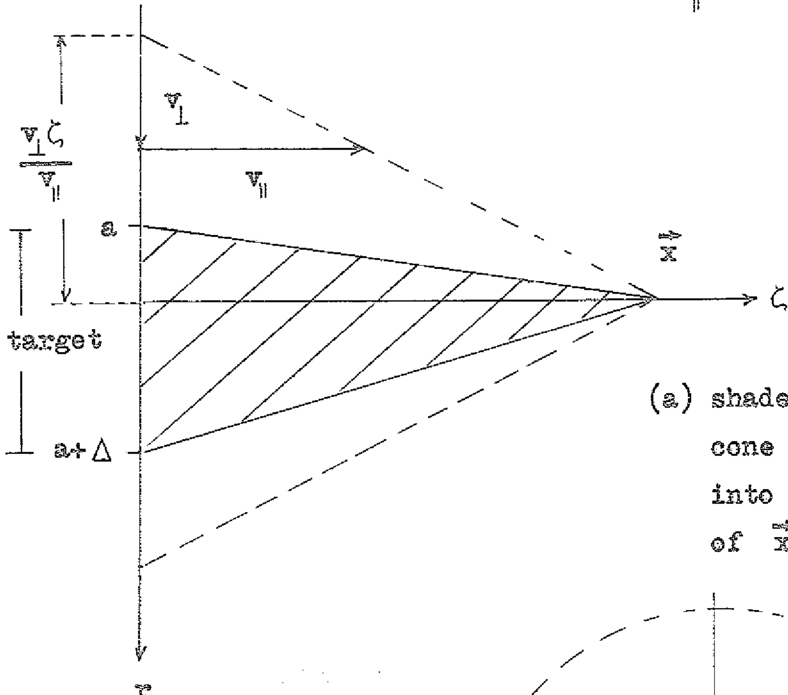
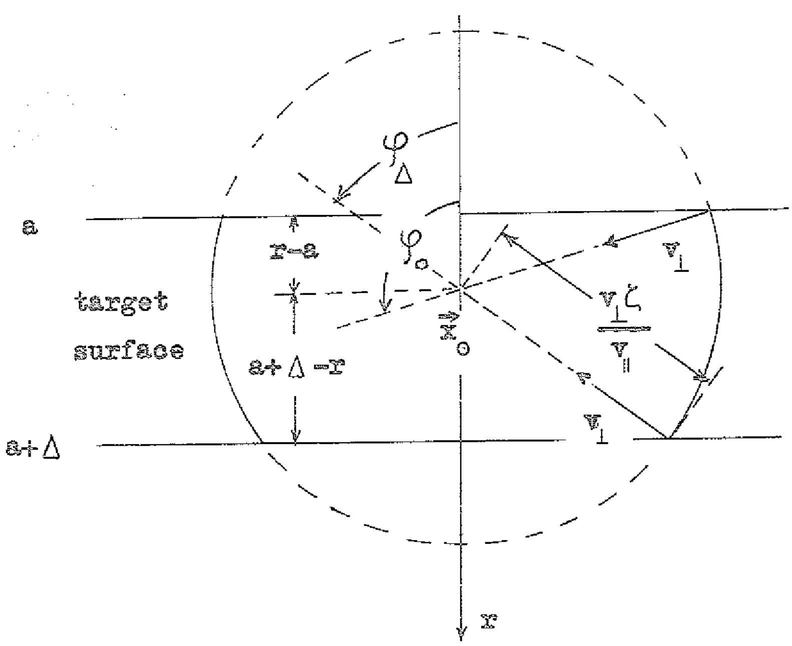


fig.2

(a) shaded area: projection of the orbit cone of the emitted impurity atoms into the r, ζ - plane for given values of $\vec{x}$ and $v_\parallel, v_\perp$



(b) basis of the orbit cone (circular line source) of the emitted impurity atoms in the target plane for given values of $\vec{x}$ and $v_\parallel, v_\perp$: definition of the angles φ_Δ and φ_o

The limits $\varphi_{\Delta,o}(\vec{x}, v_\parallel, v_\perp)$ account for the radial bounds $a \le r'(0) \le a+\Delta$ of the emitting target surface (see fig.2)

$$\begin{pmatrix} \varphi_\Delta(\vec{x}, v_\parallel, v_\perp) \\ \varphi_o(\vec{x}, v_\parallel, v_\perp) \end{pmatrix} = \arccos \begin{pmatrix} \mathrm{Min}\left(+1, \dfrac{v_\parallel}{v_\perp \zeta}(a + \Delta - r)\right) \\ \mathrm{Max}\left(-1, \dfrac{v_\parallel}{v_\perp \zeta}(a - r)\right) \end{pmatrix} \tag{10b}$$

Due to stepwise electron impact ionization of the impurity ions (with frequency ν_z) we further have

$$Q(\vec{x}, v_\parallel, v_\perp)\Big|_{Z\ge 2} = \nu_{z-1}(\zeta)\, \overline{F}(\vec{x}, v_\parallel, v_\perp)_{z-1} \tag{11}$$

Eqs.(10) and (11) presuppose that the translational state of an impurity particle does not change by electron impact ionization.

The Coulomb collisions with hydrogen ions are described by the Fokker-Planck collision operator $\nu_c\, \partial_s/\partial\tau|_{\vec{x}}$, where

$$\frac{\partial_s}{\partial\tau}\Big|_{\vec{x}} = \frac{\partial^2}{\partial w_\parallel^2} + (\alpha w_\parallel - \beta)\frac{\partial}{\partial w_\parallel} + \alpha + \frac{2\varepsilon}{w_\perp}\frac{\partial}{\partial w_\perp}\left(w_\perp \frac{\partial}{\partial w_\perp}\right) + \gamma w_\perp \frac{\partial}{\partial w_\perp} + 2\gamma \tag{12}$$

written in the normalized velocity components $w_{\parallel,\perp} = v_{\parallel,\perp}/v_{Tz}$ with $v_{Tz} = \sqrt{2kT_z/m_z}$. $\alpha, \ldots, \varepsilon$ are collision coefficients depending on the hydrogen ion distribution function as defined in reference [4] . In particular, we find for a shifted Maxwellian ion distribution function

$$\beta = 2\sqrt{\frac{m_z T_z}{m_i T_i}}\; U_{\parallel i} < 0 \tag{13}$$

where $U_{\parallel i}$ is the flow velocity of the hydrogen ions normalized to v_{Ti} .

The solution of eq.(9) poses severe difficulties because of the second derivative of $\overline{F}$ with respect to $v_\parallel$ (since the boundary condition $\overline{F}_{\zeta=0}$ is only defined for $v_\parallel \ge 0$) and the mixed derivatives in r and $v_\perp$. If, however, we ignore the thermalization between impurity and hydrogen ions (as was done in reference [1]) and confine the calculations to velocity moments of $\overline{F}$ with respect to $v_\parallel$ integrating eq.(9) over $\frac{v_\perp^2}{2}$, we obtain a form analoguous to eq.(18) from reference [1]

$$\frac{\partial}{\partial r}\left(D_\perp \left.\frac{\partial a_o}{\partial r}\right|_{v_\parallel,\zeta}\right) - V_\perp \left.\frac{\partial a_o}{\partial r}\right|_{v_\parallel,\zeta} - v_\parallel \left.\frac{\partial a_o}{\partial \zeta}\right|_{v_\parallel,r} + Q_o + \nu_c\left(\alpha v_\parallel - \beta_{eff} v_{Tz}\right)\left.\frac{\partial a_o}{\partial v_\parallel}\right|_{r,\zeta} - \nu_{eff} a_o = 0 \tag{14}$$

$$\nu_{eff} = \nu_z - \alpha\nu_c \quad , \quad \begin{pmatrix} a_o \\ Q_o \end{pmatrix} = \frac{m_z}{kT_R}\int_0^\infty dv_\perp v_\perp \begin{pmatrix} \overline{F} \\ Q \end{pmatrix} \quad , \quad T_R = \text{reference temperature}$$

$$\nu_c = Z^2 \nu_c\Big|_{Z=1} \qquad \beta_{eff} = \beta + \frac{ZeE_\parallel}{\nu_c m_z v_{Tz}} < 0$$

For radially constant plasma parameters

$$\frac{\partial}{\partial r}\left(D_\perp, V_\perp, \alpha\nu_c, \beta_{eff}\nu_c v_{Tz}, \nu_z\right) = 0$$

eq.(14) can be solved analytically. The solution reads

$$a_o\left(r,\zeta,v_\parallel \geq 0\right) = \tag{15a}$$

$$\frac{1}{2}\, a_o\left(0, v_\parallel^+(0)\right)\exp\left(-\int_0^\zeta \frac{\nu_{eff}(\xi)}{v_\parallel^+(\xi)}\,d\xi\right)\left[\operatorname{erf}\left(\frac{r-a-\int_0^\zeta \frac{V_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi}{2\sqrt{\int_0^\zeta \frac{D_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi}}\right) - \operatorname{erf}\left(\frac{r-a-\Delta-\int_0^\zeta \frac{V_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi}{2\sqrt{\int_0^\zeta \frac{D_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi}}\right)\right] +$$

$$+\frac{1}{\sqrt{\pi}}\int_0^\zeta \frac{d\tau}{v_\parallel^+(\tau)}\int_a^{a+\Delta} d\varrho\, \frac{Q_o\left[\varrho,\tau,v_\parallel^+(\tau)\right]}{2\sqrt{\int_\tau^\zeta \frac{D_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi}}\ \exp\left[-\frac{\left(r-\varrho-\int_\tau^\zeta \frac{V_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi\right)^2}{4\int_\tau^\zeta \frac{D_\perp(\xi)}{v_\parallel^+(\xi)}\,d\xi} - \int_\tau^\zeta \frac{\nu_{eff}(\xi)}{v_\parallel^+(\xi)}\,d\xi\right]$$

$$a_o\left(r,\zeta,v_\parallel<0\right) = \tag{15b}$$

$$\frac{1}{\sqrt{\pi}}\int_a^{a+\Delta} d\varrho \, \frac{a_o[\varrho,\zeta_T,0]}{2\sqrt{\int_{\zeta_T}^{\zeta}\frac{D_\perp(\xi)}{v_\parallel^-(\xi)}d\xi}} \exp\left[-\frac{\left(r-\varrho-\int_{\zeta_T}^{\zeta}\frac{V_\perp(\xi)}{v_\parallel^-(\xi)}d\xi\right)^2}{4\int_{\zeta_T}^{\zeta}\frac{D_\perp(\xi)}{v_\parallel^-(\xi)}d\xi} - \int_{\zeta_T}^{\zeta}\frac{\nu_{eff}(\xi)}{v_\parallel^-(\xi)}d\xi\right] +$$

$$+\frac{1}{\sqrt{\pi}}\int_{\zeta_T}^{\zeta}\frac{d\tau}{v_\parallel^-(\tau)}\int_a^{a+\Delta} d\varrho \, \frac{Q_o[\varrho,\tau,v_\parallel^-(\tau)]}{2\sqrt{\int_{\tau}^{\zeta}\frac{D_\perp(\xi)}{v_\parallel^-(\xi)}d\xi}} \exp\left[-\frac{\left(r-\varrho-\int_{\tau}^{\zeta}\frac{V_\perp(\xi)}{v_\parallel^-(\xi)}d\xi\right)^2}{4\int_{\tau}^{\zeta}\frac{D_\perp(\xi)}{v_\parallel^-(\xi)}d\xi} - \int_{\tau}^{\zeta}\frac{\nu_{eff}(\xi)}{v_\parallel^-(\xi)}d\xi\right]$$

In eq.(15) the contributions to a_o from outside the scrape-off layer $a \leq r \leq a+\Delta$ have been neglected assuming a drastic decrease in $Q_o^{\pm}$ +). The integrations are performed along the characteristic $v_\parallel^{\mp}(\xi)$, which is a solution of the differential equation

$$-\left.\frac{\partial v_\parallel^{\pm}(\xi)}{\partial\zeta}\right|_{v_\parallel,r} + \nu_c\left(\alpha - \frac{\beta_{eff}v_{Tz}}{v_\parallel}\right)\left.\frac{\partial v_\parallel^{\pm}(\xi)}{\partial v_\parallel}\right|_{\zeta,r} = 0 \tag{16}$$

$v_\parallel^{\pm}(\xi)$ corresponds, respectively, to positive and negative values of the velocity $v_\parallel$ at the point $\vec{x} = (\zeta,r)$ under consideration. It is clear that for $\beta_{eff} \geq 0$ the branch $v_\parallel^-(\xi)$ does not exist. Unfortunately, it is only for special space dependencies of the collision parameters that eq.(16) can be solved analytically. Special solutions have been given in reference [1] for a comparable plasma model.

It is easily verified that the solution eq.(15) is symmetrical with respect to $r = a+\Delta/2$, if $V_\perp = 0$ or $V_\perp(a+\Delta/2+\delta r) = -V_\perp(a+\Delta/2-\delta r)$ for an arbitrary radial displacement δr. In the limit $D_\perp, V_\perp \to 0$ we recover the solution of reference [1], which is valid for arbitrary radial dependencies of the plasma parameters.

+) see Addendum on p.16

Discussion of the analytical result

The interpretation of eq.(15) is as follows:

Impurity ions appearing at the point $\vec{x} = (r, \zeta)$ with a specific parallel velocity $v_\parallel$ and a specific charge state Z originate from different source points at the target and inside the scrape-off plasma. The impurity ion source $a_o(0, v_\parallel^+(0))$ at the target surface generates particles with velocities $v_\parallel \geq 0$ at $\vec{x}$. It vanishes in case of complete adsorption/neutralization of the incident impurity ions. The impurity ion sources inside the s.o.l. are either volume sources $Q_o\left[\varrho, \tau, v_\parallel^{\pm}(\tau)\right]$ or virtual surface sources $a_o\left[\varrho, \zeta_T, 0\right]$. The latter are introduced by those impurity ions, which after being born with positive velocities come to rest (at the turning point ζ_T) due to the retarding force of collisional friction with the hydrogen ions and due to the presheath electric field (if directed towards the target). These impurity ions, which are swept back to the target, emerge at $\vec{x}$ with velocities $v_\parallel < 0$.

According to our model assumptions the impurity ions are driven from the source points $\vec{x}_S = (\varrho, \zeta_S)$ to the point $\vec{x}$ under consideration by both anomalous diffusion/convection perpendicular to the m.f. and collisional friction/electrostatic acceleration (deceleration) along the m.f. lines. On its transit from a source point $\vec{x}_S$ to $\vec{x}$ an impurity ion beam is exponentially attenuated by Coulomb scattering at the hydrogen ions ($\propto \nu_c$) and electron impact ionization (ν_z) to the next higher charge state.

The probability, with which diffusion and convection from the various source points $\vec{x}_S$ contribute to the impurity ion accumulation at $\vec{x}$ is determined by the ratio $(r - \varrho - l_c^{\pm})/\, l_d^{\pm}$, where

$$l_d^{\pm} = 2\sqrt{\int_{\zeta_S}^{\zeta} \frac{D_\perp(\xi)}{v_\parallel^{\pm}(\xi)}\, d\xi} \qquad \text{and} \qquad l_c^{\pm} = \int_{\zeta_S}^{\zeta} \frac{V_\perp(\xi)}{v_\parallel^{\pm}(\xi)}\, d\xi \tag{16}$$

are characteristic diffusion and convection lengths. The latter depend on the distance $|\zeta - \zeta_S|$ (the source point coordinate ζ_S is either $0, \zeta_T$ for the surface sources or τ for the volume sources), on the transit time intervals $d\xi / v_\parallel^{\pm}(\xi)$ along the characteristic orbits (being a function of the force $m_z v_{Tz} \beta_{eff} \nu_c$ along the m.f.) and, of course, on the diffusion and convection parameters $D_\perp(\xi)$ and $V_\perp(\xi)$. It appears from eq.(15) that the probability mentioned above is a Gaussian function of $(r - \varrho - l_c^{\pm})/ l_d^{\pm}$.

In a similar way the probability, with which an impurity ion beam is attenuated due to Coulomb scattering at the hydrogen ions and electron impact ionization on its transit from the source point $\vec{x}_S$ to $\vec{x}$ is determined by the integral

$$\int_{\zeta_S}^{\zeta} \frac{\nu_{eff}(\xi)}{v_\parallel^{\pm}(\xi)} d\xi \tag{17}$$

Conclusions

In the present report an exact analytical solution has been derived for the problem of impurity transport in the vicinity of a limiter or divertor target sputtered by hydrogen ion bombardment, accounting for driving forces both parallel and perpendicular to the m.f. lines. In a following report the results of a numerical evaluation of the analytical solution will be presented. These results may serve as reference values for more complicated numerical calculations in the case of more general plasma parameter distributions.

Acknowledgements

The authors thank Dr. A. Rogister for helpful remarks concerning the drift-kinetic formalism for turbulent diffusion and convection processes. They also acknowledge useful discussions with Prof. G.Ecker.

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Appendix

A simple extension of the drift-kinetic formalism introduced in reference [5] to include fluctuations leads to the following coupled set of equations (Ω = ZeB/m = gyrofrequency , *= collision partner)

$$\bar{L}\,\bar{F} + \overline{\tilde{L}\,\tilde{F}} = \sum_{*} \bar{C}(F,F^*) - \frac{Ze}{m}\overline{\left\langle \vec{e}\,\frac{\partial f}{\partial \vec{v}} \right\rangle} \quad , \quad F + f = \text{total distribution function} \qquad (A1)$$

$$\tilde{F} = \frac{1}{\Omega}\int^{\varphi}\left(\tilde{L}\,\bar{F} + \bar{L}\,\tilde{F} + (\tilde{L}\,\tilde{F} - \overline{\tilde{L}\,\tilde{F}}) - \sum_{*}\tilde{C}(F,F^*) + \frac{Ze}{m}\widetilde{\left\langle \vec{e}\,\frac{\partial f}{\partial \vec{v}} \right\rangle}\right) d\varphi' \qquad (A2)$$

for the guiding-centre (φ- averaged) part $\bar{F}$ and the gyro(φ-) dependent part $\tilde{F}$ of the fluctuation time-averaged distribution function F . In this formalism the influence of fluctuations on the charged particle transport is linked to the nonlinear fluctuation time-averaged part $\langle \vec{e}\partial f/\partial \vec{v}\rangle$ of the kinetic equation, where $\vec{e}$, f are the fluctuating parts of the electric field and the distribution function. By definition of the gyro-averaging process, we have for any quantity A the splitting

$$A = \bar{A} + \tilde{A} \quad \text{with} \quad \bar{A} = \frac{1}{2\pi}\int_0^{2\pi} A\, d\varphi \quad \text{implying that} \quad \bar{\tilde{A}} = \tilde{\bar{A}} = 0 \qquad (A3)$$

The operators in eqs.(A1) and (A2) are usually given in a coordinate system $\vec{x}$, $\epsilon = v^2/2 + Ze\phi$, $\mu = v_\perp^2/2B$, φ (ϕ = electrostatic potential, B = m.f. strength). Transforming to a coordinate system $\vec{x}$, $v_\parallel$, $v_\perp$, φ ($v_{\parallel,\perp}$ = velocity components parallel and perpendicular to the m.f. respectively, φ = gyro-angle) we find for time-independent electromagnetic fields (stationary on a time-scale large compared to the fluctuation time)

$$\bar{L} = v_\parallel n_j \left(\left.\frac{\partial}{\partial x_j}\right|_{v_\parallel, v_\perp, \varphi} - \left.\frac{Ze}{m}\frac{\partial \phi}{\partial x_j}\frac{1}{v_\parallel}\frac{\partial}{\partial v_\parallel}\right|_{\vec{x}, v_\perp, \varphi} \right) \qquad (A4)$$

$$\tilde{L} = v_\perp s_j^o \left(\frac{\partial}{\partial x_j}\bigg|_{v_\parallel, v_\perp, \varphi} - \frac{Ze}{m}\frac{\partial\phi}{\partial x_j}\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\bigg|_{\vec{x}, v_\parallel, \varphi}\right) - \tag{A5}$$

$$- v_\parallel v_\perp (v_\parallel n_j + v_\perp s_j^o)\, s_k^o \frac{\partial n_k}{\partial x_j}\left(\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\bigg|_{\vec{x}, v_\parallel,\varphi} - \frac{1}{v_\parallel}\frac{\partial}{\partial v_\parallel}\bigg|_{\vec{x}, v_\perp, \varphi}\right) -$$

$$- \frac{\varepsilon_{krs} n_r s_s^o}{v_\perp}\left((v_\parallel n_j + v_\perp s_j^o)\left(v_\parallel \frac{\partial n_k}{\partial x_j} + v_\perp \frac{\partial s_k^o}{\partial x_j}\bigg|_\varphi\right) + \frac{Ze}{m}\frac{\partial \phi}{\partial x_k}\right)\frac{\partial}{\partial\varphi}\bigg|_{\vec{x}, v_\parallel, v_\perp}$$

C denotes the collision operator with the properties

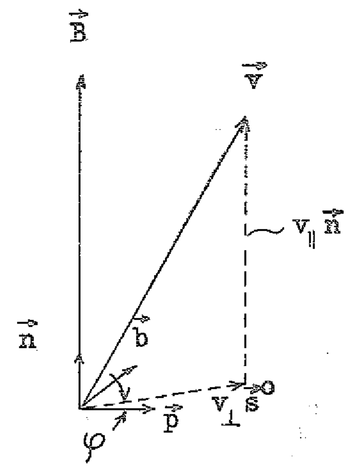
$$\tilde{C}(\overline{F},\overline{F}^*) = 0 \quad \text{and} \quad \overline{C}(\overline{F},\tilde{F}^*) + \overline{C}(\tilde{F},\overline{F}^*) = 0 \tag{A6}$$

In eqs.(A4) and (A5) tensor notations are used with δ_{ij} denoting the Kronecker symbol and ε_{ijk} being a completely antisymmetric tensor with components $0,+1,-1$ depending on whether two or three of the indices i,j,k are equal (0) or i,j,k form an even (+1) or an odd (−1) permutation of 1,2,3. According to fig.1A the velocity vector is written in the form

$$v_i = v_\parallel n_i + v_\perp s_i^o \quad \text{with} \quad s_i^o = p_i \cos\varphi + b_i \sin\varphi \tag{A7}$$

related to a right handed triple of unit vectors $n_i = B_i/B$, p_i and b_i tied to the m.f. lines.

fig.1A



Following reference [5] the drift-kinetic equation (A1) can be solved recursively, if $\tilde{F}$ is subject to an iteration of eq.(A2) assuming that the gyrofrequency is much higher than both the collision frequencies and the inverse convection time. To zero order drift-approximation one starts with $\tilde{F} = 0$. To first order one gets

$$\tilde{F} = \frac{1}{\Omega} \int \left(\tilde{L}\,\overline{F} + \frac{Ze}{m} \widetilde{\left\langle \vec{e}\,\frac{\partial f}{\partial \vec{v}} \right\rangle} \right) d\varphi \tag{A8}$$

where

$$\widetilde{\left\langle \vec{e}\,\frac{\partial f}{\partial \vec{v}} \right\rangle} = \left\langle e_j^{\perp} \left(s_j^{o} \left.\frac{\partial f}{\partial v_{\perp}}\right|_{\vec{x},v_{\parallel},\varphi} + \varepsilon_{jrs} n_r s_s^{o} \frac{1}{v_{\perp}} \left.\frac{\partial \tilde{f}}{\partial \varphi}\right|_{\vec{x},v_{\parallel},v_{\perp}} \right) \right\rangle \tag{A9}$$

assuming that the fluctuating part of the electric field $\vec{e}$ is directed perpendicular to the m.f.. If one further assumes that the wavelengths of the fluctuation spectrum are large compared to the Larmor radius, one has $f \cong \overline{f}$. In this drift-approximation of f one may neglect both the last term of eq.(A1) and the second term of eq.(A9). The anomalous part of $\tilde{F}$ then becomes

$$\tilde{F} = (p_j \sin\varphi - b_j \cos\varphi) \left\langle \frac{e_j^{\perp}}{B} \left.\frac{\partial \overline{f}}{\partial v_{\perp}}\right|_{\vec{x},v_{\parallel}} \right\rangle \tag{A1o}$$

Substitution of the first part of eq.(A8) into the term $\overline{\tilde{L}\,\tilde{F}}$ of eq.(A1) gives the usual neoclassical drift contribution to the charged particle transport perpendicular to the m.f.. By the same procedure the second part of eq.(A8), which leads to eq.(A1o), yields the anomalous transport perpendicular to the m.f.. In the s.o.l. of a tokamak plasma the anomalous contribution is expected to dominate the neoclassical one, which therefore is omitted from further consideration.

After some lengthy calculations, eqs.(A5) and (A1o) give the formula

$$\overline{\tilde{L}\,\tilde{F}} = \frac{\varepsilon_{jrl} n_r}{2} \left(\left.\frac{\partial}{\partial x_j}\right|_{v_{\parallel},v_{\perp}} - \left(\frac{Ze}{m}\frac{\partial \phi}{\partial x_j} + v_{\parallel}^2 n_k \frac{\partial n_j}{\partial x_k} \right) \frac{1}{v_{\perp}} \left.\frac{\partial}{\partial v_{\perp}}\right|_{\vec{x},v_{\parallel}} + \right. \tag{A11}$$

$$\left. + n_k \frac{\partial n_j}{\partial x_k} v_{\parallel} \left.\frac{\partial}{\partial v_{\parallel}}\right|_{\vec{x},v_{\perp}} \right) \frac{v_{\perp}}{B} \left\langle e_l^{\perp} \left.\frac{\partial \overline{f}}{\partial v_{\perp}}\right|_{\vec{x},v_{\parallel}} \right\rangle$$

On passing we note that

$$2\pi \int_{-\infty}^{+\infty} dv_{\parallel} \int_{0}^{+\infty} dv_{\perp} v_{\perp} \, \overline{\tilde{L}\,\tilde{F}} = \frac{\partial \Gamma^{a}_{j}}{\partial x_{j}} \tag{A12}$$

with the anomalous particle flux

$$\Gamma^{a}_{j} = \varepsilon_{jkl} \left\langle \frac{e^{\perp}_{k}}{B} \, 2\pi \int_{-\infty}^{+\infty} dv_{\parallel} \int_{0}^{+\infty} dv_{\perp} v_{\perp} \overline{f} \right\rangle n_{l} \tag{A13}$$

related to the fluctuating particle density and the particle drift in the fluctuating electric field. Of course, eq.(A11) does not say anything about the correlation between $e^{\perp}_{j}$, f and the time-averaged plasma parameters, their gradients and $\overline{F}$. Since a theory relating these quantities for the s.o.l. of a tokamak plasma under consideration is still missing, further assumptions relating these quantities have to be made. In the following we assume a relation of the form

$$\left\langle \frac{e^{\perp}_{l}}{B} \overline{f} \right\rangle = \varepsilon_{lmn} \left(D_{\perp} \frac{\partial \overline{F}}{\partial x_{m}} - V_{m} \overline{F} \right) n_{n} \quad , \; D_{\perp} \sim kT_{e}/eB \tag{A14}$$

which yields

$$\Gamma^{a}_{j} = - D_{\perp} \frac{\partial N}{\partial x^{\perp}_{j}} + N \, V^{\perp}_{j} \quad \text{with} \quad \begin{pmatrix} \dfrac{\partial}{\partial x^{\perp}_{j}} \\ V^{\perp}_{j} \end{pmatrix} = (\delta_{jk} - n_{j} n_{k}) \begin{pmatrix} \dfrac{\partial}{\partial x_{k}} \\ V_{k} \end{pmatrix} \tag{A15}$$

simulating a Bohm-like diffusion and a simple convection law. Ignoring the curvature of the m.f. lines

$$n_{k} \frac{\partial n_{j}}{\partial x_{k}} = 0$$

we finally obtain eq.(9).

Addendum

In eq.(15) the contributions to a_o from outside the scrape-off layer $a \le r \le a+\Delta$ have been neglected assuming a drastic decrease in Q_o for the lower charge states under consideration.

Due to the strongly reduced value of β_{eff} and the elevated densities and temperatures of electrons and hydrogen ions, the edge of the plasma core acts as a source of highly ionized impurities, which partly diffuse back into the s.o.l.. This effect can be incorporated in the present theory by adding in eq.(15) a surface source term, which describes the radial influx $\Gamma_\perp(a,\tau,v_\parallel^{\pm}(\tau))$ of multiply ionized impurities across the separatrix at $r=a$:

$$\frac{1}{\sqrt{\pi}}\int_{\substack{0\\ \zeta_T}}^{\zeta}\frac{d\tau}{v_\parallel^{\pm}(\tau)}\;\frac{\Gamma_\perp\left(a,\tau,v_\parallel^{\pm}(\tau)\right)}{2\sqrt{\displaystyle\int_\tau^\zeta\frac{D_\perp(\xi)}{v_\parallel^{\pm}(\xi)}d\xi}}\exp\left(-\frac{\left(r-a-\displaystyle\int_\tau^\zeta\frac{V_\perp(\xi)}{v_\parallel^{\pm}(\xi)}d\xi\right)^2}{4\displaystyle\int_\tau^\zeta\frac{D_\perp(\xi)}{v_\parallel^{\pm}(\xi)}d\xi}-\int_\tau^\zeta\frac{\nu_{eff}(\xi)}{v_\parallel^{\pm}(\xi)}d\xi\right)$$

corresponding to the solution for $v_\parallel \ge 0$ resp. $v_\parallel < 0$ at $\vec{x} = (r,\zeta)$.

The above formula holds quite generally and replaces the integral over the volume sources within the plasma core by a surface integral over the particle flux $\Gamma_\perp = -D_\perp \partial a_o/\partial r + V_\perp a_o$ across the bounding surface (in our case we have $\Gamma_\perp|_{r=0} = 0$ from symmetry reasons). In the wall region $r = a+\Delta$ we assume $Q_o = 0$ for all ionization states, which automatically results in nullifying the surface integral over the impurity ion flux at $r = a+\Delta$.