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Jül-2016
August 1985
ISSN 0366-0685
Als Manuskript gedruckt

Institut für Plasmaphysik, Assoziation EURATOM-KFA-Jül-2016

Zu beziehen durch: ZENTRALBIBLIOTHEK der Kernforschungsanlage Jülich GmbH
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PROFILE PARAMETERS ON TEXTOR

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Abstract

The shapes of electron and current density profiles are monitored in real-time mode in order to get rapid qualitative information on the development of a TEXTOR tokamak plasma.

The profiles are described by form parameters which relate to the signals of a 9-channel FIR-polari/interferometer in simple mathematical formulae.

These profile parameters are obtained by real-time conversion of measured quantities for display on a storage oscilloscope or on a chart recorder.

The application of the parameters is demonstrated in some examples.

Introduction

The 9-channel FIR-polari/interferometer on TEXTOR /1,2/ has been designed to qualify the profiles of the occurring electron and toroidal current densities. For practical operation of a tokamak device, it is often helpful to have real-time information on the time development of these profiles in order to quickly interpret the influence of gas puffs, additional heating, current ramps, etc. This leads to the task to describe the profile shape by means of a single parameter that can be read from a chart recorder while the experiment is running.
The description of a density profile by only one form parameter and one scaling quantity is a rather coarse one. Nevertheless, it is suitable to reveal tendencies in the profile development during the experiment.

The plasma model

The description of a density profile shape by a single parameter is not very accurate so that a simple model of the tokamak plasma should be quite sufficient (fig. 1).

Because of the low $B_p$ value in TEXTOR (0,2...0,3 without add. heating) concentrical magnetic flux surfaces are assumed. In consequence, the poloidal field $B_p$ has the form $\frac{A}{R} f(r)$ ($R$ = major radius, $r$ = minor radius) and the toroidal current density $j_t$, which is $\frac{A}{\mu_0} (\nabla \times \vec{B}) = \frac{4}{\pi} \frac{1}{r} \frac{d}{dr} (r B_p)$, is expressed by $\frac{1}{R} \left[ \frac{L}{r} \right] - \frac{R}{R_0} \left| \nabla \right| \zeta (r) \right] \quad (R_o = R$ of the magn. axis).

The flux surfaces are also surfaces of constant pressure and - because of the high thermal conductivity along the field lines - of approximately constant temperatures. This leads to the assumption that the electron density $n_e$ is a function only of the minor radius ($n_e = f(r)$). Typical curves of $n_e$ and $j_t$ vs. $R$ are shown in fig. 1.

The measured quantities.

The profile information is drawn from the signals of a 9-channel polari/interferometer employing a HCN laser at $\lambda = 337\ \mu m$ (fig. 2). The beam diameters in the torus midplane are about 1.5 cm, the time resolution is a few msec (for further information, see /1,2/). The instrument puts out time traces of 9 phase shift values and 9 Faraday-rotation angles which relate to the searched quantities $n_e$ and $j_t = \frac{A}{\mu_0} \nabla \times \vec{B}_p$ according to

$$\Delta \varphi_i = \zeta_i \int_{b_{\text{min}}}^{b_{\text{max}}} n_e \, d \zeta$$

and

$$\Delta \zeta_i = \zeta_i \int_{b_{\text{min}}}^{b_{\text{max}}} n_e B_{p,z} \, d \zeta$$

(0) marks the reference beam.
Usually it is necessary to Abel-invert the set of $\Delta \varphi_i$ and $\phi_i$ values, resp., that corresponds to a given time in the experiment in order to get the $n_e$- and $j_t$-profiles. Since this cannot be executed on a fast time scale, simple mathematical formulae have to be found which relate the profile parameters immediately to the measured quantities.

However, not all measured quantities are sensitive to $n_e$- or $j_t$-profile changes. The first task is to find simple combinations of outputs signals which can serve as a sensitive measure of the respective profile shape.

In the $n_e$ case, there is a close relation to the phase shift profile (fig. 3). Since $\Delta \varphi \sim \int n_e \text{dz}$, the $\Delta \varphi$-profile width will increase with the width of the $n_e$-profile. The $\Delta \varphi$-profile shape, in turn, can simply be characterized by the maximum value $\Delta \varphi_0$ divided by the area $F$ below the curve which is approximately given by

$$F = \sum_{i=0}^{g} \frac{\Delta \varphi_{i+} + \Delta \varphi_{i-}}{2} |x_{i+} - x_{i-}|$$

($x_i =$ distance of beam $i$ from the plasma center; $x_0 = x_1$, $x_{10} = -48$ cm, $\Delta \varphi_0 = \Delta \varphi_{10} = 0$). So, $\Delta \varphi_0/F$ is sensitive to the $n_e$-profile shape. Since $\Delta \varphi_{i+}$ will undergo only a little change at small horizontal plasma displacements, $\Delta \varphi_{i+}/F$ has the advantage to be fairly insensitive to them.

For the $j_t$ case, consider an inner segment of the torus cross section encircled by a magnetic flux surface (fig. 4). The included part $I_{t, \text{incl.}}$ of the total toroidal current $I_t$ then roughly characterizes the shape of the $j_t$-profile.

Since $I_{t, \text{incl.}}$ relates to the poloidal field $B_p$ on the mentioned flux surface according to $I_{t, \text{incl.}} \propto \oint B_p ds$, a large $B_p/I_t$ value would indicate a narrow $j_t$-profile. The $B_p$ value, in turn, can be obtained from the Faraday-rotation angles $\phi_4$ and $\phi_8$ which are sensitive to $B_p$ on the given flux surface.

Since the rotation angles scale with the electron density, they are divided by the respective phase shift values. Of course, this division does not eliminate the influence of the $n_e$-profile on the $\phi$-values, but model calculations show that occurring $n_e$-profile variations scarcely affect the $\phi_i/\Delta \varphi_i$-values.
So, quantities predominantly sensitive to $j_\ell$-profile changes are given by

$$Q_4 = \frac{\lambda_4}{I_\ell \cdot \Delta \rho_4^2} \quad \text{and} \quad Q_8 = \frac{\lambda_8}{I_\ell \cdot \Delta \rho_8^2}. $$

Because of the nearly symmetric position of the beams 4 and 8 with respect to the plasma center, $|Q_4|$ and $|Q_8|$ are of the same magnitude. A horizontal plasma displacement would increase one of the quantities and decrease the other by nearly the same amount. So, a quantity $|Q_4| + |Q_8|$ is rather insensitive to the horizontal plasma position. The toroidal geometry ($B_\phi \sim 1/R$) can be taken into account by forming $R_4|Q_4| + R_8|Q_8|.$

Definition of the profile parameters.

A suitable parameter containing basic information on the $n_e$-profile shape is obtained in the following way (fig. 5):

Starting from different profiles obtained by Abel inversion /3/ (full lines), a simple mathematical ansatz consistent with the above-mentioned plasma model is made which contains a scaling quantity ($n_{e0}$) and a shape-dependent parameter ($u$): $n_e = n_{e0} \left( 1 - \left( \frac{r}{a} \right)^u \right).$

Then one tries to match this ansatz to the different profiles by varying the two free parameters (dashed lines). If the matching works sufficiently in all cases - as it does in fig. 5 - , the form parameter should be adequate to characterize the original profiles.

The same concept holds for the $j_\ell$-profile parameter (fig. 6). In this case the ansatz is somewhat complicated, since the toroidal geometry has to be taken into account. Considering the large aspect ratio limit ($R/a \to \infty$), the ansatz simplifies to

$$j_\ell = j_{\ell 0} \left( 1 - \frac{r^2}{a^2} \right)^w. $$

Since $\mu_0 j_\ell = \frac{I}{r} \frac{\partial}{\partial r} (r B_\phi)$

in our model,

$$B_\phi = B_{\phi 0} \frac{a}{r} \left[ 1 - \left( 1 - \frac{r^2}{a^2} \right)^{w+1} \right].$$
In the general case, the plasma model requires $B_p = \frac{4}{R} f(\tau)$. So we propose the general ansatz

$$B_p = \frac{R_o}{R} B_{p0} \frac{\alpha}{\tau} \left[ 1 - \left( 1 - \frac{\tau^2}{\alpha^2} \right)^{w+1} \right],$$

which in turn yields

$$\dot{\tau} = \frac{R_o}{R} \left[ \left( 1 - \frac{\tau^2}{\alpha^2} \right)^w - \frac{R - R_o}{R} E(\tau, w) \right],$$

where

$$E(\tau, w) = \frac{1}{2w+2} \frac{\alpha^2}{\pi^2} \left[ 1 - \left( 1 - \frac{\tau^2}{\alpha^2} \right)^{w+1} \right] \in [0, \frac{1}{2}].$$

The $n_e$- profile parameter expression.

We now calculate the mathematical relation between the profile parameter and the experimental quantity $\Delta \varphi_\phi/F$.

This is done by inserting our ansatz $n_e = n_{eo} \left[ 1 - \left( \frac{\tau}{\alpha} \right)^u \right]$ into the well-known relations

$$\Delta \varphi_\phi = C_\phi \int_{\alpha}^{\infty} n_e \, d\tau$$

and

$$F = \int_{-\alpha}^{\alpha} \Delta \varphi_\phi(x) \, d(x),$$

where $\Delta \varphi_\phi(x) = C_{\phi} \int_{-\alpha}^{\alpha} n_e \, d\tau$ and $x = R - R_o$.

Since beam 6 hits the plasma center, and because of the radial symmetry,

$$\Delta \varphi_\phi = 2 C_\phi \int_{0}^{a} n_{eo}(\tau) \, d\tau$$

and

$$F = C_\phi \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} n_e \, d\tau \cdot d\tau = C_\phi \int_{\text{cross section}} \int_{\text{cross section}} n_e \cdot dA =$$

$$= C_\phi \int_{0}^{a} n_{eo}(\tau) \cdot 2\pi r \, d\tau \quad (A = \text{torus cross section}).$$

Thus,

$$\Delta \varphi_\phi = 2 C_\phi n_{eo} a \int_{0}^{a} \left[ 1 - \left( \frac{\tau}{\alpha} \right)^u \right] \, d\tau = 2 C_\phi n_{eo} a \int_{0}^{a} \left[ 1 - \left( \frac{\tau}{\alpha} \right)^u \right] \, d\left( \frac{\tau}{\alpha} \right)$$

$$= 2 C_\phi n_{eo} a \frac{u}{u + 1}.$$
and
\[ F = 2\pi C_4 n_{e_o} \int_0^a \left[ 1 - \left( \frac{r}{a} \right)^n \right] r \, dr = 2\pi C_4 n_{e_o} a^2 \int_0^1 \left[ 1 - \left( \frac{r}{a} \right)^{u+1} \right] r \left( \frac{r}{a} \right) \]
\[ = 2\pi C_4 n_{e_o} a \frac{1}{2^u} \frac{u+2}{u+1}. \]

So,
\[ \frac{\Delta \varphi_6}{F} = \frac{2}{\pi \alpha} \frac{u+2}{u+1}. \]

Solving this equation for \( u \) yields
\[ u = \frac{\frac{4}{\pi \alpha} - \frac{\Delta \varphi_6}{F}}{\frac{2}{\pi \alpha} \frac{1}{u+1}}. \]

It must be noted that \( F \) is not a directly measured quantity. We approximate \( F \) by
\[ \sum := \sum_{i=0}^{j-1} \frac{\Delta \varphi_{i+1} + \Delta \varphi_i}{2} \left| X_{i+1} - X_i \right|. \]

(\( x_0 = x_4, x_4 = -48 \text{ cm}, \Delta \varphi_0 = \Delta \varphi_4 = 0 \), see before) and use
\[ u \approx \frac{\frac{4}{\pi \alpha} \sum - \Delta \varphi_6}{\Delta \varphi_6 - \frac{2}{\pi \alpha} \sum}. \]

A computer simulation shows the validity of this expression (fig. 7). In addition, fig. 7 demonstrates the influence of horizontal plasma displacements. For customary values of \( u \) (\( 1 < u < 3 \)), the effect of small plasma shifts on the approximative \( u \) value can be neglected.

The \( J_+ \)-profile parameter expression.

The calculation of the relation between the parameter \( w \) and the experimental data
\[ R_{4l} \left| Q_4 \right| + R_{8l} \left| Q_8 \right| = R_4 \frac{\Delta q_4}{I_4} - R_8 \frac{\Delta q_8}{I_8} \]
is executed in a similar way.

We start with the ansatz
\[ B_p = B_{p_0} \frac{R_0}{R} \frac{c_l}{\tau} \left[ 1 - \left( 1 - \left( \frac{r}{a} \right)^2 \right)^{u+1} \right] \]
for the poloidal field and assume, for simplicity, \( \kappa_e = \kappa_{e_0} \left[ 1 - \left( \frac{r}{a} \right)^2 \right] \).
This is possible because of the nearly vanishing influence of the \( n_e \)-profile shape on \( Q_4 \) and \( Q_8 \) (see fig. 8).
It is known that
\[ d_i = C_\odot \sum_{k} \alpha_{c} B_{r,k} \, dz = \sum_{k} \alpha_{c} B_{r,k} \frac{x}{r} \, dz, \]
\[ \Delta \varphi_i = C_\odot \sum \alpha_{c} \, dz, \]
and
\[ I_t = \frac{1}{\mu_0} \oint_{\text{plasma}} \left. B_p \, d\mathbf{s} \right|_{r=a} = \frac{2\pi}{\mu_0} \int_{\pi/2}^{\pi} B_p \left|_{r=a} \right. \, \alpha \, d\theta, \]
where \( \theta \) is the poloidal angle.

At first we calculate
\[ \Delta \varphi_i = C_\odot \sum \alpha_{c} \int_{-\sqrt{\alpha^2 - x_i^2}}^{+\sqrt{\alpha^2 - x_i^2}} d\zeta = 2 C_\odot \alpha_{c} \sum \int_{0}^{\sqrt{1 - (\frac{\zeta}{a})^2}} \left[ 1 - (\frac{\zeta}{a})^2 \right] d\zeta.
\]

With the substitution \( \tau' = \tau/a \), \( x' = x/a \), \( z' = z/a \) we obtain
\[ \Delta \varphi_i = 2 C_\odot \alpha_{c} \int_{0}^{\sqrt{1 - x_i'/x_i^2}} \left[ 1 - (\tau'/x_i')^2 \right] d\tau' = 2 C_\odot \alpha_{c} \int_{0}^{\sqrt{1 - x_i'/x_i^2}} \left[ 1 - (\tau'/x_i')^2 \right] d\tau' = 2 C_\odot \alpha_{c} \int_{0}^{\sqrt{1 - x_i'/x_i^2}} \left[ 1 - (\tau'/x_i')^2 \right] d\tau'.
\]

The relation \( \tau'^2 + x'^2 = \frac{1}{\sqrt{1 - x_i'^2}} \) yields
\[ \tau' \, d\tau' = x' \, dx' + z' \, dz'.
\]

\[ \left. \frac{d\zeta}{\sqrt{1 - x_i'^2}} \right|_{x_i'} = \left. \frac{d\tau'}{\sqrt{1 - x_i'^2}} \right|_{x_i'}, \text{ and so}
\]
\[ \Delta \varphi_i = 2 C_\odot \alpha_{c} \int_{0}^{\sqrt{1 - x_i'/x_i^2}} \left[ 1 - (\tau'/x_i')^2 \right] d\tau' = 2 C_\odot \alpha_{c} \int_{0}^{\sqrt{1 - x_i'/x_i^2}} \left[ 1 - (\tau'/x_i')^2 \right] d\tau'.
\]

We then calculate
\[ I_t = \frac{1}{\mu_0} \oint_{\text{plasma}} B_p \left|_{r=a} \right. \, \alpha \, d\theta = \frac{2\pi}{\mu_0} B_{po} \gamma_{po} \pi \int_{0}^{\pi} \frac{R_o}{R_o + \xi} \, d\theta = \frac{2\pi}{\mu_0} B_{po} \gamma_{po} \pi \int_{0}^{\pi} \frac{R_o}{R_o + \xi} \, d\theta = \frac{2\pi}{\mu_0} B_{po} \gamma_{po} \pi \int_{0}^{\pi} \frac{R_o}{R_o + \xi} \, d\theta.
\]

The most difficult term is
\[ d_i = C_\odot \sum \alpha_{c} B_{r,k} \frac{x_i}{r} \, dz.
\]

In analogy to the \( \Delta \varphi_i \) calculation, we obtain
\[ d_i = 2 C_\odot \alpha_{c} B_{po} \gamma_{po} \alpha \int_{x_i}^{R_o} \frac{R_o}{R_o + \xi} \frac{1}{\frac{1}{x_i} \left[ 1 - (\frac{x_i}{x_i})^{w+1} \right] + \frac{1}{x_i} \left[ 1 + (\frac{x_i}{x_i})^{w+1} \right]} \, d\xi = 2 C_\odot \alpha_{c} B_{po} \gamma_{po} \alpha \int_{x_i}^{R_o} \frac{R_o}{R_o + \xi} \frac{1}{\frac{1}{x_i} \left[ 1 - (\frac{x_i}{x_i})^{w+1} \right] + \frac{1}{x_i} \left[ 1 + (\frac{x_i}{x_i})^{w+1} \right]} \, d\xi.
\]

We now restrict our calculations on integer \( w \) values in order to continue an analytical processing: In this case,
\[ \left[ 1 - (\frac{x}{x_i})^{w+1} \right] = \frac{x_i}{x_i} \left[ 1 - (\frac{x}{x_i})^{w+1} \right] = \frac{x}{x_i} \left[ 1 + (\frac{x}{x_i})^{w+1} \right].
\]
and the integral becomes

\[ \int_0^1 \left[ \frac{1}{x_i^4} + \sum_{n=1}^{w+1} \frac{(-1)^n (w+1)}{x_i^{4n+1}} \right] \frac{r' dr'}{x_i^{1/2}} = \frac{\tau_i}{R_i} \]

Definite integrals of the form \( \int_0^1 \frac{r' dr'}{x_i^{1/2}} \) are solved by

\[ \sqrt{1-x_i^2} \sum_{k=0}^{\infty} \frac{(2k)!}{(2w+1)! (k!)^2} \left( \frac{4}{x_i^2} \right)^{w-k} \]

Thus, \( \frac{\tau_i}{R_i} = \sqrt{1-x_i^2} \left[ w + 1 + \sum_{n=1}^{w+1} \frac{(-1)^n}{(w+1)} \cdot \frac{(w+2)!}{(w+1-n)! (2w+1)!} \sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^2} \left( \frac{4}{x_i^2} \right)^{w-k} \right] \)

Defining \( J_i^* \) by the \( \Delta \eta_i \) -integral

\[ \frac{\partial^2 \xi_i}{I_t \Delta \eta_i} \]

is proportional to \( \frac{\tau_i}{J_i^*} \). We have calculated the expressions for different customary \( w \) values (tab. 1). Notice that no \( (1-x_i^2)^{-1} \) terms are necessary. One always obtains the form \( a_0 + a_1 x_i^2 + a_2 x_i^4 + \ldots \).

Since \( x_4 \approx 0.40 \) and \( x_8 \approx -0.35 \) (at \( a=50 \) cm), terms with exponents larger than 2 may be neglected. The general form of \( J_i/J_i^* \) then is \( a_0(w) - a_1(w) x_i^2 \) with \( a_0, a_1 \) listed in tab. 2. The dependence of \( a_0 \) and \( a_1 \) on \( w \) can be approximated by linear regression:

\[ a_0(w) = 1.087 + 0.659 w \]
\[ a_1(w) = -0.489 + 1.563 w \]

So,

\[ \frac{\partial^2 \xi_i}{I_t \Delta \eta_i} = \frac{G_s}{G_t} \cdot \frac{\mu_0}{2 \pi} \sqrt{R_o^2 - a^2} \cdot \left[ \sum_{n=1}^{w+1} \frac{(-1)^n (w+2)!}{(w+1-n)! (2w+1)!} \sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^2} \left( \frac{4}{x_i^2} \right)^{w-k} \right] \]

and with \( R_i = R_0 + x_i \), \( R_0' = \sqrt{R_o^2 - a^2} \)

\[ R_i \frac{\partial^2 \xi_i}{I_t \Delta \eta_i} = R_0' \frac{\partial^2 \xi_i}{I_t \Delta \eta_i} \approx \]

\[ \frac{G_s}{G_t} \cdot \frac{\mu_0}{2 \pi} \frac{k_0}{a} \left[ \left( 1.087 + 0.659 w \right) \frac{x_i^2 - x_8^2}{2 \pi} - (-0.489 + 1.563 w) \frac{x_i^2 - x_8^2}{2 \pi} \right] \]

Up to this point, only integer \( w \) values have been considered. Because of their approximative character, the last formulae hold also for non-integer \( w \). The last step is to invert (*) with respect to \( w \):

\[ w \approx \frac{(R_4 - R_8) \frac{\partial^2 \xi_i}{I_t \Delta \eta_i}}{(R_4 - R_8) \frac{\partial^2 \xi_i}{I_t \Delta \eta_i}} \cdot \frac{G_s}{G_t} \cdot \frac{2 \pi \alpha}{\mu_0 R_o^2} \cdot \frac{1.087 a^2 (x_i - x_8)}{0.659 a^2 (x_i - x_8)} \]

\[ w \approx 0.659 a^2 (x_i - x_8) - 1.563 (x_i^3 - x_8^3) \]
Insertion of

\[ C_1 = 1.51 \cdot 10^{-15} \text{ fringes cm}^2 \]
\[ C_2 = 1.698 \cdot 10^{-15} \text{ deg cm}^2 \text{ kG}^{-1} \]
\[ \mu_0 = 4\pi \cdot 10^{-1} \text{ cm kG kA}^{-1} \]

and

\[ R_0 = 168 \text{ cm} \quad \text{(at } a \approx 50 \text{ cm)} \]
\[ x_4 = +20.0 \text{ cm}, \Rightarrow R_4 = 195.0 \text{ cm} \]
\[ x_8 = -17.5 \text{ cm}, \Rightarrow R_8 = 157.5 \text{ cm} \]

yields

\[ w \approx \frac{(5.16 \frac{x_4}{I_e \Delta \varphi_4} - 4.17 \frac{x_8}{I_e \Delta \varphi_8}) a^4 - 40.76 a^2 - 6533}{24.71 a^2 - 2088} \]

where

\[ \Delta \varphi \text{ is expressed in deg} \]
\[ \Delta \varphi \text{ in fringes} \]
\[ I \text{ in kA} \]
\[ a \text{ in cm}. \]

This formula may be expanded at \( a = 50 \text{ cm} \), since the plasma radius \( a \) does not decrease much below 45 cm in TEXTOR:

\[ w \approx 790 \frac{x_4}{I_e \Delta \varphi_4} - 640 \frac{x_8}{I_e \Delta \varphi_8} - 11.20 + 0.17 a \]

(at \( a \approx 50 \text{ cm}, w \approx 3 \)).

The validity of this expression, and its dependence on horizontal plasma displacements, have been checked by computer simulation (fig. 9).

The analog computing device.

The circuit used to put out the \( n_e \) profile parameter according to

\[ \nu = \frac{4}{\pi \alpha} \sum - \Delta \varphi_6 \]
\[ \Delta \varphi_6 - \frac{2}{\pi \alpha} \sum' + \Delta \varphi_6 \]

is shown in fig. 10.

It is fed by two input voltages, namely

\[ U_{\varphi_6} = \frac{1}{0.755} \sqrt{\frac{\Delta \varphi_6}{\text{fringes}}} \]

and

\[ U_{\sum} = \frac{1}{55.0} \sqrt{\frac{\sum}{\text{fringes cm}}} \].
Thus,
\[ u \approx \frac{-2}{\pi (a/cm)} \frac{55.0}{0.755} \frac{U_\Sigma}{U_{\varphi_6}} + \frac{U_{\varphi_6}}{55.0} \frac{U_\Sigma}{0.755} \]

The operational amplifier OA1 multiplies \( U_\Sigma \) by a scaling factor \( \frac{55.0}{0.755} \) and a variable factor \( \frac{-2}{\pi (a/cm)} \). The factor \( \frac{\alpha}{cm} \) can be varied from 39 to 51 by means of a potentiometer. So, OA1 puts out a signal \(- f \cdot U_\Sigma\) with \( f \approx 1\). OA2 adds \( U_{\varphi_6} \) and inverts the result, yielding \( f \cdot U_\Sigma - U_{\varphi_6} \). This is the denominator of the \( u \) expression, being usually negative. It is not directly connected with the denominator input of the integrated circuit divider IC1 but fed through a zero suppression circuit, consisting of OA4 and OA5. This circuit replaces all voltages greater than \(- 0.15 \text{ V} \) (including 0) by \(- 0.15 \text{ V} \), thus avoiding unreasonable division results as long as signals are small.

The numerator is created by OA3 which subtracts the denominator signal from \(- f \cdot U_\Sigma\). Simultaneously, the result is multiplied by \( \frac{1}{10} \) to compensate for the divider putting out 10 times the division result. OA3 is offset-controlled in order to ensure a proper zero line.

The \( j_t \) profile parameter circuit is depicted in figs. 11a and b. It executes the calculation
\[ \omega \approx \left( 790 \frac{d\varphi}{d\varphi_t} - 640 \frac{d\varphi}{d\varphi_t} \right) \frac{I_t}{11.20 + 0.17 \alpha} \]
where \( \varphi \) must be given in deg, \( \Delta \varphi \) in fringes, \( I_t \) in kA, and \( \alpha \) in cm.

The input voltages are
\[ U_{\varphi_{4,8}} = \frac{1}{2} V \cdot \frac{d\varphi_{4,8}}{\text{deg}} \]
\[ U_{\varphi_{4,8}} = \frac{1}{0.755} V \cdot \frac{\Delta \varphi_{4,8}}{\text{fringes}} \]
and
\[ U_\Sigma = \frac{1}{66.1} V \cdot \frac{I_t}{\text{kA}} \]
Thus, \( w \approx (79.0 \frac{U_{q4}}{U_{q4}} - 640 \frac{U_{q8}}{U_{q8}} \frac{2}{0.755} \frac{V}{U_{I}} - 11.20 + 0.17 \frac{a_i}{cm} \)

\( \approx (31.6 \frac{U_{q4}}{U_{q4}} - 25.6 \frac{U_{q8}}{U_{q8}}) \frac{V}{U_{I}} - 11.20 + 0.17 \frac{a}{cm} \).

Fig. 11a shows the calculation of the parenthesis value.

The rotation angles \( \phi_4 \) and \( \phi_8 \) have nonnegligible offset values that must be subtracted for a correct calculation. Because of this, \( U_{q4} \) and \( U_{q8} \) are put on offset suppression circuits before wiring them to the numerator input of the respective dividing IC. The offset suppression consists of a 100 nF capacitor, a 5V Reed relais and a high input impedance operational amplifier LF 356. The latter works as a simple impedance converter.

Normally, the Reed relais switch is closed, connecting the (+) input of the LF 356 to ground potential and charging the 100 nF capacitor with the respective offset voltage. One second before the tokamak discharge, the switch opens for 8 seconds and puts the (+) input to a potential \( V \) minus offset value. Because of the high input impedance \((>10^9 \Omega)\) of the operational amplifier, the capacitor holds its charge and so stores the offset voltage.

The trigger input of the Reed relais is galvanically decoupled by means of a HCPL 2601 optocoupler.

\( U_{q4} \) and \( U_{q8} \) are fed through zero suppression circuits (cf. fig. 10) which precede the denominator inputs of the dividing ICs. Since the phase shifts are positive, all voltages less than 0.40 V (corresponding to 0.3 fringes) are replaced by 0.40 V.

The two AD 535 integrated circuit dividers put out \( 10 \times \frac{U_{q4}}{U_{q4}} (i = 4,8) \). Since the following division by \( U_{I} \) generates another factor of 10, the operational amplifier OA I calculates \(-(0.316 \frac{U_{q4}}{U_{q4}} - 0.256 \frac{U_{q8}}{U_{q8}})\).
Fig. 11b shows the division by $U_I$ (including a zero suppression for $U_I < 1.0 \ V$, corresponding to $66 \ kA$) and further signal processing. OA II adds 11.20, subtracts 0.17 (a/cm) and inverts the result, yielding $w$. Here, $\frac{a}{cm^2}$ can be varied between 52 and 40 by means of a potentiometer.

Between tokamak shots, the value of the parenthesis in the $w$ formula is zero, causing the output voltage of the circuit to be $-11.20 + 0.17 \ a \approx -2.65$. Furthermore, the noise on $U_4$ and $U_8$ is amplified considerably because of the small denominator threshold voltages. Therefore, the last two operational amplifiers have been added to the circuit. They suppress negative $w$ values and so generate a clean zero line.

In order to accommodate the output voltage to the CAMAC module used to digitize the profile parameter, $w$ is further divided by 2 by the last operational amplifier.

Application of the parameters.

An experiment suitable to demonstrate the application of the profile parameters is characterized by the upper traces of fig. 12, showing the time development of loop voltage $U_L$, plasma current $I_p$, line-averaged electron density $n_e$, and electron temperature of the plasma center $T_e$. To induce changes in plasma state, a current ramp between 700 and 1200 msec is generated.

The profile parameter traces show $n_e$ parameter values typically between 1 (narrow) and 3 (broad), and $j_t$ parameter values typically between 2 (broad) and 5 (narrow). The startup phase of the tokamak discharge ($\approx 300$ msec) is characterized by contracting $n_e$ and $j_t$ profiles. In the following current plateau phase, the distributions adjust themselves to relatively broad profile shapes ($\approx 500$ msec). The current ramp triggers a second contraction of the profiles, leading to narrow shapes ($\approx 1150$ msec). Subsequently, the profile parameters release to medium values ($\approx 2000$ msec). Towards the end of the discharge, the plasma current runs down, and the profiles contract once more.
Using the scaling quantities $F$ (~ total electron number, see above) and $I_t$ (total toroidal current) and the given profile form parameters, approximate $n_e$ and $j_t$ profiles can be calculated and compared with those obtained by Abel inversion.

This is done in fig. 13 for three different times. One recognizes that, in all cases, the mean widths of the profiles are approximated very well. In case of the $n_e$ profiles, even the peak values fit in a rather satisfactory way, whereas the peak values of the approximate $j_t$ profiles are very uncertain, being sensitive to the choice of the plasma radius.

Fig. 14 shows the behaviour of the parameters during a smooth discharge with constant parameter values in the plateau phase. The $n_e$ parameter settles down at 2, indicating a $1 - r^2$ profile which is commonly used for calculations requiring $n_e$ profile information.

As a special application of the profile parameters, the influence of current ramps with different slopes is demonstrated in real-time (fig. 15). Profile contractions due to the skin effect are clearly seen. Even the fact that faster current ramps lead to narrower current density profiles becomes quite obvious.

**Summary.**

It has been demonstrated that the electron and toroidal current density profiles of a tokamak device can be characterized in a wide range by two parameters. The characterisation is restricted to profiles which are not essentially hollow. Certain signals of a multi-channel FIR-polarimeter/interferometer have been found which are sensitive to variations of the respective profile shape. Simple approximate formulae have been deduced which relate the parameters to the selected signals. An analog computing device has been constructed which uses the formulae to convert the signals into parameter values in real-time mode.

The actual choice of profile-dependent input signals is obviously dependent on the given measuring device. In the case of the $n_e$ profile, it should be suffi-
cient to employ a two-channel instrument with one probing beam traversing the plasma center and a second beam passing at about half the minor radius. Then, the peakedness of the $n_e$ profile might be expressed by the phase shift ratio. In the case of the $j_t$ profile, only one polar/interferometer beam - passing at about half plasma radius - is necessary. However, in both cases the signals would be rather sensitive to displacements of the plasma column.

It has been shown that the parameter signals used on TEXTOR are suitable to get a review on the influence of running-condition changes on the profiles. The parameters are helpful additional information for a quick interpretation of such modifications.

References

/1/ Soltwisch, H., Equipe TFR, Infrared Physics 21 (1981) 287

/2/ Soltwisch, H., Nucl. Fus. 23 (1983) 1681


### Tab. 1

<table>
<thead>
<tr>
<th>w</th>
<th>$\frac{J_i}{J_i^*}$ expressions for different w values</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1$</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{1}{5} (9 - 4x_1^2)$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{35} (87 - 76x_1^2 + 24x_1^4)$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{105} (325 - 420x_1^2 + 264x_1^4 - 64x_1^6)$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{1155} (4215 - 7180x_1^2 + 6744x_1^4 - 3264x_1^6 + 640x_1^8)$</td>
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### Tab. 2

<table>
<thead>
<tr>
<th>w</th>
<th>$a_0(w)$</th>
<th>$a_1(w)$</th>
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<tr>
<td>0</td>
<td>$1$</td>
<td>$0$</td>
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<tr>
<td>1</td>
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<td>$0.800$</td>
</tr>
<tr>
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<td>$2.486$</td>
<td>$2.171$</td>
</tr>
<tr>
<td>3</td>
<td>$3.095$</td>
<td>$4.000$</td>
</tr>
<tr>
<td>4</td>
<td>$3.649$</td>
<td>$6.216$</td>
</tr>
</tbody>
</table>
model applied to the TEXTOR plasma ($\beta_p \ll 1$)

a) concentrical magn. flux surfaces ($B_p = \frac{1}{R} f(r)$)
b) $n_e = f(r)$

\[ j_t = \frac{1}{R} \left[ f(r) - \frac{R - R_0}{R} e(r) \right] \]

\[ n_e = f(r) \]
Fig. 2 9-channel combined polarimeter/interferometer
($\lambda_{\text{HCN}} \approx 337$ $\mu$m):

Phase shift

$$\Delta \varphi_i = C_1 \int n_e dz \quad (i)$$

Faraday rotation angle

$$\alpha_i = C_2 \int n_e B_{p2} dz \quad (i)$$

$$R_1 = 222.5 \text{ cm}$$
$$R_2 = 215 \text{ cm}$$
$$R_3 = 205 \text{ cm}$$
$$R_4 = 195 \text{ cm}$$
$$R_5 = 185 \text{ cm}$$
$$R_6 = 175 \text{ cm}$$
$$R_7 = 165 \text{ cm}$$
$$R_8 = 157.5 \text{ cm}$$
$$R_9 = 144 \text{ cm}$$
**fig. 3**  
**electron density profile**

Phase shift  
\[ \Delta \varphi \sim \int n_e \, dz \]

\[ \Delta \varphi_0 \]

\[ \frac{F}{F} = \text{measure for profile form} \]

**fig. 4**  
**toroidal current density profile**

Magnetic flux surfaces (simplified model)

\[ \phi_{\text{flux surface}} \cdot \hat{B}_p \cdot ds = \mu_0 I_{t, \text{included}} \]

- \( B_p \) increases with the included part of the total toroidal current \( I_t \)

Faraday-rotation angle  
\[ \alpha_i \sim \int n_e \cdot B_{p,z} \, dz \]

Measure for profile form:  
\[ \frac{\alpha_i}{I_t \cdot \Delta \varphi_i}, \quad \frac{\alpha_8}{I_t \cdot \Delta \varphi_8} \]
**fig. 5** $n_e$ - profile parameter

$n_e \approx n_{e0} \left(1 - \left(\frac{r}{a}\right)^{1.7}\right)$

ansatz $n_e(r) \approx n_{e0} \left(1 - \left(\frac{r}{a}\right)^u\right)$, $u = \text{profile parameter}$

**fig. 6** $j_t$ - profile parameter

$j_t \approx j_{to} \frac{R_0}{R} \left[\left(1 - \frac{r^2}{a^2}\right)^{4.6} - \frac{R-R_0}{R} \epsilon(r)\right]$  

ansatz $j_t(r,R) \approx j_{to} \frac{R_0}{R} \left[\left(1 - \frac{r^2}{a^2}\right)^{2.9} - \frac{R-R_0}{R} \epsilon(r)\right]$  

$j_t \approx j_{to} \frac{R_0}{R} \left[\left(1 - \frac{r^2}{a^2}\right)^w - \frac{R-R_0}{R} \epsilon(r,w)\right]$,  

$w = \text{profile parameter}, \quad \epsilon = \text{correction term} < \frac{1}{2}$
fig. 7

Validity of the \( u \) expression.

\( a = \text{plasma radius} \)
\( h = \text{horizontal plasma displacement} \)

\begin{align*}
\text{a} & = 50 \text{ cm} \\
\text{a} & = 45 \text{ cm}
\end{align*}
Computer calculations on the influence of the $n_e$-profile shape (as expressed by $u$) on $Q_4$ and $Q_8$ at different $w$ values.

The plasma radius is assumed to be 50 cm.
Validity of the \( w \) expression.

\( a = \) plasma radius
\( h = \) horizontal plasma displacement

\( a = 50 \text{ cm} \)

\( a = 45 \text{ cm} \)
fig. 10

u parameter circuit diagram
fig. 13

profiles by Abel inversion

approximative profiles
fig. 14

**ARBITRARY UNITS**

- $U_L$  
  $1 \text{V}$

- $I_p$
  $340 \text{kA}$

- $\bar{n}_e$
  $2.2 \cdot 10^{13} \text{cm}^{-3}$

- $T_{eo}$
  $800 \text{eV}$

**NE-PROFILE PARAMETER**

**JT-PROFILE PARAMETER**

broad profiles

broad profiles