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The Muon Anomalous Magnetic Moment: A Probe for Physics beyond the Standard Model of High Energy Physics

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The search for physics beyond the standard model, either in direct collider or in high precision low energy experiments is a most active endeavour in particle physics. A prime candidate for the detection of this new physics is the anomalous magnetic moment of the muon. In this project, we present a lattice QCD calculation of the leading order hadronic contribution of the lepton anomalous magnetic moments, i.e. for the electron, the muon and the tau. Such lattice calculations are indispensable as input for the non-perturbative contributions as required in newly planned experiments for measuring these anomalous magentic moments.

1 Introduction

With the latest triumph of the Higgs boson discovery at the Large Hadron Collider (LHC)[1,2] the standard model (SM) of elementary particle physics can be considered to be completely verified. However, this success story of the SM leaves even more mysterious the unanswered questions in particle physics such as the quark mass hierarchy, the matter anti-matter asymmetry of the universe, the large amount of charge and parity (CP) violation and the so far undetected constitution of dark matter. All these experimental observations cannot be explained within the SM and need extensions of the SM or even a completely new theory in which the SM is embedded.

There are, in principle, two ways to find hints of this anticipated new theory. The first is through very high energy particle accelerators such as the LHC at CERN which can lead to a direct experimental observation of new states and particles. The second way is through high precision experiments from which deviations from SM predictions could be detected. Clearly, to find such deviations, the theoretical computations within the SM has to match the experimental accuracy in order to demonstrate a failure of the SM without doubt. This article is concerned with a prime candidate for detecting such a breakdown of the SM, namely the muon anomalous magnetic moment as will be described and explained in more detail below.

The known interactions in the SM are the electromagnetic and weak forces which can be analysed soundly by perturbation theory and which are responsible for, e.g., electromagnetic effects and radioactive decays. On the other hand, the strong force between quarks and gluons is responsible for all baryonic form of matter surrounding us, namely protons and neutrons: only a few percent of the mass of protons and neutrons stems from the quark masses (gluons are massless), the predominant part comes from the strong interaction itself by binding energy. In fact, all nuclear matter is governed by the strong force and hence its understanding is of crucial importance to understand nuclear reactions. One characteristic feature of the strong force is that at large distances the constituent particles, the quarks and gluons, interact strongly such that perturbative methods cannot be applied.

The strong force is also essential for the early stage of the universe where quarks and gluons were not bound in hadrons but interacted within a plasma of particles. To understand the transition of this plasma to our observed universe today is of fundamental importance to reveal the origin of our world and hence of our sheer existence. Also, important nuclear processes like the triple-alpha process (nuclear fission of three Helium nuclei) are ultimately determined by the strong force.

The strong interaction between quarks and gluons requires a tool for evaluating its theoretical model, quantum chromodynamic, which is of non-perturbative nature. This can be achieved by discretising space and (Euclidean) time and put QCD on a 4-dimensional space-time crystal leading to the Lattice QCD (LQCD) formulation of the theory of the strong interaction. Performing the quantisation of LQCD with Feynman's path integral allows for numerical simulations of the theory by means of Markov chain Monte Carlo methods. This approach has been very successful and led to e.g., the evaluation of the low lying hadron spectrum, resonances, the spin composition of the proton, the chiral condensate as order parameter of spontaneous chiral symmetry breaking and the computation of fundamental parameters of the theory, see, e.g. Refs. 3–10 for our own work within the European Twisted Mass Collaboration (ETMC).

In the following, we will concentrate on the anomalous magnetic moment of the muon which is a very promising probe for physics beyond the SM. It receives important non-perturbative contributions and needs therefore input from LQCD.

2 The Physical Point

In the past, it has been very difficult to perform computations in LQCD with physical values of the employed quark masses. The reason is that the used algorithms scale with some power of the inverse quark mass and it turns out that the physical value of the light quark mass is so small that numerical simulations directly at the physical quark mass were prohibitively expensive.

However, the last years have seen a dramatic improvement in algorithms[11]. In addition, the supercomputer architectures have developed tremendously, the JUQUEEN (JSC), Hazel Hen (HLRS) and SuperMuc (LRZ) machines at the Gauss Centre being prime examples. In addition, the LQCD application is highly appropriate for the massively parallel supercomputer architectures. In Fig. 1 we show a strong scaling plot for our particular code. We show the speedup of the iterative solver, the so-called *conjugate gradient solver*, used in our software relative to 128 nodes of JUQUEEN for fixed problem size. The most important driver routines are improved using XLC intrinsics for QPX and low level communication routines leading to more than 30% of peak performance. We observe super-linear scaling up to 1024 nodes, followed by some flattening due to too small local problem size. The dashed line represents linear scaling.

These developments led eventually to a situation where simulations directly at the physical value of the light quark masses, usually called the physical point, became possible. Within our collaboration, which uses the twisted mass formulation of lattice QCD[12,13],

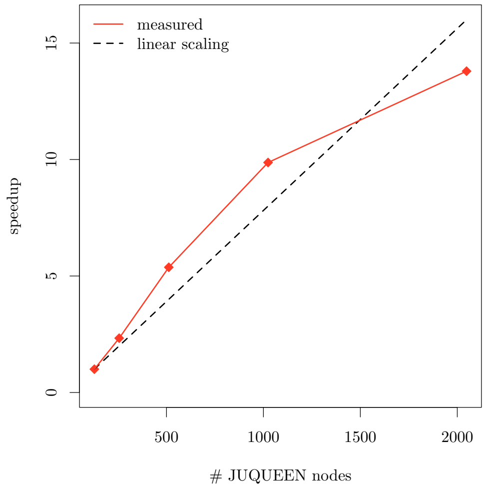

Figure 1. The speedup of our code on JUQUEEN relative to 128 nodes with problem size fixed (strong scaling). The dashed line represents linear scaling.

we were indeed able to perform such simulations[14,15] with first results for light meson and baryon physics. The focus of this article is, however, the muon anomalous magnetic moment which we will discuss now.

3 The Muon Anomalous Magnetic Moment

The muon is a so-called *lepton* which has very similar properties as the electron, for example the charge and the spin, but it is about 200 times heavier. The muon has an intrinsic spin of 1/2 on the classical level, see left panel of Fig. 2. However, an elementary particle such as the muon is described by a *quantum field theory*. In this approach to high energy physics spontaneous generations of particle anti-particle pairs out of the vacuum are a very natural element. For example, the generation of electron-positron pairs (the positron being the anti-particle of the electron) inside the muon, leads to a deformation of the muon. As a consequence, also a deviation from the classical value of the spin from 1/2 occurs. This is illustrated by the right panel of Fig. 2.

In reality, the situation is, of course, much more complicated than shown in the schematic pictures of Fig. 2. There are much more interactions taking place and not only electrons are generated, but also quarks and gluons and the W- and Z-bosons which interact through the weak forces.

What makes the deviation from the value 1/2 very attractive is the fact that the spin of the muon or the electron can be measured extremely precisely with 7 (in case of the muon) to 10 (in case of the electron) significant digits. This precision opens a window to detect possible particle anti-particle pair creations of particles that are not within the standard model and come from some new, so far completely unknown new physics. Since, theoretically, the chance to detect such new particles is proportional to the mass squared of the underlying lepton and since the mass of the muon is about 200 times larger than the

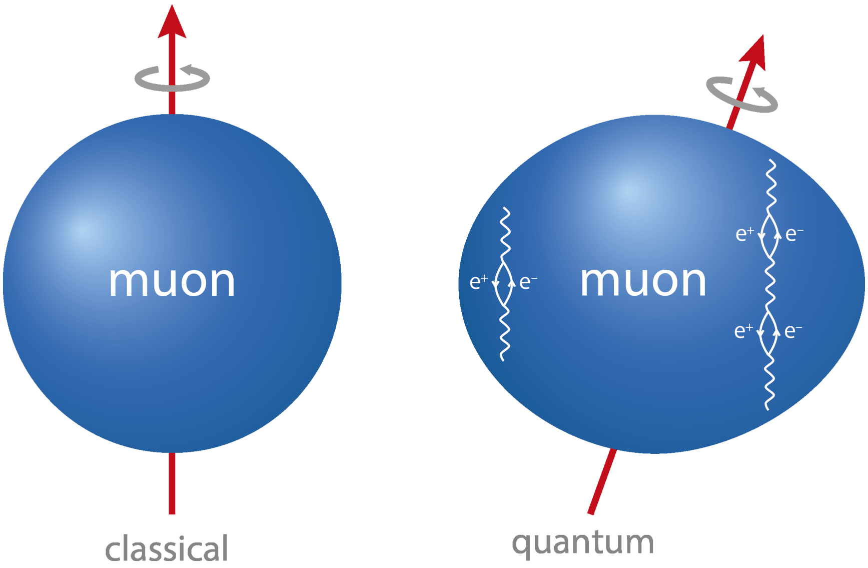

Figure 2. The spin of the muon. Left: on the classical level, the spin of the muon is 1/2. Right: when quantum corrections appear (here the generation of electron positron pairs), the muon is deformed and a deviation of the spin from the value 1/2 can be computed theoretically to a very high accuracy. This deviation has been indeed detected experimentally to a precision that matches the theoretical calculation.

one of the electron, the muon is an ideal laboratory to search for new physics beyond the standard model.

To this end, the theoretical SM prediction must match the experimental precision with all contributions from the SM forces well controlled. And, it is here where LQCD comes into the game since it is only through the *ab initio*, non-perturbative lattice calculations that the non-perturbative strong force contribution can be computed. Thus, experiments eagerly await results from LQCD for the non-perturbative strong force contribution to the spin of the muon.

Similar to the muon, also the τ-lepton and, as mentioned already, the electron have anomalous magnetic moments. In this project we have therefore set out to determine the most important non-perturbative contribution to anomalous magnetic moments of these leptons. It turns out that this is the leading-order light quark hadronic contribution for all three leptons, the electron, the muon and the τ, denoted as a_e, a_μ and a_τ, respectively. To this end, we have followed our earlier work, described in Ref. 17 and have concentrated on the anomalous magnetic moment of the muon. The extension of this work to the case of the electron and τ leptons has been straightforward and required essentially to tune in the correct masses of these leptons.

Another, very important new ingredient has been that these anomalous magnetic moments were computed at the physical value of the light quark masses, avoiding in this way the demanding and rather uncontrolled extrapolation to the physical point. Nevertheless, we will compare the results obtained at the physical point with the ones that were obtained from ensembles at unphysically large pion masses and which were then extrapolated to

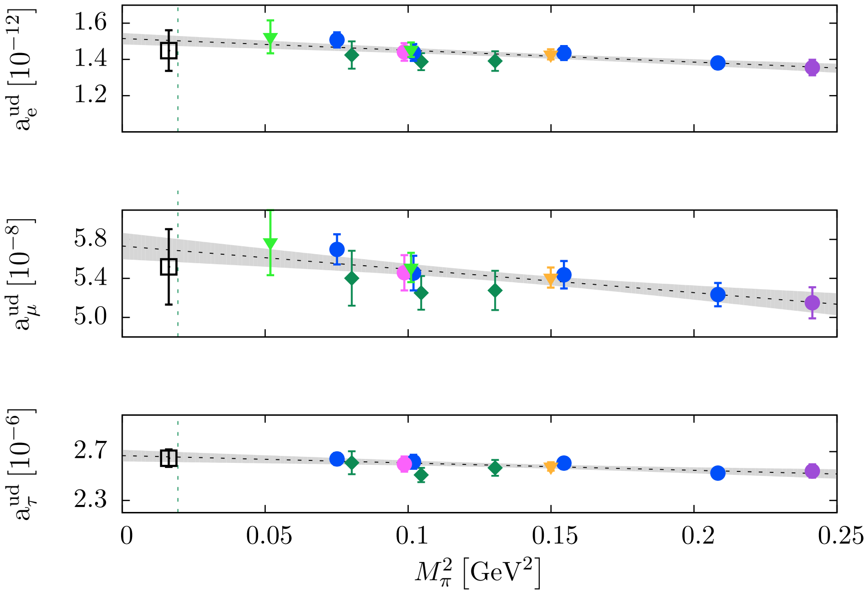

Figure 3. Comparison of the chiral extrapolation of the light quark contributions to the three lepton anomalous magnetic moments obtained from $N_f = 2+1+1$ simulations to the values pion mass (black square). The dark green diamonds correspond to $a = 0.086$ fm and $L = 2.8$ fm and the circles to $a = 0.078$ fm, the violet one stands for $L = 1.9$ fm, the blue ones for $L = 2.5$ fm, and the pink for $L = 3.7$ fm. The orange triangle shows the value obtained for $a = 0.061$ fm and $L = 1.9$ fm and the light green triangle denotes $a = 0.061$ fm and $L = 2.9$ fm.

the physical point. In Fig. 3 we show the data for the three lepton anomalous magnetic moments as a function of m_π^2 comparing the results of Refs. 17, 18 with the new result at the physical point.

For our results at unphysically large pion masses we have used a particular technique developed by us[21,22,17,18]. This amounts to a particular definition of the vacuum polarisation function as in

$$a_{\rm l}^{\rm hvp} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{H^2}\frac{H_{\rm phys}^2}{m_l^2}\right) \Pi_{\rm R}(Q^2)\,, \tag{1}$$

with the hadronic scale $H = M_V$, the lowest lying vector meson state, and m_l the lepton mass.

When determining the lepton anomalous magnetic moments the chiral extrapolation to the physical pion mass can lead to a severe systematic error. This uncertainty is avoided when using ensembles at the physical point[15]. We have computed the light quark contributions to the lepton anomalous magnetic moments on 800 configurations of the new physical ensemble. We find full agreement with our previous results for the light quark contribution originating from a chiral extrapolation of our $N_f = 2$ simulations where only the light up and down quarks were considered. Using unphysically large pion masses, we also performed simulations by including the strange and the charm quarks as dynamical degrees of freedom, a situation we denote as $N_f = 2+1+1$ simulation. The data of these

$N_f = 2+1+1$ calculations as well as the corresponding chiral extrapolations are also depicted in Fig. 3 as dashed lines with shaded error band. We give the extrapolated values for $N_f = 2$ and $N_f = 2+1+1$ in Tab. 1.

	physical point	extr. $N_f = 2$	extr. $N_f = 2+1+1$
$a_{\mathrm{e}}^{\mathrm{hvp}} \cdot 10^{12}$	$1.45(11)$	$1.51(04)$	$1.50(03)$
$a_{\mu}^{\mathrm{hvp}} \cdot 10^{8}$	$5.52(39)$	$5.72(16)$	$5.67(11)$
$a_{\tau}^{\mathrm{hvp}} \cdot 10^{6}$	$2.65(07)$	$2.65(02)$	$2.66(02)$

Table 1. Comparison of the values for $a_{\mathrm{e}}^{\mathrm{hvp}}$, a_{μ}^{hvp}, and a_{τ}^{hvp} obtained at the physical point with the results of the linear extrapolations from our definition Eq. 1 on the $N_f = 2$ and $N_f = 2+1+1$ ETMC ensembles without clover term.

An important aspect of our work has been to carry through a careful quantitative determination of systematic uncertainties which arise in our calculation for the data of the lepton anomalous magnetic moments when not simulating at the physical point. These systematic effects originate from the chiral extrapolation, the continuum limit, the fit range for the vector meson mass and the form of the fit function. These investigations are described in detail in Ref. 18. We also compared the approach of using Padé fits as proposed in Ref. 19 with our standard fits[18]. As discussed in Ref. 20 we could, however, not find a clear advantage of using the Padé approach which led us therefore to stay with our standard fit function.

4 Conclusion and Outlook

In this project, we accomplished a computation of the leading order hadronic contributions to the lepton anomalous magnetic moment directly at the physical values of the light quarks masses. This setup avoids an extrapolation of previous results, obtained at unphysically large quark masses, to the physical point which has been afflicted with a hard to quantify systematic uncertainty. However, we found a very satisfactory agreement with our present physical point results and the earlier calculation. This is very reassuring that the extrapolations performed previously were correct.

Our results can be considered as a most important input to the newly planned experiments at Fermilab[24] and KEK[25] which aim a factor of four higher precision. To match this substantially improved accuracy, in the LQCD calculations more values of lattice spacing, larger volumes and electromagnetic and isospin breaking effects have to be accounted for. Steps in this direction are in progress. In addition, simulations at the physical point with active light up and down as well as the heavier strange and the charm quarks are ongoing such that a fully physical condition can be reached. Thus, our LQCD calculations are taken up the challenge to provide precise enough input for experiments which provide a most promising prospect to see, whether the muon anomalous magnetic moment indeed allows to detect physics beyond the standard model.

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