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Towards Multidimensional Hydrodynamic Simulations of Stars

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With the Seven-League Hydro (SLH) code, designed to model hydrodynamic flows in stellar interiors, simulations of dynamical processes in multidimensional hydrodynamic simulations are carried out. After a description of the features of SLH, we present several examples of its applications to problems in stellar astrophysics.

1 Introduction

Stars are made of gas held together by self-gravity. In their interiors, nuclear reactions take place that produce energy and provide the pressure gradient to stabilise them against gravitational collapse[a]. The energy produced near the centre of the star is transported outwards by radiation or convection. It is radiated into space from the stellar surface, which makes the star shine. In addition, effects such as rotation, stellar winds, magnetic fields, and the interaction with a binary companion may contribute to the structure and evolution of the star. Clearly, stellar structure and evolution is a multi-physics problem. Moreover, the wide range of relevant spatial and temporal scales challenges numerical simulations of processes in stars. Hydrodynamical instabilities may occur in thin layers and act on very short dynamical time scales, while some burning stages in stars may take gigayears to complete.

The classical approach to these challenges is to reduce the dimensionality of the problem by assuming spherical symmetry of non-rotating stars. The concept of shellular rotation[1,2] allows to treat even rotating stars in a one-dimensional formulation. In addition, many stellar models assume hydrostatic equilibrium. They have been very successful in explaining stellar structure over a wide range of masses and the different evolutionary

[a]In some compact stellar objects, pressure is provided by quantum-mechanical or nuclear physics effects.

phases of stars. The simplifications made to cover the entire stellar live in one simulation, however, restrict the predictivity of the models. Intrinsically multidimensional processes and those acting on short time scales enter as parametric descriptions. The thus introduced free parameters can be used to fit observations. To arrive at *predictive* models and to turn the qualitative description of stellar evolution into a reliable quantitative theory, such parametric formulations have to be tested and verified or improved and the parameters have to be fixed based on *multidimensional hydrodynamic simulations.* Ideally, certain dynamic phases of stellar evolution can be covered completely in multidimensional approaches.

For this, special numerical approaches are required. These are briefly reviewed in Sec. 2 and their implementation into an astrophysical simulation code is described in Sec. 3. Sec. 4 gives examples of applications to problems in stellar astrophysics.

2 Low-Mach Number Hydrodynamics for Simulations in Stellar Astrophysics

Numerical approaches to modelling flows in stars have to deal with fluid flows at low Mach numbers $M = |\boldsymbol{v}|/c_{\mathrm{s}}$, where $\boldsymbol{v}$ is the velocity and c_{s} is the speed of sound. Most multidimensional hydrodynamics codes use explicit time-stepping to advance the solution in time. This has the disadvantage that the time step Δt is limited for reasons of *stability* by the so-called Courant-Friedrichs-Lewy (CFL) condition. In the context of the Euler equations of fluid dynamics this translates to

$$\Delta t_{\text{explicit}} \leq \mathrm{CFL}\frac{\Delta x}{|u + c_{\mathrm{s}}|} \overset{u \ll c_{\mathrm{s}}}{\approx} \mathrm{CFL}\frac{\Delta x}{c_{\mathrm{s}}},$$

with the local fluid velocity u and the grid spacing Δx. CFL is a dimensionless constant of order 1, whose value depends on the exact time-stepping method being used. *Implicit* methods are not subject to such a restriction for stability. Still the time step cannot be arbitrarily large for reasons of *accuracy.* A common choice is to let the fluid progress one grid cell per time step, resulting in

$$\Delta t_{\text{implicit}} \approx \mathrm{CFL}\frac{\Delta x}{|u|}.$$

This implies that the implicit method can use time steps larger by a factor of the inverse Mach number compared to an explicit time discretisation, making it the favoured choice for simulations in the low-Mach number regime.

In addition, conventional numerical hydro solvers show *excessive dissipation* for flows at Mach numbers below 10^{-2}. This is because in many cases the numerical dissipation terms needed for stabilising finite-volume schemes do not scale with Mach number in the same way as the physical flux terms[3,4]. They completely dominate the numerical flux at low Mach numbers. An illustrative example of this issue is the *Gresho vortex* – a stationary solution to the Euler equations. The top row in Fig. 1 shows the poor performance of the unmodified Roe solver[5] in resolving flows at low Mach numbers. Other approximate Riemann solvers have similar problems. Yet the approximations many of the common low-Mach schemes are based on (e.g. Boussinesq or anelastic) strongly restrict their applicability by modifying the equations to be solved numerically. They thus apply only to certain limits in the flow regime. A way to deal with the low-Mach number problem while

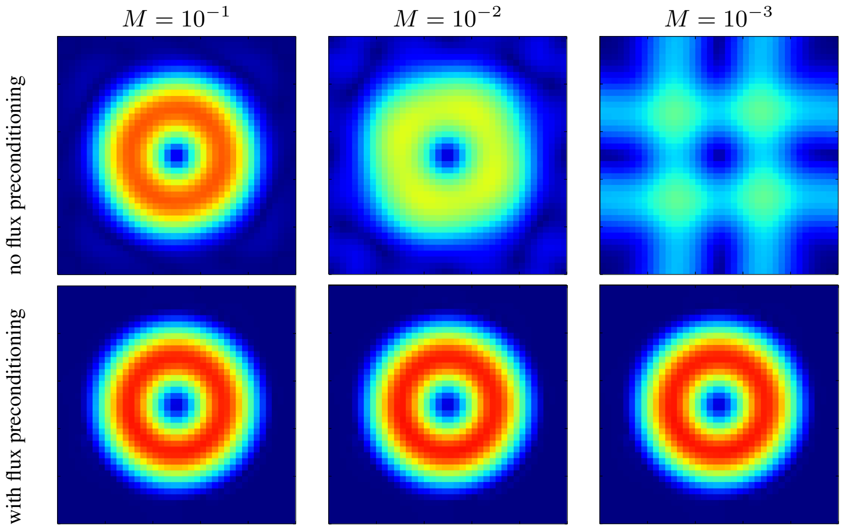

Figure 1. Mach number of the Gresho vortex at $t^* = 2$ (non-dimensional units) computed with Roe's method and linear reconstruction of the interface values.

still solving the complete compressible Euler equations of hydrodynamics is *flux preconditioning*[6]. A special preconditioning matrix is required for astrophysical applications[3]. The result of this modification is depicted in the bottom row of Fig. 1. This technique allows for the construction of an *all-Mach number hydro solver.*

3 The Seven-League Hydro Code

For our simulations we employ the Seven-League Hydro (SLH) code[7,3]. It solves the Euler equations of fluid dynamics in a finite-volume discretisation including an astrophysical equation of state[8], radiative transfer in the diffusion limit, gravity, and a nuclear reaction network[9,10]. It supports one, two, or three spatial dimensions with a very flexible approach to geometry[11]. In combination with a highly efficient implicit time discretisation method and well-balancing techniques to guarantee the correct representation of hydrostatic equilibria, the SLH code is a versatile tool to be applied for constructing the next generation of stellar models. It is particularly well suited for simulations of flows in stellar interiors at low Mach numbers – a regime where many other hydrodynamics codes have severe difficulties. For this, SLH offers several low-Mach number hydro solvers. In addition to the aforementioned preconditioning technique[7,3], other numerical flux functions suited for low-Mach number flows, e.g. the AUSM$^+$-up method[12] are implemented.

The SLH code uses either pure MPI or hybrid MPI/OpenMP parallelisation. The computational grid is distributed among the MPI processes using domain decomposition with ghost cells. SLH takes care that OpenMP threads use an optimal domain decomposition and that they always work on the same part of memory. Tests on JUQUEEN showed that it is most efficient to use one task per core and four threads per core. We performed a series of short scaling tests on JUQUEEN. The result of strong scaling tests on JUQUEEN

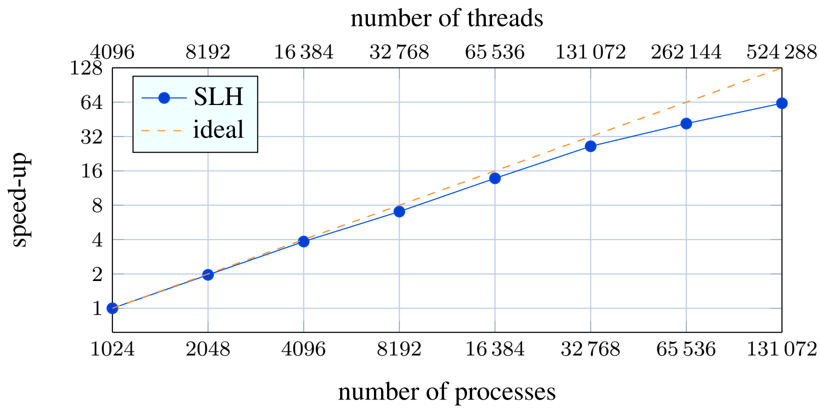

Figure 2. Strong scaling results on JUQUEEN. Tests were computed on a 512^3 grid with the Taylor–Green vortex as a typical three-dimensional test problem. All runs were performed with 16 MPI tasks per node and 4 threads per task using the BiCGSTAB(5) solver.

are shown in Fig. 2. It demonstrates excellent scaling behaviour of our code up to several 100 000 threads. During the *2016 Extreme Scaling Workshop*[14] we were able to test scaling of SLH to the full JUQUEEN machine. The result[15] shows very good scaling behaviour with 88% efficiency on the full machine compared to half of the machine.

4 Application to Stellar Astrophysics Problems

4.1 Example 1: Silicon Burning

The last instances of the evolution of a massive star before its core collapses giving rise to a supernova explosion are dominated by silicon burning. Of particular interest is the morphology of the star's centre composed of iron-group elements and of the silicon-burning layer around it. The key to successful simulations of core-collapse supernovae seems to be the multidimensional nature of the involved flows and hydrodynamical instabilities, which result in asymmetries in the explosion phase. These are seeded by asymmetries in the initial conditions[16–18]. It is therefore desirable to start such simulations from realistic multidimensional hydrodynamical models of the progenitor evolution.

Such simulations were successfully carried out with SLH. After ensuring that the mapping of a one-dimensional stellar model computed with MESA[19] into a multidimensional SLH setup preserves the stratification of physical quantities and the location of convective shells, test runs in two spatial dimensions were performed (see Fig. 3, left panel).

Very recently, a pilot run in three spatial dimensions was carried out. Although the spatial resolution was not very high, it clearly showed the expected convective flow pattern in the silicon-burning layer. An illustration is given in Fig. 3, right panel. Increasing neutronisation of the central material and a contraction of the core was observed. The model approaches core collapse.

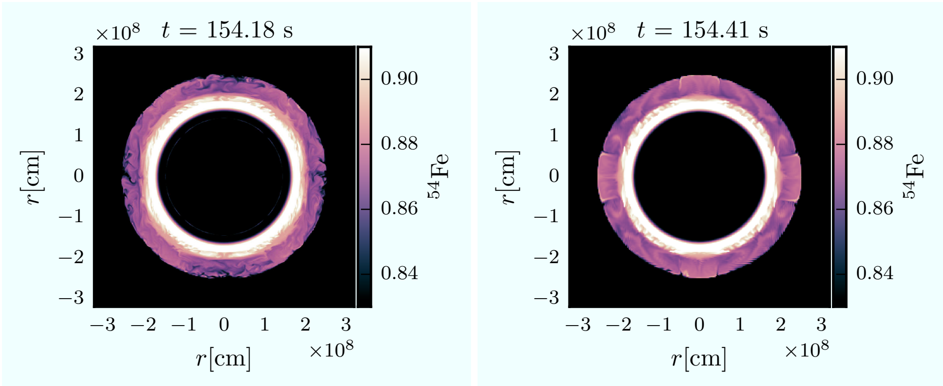

Figure 3. Simulations of the silicon burning phase in a 15 solar mass star in two spatial dimensions with 512^2 cells (left panel) and 3D with 256^3 cells (right panel). The colour coded abundance of ^{54}Fe is shown in both panels. Regions of fully developed convective burning can be identified in the purple region between 1.9×10^8 cm and 2.6×10^8 cm.

4.2 Example 2: Dynamical Shear Instabilities

Mixing processes due to unstable shear flows in rotating stars are believed to play an important role in stellar evolution. As they are intrinsically multidimensional phenomena, the incorporation of such shear instabilities into conventional 1D evolution codes is only possible using very simplifying approximations. Our objective is therefore to simulate consistent, three-dimensional models of rotating stars that feature a shear unstable region. A first example of such an approach combining two-dimensional SLH simulations with initial setups derived by Dr. Raphael Hirschi and collaborators (Keele University, UK) from 1D classical stellar evolution calculations is given in a study that has recently been published[20].

Setting up a three-dimensional simulation is more challenging, because a 1D stellar profile only guarantees stability against other effects than shear *on average* and only *in radial direction*. Thus, in three-dimensional mappings additional unstable regions (e.g. convective zones) may occur at different latitudes. To avoid this, a new initial setup was developed based on a 20 $M_\odot$ star with a slightly modified profile of the angular velocity. We thus obtain a model with a spatially isolated region around the equator that is predicted to be unstable solely to dynamical shear.

It is important to find a suitable resolution and spatial domain that still ensure accurate results while using a reasonable amount of computing time. A preliminary 3D simulation with a setup meeting these requirements is shown in Fig. 4. The formation of a large eddy is clearly visible. In the further evolution, the eddy will decay into smaller structures while redistributing the angular velocity. This results in a new, stable configuration.

4.3 Example 3: Convective Boundary Mixing in Stars

A fundamental problem of stellar evolution is to characterise a convective boundary through dynamical and thermodynamical conditions, and to foretell the consequent entrained mass. We address this problem by using a so-called box-in-a-star approach: plane

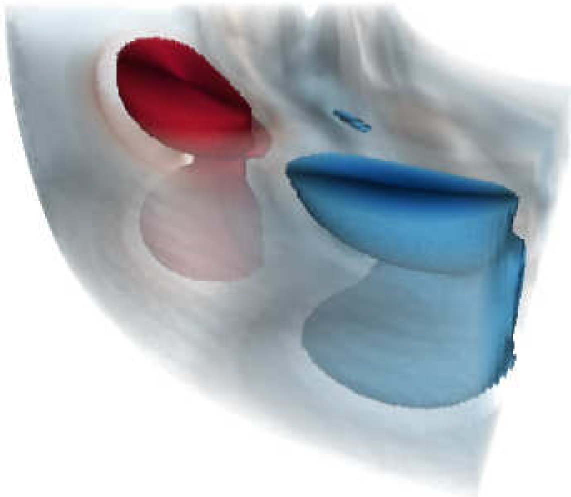

Figure 4. Shear instability in a $20\,M_\odot$ stellar model. Shown is the radial velocity (red: negative, blue: positive) at $t = 20\,\mathrm{min}$ after initialisation of the simulation for a wedge around the equatorial plane. The resolution is $128(r) \times 64(\vartheta) \times 128(\varphi)$ grid cells.

parallel simulations of regions within a star containing a convective layer (see Fig. 5). Performing these kinds of simulations for convective stratifications characterised by a finite number of physical parameters – mean molecular weight of convecting and bounding fluids, degree of temperature gradient and aspect ratio, for example – will enable us to quantify the rate of mixing across upper and lower convective boundaries and to identify the key transport processes and hydrodynamic instabilities responsible for the mixing under the various conditions.

A method for connecting the results of such multidimensional simulations to one-dimensional parametrisations in stellar evolution models[21] involves calculating the so-called bulk-Richardson number of a convective boundary, which is the ratio of potential energy of the stable fluid to the kinetic energy of the convective flow in the boundary layer. The entrainment rate is theorised to be a function of this ratio including two coefficients, which are to be determined by experiment or simulation. It is our goal to determine these coefficients as a function of stratification and, hence, to confirm that this method can work and calibrate its coefficients, or to reject the theory and propose something new.

Due to the large computational requirements of a study in three spatial dimensions (3D), a 2D setup is currently used for a parameter study. Although the mixing is expected to be different in the two cases, 2D simulations can be used to test the robustness of the entrainment theory based on the bulk-Richardson number. An example is shown in Fig. 5.

5 Concluding Remarks

We have described the Seven-League Hydro code that features a number of methods optimised for simulating fluid-dynamical processes in stellar interiors. Particular attention was paid to the development and implementation of methods that are able to follow flows

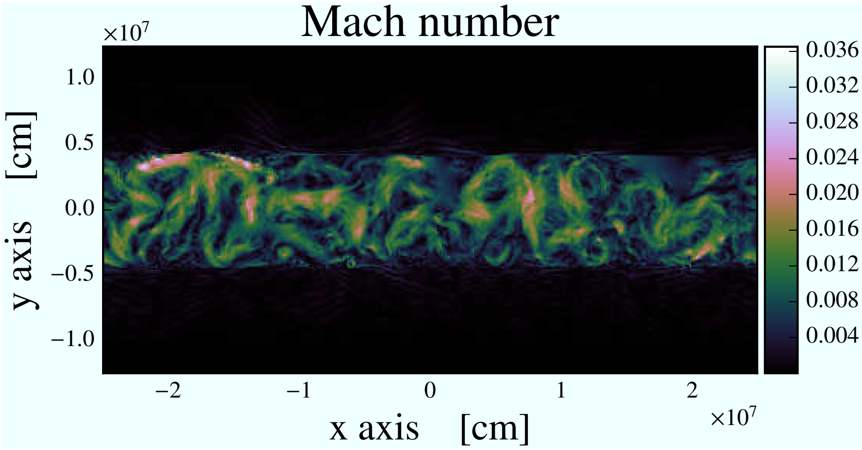

Figure 5. Mach number of the convective layer between two stable layers.

at very low Mach numbers and to a parallelisation that allows to perform simulations on massively parallel supercomputers, such as JUQUEEN. Several examples of application to problems in stellar astrophysics were given.

Acknowledgements

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