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Quark Physics from Lattice QCD

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We report here on two major physical results, that have been obtained using the computer resources of JUQUEEN, both published in Physical Review Letters. The first one¹ concerns the mass difference between the up and down quarks, while the second one² is a determination of the quark content of the nucleon, where we have for the first time in the literature given individual up- and down- quark contents of the proton and the neutron.

1 Up and Down Quark Masses

1.1 Motivation

The up (u) and down (d) quark masses are two fundamental parameters of the Standard Model of Particle Physics. These masses cannot be directly determined through experiment because of the confinement of quarks within hadrons. Lattice QCD provides an *ab initio* approach to the non-perturbative calculation of QCD correlation functions. This method can be used to determine the light quark masses from the experimental values of hadron masses. In earlier work³ we determined precisely m_{ud} , the average of the up and down quark masses, using lattice QCD simulations at the physical values of the quark masses. This quantity has also been studied by many other lattice collaborations and considerable progress has been made on its determination. This time we aimed for the calculation of the light-quark mass difference $\delta m = m_u - m_d$. This quantity is more difficult to obtain than m_{ud} . Its small effect on hadron masses, of order $\mathcal{O}(\delta m/\Lambda_{QCD}) \sim \mathcal{O}(1\%)$, is expected to be comparable in size to the leading $\mathcal{O}(\alpha)$ electromagnetic (EM) corrections, usually not included in lattice simulations.

1.2 Methodology

Our strategy was to first generate fully dynamical $N_f = 2 + 1$ QCD configurations with quark masses down to the isospin symmetric “physical point” of the average up- and down quark mass m_{ud} . We then introduced in the valence sector mass and QED isospin breaking in a controlled fashion. In one set of ensembles we tuned the renormalised valence quark masses so that they are as close as possible to the renormalised sea quark masses so as to minimise the necessary partial quenching. In a second set of valence ensembles, we tuned the isospin splitting to be close to the physical one. This setup ensures that in a power counting scheme where

$$\alpha \sim m_{ud}/\Lambda_{QCD} \sim -\delta m/\Lambda_{QCD}$$

with the isospin breaking parameter $\delta m = m_u - m_d$, the effects we ignore are strictly of second order. At the physical point, all of these expansion parameters are $\sim 1\%$, ensuring

that our approximation makes sense. We of course estimated the size of these neglected contributions to all our observables and reported them in the error budget.

The light quark mass difference δm can be directly computed by going to the physical point and reading off the quark masses there. There are two complications: First, one has to define the physical point in terms of physically observable quantities. Second, one has to measure a properly renormalised mass.

Regarding the first point, we defined the physical point through fixing the charged pion mass and both the neutral and charged kaon masses, the Thomson limit value of the electromagnetic coupling α and either the mass of the Ω^- or the Ξ baryon. Regarding the second point we note that up to the order we are working with, the quark mass difference δm is proportional to the partially quenched meson mass difference

$$\Delta M^2 = M_{dd}^2 - M_{uu}^2 = 2B_2\delta m + \dots$$

with a proportionality constant B_2 that was determined by our collaboration in a previous paper⁴.

Since we use both neutral and charged kaon masses to determine the physical point and it is natural (in a leading order PQ χ PT+QED expansion) to parameterise their square mass difference as

$$\Delta M_K^2 = C_K\alpha + D_K\Delta M^2$$

we can read off the quark mass difference from this and the previous relation (see Fig. 1).

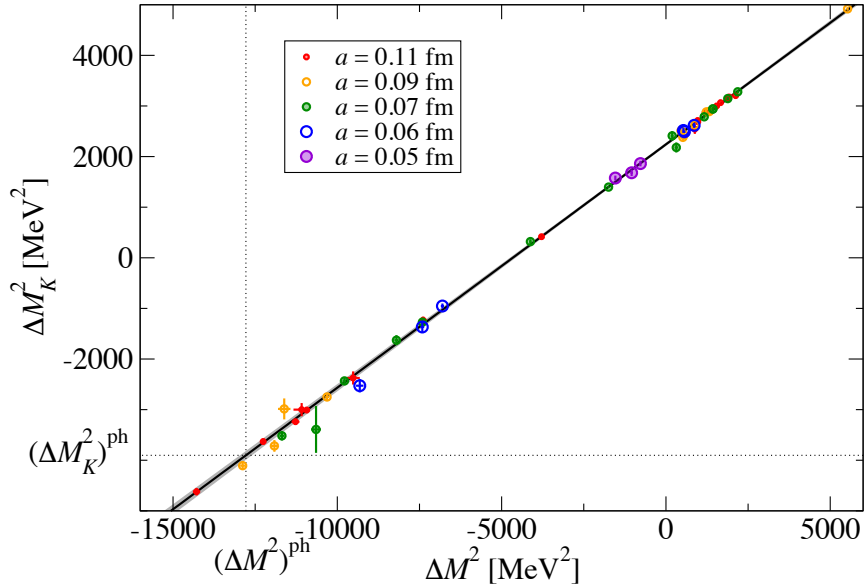


Figure 1. Example of a fit of the dependence of ΔM_K^2 on α and ΔM^2 . Here, ΔM_K^2 is plotted as a function of ΔM^2 . The dependence of the lattice results on all other variables has been subtracted using the fit. The fit has a correlated χ^2/dof equal to 1.59. It is plotted as a solid curve, with its 1σ band.

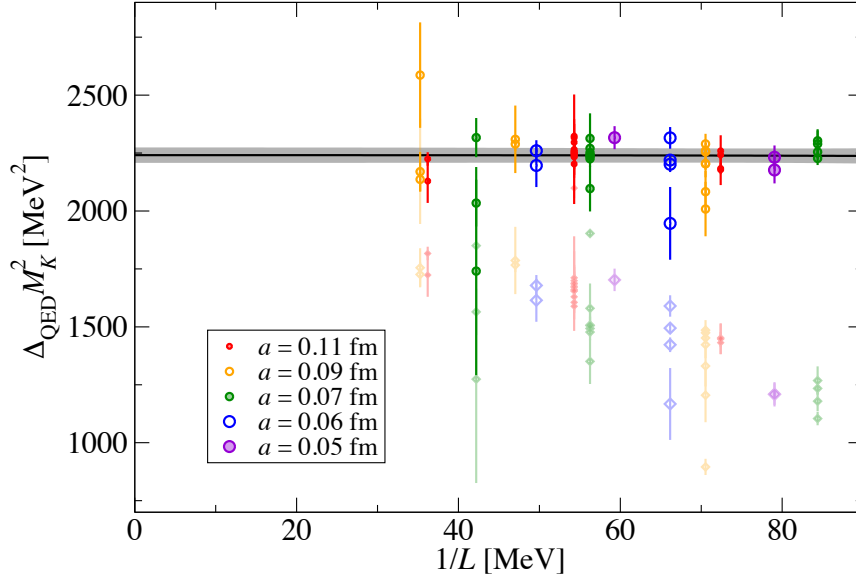


Figure 2. Same fit as in Fig. 1. Here $\Delta_{\text{QED}}M_K^2$ is plotted as a function of $1/L$. The grayed symbols show the full volume dependence of the data. For the plain symbols, the universal $O(1/L)$ and $O(1/L^2)$ finite volume effects from Ref. 5 have been subtracted and the fit to the $O(1/L^3)$ correction is plotted.

Since our data are precise enough to be sensitive to some subleading effects, the parameters C_K and D_K are not constants but do depend on lattice volume, lattice spacing and the quark masses. In particular, the QED part contains power law finite volume corrections that have to be subtracted. The exact form of these subtraction terms up to order $O(1/L^2)$ has been worked out by our collaboration in Ref. 5 and does depend on the aspect ratio T/L of the lattice (see Fig. 2).

1.3 Results

As a direct result of the previously described fit we obtain a value for the QED contribution to the square mass difference of the charged and neutral kaons

$$\Delta_{\text{QED}}M_K^2 = 2186(26)(68)(219) \text{ MeV}^2$$

where we give separately statistical, systematic and an estimate of the partial quenching error. As described previously, this allows us to compute the light quark mass difference (here given in the $\overline{\text{MS}}$ scheme)

$$\delta m = m_u - m_d = -2.41(6)(4)(9) \text{ MeV}.$$

From this result and previous results for the quark mass average published by our collaboration³, we could further give masses for the individual light quarks as well as their ratios. In particular the result for the up quark mass (also in the $\overline{\text{MS}}$ scheme)

$$m_u = 2.27(6)(5)(4) \text{ MeV}$$

implies that the $m_u = 0$ solution to the strong CP problem is very strongly ruled out.

The violation of Dashen's theorem can be characterised by the parameter

$$\varepsilon = \frac{\Delta_{\text{QED}} M_K^2 - \Delta_{\text{QED}} M_\pi^2}{\Delta M_\pi^2}.$$

Since the leading $\mathcal{O}(\delta m)$ corrections to ΔM_π^2 vanish, $\Delta_{\text{QED}} M_\pi^2 = \Delta M_\pi^2$ up to higher order corrections. Therefore, at the level of precision considered in this project, we can take the experimental value for ΔM_π^2 , which together with our result for $\Delta_{\text{QED}} M_K^2$ yields

$$\varepsilon = 0.73(2)(5)(17).$$

1.4 Outlook

As one can see from these results, partial quenching can be the dominant source of error. It is therefore desirable to eliminate this error with a full calculation in QCD+QED. This is work-in-progress.

2 Quark Content of the Nucleon

2.1 Motivation

The second work that was completed recently was the determination of the scalar quark contents of the nucleon

$$f_q^N = m_q \frac{\langle N | \bar{q}q | N \rangle}{2M_N^2},$$

where q can be a quark of any kind. Although they cannot directly be accessed in experiment, they are scheme and scale-independent quantities that allow to translate quark-level couplings into effective, scalar couplings with a nucleon. They are related to a wide variety of observables such as pion and kaon-nucleon scattering amplitudes, quark-mass ratios or quark-mass contributions to nucleon masses. Their knowledge is also very important for Dark Matter (DM) searches, as they allow to convert DM-quark couplings into spin-independent, DM-nucleon cross sections.

In order to compute the quark content of the nucleon, we employed the Feynman-Hellman theorem that relates the quark content to the derivative of the nucleon mass with respect to the quark mass

$$f_q^N = \frac{m_q}{M_N} \frac{\partial M_N}{\partial m_q}$$

at the physical point. We performed this procedure on fully unquenched $N_f = 2 + 1$ configurations that also were the basis of the previous subproject. The physical point is determined using isospin averaged pion, kaon and Ω masses.

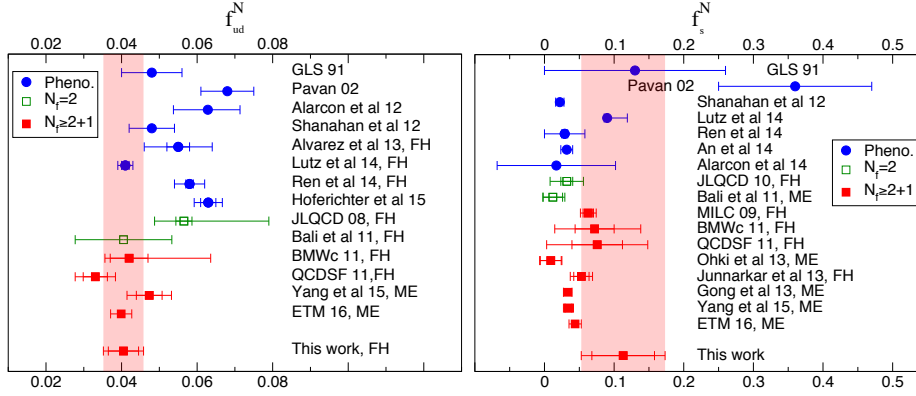


Figure 3. Comparison of our results for f_{ud}^N (left panel) and for f_s^N (right panel) with values from the literature. For lattice based determinations “FH” denotes studies that use the Feynman-Hellmann theorem while “ME” denotes direct computations of the matrix element.

2.2 Methodology

We have computed the derivative of the nucleon mass with respect to both light and strange quark masses. In these calculations, it was usual to replace the quark mass with the squared pion mass as to leading order in χ PT they are proportional. Since we had at our disposal the renormalised quark masses^{3,1}, we decided to instead determine the quark content directly with the quark masses.

Also, isospin splitting is usually ignored in the calculations of the scalar quark content of baryons. In our paper we demonstrated how the individual up- and down- quark contents of both the proton and the neutron can be obtained again up to second order corrections in our power counting with

$$\alpha \sim m_{ud}/\Lambda_{\text{QCD}} \sim -\delta m/\Lambda_{\text{QCD}}$$

with the help of our result on the QED contribution to the nucleon splitting from Ref. 5.

2.3 Results

Reading off the slope of the nucleon mass at the physical point with respect to light and strange mass variations gives us the quark content of the isospin averaged nucleon as

$$f_{ud}^N = 0.0405(49)(35) \quad f_s^N = 0.113(45)(40)$$

and, with the help of the QED contribution to the nucleon splitting from Ref. 5, we obtain individual light quark contents of the proton and neutron of

$$\begin{aligned} f_u^p &= 0.0139(13)(12) & f_u^n &= 0.0116(13)(11) \\ f_d^p &= 0.0253(28)(24) & f_d^n &= 0.0302(28)(25), \end{aligned}$$

where in all cases the first error is statistical and the second is systematic. A comparison of our results to other determinations in the literature is given in Fig. 3.

2.4 Outlook

As the strange quark content is the more important one for dark matter experiments and in addition the charm quark content might be relevant, too, it is evident that more effort should be put into determining them with precision. The current set of configurations is not sufficient to achieve this goal, but with an extension of our $N_f = 4 \times 1$ data set, that was underlying⁵, towards the physical point, a determination of the nucleon strange quark content with a precision of $\sim 10\% - 20\%$ will be possible. This is also a work-in-progress.

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References

1. Z. Fodor *et al.*, *Up and down quark masses and corrections to Dashen's theorem from lattice QCD and quenched QED*, Phys. Rev. Lett. **117**, no.8, 082001, 2016, doi:10.1103/PhysRevLett.117.082001, arXiv:1604.07112 [hep-lat].
2. S. Durr *et al.*, *Lattice computation of the nucleon scalar quark contents at the physical point*, Phys. Rev. Lett. **116**, no.17, 172001, 2016, doi:10.1103/PhysRevLett.116.172001, arXiv:1510.08013 [hep-lat].
3. S. Durr *et al.*, *Lattice QCD at the physical point: light quark masses*, Phys. Lett. B **701**, 265, 2011, doi:10.1016/j.physletb.2011.05.053, arXiv:1011.2403 [hep-lat].
4. S. Durr *et al.* [Budapest-Marseille-Wuppertal Collaboration], *Lattice QCD at the physical point meets SU(2) chiral perturbation theory*, Phys. Rev. D **90**, no.11, 114504, 2014, doi:10.1103/PhysRevD.90.114504, arXiv:1310.3626 [hep-lat].
5. S. Borsanyi *et al.*, *Ab initio calculation of the neutron-proton mass difference*, Science **347**, 1452, 2015, doi:10.1126/science.1257050, arXiv:1406.4088 [hep-lat].