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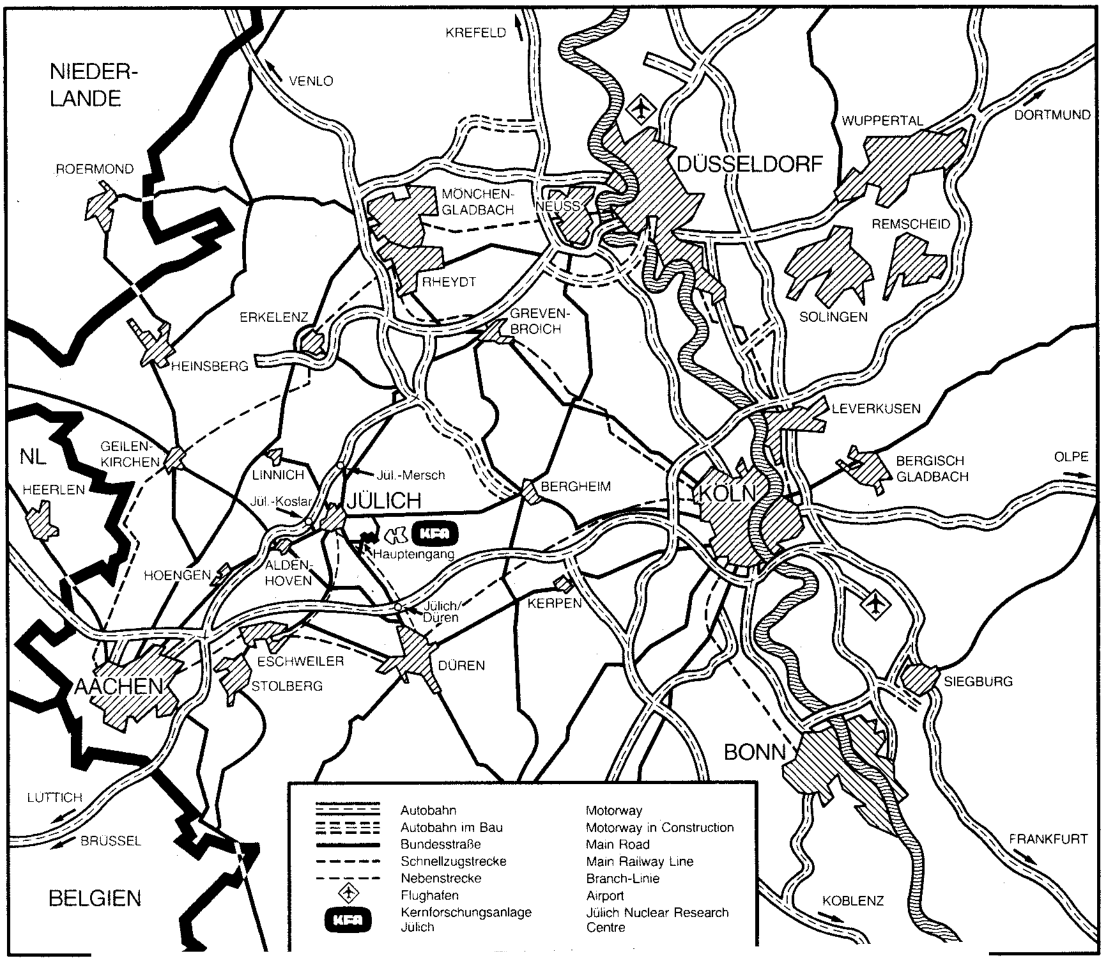

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Abstract

For current profiles with a slope discontinuity at ε_2 close to ε_1 $[q(\varepsilon_1) = 1]$ and a small shear $\hat{s}(\varepsilon_1)$, the jump of the derivative of the magnetohydrodynamic (MHD) solution across the singular q=1 layer is of the form $[\Psi']_{\varepsilon_1} \propto \hat{s}_1[1 - 4f(\mu)\, \varepsilon_1^2/\hat{s}_1^2]$; ε is the finite inverse aspect ratio; $\mu \equiv \hat{s}_1\, \varepsilon_1/8\, (\varepsilon_2 - \varepsilon_1)$ measures the ratio of the two small parameters. The growth rate of the sawtooth trigger can accordingly pass from resistive to MHD values for slight variations either of $\hat{s}_1$ or of $(\varepsilon_2 - \varepsilon_1)/\varepsilon_1$. The above profile properties have been observed in sawtoothing TEXTOR discharges.

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In TEXTOR[1] and in TEXT[2], sawteeth oscillations[3] have been proven to occur whilst the axial safety factor is well below unity. Their period is of the order of 15-20 msec in TEXTOR where precursors, identified as 1/m = 1/1 modes, are observed to develop within a few milliseconds before the central temperature crashes. Since the tearing mode is known to be always unstable when $q_0 < 1$, the question arises, in view of the long magnetic diffusion time-scale, as to how this instability can be characterized by very different growth rates in the course of one sawtooth period.

The growth rate of the tearing mode obtains from a singular layer calculation[4,5] in which the inverse of the jump of the logarithmic derivative of the mhd solution $(1/\Delta')$ defines the boundary condition: $\gamma \equiv \gamma\ (1/\Delta')$. The parameter $1/\Delta'$ must therefore be an extremely sensitive function of the current profile to overcome the difference of orders of magnitude between the acceleration time of the linear growth rate (~1 msec)[6] and the magnetic diffusion time (~ 60 msec at the q=1 surface in TEXTOR). Campbell et al.[7] indicate similarly that the suppression of sawteeth for large periods by means of additional heating in JET was probably achieved with only small changes in the current profiles. It is shown in this work that small variations of the latter will indeed lead to spectacular changes in $1/\Delta'$ provided they are characterized by a rapid change of slope next to, and by a small shear at the q=1 surface. This follows from that the well known logarithmic solution will no longer be genuinely antisymmetric - and therefore will contribute to Δ' - in the limit where dJ/dr is discontinuous at r_1. As it turns out, the required profiles are quite close to those observed.

Although the cylindrical and the toroidal theories differ, the explanation of two time-scales tearing growth can be provided in the framework of either of them but always crucially hangs on keeping finite aspect-ratio terms. The starting equation can be written in the form

$$(1 - \lambda\rho^2) \frac{d}{d\rho} (\rho^3 F^2 \frac{d}{d\rho} \frac{\psi}{F}) = 2\lambda\rho^4 F^2 \frac{d}{d\rho} (\frac{\psi}{F}) + \lambda g \frac{\psi}{F} \qquad (1a)$$

where ψ is the radial component of the magnetic perturbation,

$$g = \rho^3 F^2 \; [\zeta - \frac{2\xi}{F} (1 + \frac{1}{q})] \; , \; F = (1-q)/q \qquad (1b)$$

and for a cylinder[8,9], respectively a torus[10,11]: $\zeta_C = 1$, $\zeta_T = 0(1)$, and $\xi_C = 1$, $\xi_T = - 13/16$ (the last result obtains by considering that the Shafranov shift $\Delta \simeq \varepsilon r/8$ inside the q=1 surface). $\rho = r/a$ is the normalized radius, $\lambda = (a/R)^2$ is the square of the inverse aspect ratio, $q(\rho) = \sqrt{\lambda}\rho B_\phi/B_\Theta$ is the safety factor; a, R, B_ϕ and B_Θ are respectively the plasma minor and major radii and the toroidal and poloidal components of the confining magnetic field. Equation (1a) has a singularity at $\rho=\rho_1$ where the safety factor is unity. The derivative of ψ will consequently be logarithmically divergent unless

$$d(\rho^3 dF/d\rho)/d\rho \propto \rho^2 dJ/d\rho = 0 \qquad (\rho = \rho_1)$$

where J is the current density:

$$J = (\sqrt{\lambda} c B_\phi/4\pi a) \; d(\rho^2/q)/\rho d\rho. \qquad (2)$$

It is easily shown by balancing the different terms of Eq. (1a) that the singular turning point associated with g can be dealt with by expansion if $(16\ \lambda\rho/F')_{\rho_1} \ll 1$. (This condition, not surprisingly, is that for which the Bessel functions which solve the singular turning point equation can be expanded over the whole domain $0 \leqslant \rho \leqslant 1$.) Fortunately, this limitation is here of no concern as we shall find that $1/\Delta'$ gets the required sensitivity to variations of the current profile already when $F'_{\rho_1} \sim 2\sqrt{\lambda}$. The general solution to Eq. (1a) can then be written in the expanded form $\psi = \psi^0 + \lambda\psi^1$ where, upon requesting that $\lim\,(\rho\to 0)\psi$ be finite and that $\lim(\rho\to\infty)\ \psi = 0$,

$$\psi^{(0)-} = F \quad ; \quad \psi^{(0)+} \equiv 0 \ ; \tag{3}$$

$$\psi^{(1)-} = c_1^{(1)-}\, F + F \int_0^{\rho} \frac{d\rho'}{(\rho^3F^2)_{\rho'}} \int_0^{\rho'} d\rho''\ g \tag{4a}$$

$$\psi^{(1)+} = c_2^{(1)+}\, F \int_{\infty}^{\rho} \frac{d\rho'}{(\rho^3F^2)_{\rho'}} \tag{4b}$$

for $\rho \leqslant \rho_1$, respectively $\rho \geqslant \rho_1$. Different representations are needed in the two domains because $F(\rho_1) = 0$.

Continuity at $\rho = \rho_1$ requires that

$$c_2^{(1)+} = \int_0^{\rho_1} d\rho\ g \quad \text{while} \quad \lim_{\rho\to\rho_1} \psi = -\,\lambda c_2^{(1)+}/(\rho^3F')_{\rho_1} \tag{4c}$$

In order to calculate the derivative on both sides of the singular layer, we have adopted the following parametric model of the current density profile:

$$J=J_0 \text{ for } \rho\leqslant\rho_0 \ ; \quad J=\alpha J_0 \text{ for } \rho_0<\rho,\ \rho_1<\rho_2 \ ; \quad J=\alpha J_0(\rho_2/\rho)^4 \text{ for } \rho_2\leqslant\rho; \tag{5}$$

we have accordingly in the three regions

$$F = (1-q_o)/q_o \quad (I) \quad ; \quad F = (\rho_1^2/\rho^2-1)\ (q_o-\alpha)/q_o \quad (II) \quad ;$$

$$F = (\rho_1^2/\rho^2-1)\ (q_o-\alpha)/q_o - (\rho_2^2/\rho^2-1)^2\ \alpha/q_o \quad (III)\ , \qquad (6)$$

where $\rho_o^2/\rho_1^2 = (q_o-\alpha)/(1-\alpha)$ (hence $\alpha \leqslant q_o$). The results of a tedious calculation are:

$$\lim_{\rho\to\rho_1} \frac{\partial\psi}{\partial\rho}^{(1)-} = \frac{\rho_1^2}{4} F'_{\rho_1} \left[\zeta\left(\frac{q_o-\alpha}{1-\alpha} - \frac{1}{2}\right) + \xi\left(\frac{2}{1-\alpha} + \frac{q_o+\alpha}{q_o-\alpha}\,\frac{1-q_o}{1-\alpha} - \frac{1+q_o}{1-q_o}\,\frac{q_o-\alpha}{1-\alpha}\right)\right] ; \qquad (7a)$$

$$\lim_{\rho\to\rho_1} \frac{\partial\psi}{\partial\rho}^{(1)+} = c_2^{(1)+}\ F'_{\rho_1}\ \frac{s}{\rho_1^2}\left(t + \frac{u}{2\sqrt{-v}}\ \ln\frac{1-\sqrt{-v}}{1+\sqrt{-v}}\right) \quad \text{if } \alpha_o^2 \leqslant \alpha\, q_o \qquad (7b)$$

$$\lim_{\rho\to\rho_1} \frac{\partial\psi}{\partial\rho}^{(1)+} = c_2^{(1)+}\ F'_{\rho_1}\ \frac{s}{\rho_1^2}\left[t + \frac{u}{\sqrt{v}}\left(\tan^{-1}\frac{1}{\sqrt{v}} - \frac{\pi}{2}\right)\right] \quad \text{if } \alpha_o^2 \geqslant \alpha\, q_o \qquad (7c)$$

where we have defined the parameter $\alpha_o > \alpha$ through the relation $\rho_2^2 - \rho_1^2 = (\alpha_o - \alpha)\ \rho_1^2/2q_o$, and the quantities

$$s = q_o^2/2(q_o-\alpha)^2\ (\alpha_o^2 - \alpha q_o)\ ;\ t = \alpha(\alpha_o + 2\alpha + \alpha\alpha_o/q_o) + \alpha_o^2 - \alpha q_o\ ;$$

$$u = \alpha(\alpha_o + \alpha)^2/\alpha_o \qquad ;\ v = \alpha(\alpha_o^2 - \alpha\, q_o)/\alpha_o^2(q_o - \alpha) \qquad (8)$$

Furthermore

$$c_2^{(1)+} = \rho_1^4 \left(\frac{q_o-\alpha}{q_o}\right)^2 \left(\frac{1-q_o}{1-\alpha}\right) \left[- \frac{\zeta}{2} - \frac{\alpha\xi}{q_o-\alpha} + \left(\frac{\zeta}{2} - \xi\right) \frac{1-\alpha}{1-q_o} \ln \frac{1-\alpha}{q_o-\alpha}\right] \qquad (9)$$

It is clear from these results that the jump of $\lambda\psi^{(1)'}$ across the singularity will compete best with the jump of $\psi^{(o)'} = - F'_{\rho_1}$ when $\alpha\to q_o$ and $\alpha_o^2 \to \alpha q_o$ simultaneously. Noting that $(q_o - \alpha)/q_o = \hat{s}_1/2$, we thus let

$$q_o - \alpha = \mu(\alpha_o - \alpha) \qquad [\mu = \hat{s}_1\rho_1/8\,(\rho_2 - \rho_1)] \qquad (10)$$

with $\mu \geqslant 2$ if $\alpha_o^2 \leqslant \alpha q_o$ and $\mu \leqslant 2$ if $\alpha_o^2 \geqslant \alpha q_o$. We obtain in leading order

$$\frac{1}{\Delta'} = - \frac{\xi\,(\lambda\rho_1^3/2)}{\hat{s}_1 + 4\xi\,\lambda\rho_1^2\, f(\mu)/\hat{s}_1} \qquad (11)$$

where, respectively for $\mu \geqslant 2$ and for $\mu \leqslant 2$,

$$f(\mu) = \frac{\mu}{2-\mu}\left[2 - \sqrt{\frac{\mu}{\mu-2}}\,\ln \frac{(\sqrt{\mu} + \sqrt{\mu-2})^2}{2}\right] \qquad (12a)$$

$$f(\mu) = \frac{2\mu}{2-\mu}\left[1 + \sqrt{\frac{\mu}{2-\mu}}\left(\tan^{-1}\sqrt{\frac{\mu}{2-\mu}} - \frac{\pi}{2}\right)\right] . \qquad (12b)$$

$f(\mu)$ is a monotonically increasing function with $f(0) = 0$, $f(2) = 2/3$, $\lim\,(\mu\to\infty)\, f(\mu) = \ln 2\mu$. The second term in the denominator of Eq. (11) does not appear in the literature. It arises, see Eq. (4b), from the second lowest order homogeneous solution to Eq. (1), the (constant) amplitude factor of which is, in view of the boundary conditions, proportional to ε_1^2. This explains why, as shown before, the expansion is still valid though the two terms in (11) may be of the same order of magnitude. This arises if, at the q=1 surface, the shear parameter $\hat{s}_1$ is not appreciably larger than the inverse aspect ratio $\varepsilon_1 = \sqrt{\lambda}\ \rho_1$, and if the radius ρ_2, where the change of slope in the current density profile occurs, is sufficiently close to ρ_1 [$f(\mu)$ finite or large]. The variation of the toroidal $1/\Delta'$, after normalizing the gradient to the ion Larmor radius of a 1 keV, 2 Tesla, Deuterium TEXTOR plasma (a/R = 46 cm/175 cm) is shown in Fig. 1 for $\rho_1 = 0.43$ (r_1 = 19.8 cm) and, respectively, $\mu = 2$ and $\mu = 0$ (the limit $\mu = 0$ and the usual approximation $\varepsilon \equiv 0$ coincide).

Calculation of the growth rate as function of $\hat{s}_1$ has been performed with a resistive fluid model of the singular layer [Eq. II-11 of Ref. 5 where one lets the electron diamagnetic drift frequency $\omega_e^* \rightarrow 0$] and the above TEXTOR parameters, assuming moreover that $N(\rho_1) = 5 \times 10^{13}\ cm^{-3}$ (density) and $Z(\rho_1) = 2$ (effective charge). The numerical results in toroidal geometry are shown in Fig. 2 ; asymptotic curves obtained in two different limits, viz.

$$\gamma\tau_e = (\sqrt{0.51}\ \hat{s}_1/k\ \beta_i^*)^{2/3} \tag{13a}$$

$$\gamma\tau_e = 1.16\ \frac{(-a/a_i)\ (\lambda\rho_1^3/4)}{-(16/13) + 4\ \lambda\rho_1^2\ f(\mu)\ \hat{s}_1^{-2}}\ , \tag{13b}$$

are given for $\mu = 2$. Here $\beta^* = 4\pi N\ T_i m_i/m_e B^2$ and $k = Rr_1/a_i\ c_e\ \tau_e$ ($c_e = \sqrt{T_e/m_e}$; $a_i = c_i/\Omega_i$; τ_e is the Braginskii electron collision time). It clearly appears that for a small absolute variation of $\hat{s}_1$, say from 2×10^{-3} to 1×10^{-1}, the growth rate jumps from 0.3

msec^{-1} - this can be tolerated over a sawtooth period - to 20 msec^{-1}: this corresponds to the time-scale of the central temperature crash. This variation of $\hat{s}_1$ is possible in a sawtooth period since the magnetic diffusion time-scale is about 60 msec at the q=1 surface of TEXTOR. The acceleration time-scale of the growth rate in the regime described by Eq. (13b) is $[d\ln \gamma/dt]^{-1} = (0.034 - 16\ \hat{s}_1^2/13)[0.068\ d\ln \hat{s}_1/dt]^{-1} \simeq 0.32 \times 0.1 \times 60$ msec $\simeq 1.9$ msec for $\hat{s}_1 = 0.1$ and goes to zero when $\hat{s}_1 \rightarrow 0.166$; this critical value increases with μ. It is stressed that similar variations of γ can also be obtained by changing $(\rho_2 - \rho_1)/\rho_1$. We note that the conventional theory (corresponding to $\mu = 0$) predicts only one time scale: a slow one in toroidal geometry ($\gamma\tau_e \sim 0.01$) and a fast one in cylindrical geometry ($\gamma\tau_e \sim 0.31$).

The conclusion from the above analysis is thus that the growth rate of the precursors can change by orders of magnitude during one sawtooth period provided the following two conditions are fulfilled:

$$\hat{s}_1/2 \equiv 1 - 2\pi\ RJ_1/c\ B_\phi \ll 1 \qquad \text{and} \qquad (\rho_2 - \rho_1)/\rho_1 \ll 1. \qquad (14)$$

[the relation $\rho_1^2 \simeq 1/2\ q_a$ also obtains here, but depends on the model profile assumed outside $\rho = \rho_1$ (Eq. 5)]. These constraints are well verified by the Faraday rotation measurement of the current density profile in TEXTOR (Figs. 5 and 6 of Ref. 1). To obtain a quantitative comparison, we have calculated the parameters entering our model for the discharge 15088 [$q_0 = 0.69$ (measured), $B = 1.49$ T, $q_a = 2.1$] assuming the two terms in the denominator of Eq. (11) to be equal and, again, $\mu = 2$. Thus

$$\rho_1 = 0.49 \quad ; \quad \alpha = 0.62 \quad ; \quad \rho_o = 0.21 \quad ; \quad \rho_2 = 0.50 \; .$$

Figure 3 shows both the experimental and the theoretical current density profiles. The theoretical results have been corrected for toroidal effects by considering that $J \propto B \propto 1/R$ in a force free field $(\vec{J} \times \vec{B} = 0)$. The agreement is obviously remarkable both as concerns the values of the plateau current (J_1) and the experimental and theoretical radii r_2 which are both close to r_1. Fig. 6 of Ref. /1/ shows moreover that in non-disruptive discharges, $0.75 \leqslant 2\pi R J_1/c B_\phi \leqslant 0.8$ in additional support to theory.

Wesson et al.[12] have suggested that the instability involved in the very fast collapse phase of the sawteeth is an ideal m=1 "quasi-interchange" mode. The idea has been farther explored in Ref. 13. Objections are that the model would apply to flat current profiles only and fails to account for the suddenness of the onset. Both difficulties can be overcome with our model since it applies independently of the value of q_o (which seems more appropriate to the experimental situation) and since the acceleration of the growth rate becomes infinite either if the shear $\hat{s}_1$ or if the parameter $(\rho_2 - \rho_1)/\rho_1$ reach certain values (we recall that ρ_2 is the radius where the slope of the current profile is assumed to be discontinuous).

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Figure Captions

Fig.1: Variation of the jump, in toroidal geometry, of the inverse of the logarithmic derivative of the magnetohydrodynamic solution across the singular q=1 layer (normalized to the ion Larmor radius) versus the parameter $(q_0 - \alpha)/q_0 = \hat{s}_1/2$ for $\mu = 2$ and $\mu = 0$ (this case coincides with the $\varepsilon = 0$ theory).

Fig.2: The normalized growth rate $\gamma\tau_e$ versus the parameter $(q_0 - \alpha)/q_0 = \hat{s}_1/2$ for $\mu = 0$ and $\mu = 2$. The dashed curves are obtained by asymptotic expansion of the dispersion relation.

Fig.3: Comparison of the measured current density profile with that predicted by theory for the parametric model of Eq. (5) (TEXTOR shot 15088).

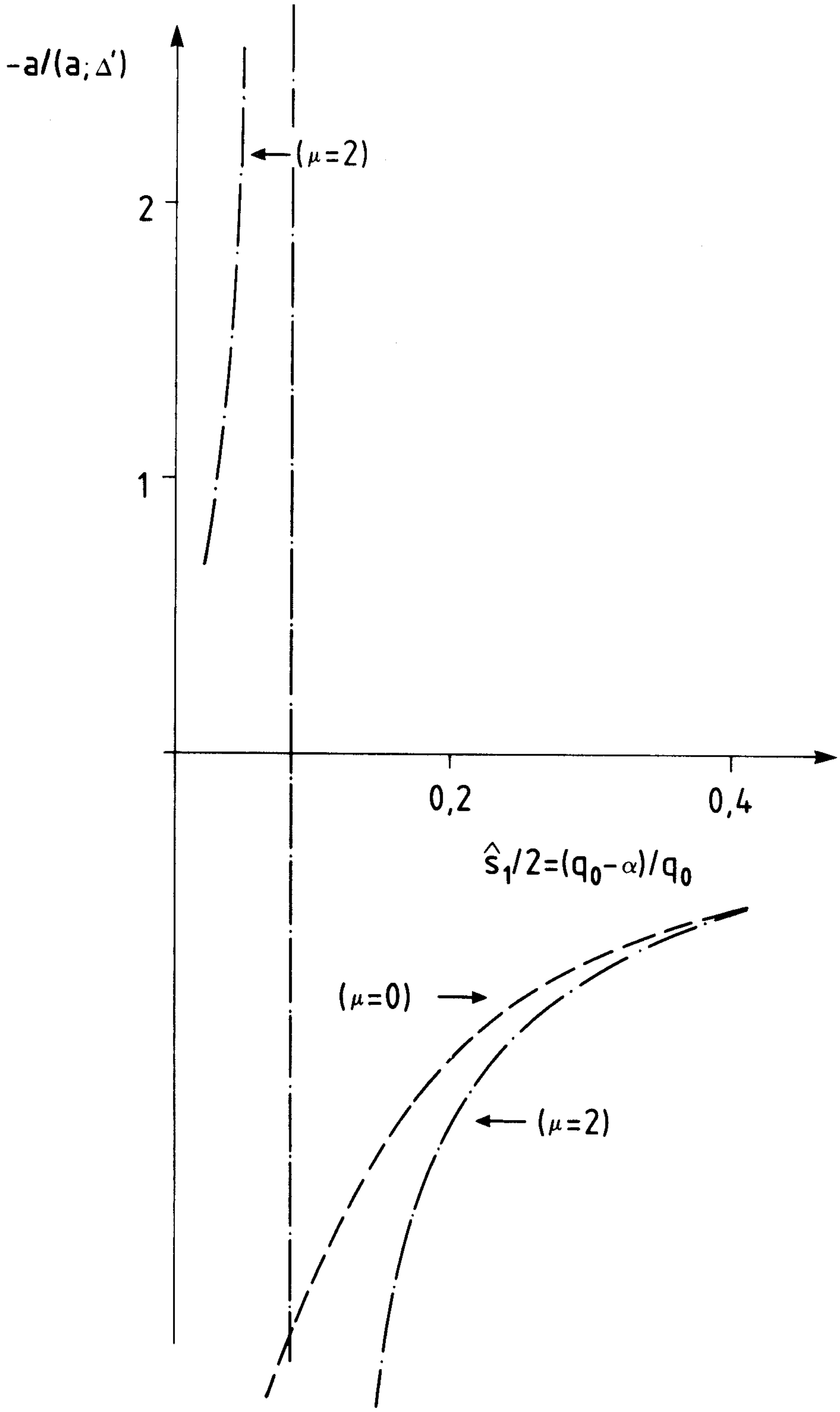

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Fig.1

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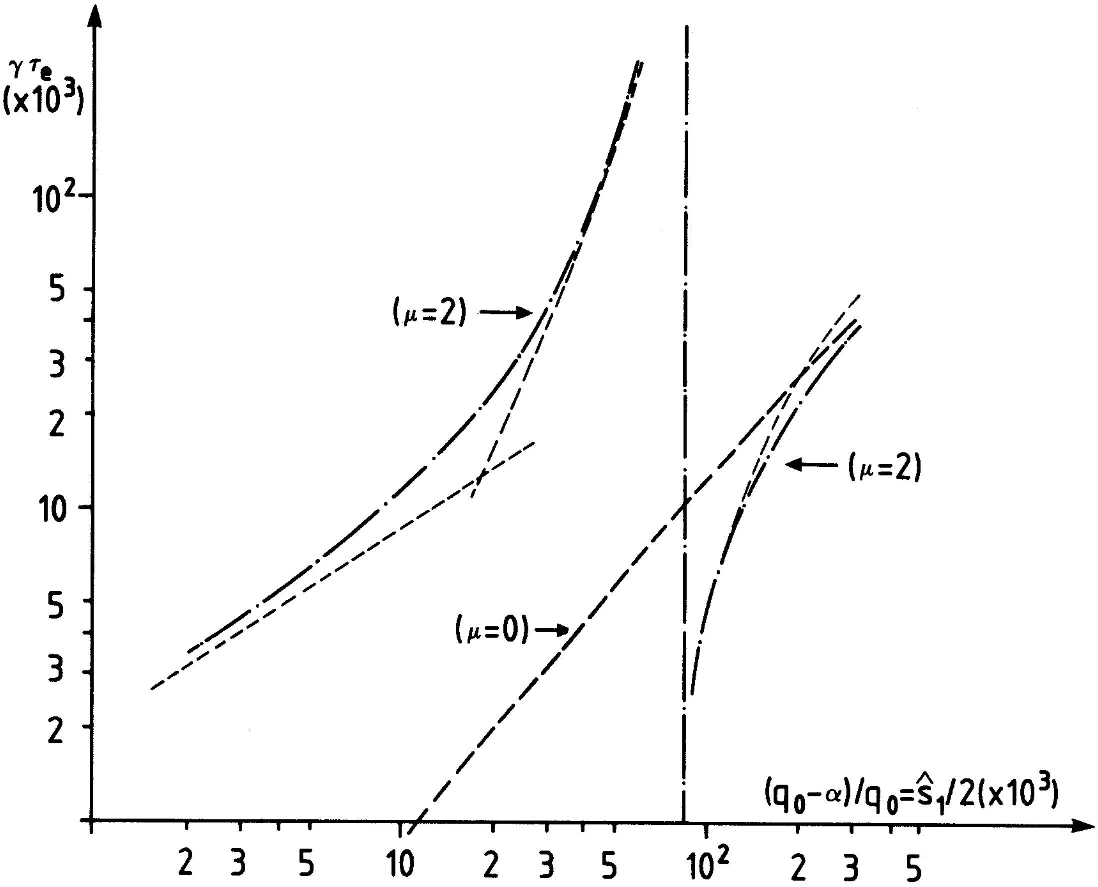

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Fig.2

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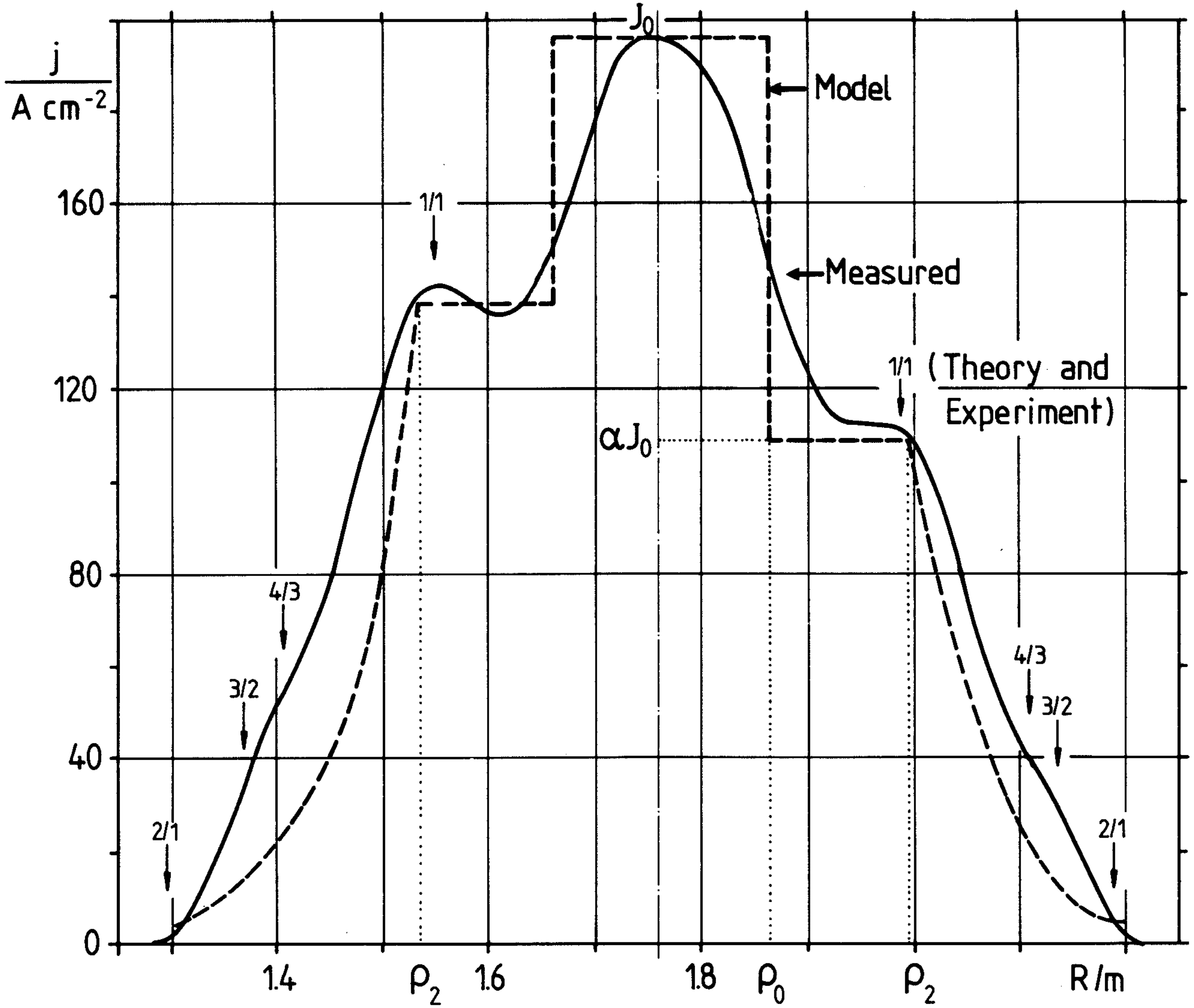

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Fig. 3

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