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Helium Particle Confinement
and Exhaust Efficiency**

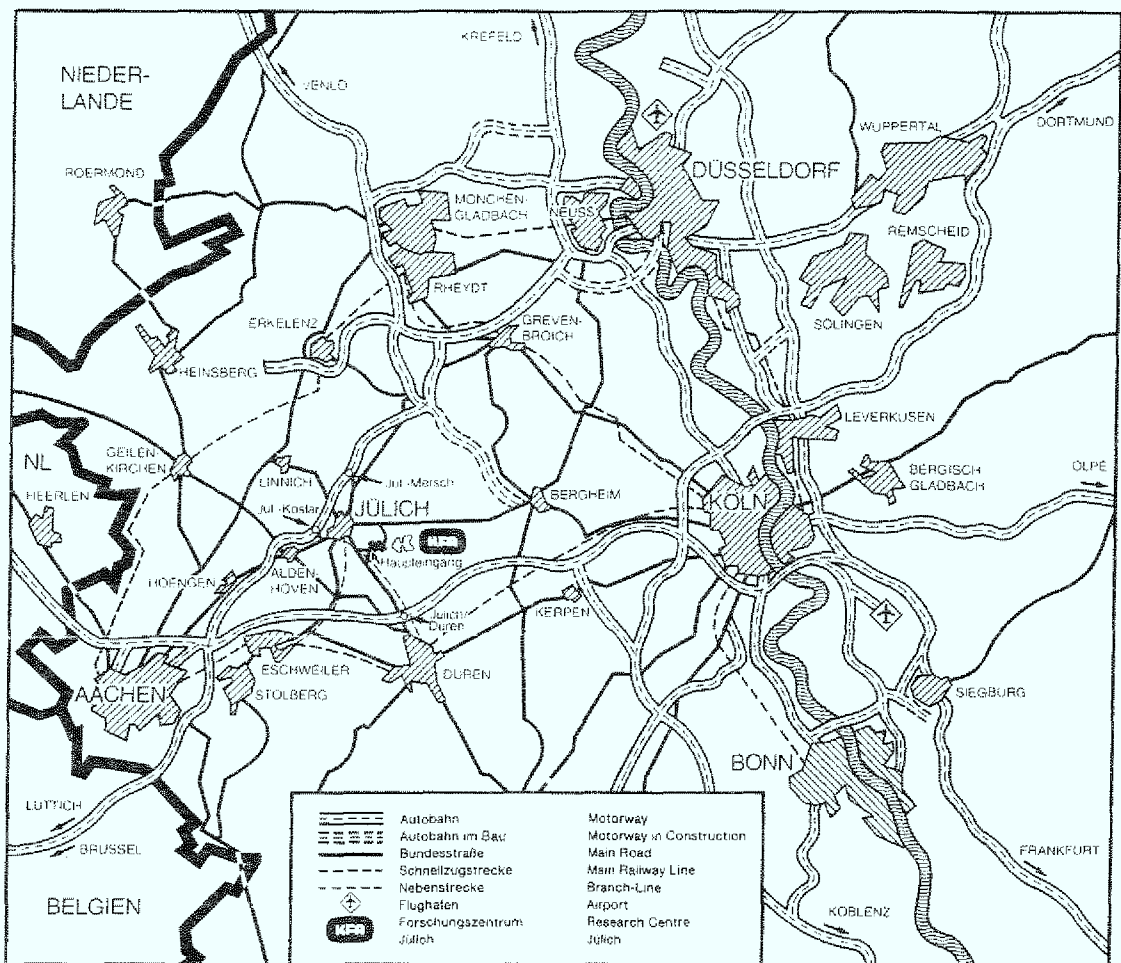
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Burn Condition for Fusion Plasmas, Helium Particle Confinement and Exhaust Efficiency

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Abstract

The burn condition for D-T plasmas is derived in a form which accounts for the stationary helium concentration being selfconsistently determined by the coupling of the fusion power and helium production. The ratio of the global α -particle confinement time τ_α^* (definition in the text) to the energy confinement time τ_E is taken as a quantity characterizing the prevailing confinement and exhaust regime. It is shown that for an ignited and stationary burning D-T plasma this ratio needs to remain sufficiently below 15 (or below 10 for typical impurity concentrations). This poses a lower limit to the required helium exhaust efficiency and indicates that operational regimes might be desirable where the internal particle confinement time does not diverge too far from and in excess of the energy confinement time, and where the boundary layer — e.g. through high particle density and outward flow — acts as a sufficient shield against recycled α -particles.

It is further demonstrated that for any given temperature (and for otherwise equal overall exhaust and recycling conditions) either burning can be achieved at two quite different values of the fusion parameter (triple product $n \cdot T \cdot \tau_E$) and ash concentrations (one at low concentrations in the range 5 – 20 %, the other in the range of 20 – 35 %) or not at all.

A similar evaluation applied to the D-³He fusion reaction leads to even more stringent limits on the ratio τ_α^*/τ_E , which must be smaller by a factor 3 to 4 compared to the D-T case.

0. Introduction

Helium — as the ash of D-T-fusion reactions — and its effect on the pumping requirements, on the burn condition, or on the minimum reactor size, has already been considered in several publications (e.g. /1-3/).

In these previous studies or as their result it is generally postulated that the relative concentration of helium in a quasistationary burning plasma should not exceed the range of about 5%–10%; such concentrations are considered to be unavoidable, but tolerable in view of fuel dilution (at a given density or pressure limit) and radiation losses.

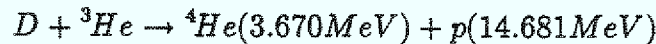
In this paper we discuss a different approach in order to obtain a correlation between burn condition, exhaust requirement, and the ratio between energy confinement time τ_E and global helium particle confinement time τ_α^* (a definition is given in section 5). The main difference is our accounting for the coupling of the α -power production rate P_α and the effective ash (and impurity) concentration in the plasma, as e.g. measured by an effective charge number Z_{eff} . The existence of such a correlation is obvious since the fusion power and the α particles are consequences of the same reaction, i.e. the D-T fusion process



Instead of prescribing a constant concentration $f_{He} = n_{He}/n_e$, we therefore solve for f_{He} from the global power and α -particle balance equations (section 2). A trivial result of this correlation is the fact that without any outward transport and removal of the He ashes the growth rate of its relative concentration is typically $0.01 \cdot \tau_E$; consequently our consideration on the required stationary burn condition addresses burn times in excess of about ten energy confinement times (which would be too short for an economic pulsed reactor operation).

One result is the existence (if any) of two solutions for f_{He} . This means that an ignited plasma can burn in either a “low He concentration mode” with $f_{He} \sim 5 - 15\%$, or in a “high He concentration mode” with $f_{He} \sim 15 - 25\%$ and much higher required ignition parameters $n \cdot T \cdot \tau_E$ in the latter case. The range of allowed temperatures T and accessible ignition parameters is reduced with increasing ratio τ_α^*/τ_E and finally vanishes. Enhanced shrinking of the accessible parameter space occurs if additional impurities — from plasma wall interaction — are included, e.g. by adding one impurity species with charge number Z and a constant concentration $f_Z = n_Z/n_e$. This is discussed in section 3.

For comparison we have carried out a similar analysis based on the fusion reaction



which is often considered as a far goal in view of reduced neutron release and other positive safety prospects. This is discussed in section 4. Here both fusion products contribute to the fuel dilution and the burn temperature is much higher. The result is that the requirements for particle outward transport and for an efficient ash removal system are even more severe.

1. Basic results of confinement studies and motivation

Starting point of our consideration is the generally observed correlation between energy confinement time τ_E and particle confinement time τ_p or, in other words, between the coefficients of radial heat conduction and of particle diffusion. This correlation appears to be a characteristic feature of specific confinement regimes /4 – 7/, with a tendency for a disproportional increase of particle confinement in cases of improved energy confinement. Recent experimental data for χ_e/D_e range between 7, reported by the JET-team /4,5/, and 3, reported by the ASDEX-team /6/ (this difference is partly due to the fact that for the analysis of the ASDEX data an additional inward particle flow term has been introduced to describe the overall particle balance). For “Advanced Stellarators”, a ratio of about 5 is predicted from theoretical analysis /7/. We will assume for the following that this ratio is a constant.

If the corresponding transport coefficients η (e.g. $\eta = D_e$ or $\eta = \chi_e$) are constants, then the time dependence of a transported quantity $\psi(r, t)$ (e.g. particle- or energy density) can be written as

$$\psi(r, t) = \psi(r) \cdot \exp(-t/\tau) \quad . \quad (1.1)$$

Then the relationship between η and the confinement time τ reads:

$$\eta \sim \frac{1}{\tau} \quad (1.2)$$

and assuming $D_\alpha \approx D_e$, we define:

$$\rho := \frac{\tau_\alpha^*}{\tau_E} \approx \frac{\chi_e}{D_e} \quad (1.3)$$

Therefore, in what follows we shall analyse how the ratio ρ between the α -particle confinement time τ_α^* and the energy confinement time τ_E enters and modifies the burn condition and, in particular, examine whether there exists a critical value of this ratio beyond which stationary burning is impossible. Consequently, we take this ratio (and not any longer the relative helium concentration) as free parameter.

As a first approach, and in order to obtain closed form relations between characteristic global quantities, we use a zero dimensional model for our analysis. Although profile effects can easily be included by an appropriate parametrisation they have no qualitative and only minor quantitative consequences for our results. Therefore, and in order to keep the number of parameters as small as possible, we leave these effects out of the present study.

In order to evaluate the results of the following sections, which are first given in terms of ρ , with regard to more controllable reactor parameters such as the removal efficiency ϵ_{rem} , refueling efficiency γ or wall recycling coefficient R , we will derive a definition of the global particle confinement time τ_α^* for α particles in section 5. For the time

being, we note that the global α particle confinement time τ_α^* used in the next sections is defined as:

$$\tau_\alpha^* := \frac{N_\alpha}{\Phi_{\alpha,out} - \Phi_{\alpha,ret}} \quad (1.4)$$

Here N_α is the number of α particles in the burning region with volume V , and $\Phi_\alpha = \Phi_{\alpha,out} - \Phi_{\alpha,ret}$ is the net flux of α particles leaving this region through the surface S (fig. 1), defined as the difference between outgoing and returning flux $\Phi_{\alpha,out}$ and $\Phi_{\alpha,ret}$, respectively.

Often, the global particle confinement τ_p^* is experimentally determined in terms of $N/\dot{N}$ after all sources are "turned off". However, since we are considering here (quasi-) stationary discharge conditions, we rather apply the equivalent definition (1.4) (a la Gauss theorem) on the global α -particle confinement time τ_α^* in such way, that the stationary fuel (D-T) density profile be maintained through an adequate external fueling process which replaces both, the burned fuel particles and that part of the fuel particles which is necessarily being pumped away together with the helium ashes. Furthermore the helium particles, which are pumped away, are replaced by the fusion process as a source term. In fact, it is the purpose of our investigation to calculate the stationary He-concentration (and its influence on the burn condition) which is established as a result of the global α particle confinement time τ_α^* (see section 5).

2. Global relations for a burning D/T plasma

In recalling the basic equations describing an ignited D/T plasma, namely the power- and particle balance equations, we follow ref. /8/, where the effect of impurities on the ignition condition has been addressed.

We make the following simplifying assumptions:

- 1.) a homogeneous plasma where all species have the same temperature T :
 $T = T_e = T_i = T_{He} = T_Z$
- 2.) the impurity concentration consists of a single species, in addition to the helium ash which is accounted for separately.
- 3.) the α -particles are fully thermalized before participating in the loss processes as a component of the thermal plasma; i.e. they transfer their whole initial energy of $E_\alpha = 3.52$ MeV to the plasma.
- 4.) the pressure of the energetic non-thermalized α -particles is neglected in the pressure balance.

Let again $f_Z = n_Z/n_e$ and $f_{He} = n_{He}/n_e$ be the concentration of impurities (charge state $\hat{Z}$, nuclear charge number Z) and α particles (He^{++}) relative to the electron density n_e , respectively. Because we will exclusively be dealing with low Z impurities in this report, and with plasma temperatures ~ 10 keV and larger, we assume $\hat{Z} = Z$

throughout this paper, i.e. fully stripped impurities. We assume $n_D = n_T$, i.e. equal deuterium and tritium densities, and set $n_i = n_D + n_T$. From quasi-neutrality, we have for the dilution parameter

$$f_i := \frac{n_i}{n_e} = 1 - Z \cdot f_Z - 2 \cdot f_{He}, \quad (2.1)$$

and for the total particle density:

$$n_{tot} := n_e + n_i + n_Z + n_{He} = n_e [2 - (Z - 1)f_Z - f_{He}] =: n_e \cdot f_{tot}. \quad (2.2)$$

The ignition condition expresses the balance between α -power production P_α and losses P_t and P_{rad} (P_t : heat transport losses due to conduction and convection, P_{rad} : radiation losses) and is written in the form:

$$\frac{1}{4} n_e^2 \cdot f_i^2 E_\alpha \cdot \langle \sigma v \rangle = \frac{3}{2} \cdot \frac{1}{\tau_E} \cdot n_e \cdot f_{tot} \cdot T + R_{rad}(T, f_Z, f_{He}) \cdot n_e^2. \quad (2.3a)$$

If a different definition of τ_E is used, such that τ_E implicitly also includes the radiation losses the ignition condition is

$$\frac{1}{4} n_e^2 \cdot f_i^2 E_\alpha \cdot \langle \sigma v \rangle = \frac{3}{2} \cdot \frac{1}{\tau_E} \cdot n_e \cdot f_{tot} \cdot T. \quad (2.3b)$$

For the radiation loss rate R_{rad} we take the expression

$$\begin{aligned} R_{rad} &= R_b + R_Z = f_i \cdot R_{b,1}(T) + f_{He} \cdot R_{b,2}(T) + f_Z \cdot R_{rad,Z}(T) \\ &= C_B \cdot T^{\frac{1}{2}} \left[f_i \cdot g_{ff}\left(\frac{1}{T}\right) + f_{He} \cdot 4 \cdot g_{ff}\left(\frac{4}{T}\right) \right] + f_Z \cdot R_{rad,Z}(T) \end{aligned} \quad (2.4)$$

Here C_B is the constant for bremsstrahlung: $C_B = 1.537 \cdot 10^{-29} \text{ eV}^{\frac{1}{2}} \cdot \text{Amp} \cdot \text{cm}^3$, g_{ff} are the Gaunt factors taken from ref./9/. We have used the abbreviation $R_{b,k}(T) = C_B \cdot T^{\frac{1}{2}} \cdot k^2 \cdot g_{ff}\left(\frac{k^2}{T}\right)$, while $f_Z \cdot R_{rad,Z}(T)$ is the cooling rate for impurities of nuclear charge number Z taken from ref./10/. For the temperature range $5 < T [\text{keV}] < 100$ and the low Z impurities considered here these corona approximation rates are very close to the bremsstrahlung rate alone: $f_Z \cdot R_{rad,Z}(T) \sim f_Z \cdot R_{b,Z}(T) = f_Z \cdot C_B \cdot T^{\frac{1}{2}} \cdot Z^2 \cdot g_{ff}\left(\frac{Z^2}{T}\right)$. The fusion reaction rate $\langle \sigma v \rangle$ is the average over a Maxwellian energy distribution and is taken from ref./11/.

Eq. (4) is then rewritten in the form

$$n_e \cdot \tau_E = \frac{\frac{3}{2} \cdot f_{tot} \cdot T}{\frac{\langle \sigma v \rangle}{4} \cdot f_i^2 \cdot E_\alpha - R_{rad}(T)} \quad (2.5a)$$

Inserting the total pressure $p_{tot} := n_{tot} \cdot T$, the fusion product $p_{tot} \cdot \tau_E$ becomes

$$p_{tot} \cdot \tau_E = \frac{\frac{3}{2} \cdot f_{tot} \cdot T^2}{\frac{\langle \sigma v \rangle}{4} \cdot f_i^2 \cdot E_\alpha - R_{rad}(T)}. \quad (2.5b)$$

The ignition parameters eqs. (2.5a, 2.5b) are usually plotted as functions of T for fixed Z , f_Z and f_{He} (figs. 2 and 3), but f_{He} and the α power source P_α are not independent of each other and depend strongly on T .

The α -particle balance reads (with $\rho = \tau_\alpha^*/\tau_E$):

$$\frac{1}{4} \cdot n_e^2 \cdot f_i^2 \cdot \langle \sigma v \rangle = \frac{n_{He}}{\tau_\alpha^*} = \frac{n_e \cdot f_{He}}{\rho \cdot \tau_E} \quad (2.6)$$

or

$$n_e \cdot \tau_E = \frac{4 \cdot f_{He}}{\rho \cdot f_i^2 \langle \sigma v \rangle} \quad (2.7)$$

Equating (2.5a) and (2.7) and eliminating f_i by using eq. (2.1) yields the cubic equation for f_{He} :

$$a_0 + a_1 \cdot f_{He} + a_2 \cdot f_{He}^2 + a_3 \cdot f_{He}^3 = 0 \quad (2.8)$$

with

$$a_0 = -\frac{3}{2}T \cdot [f_Z^3(-Z^3 + Z^2) + f_Z^2(4Z^2 - 2Z) + f_Z(-5Z + 1) + 2] \quad (2.8.1)$$

$$a_1 = -\frac{3}{2}T \cdot [f_Z^2(4Z - 5Z^2) + f_Z(14Z - 4) - 9] + \quad (2.8.2)$$

$$+ \frac{E_\alpha}{\rho} \cdot [f_Z^2 \cdot Z^2 - f_Z \cdot 2 \cdot Z + 1] + \\ + \frac{4}{\langle \sigma v \rangle \cdot \rho} \cdot [(f_Z \cdot Z - 1) \cdot R_{b,1} - f_Z \cdot R_{rad,Z}(T)]$$

$$a_2 = -\frac{3}{2} \cdot T \cdot [f_Z(-8Z + 4) + 12] + \quad (2.8.3)$$

$$+ \frac{E_\alpha}{\rho} \cdot [f_Z \cdot 4 \cdot Z - 4] + \\ + \frac{4}{\langle \sigma v \rangle \cdot \rho} \cdot [2 \cdot R_{b,1}(T) - R_{b,2}(T)]$$

$$a_3 = -\frac{3}{2} \cdot T \cdot [-4] + \quad (2.8.4)$$

$$+ \frac{E_\alpha}{\rho} \cdot [4]$$

3. Results and Discussion

The algebraic equation (2.8) can be solved in closed form and has three (not necessarily different) solutions.

We are only interested in real solutions with $0 \leq f_{He} < 0.5 \cdot (1 - Z \cdot f_Z)$ which are physically relevant.

Inserting the f_{He} found that way in the eq. (2.5) or (2.7), we have expressed the ignition parameter (2.8) as function of T , Z , f_Z and ρ . In what follows we discuss 4 different cases.

Case 1: no impurities ($f_Z = 0$), radiation losses implicitly accounted for in τ_E , i.e. we set $R_{rad} = 0$ in (2.3)

In this case the coefficients of the polynomial equation (2.8) simplify to

$$a_0 = -3 \cdot T, \quad (3.1.1)$$

$$a_1 = \frac{E_\alpha}{\rho} + \frac{27}{2} \cdot T, \quad (3.1.2)$$

$$a_2 = (-4) \cdot \left(\frac{E_\alpha}{\rho} + \frac{9}{2} \cdot T \right), \quad (3.1.3)$$

$$a_3 = 4 \cdot \left(\frac{E_\alpha}{\rho} + \frac{3}{2} \cdot T \right) \quad (3.1.4)$$

Beside the unphysical double root $f_{He} = 0.5$, the only solution is

$$f_{He} = \frac{3 \cdot T}{\frac{E_\alpha}{\rho} + \frac{3}{2} \cdot T}. \quad (3.1.5)$$

We will discuss this solution together with the more general case 2, and note here only that from the condition $f_{He} < 0.5$ we have:

$$T < T_{crit} = \frac{E_\alpha}{4.5 \cdot \rho} = \frac{782.2}{\rho} \quad (3.1.6)$$

The limit in T can clearly be seen in fig. 4 where the ignition parameter $n_e \cdot \tau_E$, plotted as function of T , has a singularity and the range of accessible temperatures is bounded from above by T_{crit} , if $\rho = \text{constant} > 0$. This differs from the case shown in fig. 3 with $f_{He} = \text{constant} \geq 0$ where no such singularity appears.

Case 2: $f_Z \neq 0$, radiation losses implicitly accounted for in τ_E

The coefficients of the polynomial (2.8) now take the form given in eqs. (2.8.1) to (2.8.4), except that the last terms in a_1 and a_2 resp., which begin with $4/(\sigma v) \cdot \rho$, must be omitted.

Again we find an unphysical double root at the upper bound for f_{He} , i.e. at $0.5 \cdot (1 - Z \cdot f_Z)$. The only valid solution is now given by:

$$f_{He} = \frac{3 \cdot T \cdot (1 - 0.5 \cdot (Z - 1) \cdot f_Z)}{\frac{E_\alpha}{\rho} + \frac{3}{2} \cdot T}. \quad (3.2.1)$$

The equation (3.1.6) for the critical temperature generalizes to:

$$T < T_{crit} = \frac{E_{\alpha} \cdot (1 - Z \cdot f_Z)}{(4.5 + 3 \cdot f_Z - \frac{3}{2} \cdot Z \cdot f_Z) \cdot \rho} \quad (3.2.2)$$

which (see fig. 5) e.g. for a 2 % carbon ($Z=6$) contamination would lead to $T_{crit} = 707.2/\rho$, or for a concentration of 4 % instead to $T_{crit} = 628.0/\rho$.

Case 3: $f_Z = 0$, but radiation losses explicit, i.e. not included in τ_E .

Excluding the radiation losses from the definition of the confinement time τ_E has the immediate effect that the range of possible temperatures for ignition becomes finite since the denominator in eqs. (2.5) must be positive. The most striking difference to cases with a constant f_{He} as parameter is the fact that now there are either exactly two physically valid solutions, i.e. solutions in the range $0 \leq f_{He} < 0.5$, or none.

Fig. 6 shows the relative helium concentrations as function of T and parameter ρ . It is seen that there are two branches, a lower one with f_{He} in the (5 – 20) % range and an upper one in the (20 – 35) % range. The helium concentrations are used in fig. 7 to plot the ignition parameters $n_e \cdot \tau_E$ and $p_{tot} \cdot \tau_E$ as functions of T . We further note that the temperature range in which ignition is possible has narrowed from 4.5 keV – 889 keV to only 6 keV – 56 keV for $\rho = 5$, and to 9 keV – 20 keV for the case $\rho = 13$. It will totally disappear for $\rho_{crit} \approx 15$. The presence of α particles affects the ignition condition almost exclusively due to fuel dilution, whereas the additional bremsstrahlung losses have only minor effects. Fig. 7 also demonstrates the expected result that the minimal possible ignition parameter ($n_e \cdot \tau_E$ or $p_{tot} \cdot \tau_E$) increases with ρ . This will be discussed below in case 4. But at the same time, and perhaps less expected, the temperature T_{min} at which this minimum occurs, becomes smaller with increasing values of ρ . For example we find $T_{min} = 25$ keV for the helium free case ($\rho = 0$), but $T_{min} = 15$ keV for $\rho = 13$. This is a consequence of the temperature dependence of f_{He} and of dilution being the dominant effect.

Case 4: $f_Z \neq 0$, radiation losses explicit, i.e. not included in τ_E .

Using again ρ as parameter we find (fig. 8, $f_Z = 2$ %, $Z=6$: carbon) a much faster decrease of the ignition region, with a critical ρ of $\rho_{crit} \sim 9.5$. The above mentioned two branches of helium concentrations are now in the ranges (5 – 15) % and (15 – 25) % respectively. The interdependence of impurity contamination and helium removal requirements is more clearly seen in fig. 9, where the critical parameter ρ_{crit} is plotted as function of f_Z for a few selected light impurities ($Z=4$: Be, $Z=5$: B, $Z=6$: C and $Z=8$: O).

We recover the value $\rho_{crit} \approx 15$ for the ideal case 3 with no impurities on the ordinate, and also maximum allowable impurity concentrations in the case of perfect helium removal $\rho_{crit} = 0$., namely $f_O \sim 7\%$, $f_C \sim 10\%$, $f_B \sim 12.5\%$ and $f_{Be} \sim 16.5\%$ on the abscissa. We note that e.g. for a carbon concentration of 4% the ratio $\rho = \tau_{\alpha}^*/\tau_E$ must be made smaller than 5 since otherwise no burning is possible. But much smaller values of ρ already can lead to a drastic increase of the smallest possible ignition parameter

$n_e \cdot \tau_E$ or $p_{tot} \cdot \tau_E$ in the accessible temperature range. Fig. 10 clearly shows how this minimum parameter increases with both the impurity concentration f_Z and the confinement time ratio ρ .

4. Burn Conditions for a D-³He reactor

As an alternative to the D-T fusion reaction also the D-³He fusion process has recently attracted renewed interest as a possible candidate for an advanced fusion reactor in the future.

Despite several disadvantages (much lower reaction rate $\langle \sigma v \rangle$ which peaks at much higher plasma temperature, probably larger fast ion losses due to trapping in the magnetic ripple, etc.) its striking advantage is the fact that — by neglecting D-D reactions — 100 % of the energy (compared to only 20 % in the D-T case) is released in form of charged particles and can be used to maintain the plasma energy balance directly. Furthermore, the low neutron wall loading from D-D reactions, the low nuclear decay heat and the avoidance of tritium breeding are very attractive perspectives from a maintenance and safety point of view [12, 13].

However if dilution due to ash contamination poses a problem already in a D-T reactor, this should be even more severe for the D-³He concept.

In order to assess this problem we have carried out the same analysis as in sections 2 and 3 for a burning D-³He plasma.

First, the following redefinitions and modifications have to be made to the equations of section 2:

$$n_i = n_D + n_{^3\text{He}} \quad , \quad \text{and} \quad n_D = n_{^3\text{He}} \quad (4.1)$$

The He concentration f_{He} is now replaced by an "ash concentration" f_{ash} (and the α -particle confinement time τ_α^* is replaced by the "ash particle confinement time" τ_a^*).

We have

$$f_{ash} = \frac{n_{ash}}{n_e} \quad \text{with} \quad n_{ash} = n_p + n_{^4\text{He}} \quad \text{and} \quad n_p = n_{^4\text{He}} \quad (4.2)$$

It follows for the dilution parameter

$$f_i = \frac{n_i}{n_e} = \frac{2}{3} - f_{ash} - \frac{2}{3} \cdot Z \cdot f_Z \quad , \quad (4.3)$$

and the total particle density is now independent of f_{ash} :

$$f_{tot} = \frac{n_{tot}}{n_e} = \frac{5}{3} - \left(\frac{2}{3} \cdot Z - 1 \right) \cdot f_Z \quad . \quad (4.4)$$

The bremsstrahlung losses due to fuel and ash particles with the definitions (2.4) now take the form:

$$R_b = (f_i + f_{ash}) \cdot 0.5 \cdot [R_{b,1}(T) + R_{b,2}(T)] \quad , \quad (4.5)$$

and the particle balance eq. (2.6) becomes

$$\frac{1}{4} \cdot n_e^2 \cdot f_i^2 \cdot \langle \sigma v \rangle = \frac{n_{ash}}{2 \cdot \tau_a^*} \quad , \quad (4.6)$$

Finally, the fusion energy E_α carried by the α -particle before must now be replaced by the total yield

$$E_a = (3.67 + 14.681) MeV = 18.351 MeV.$$

This leads to the following coefficients $b_0, \dots, b_3$ in the cubic equation (analogous to eq. 2.8) for f_{ash} :

$$b_0 = -\frac{3}{2} \cdot T \cdot f_{tot} \cdot \left[\frac{2}{3} \cdot (Z \cdot f_Z - 1) \right]^2 \quad (4.7.1)$$

$$\begin{aligned} b_1 = & -\frac{3}{2} \cdot T \cdot f_{tot} \cdot \left[\frac{4}{3} \cdot (Z \cdot f_Z - 1) \right] + \\ & + \frac{E_a}{\rho} \cdot \frac{1}{2} \cdot \left[\frac{2}{3} \cdot (Z \cdot f_Z - 1) \right]^2 + \\ & + \frac{4}{\langle \sigma v \rangle \cdot \rho} \cdot \frac{1}{2} \cdot \left\{ \frac{2}{3} \cdot (Z \cdot f_Z - 1) \cdot \frac{1}{2} \cdot [R_{b,1}(T) + R_{b,2}(T)] - f_Z \cdot R_{rad,Z} \right\} \end{aligned} \quad (4.7.2)$$

$$\begin{aligned} b_2 = & -\frac{3}{2} \cdot T \cdot f_{tot} + \\ & + \frac{E_a}{\rho} \cdot \frac{1}{2} \cdot \left[\frac{4}{3} \cdot (Z \cdot f_Z - 1) \right] \end{aligned} \quad (4.7.3)$$

$$b_3 = \frac{E_a}{\rho} \cdot \frac{1}{2} \quad (4.7.4)$$

For the definition of τ_E with implicit accounting for radiation losses the solution (corresponding to eq. (3.2.1)) becomes:

$$f_{ash} = \frac{3 \cdot T \cdot f_{tot}}{\frac{E_a}{\rho}} \quad (4.8)$$

If instead radiation losses are excluded from τ_E , then the resulting burn condition contours are shown in fig. 11 (to be compared with fig. 7a) for the ash contaminations derived from eqs. (4.7) but an otherwise pure D-³He plasma ($f_Z = 0$), and in fig. 12 (to be compared with fig. 8a) for an additional impurity contamination of 2 % carbon ($f_Z = 0.02$, $Z = 6$). As expected the fusion region is shifted to higher temperatures and fusion products, but, more important in our present study, also to much (a factor 3 – 4) smaller critical values of the confinement time ratio ρ . This is a clear remainder of the fact that fuel dilution due to the ashes would be an even more severe problem for this fusion reactor type.

5. Definition of the global alpha particle confinement time τ_α^*

In deriving the global α -particle confinement time τ_α^* we distinguish between two "kinds" of α particles (see fig. 1) which are characterized by the location of their sources. One species (α_1) consists of the first generation α -particles, i.e. of particles born mainly in the plasma core by fusion events before they cross the boundary S of the burning region in outward direction for the first time. Let their total number be $N_{\alpha 1}$ and their mean lifetime be $\tau_{\alpha 1}$; in that sense $\tau_{\alpha 1}$ may be called the "internal α -particle confinement time". Then, their flux is (from Gauss theorem) $\Phi_{\alpha 1} = N_{\alpha 1}/\tau_{\alpha 1}$ and we have:

$$\frac{dN_{\alpha 1}(t)}{dt} = -\frac{N_{\alpha 1}}{\tau_{\alpha 1}} + \frac{N_D \cdot N_T \cdot \langle \sigma v \rangle}{V} \quad (5.1a)$$

Here N_D and N_T are the D and T particle numbers respectively in the volume V bounded by S, $\langle \sigma v \rangle$ is again the fusion reaction rate. If the α particle production process is terminated at $t=0$, then the solution is

$$N_{\alpha 1}(t) = N_{\alpha 1}(0) \cdot \exp(-t/\tau_{\alpha 1}) \quad \text{with } N_{\alpha 1}(0) = \frac{\tau_{\alpha 1} \cdot N_D \cdot N_T \cdot \langle \sigma v \rangle}{V} \quad (5.1b)$$

The second "species" (α_2) of total particle number $N_{\alpha 2}$ comprises all higher generations, i.e. particles returning to the burning region due to a finite "effective" recycling coefficient R_{eff} . We allow for a mean residence time $\tau_{\alpha 2}$ in the burning plasma for these particles which may be different from $\tau_{\alpha 1}$, because these particles are created at the reactor's inner wall components (limiter or divertor) or return from a nonperfect pumping system. They are mainly subject to transport processes in the outer zone of the confinement region and we refer to $\tau_{\alpha 2}$ as "external α -particle confinement time". R_{eff} , as defined here, is the ratio of the α particle flux $\Phi_{\alpha, out}$ crossing the boundary S in outward direction to the flux $\Phi_{\alpha, ret}$ of α particles crossing this surface in inward direction. Clearly, this definition of R_{eff} also includes screening (of α particles) in the boundary layer, i.e. for a "refueling efficiency γ " less than one (and perhaps close to zero). In order to derive the particle conservation equation for α_2 particles, we use the fact that according to the previous definitions the flux $R_{eff} \cdot \Phi_{\alpha 1, out}$ of the returning α_1 particles is the source for the first generation of α_2 particles.

Assuming the same recycling coefficient R_{eff} for the first and the higher generations of α -particles, we have:

$$\frac{dN_{\alpha 2}(t)}{dt} = -\frac{N_{\alpha 2}(t)}{\tau_{\alpha 2}} + R_{eff} \left(\frac{N_{\alpha 2}(t)}{\tau_{\alpha 2}} + \frac{N_{\alpha 1}(t)}{\tau_{\alpha 1}} \right) \quad (5.2a)$$

In the stationary case, the solutions from eq. (5.1a and (5.2a) are

$$\begin{aligned} N_{\alpha 1}(t < 0) &= N_{\alpha 1}(0) \\ N_{\alpha 2}(t < 0) &= \delta \cdot N_{\alpha 1}(0), \quad \text{with } \delta = \frac{R_{eff}}{(1 - R_{eff})} \cdot \frac{\tau_{\alpha 2}}{\tau_{\alpha 1}} \end{aligned} \quad (5.2b)$$

Adding $N_{\alpha 1}(t < 0)$ and $N_{\alpha 2}(t < 0)$ and inserting from (5.1b) we find for the total number $N_{\alpha} = N_{\alpha 1} + N_{\alpha 2}$ of α -particles in the burning zone

$$N_{\alpha} = \frac{N_D \cdot N_T \cdot \langle \sigma v \rangle}{V} \cdot \left[\tau_{\alpha 1} + \frac{R_{eff}}{1 - R_{eff}} \cdot \tau_{\alpha 2} \right] \quad (5.3)$$

which clearly shows that the global confinement time τ_{α}^* of α -particles in the burn region (responsible for dilution and radiation) is given by the expression in the parenthesis of eq. (5.3). We obtain the same result by considering the initial decay time for the time-dependent case:

$$N_{\alpha}(t) = N_{\alpha 1}(0) \cdot [(\delta - \beta) \cdot \exp(-t/\tau_{\alpha 2}^*) + (1 + \beta) \cdot \exp(-t/\tau_{\alpha 1})] \quad (5.4)$$

with $\tau_{\alpha 2}^* = \tau_{\alpha 2}/(1 - R_{eff})$ and $\beta = R_{eff}/(1 - R_{eff}) \cdot \tau_{\alpha 2}/(\tau_{\alpha 1} - \tau_{\alpha 2}^*)$ (β may be positive or negative, depending on $\tau_{\alpha 1}$, $\tau_{\alpha 2}$ and R_{eff}).

We are dealing, in general, with two characteristic times $\tau_{\alpha 2}^*$ and $\tau_{\alpha 1}$, which excludes the possibility of writing the α particle density $n_{He}(t)$ in the form (1.1). Nevertheless we can define an effective confinement time τ_{α}^* at time $t=0$ after shut-down:

$$\Phi_{\alpha, out} - \Phi_{\alpha, ret} = - \left. \frac{dN_{\alpha}(t)}{dt} \right|_{t=0} =: - \frac{N_{\alpha}(0)}{\tau_{\alpha}^*} \quad (5.5)$$

Taking the derivative of (5.4) at $t=0$ and making use of $N_{\alpha}(0) = N_{\alpha 1}(0) \cdot (1 + \delta)$ we find the equation

$$\frac{1}{\tau_{\alpha}^*} \cdot (1 + \delta) = \frac{1}{\tau_{\alpha 2}^*} \cdot [\delta - \beta] + \frac{1}{\tau_{\alpha 1}} \cdot [1 + \beta] \quad , \quad (5.6)$$

which, after inserting for δ, β and $\tau_{\alpha 2}^*$, yields again

$$\tau_{\alpha}^* = \tau_{\alpha 1} + \frac{R_{eff}}{(1 - R_{eff})} \cdot \tau_{\alpha 2} = \tau_{\alpha 1} + \frac{(1 - \varepsilon_{eff})}{\varepsilon_{eff}} \cdot \tau_{\alpha 2} \quad (5.7)$$

We note that this τ_{α}^* is the same quantity as found before in eq. (5.3) and as used in the particle balance equation (2.6) in section 2.

Here we have made use of the (effective) exhaust efficiency $\varepsilon_{eff} = 1 - R_{eff}$, which is defined in ref. /14/ consistently with our present derivation as the probability for a particle — in our case for an α -particle — to be pumped away (e.g. by a pump-limiter, by divertor-pumps or by surface pumps) whenever it crosses the boundary of the burning region in outward direction and enters the outer plasma layer. For pump limiters, values of ε_{eff} in excess of 10% may be achievable /14/. Similar values of ε_{eff} for divertors may require to apply particle collecting scoops /15/.

Clearly, for $\tau_{\alpha 1} = \tau_{\alpha 2} =: \tau_{\alpha}$ equation (5.7) reduces to the standard definition $\tau_{\alpha}^* = \tau_{\alpha}/(1 - R_{eff})$ of the "global" (or "effective") particle confinement time.

The factor $(1 - \varepsilon_{eff})/\varepsilon_{eff}$ is the mean number of recycling events back into the burning region experienced by an α particle, before it is pumped away. As we shall discuss next, ε_{eff} depends on the exhaust efficiency of the pumping stations, ε_p , on the surface recycling coefficients, R , and on the refueling efficiency γ .

In general we may consider three loss mechanisms for the flux Φ_{out} of particles which leave the volume V through surface S (fig. 1).

The particles may either be collected by the pumping system, with collection efficiency ε_{coll} : $\Phi_{coll} := \varepsilon_{coll} \cdot \Phi_{out}$ or eventually hit the wall: $\Phi_{wall} := (1 - \varepsilon_{coll}) \cdot \Phi_{out}$. A fraction ε_{rem} (removal efficiency) of Φ_{coll} will be pumped away:

$$\Phi_{pump} := \varepsilon_{rem} \cdot \varepsilon_{coll} \cdot \Phi_{out} =: \varepsilon_p \cdot \Phi_{out} \quad , \quad (5.8)$$

which defines the exhaust efficiency $\varepsilon_p := \varepsilon_{rem} \cdot \varepsilon_{coll}$ for the pumping system.

A flux $\Phi_{rec} := R \cdot (1 - \varepsilon_{coll}) \cdot \Phi_{out}$ will recycle from the wall (R : wall recycling coefficient), and another part $\varepsilon_w \cdot \Phi_{out} := (1 - R) \cdot (1 - \varepsilon_{coll}) \cdot \Phi_{out}$ will be absorbed by the walls. The fraction R_{ret} of the flux Φ_{out} , which is not absorbed by the first wall system is

$$R_{ret} := 1 - \varepsilon_{ret} := 1 - (\varepsilon_w + \varepsilon_p) = R \cdot (1 - \varepsilon_{coll}) + \varepsilon_{rem} \cdot \varepsilon_{coll} \quad , \quad (5.9)$$

So far we had not yet accounted for the possibility that a significant fraction of the returning flux Φ_{ret} of the α -particles may be screened by the boundary layer and transported back to the wall components before it reenters the burn zone. By addressing this process now we define the boundary layer as that zone where practically no fusion processes occur (shaded area of fig. 1) and introduce a finite screening efficiency ε_{scr} . This screening efficiency ε_{scr} will further reduce the flux $\Phi_{ret} = R_{ret} \cdot \Phi_{out}$, so that only the flux $\gamma \cdot R_{ret} \cdot \Phi_{out}$ will reenter the burning volume V , with $\gamma = 1 - \varepsilon_{scr}$ being the refueling efficiency. The fraction $\varepsilon_{scr} \cdot R_{ret} \cdot \Phi_{out}$ will undergo the recycling process again, so that finally the total flux Φ_{ret} of particles reentering the volume V through surface S reads:

$$\Phi_{ret} = \Phi_{out} \cdot R_{ret} \cdot \gamma \cdot \frac{1}{1 - (1 - \gamma) \cdot R_{ret}} \quad , \quad (5.10)$$

the last factor describing flux enhancement due to local recycling in the above defined boundary layer.

We thus have for the effective recycling coefficient R_{eff} and effective exhaust efficiency used in formulars (5.2 – 5.7) the expressions:

$$R_{eff} = 1 - \varepsilon_{eff} = R_{ret} \cdot \gamma \cdot \frac{1}{1 - (1 - \gamma) \cdot R_{ret}} \quad . \quad (5.11)$$

Inserting these relations into eq. (5.7) yields a general expression for the global α -particle confinement time τ_α^*

$$\tau_\alpha^* = \tau_{\alpha 1} + \frac{(1 - \varepsilon_{scr}) \cdot (1 - \varepsilon_{ret})}{\varepsilon_{ret}} \cdot \tau_{\alpha 2} \quad . \quad (5.12)$$

In order to interpret this equation, one may discuss two special cases.

In the case of negligible screening, i.e. for $\varepsilon_{scr} \ll 1$, and with $\varepsilon_{eff} \approx \varepsilon_{ret}$, eq. (5.12) becomes identical with eq. (5.7). Moreover, since for stationary wall conditions ε_w approaches zero, ε_{ret} can be replaced by the exhaust efficiency ε_p . Finally, for $\varepsilon_p \ll 1$, we may write

$$\tau_{\alpha}^* \approx \tau_{\alpha 1} + \frac{\tau_{\alpha 2}}{\varepsilon_p} \quad (5.13)$$

In case of strong boundary screening ($\gamma \ll 1$), on the other hand and taking again $\varepsilon_w \ll 1$, $\varepsilon_p \ll 1$, we obtain

$$\tau_{\alpha}^* \approx \tau_{\alpha 1} + \frac{\gamma}{\varepsilon_p} \cdot \tau_{\alpha 2} \quad (5.14)$$

Obviously, this case is the more desired one in order to keep τ_{α}^* small. However, even then the screening of the recycled α -particles by the boundary layer becomes only effective as far as it is accompanied by a non-negligible value of the exhaust efficiency ε_p . So it depends on the balance between the two competing quantities γ and ε_p — if both are approaching small values — whether the second term of eq. (5.14) leads to a dominating or rather to a negligible contribution to the global α -particle confinement time τ_{α}^* and thus to the helium concentration.

This section has shown, that the global α -particle confinement time τ_{α}^* consists of two different terms. We may recall that the first term, $\tau_{\alpha 1}$, is mainly governed by the chosen plasma confinement regime, whereas the second term is determined by both, the above mentioned balance between recycling and screening, and, with respect to the value of $\tau_{\alpha 2}$, also by the properties of the confinement regime. Since the transport properties of the burning core and of the boundary layer may be coupled to each other, an overall optimization is required in order to fulfill the burn condition as given in this paper.

Conclusions

It has been shown that the ratio ρ between the global α -particle confinement time τ_{α}^* and the energy confinement time τ_E is a critical quantity which determines the required ignition parameter $n \cdot T \cdot \tau_E$ of a (quasi-) stationary D-T plasma. The operational value of ρ should be kept well below a critical value ρ_{crit} lying in the range of 7 – 15 for D-T burning and in the range of 2 – 5 for D-³He burning; the exact value of ρ_{crit} depends on the species and the concentration of wall-released impurities. Since the required value of the ignition parameter increases with growing values of ρ , any gain in τ_E obtained from improved confinement has to be weighted against a possible rise in ρ in case this gain is accompanied by a disproportional “gain” in $\tau_{\alpha 1}$ and/or $\tau_{\alpha 2}$.

Therefore, in order to evaluate the potential of specific confinement and exhaust concepts, such as pumped divertors, pump limiters or ergodisation schemes, it is suggested to address the achievable value of ρ experimentally in present day machines. This might yield some basis for an extrapolation to a future burn experiment.

Furthermore, by analysing the processes which determine τ_{α}^* , we have established a relationship between the exhaust efficiency, ε_p , the recycling coefficient, R , of the first wall system, the refueling efficiency, γ , and the “internal” and “external” α -particle confinement times, $\tau_{\alpha 1}$ and $\tau_{\alpha 2}$. The quantities ε_p , R and γ and their combination describe the effective recycling of ash-particles (^4He alone or both, ^4He and p) back into the burn region due to imperfect pumping and shielding.

Acknowledgement

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Figure Captions

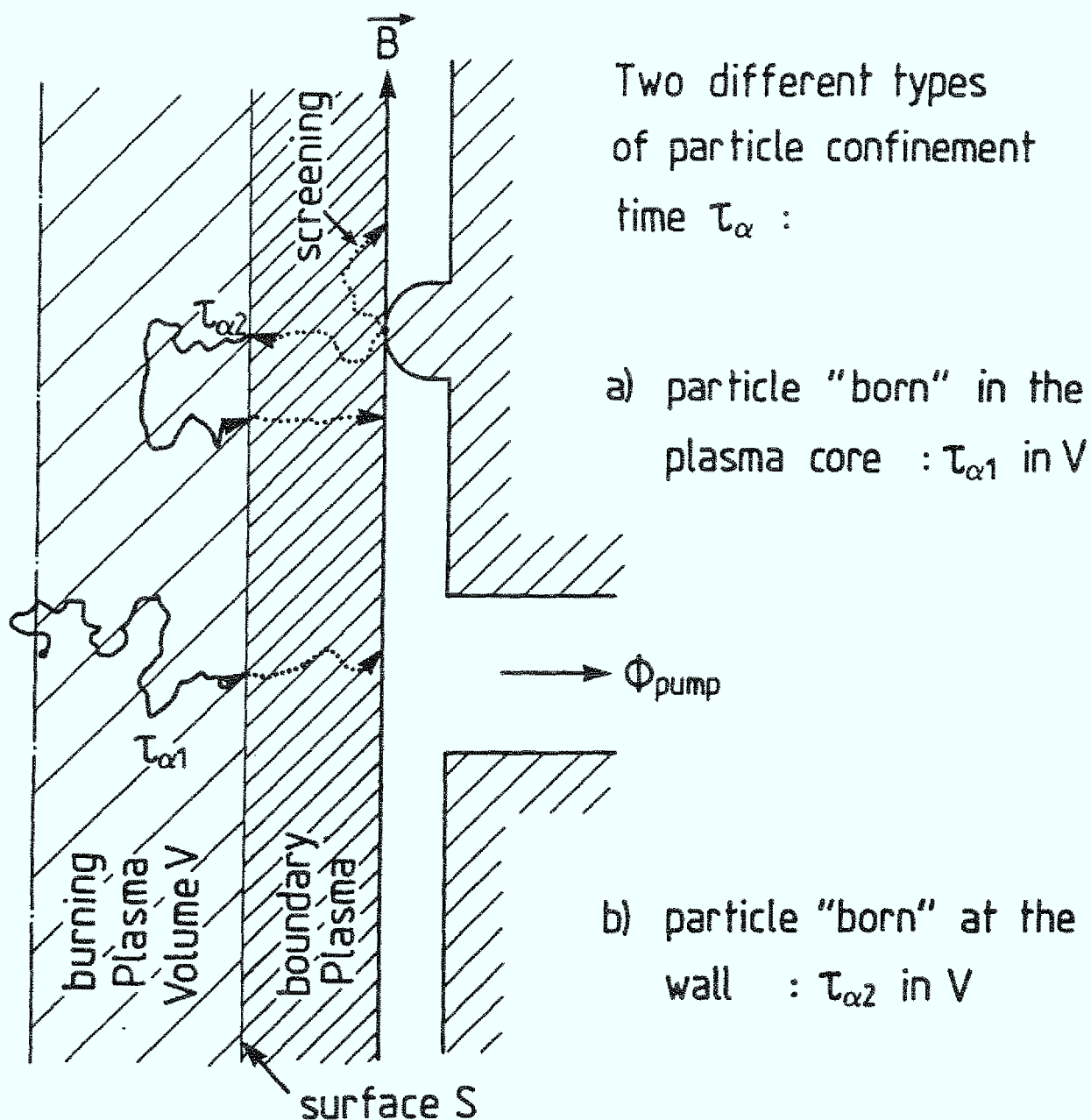
- fig. 1 model and definitions
- fig. 2 ignition parameter $n_e \cdot T \cdot \tau_E$ vs. T for constant helium concentrations $f_{He} = 0.0, 0.3, 0.025$
- fig. 3 dito but radiation losses implicitly accounted for in the definition of τ_E
- fig. 4 ignition parameter (radiation losses implicit in τ_E) $n_e \cdot \tau_E$ vs. T for constant confinement time ratios $\rho = 0, 5, 10, 20, 40$
- fig. 5 critical temperature as function of ρ for several light impurity contaminations
- fig. 6 relative helium concentration in a stationary burning D-T plasma as function of T and ρ (radiation losses explicit)
- fig. 7ab Ignition parameters $n_e \cdot \tau_E$ and $p_{tot} \cdot \tau_E$ in a pure (no impurities except 4He) burning D-T plasma as function of T and ρ
- fig. 8ab like figs. 7ab, but additional 2 % carbon ($Z=6$) contamination
- fig. 9 critical confinement time ratio as function of impurity concentration f_Z for Be, B, C and O
- fig.10 minimum ignition parameter $p_{tot} \cdot \tau_E$ as function of confinement time ratio for different concentrations of carbon impurity ($Z=6$).
- fig.11 ignition parameter $n_e \cdot \tau_E$ in a pure (no impurities except ashes) burning D- 3He plasma as function of T and ρ (to be compared with fig. 7a)
- fig.12 like fig. 11, but additional 2 % carbon ($Z = 6$) contamination (to be compared with fig. 8a)

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Fig.1

Helium Exhaust: Model and Definition



$$\tau_{\alpha}^* = \tau_{\alpha 1} + \frac{(1 - \epsilon_{\text{ret}})(1 - \epsilon_{\text{scr}})}{\epsilon_{\text{ret}}} \tau_{\alpha 2}$$

Fig. 2

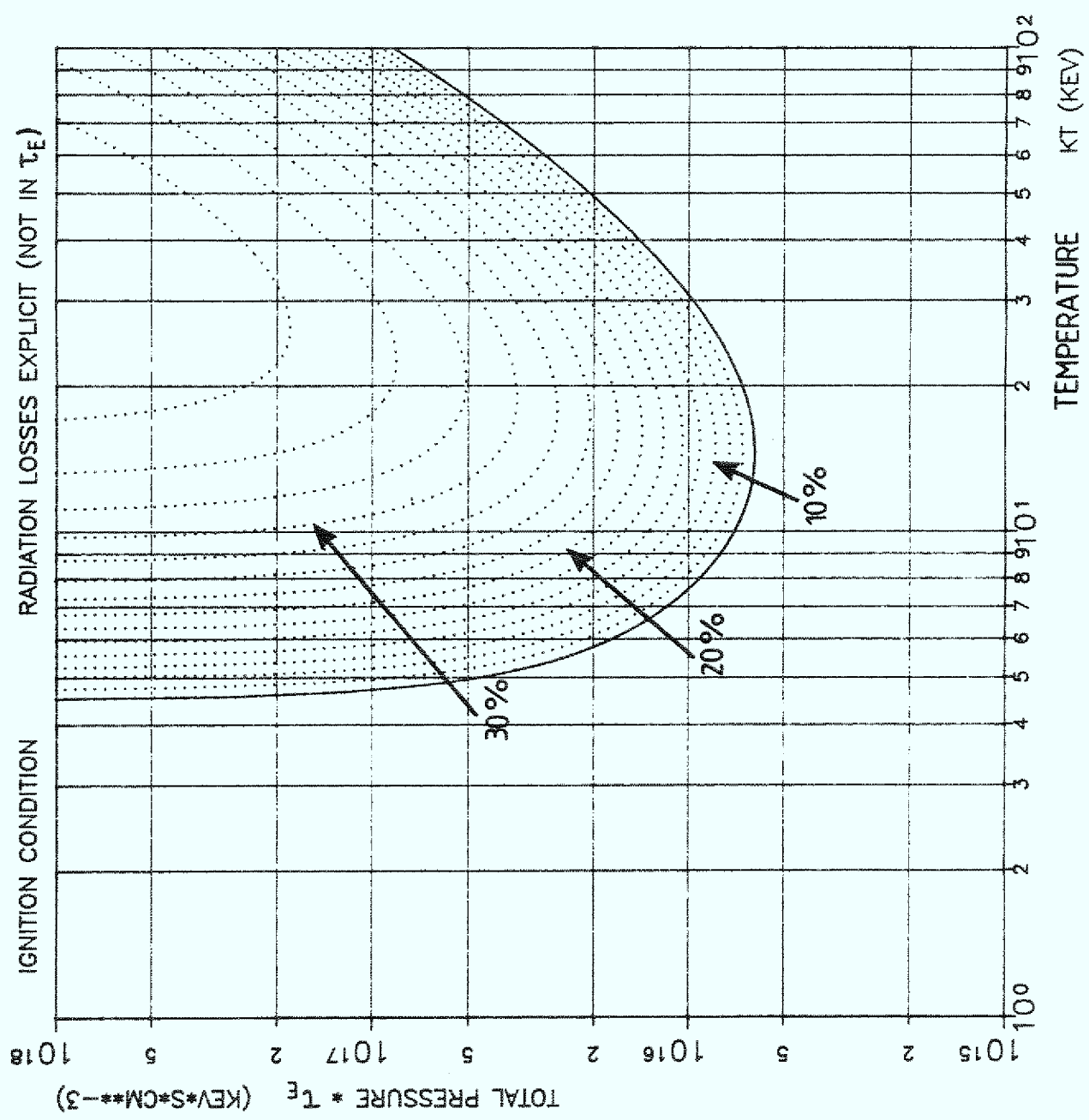


Fig. 3

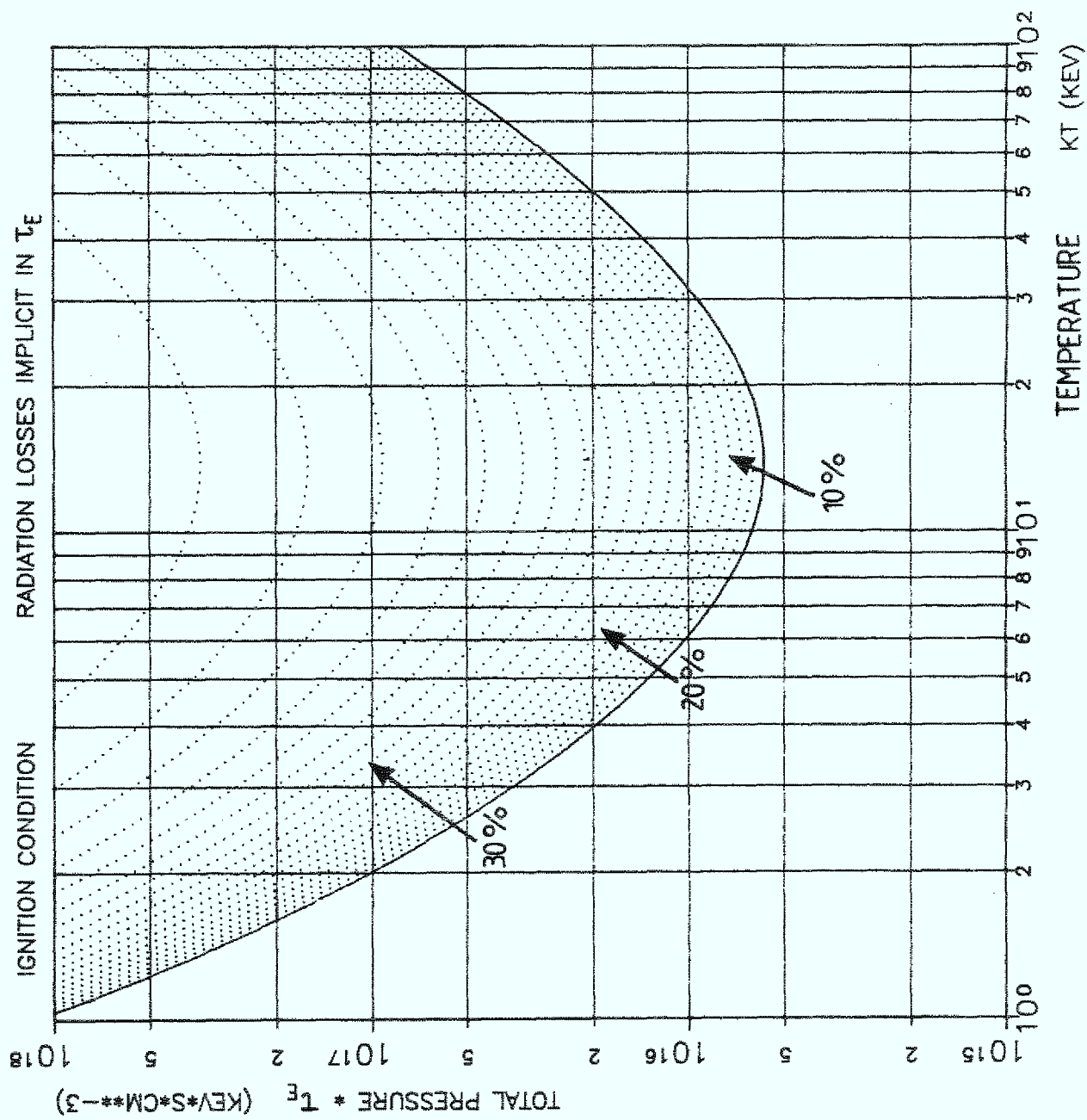


Fig.4

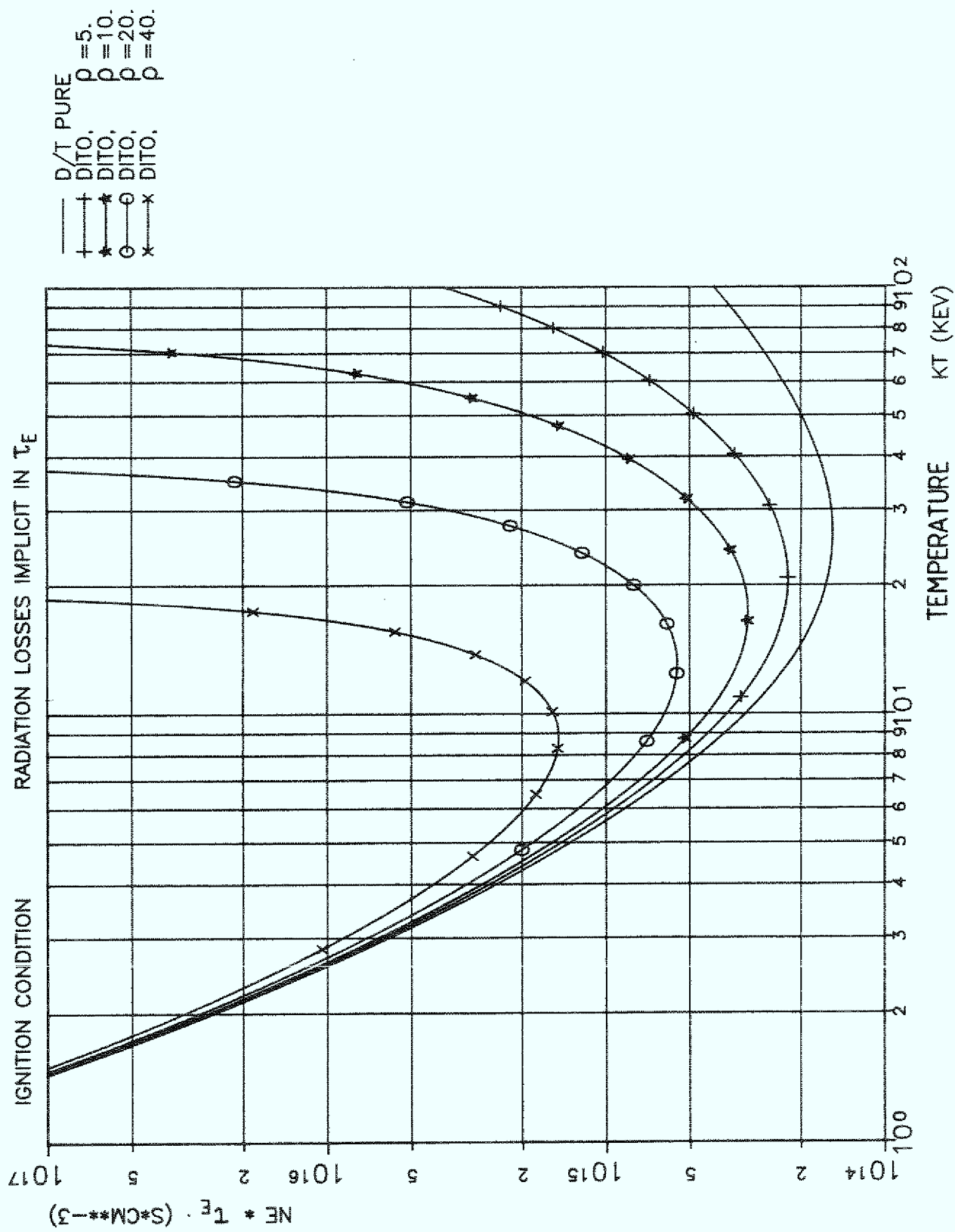


Fig.5

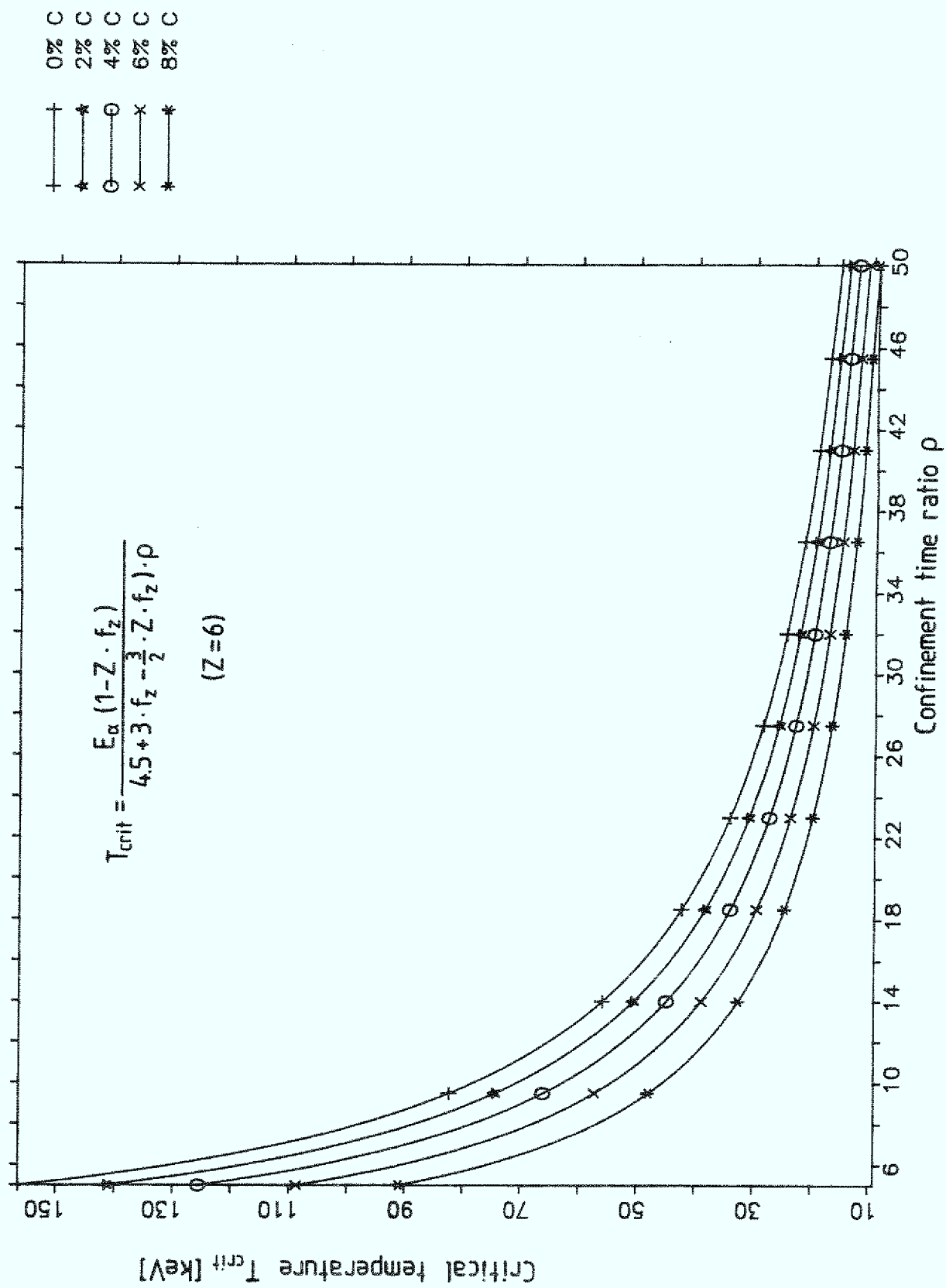


Fig.6

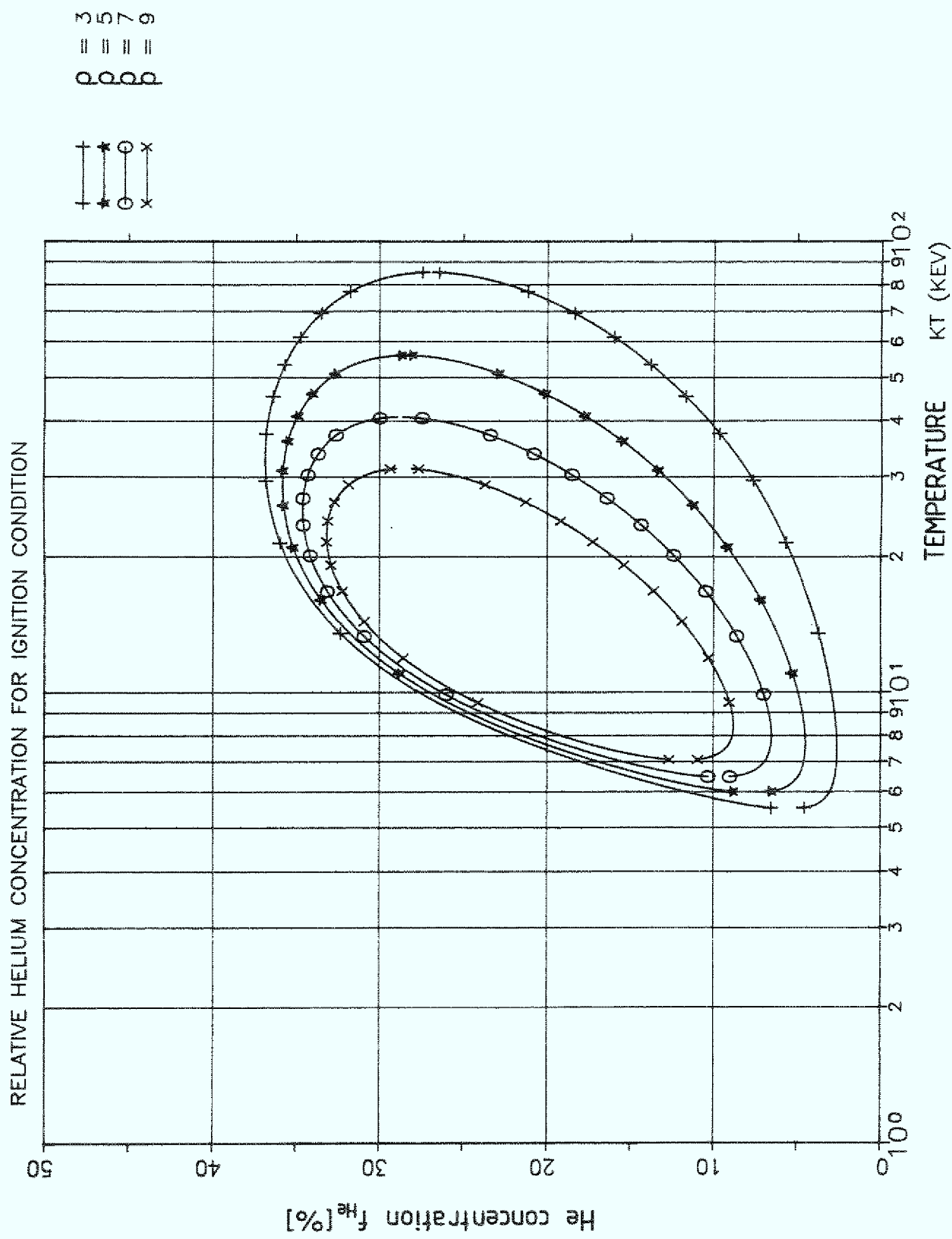


Fig.7a

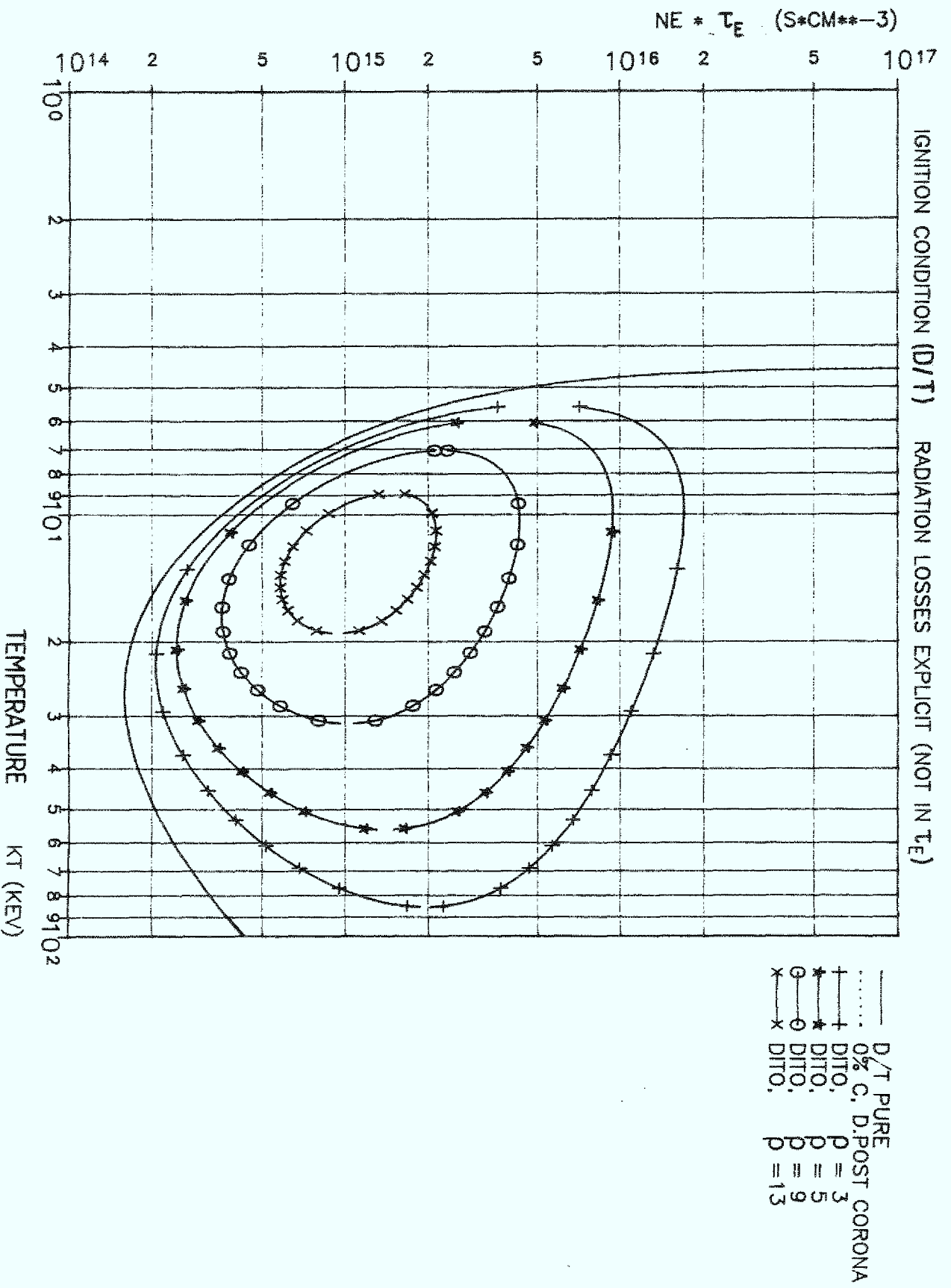


Fig. 7b

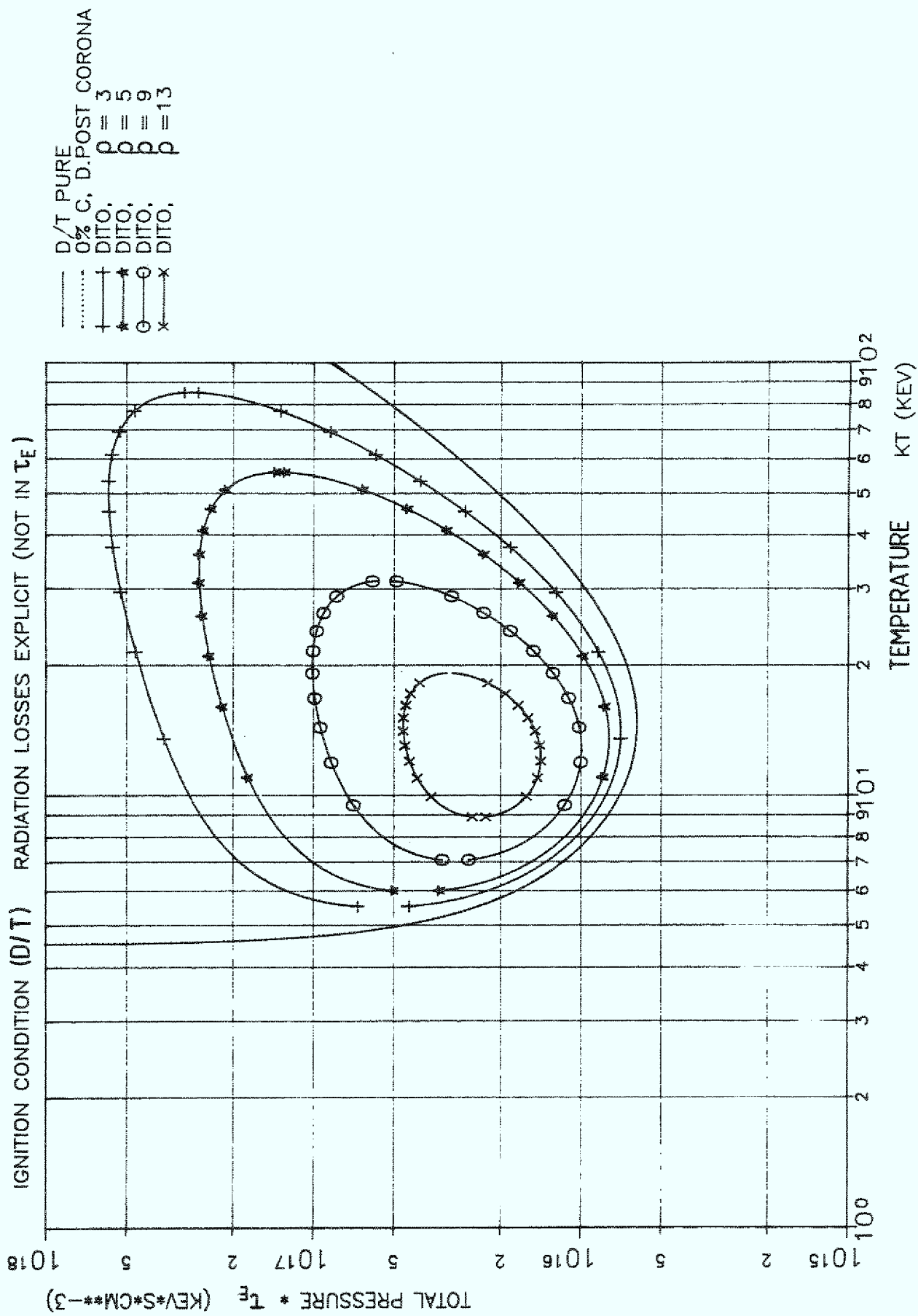


Fig.8a

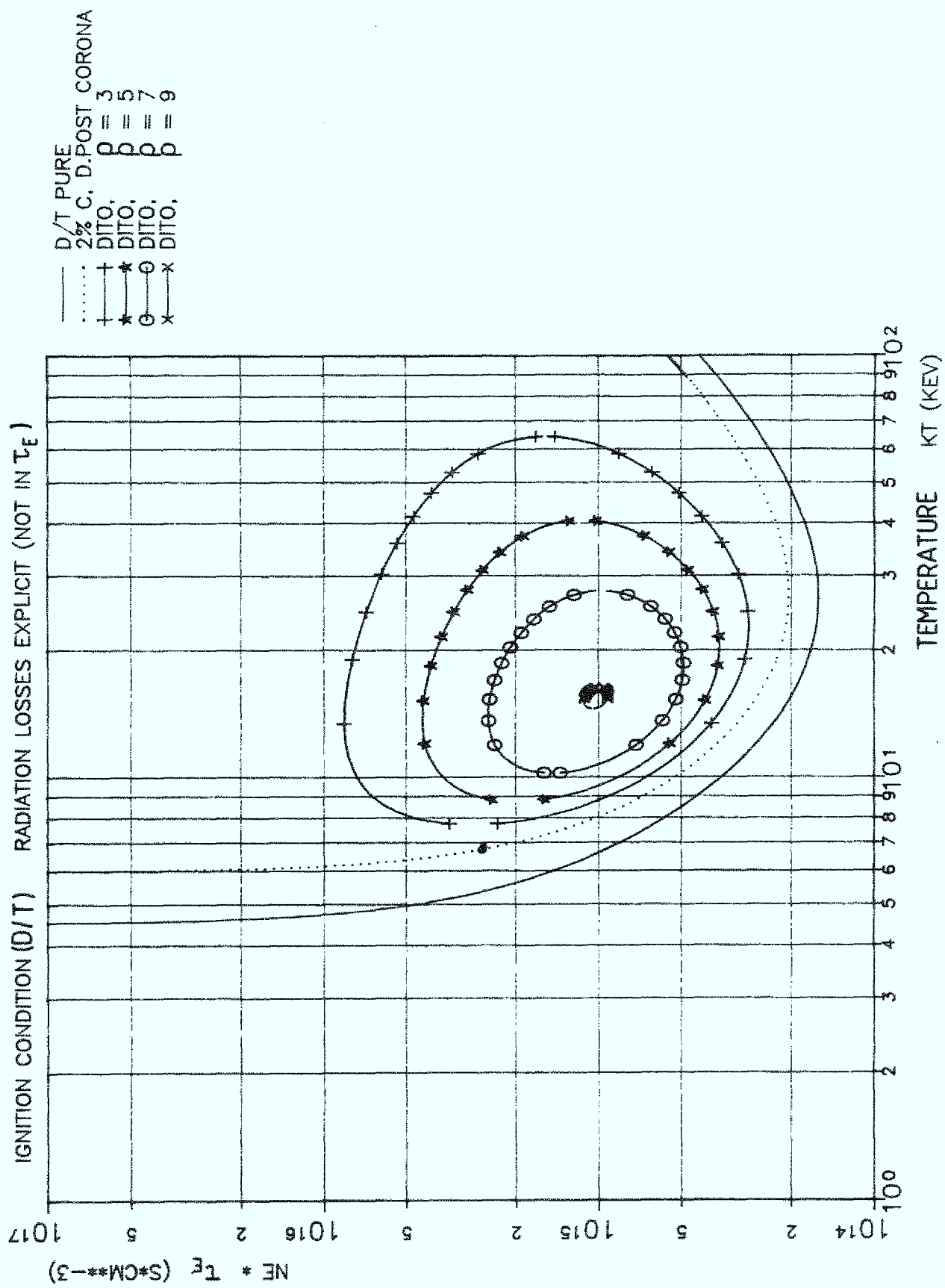


Fig.8b

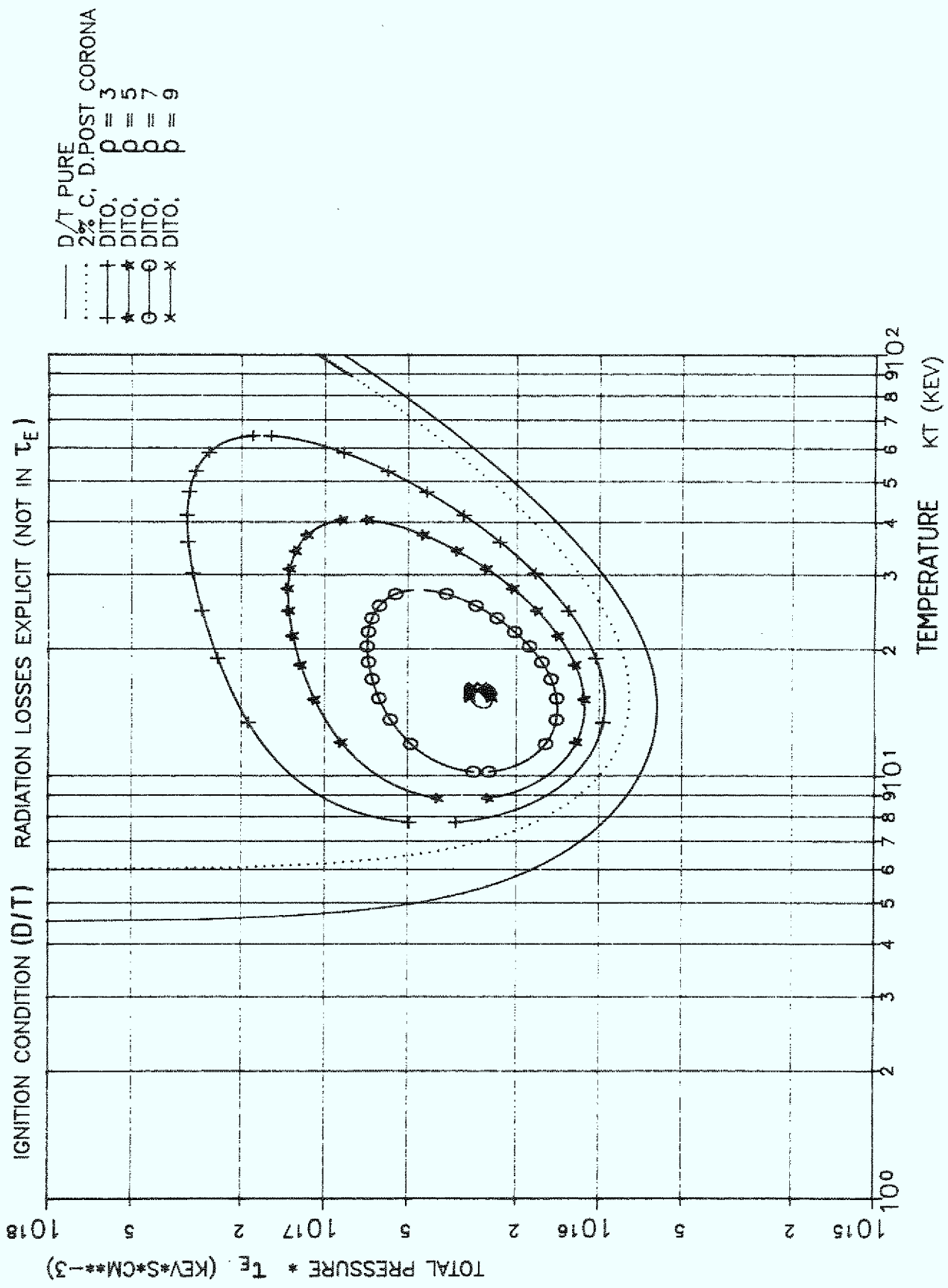


Fig.9

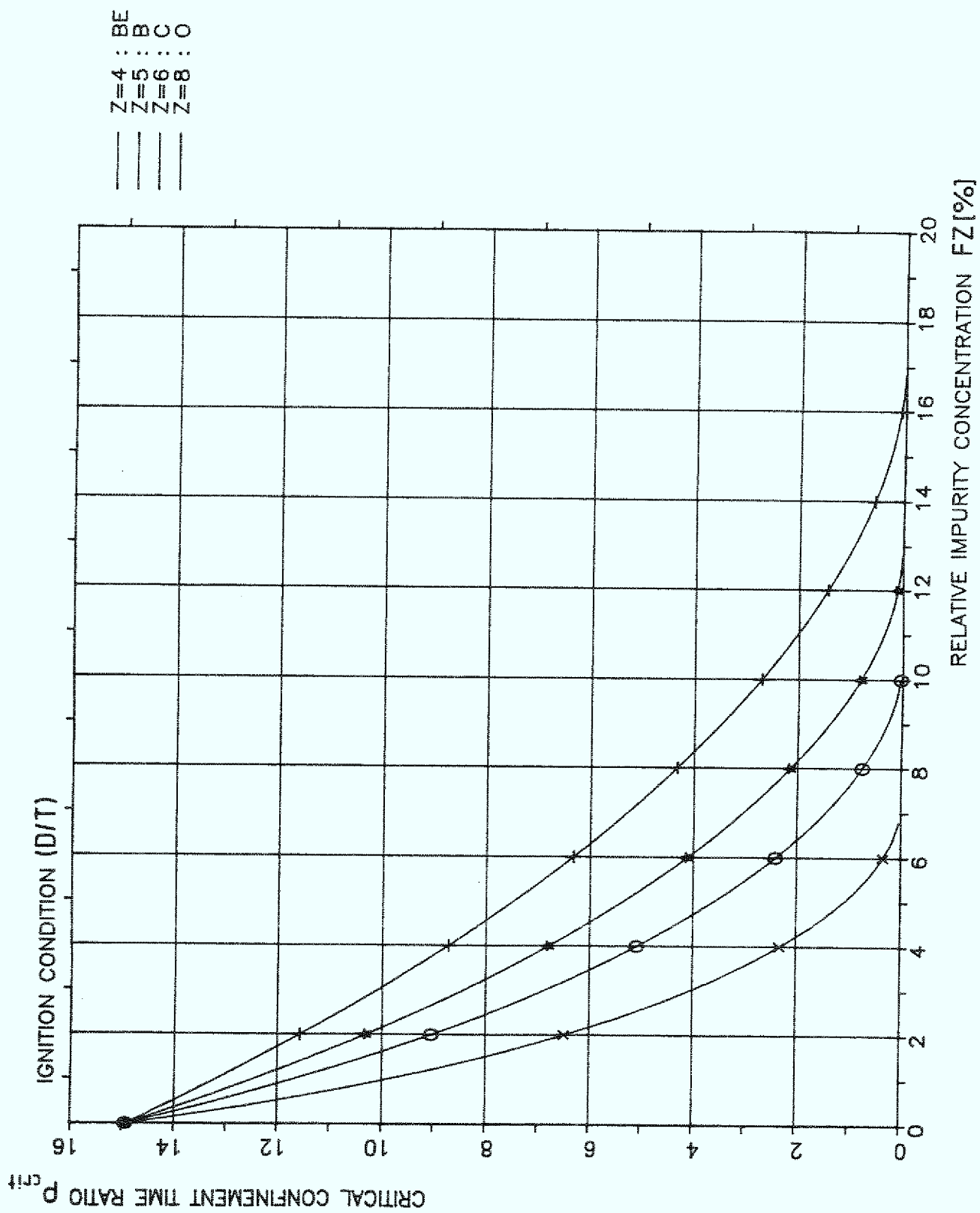


Fig.10

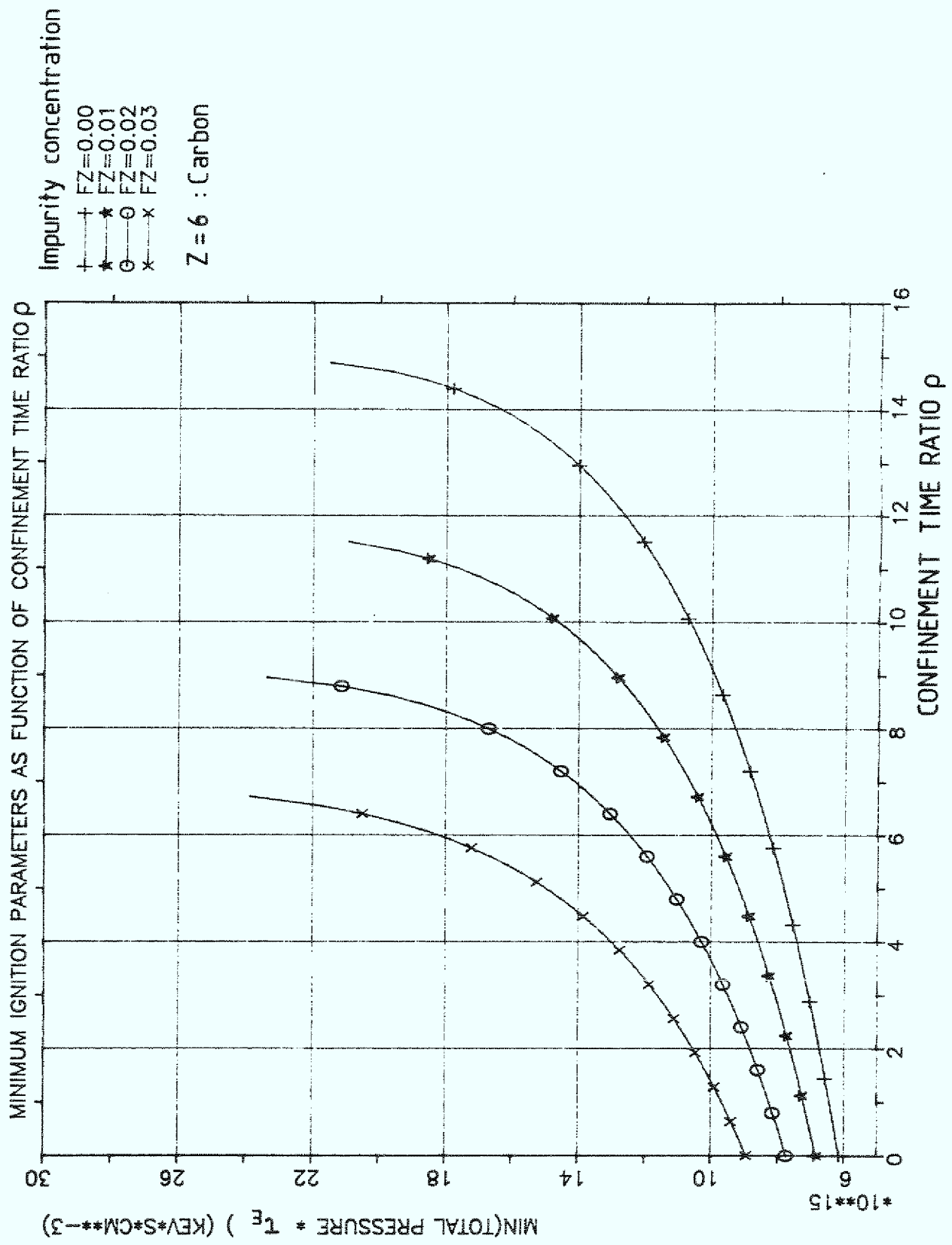


Fig.11

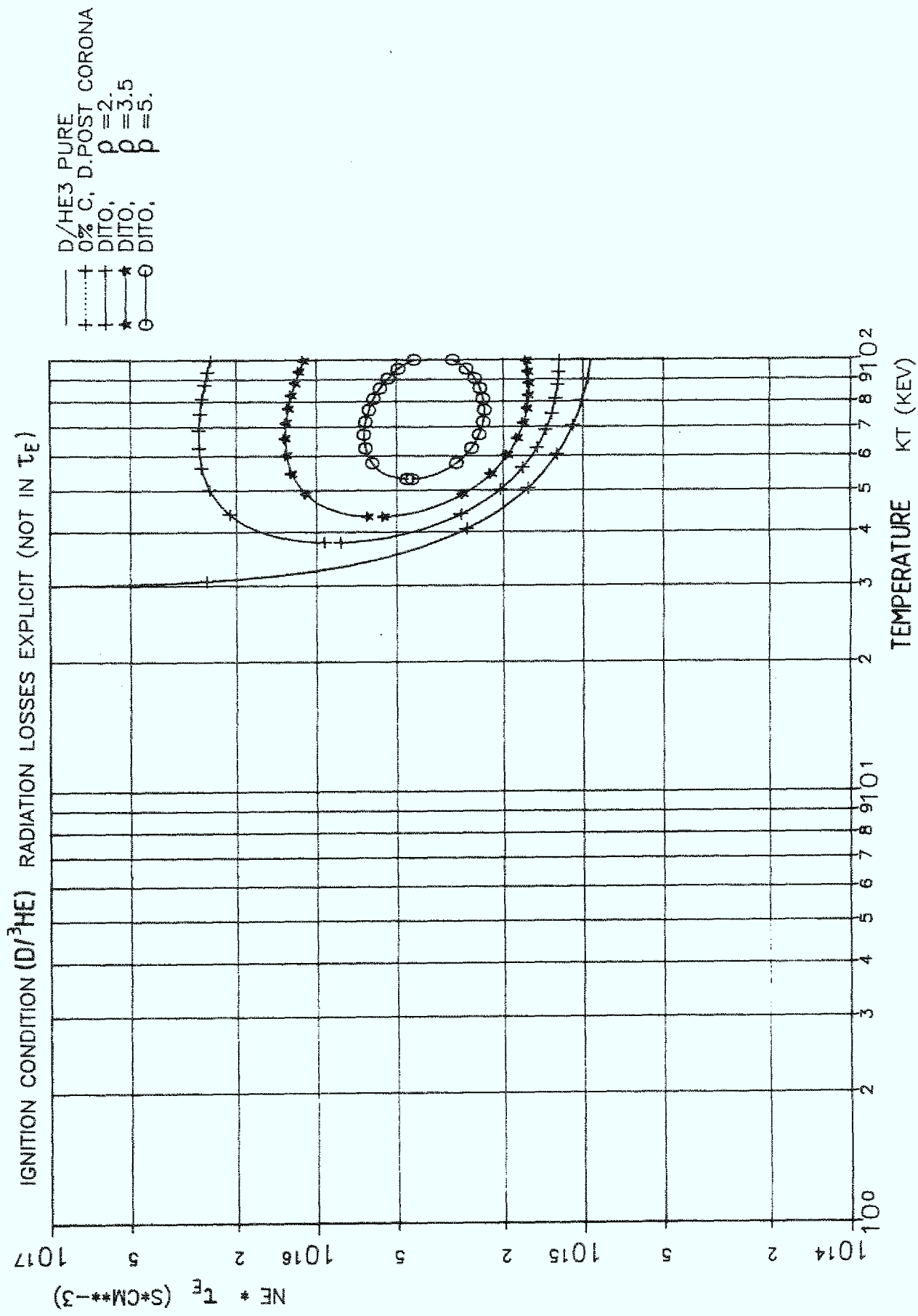


Fig.12

