Testing quantum fault tolerance on small systems

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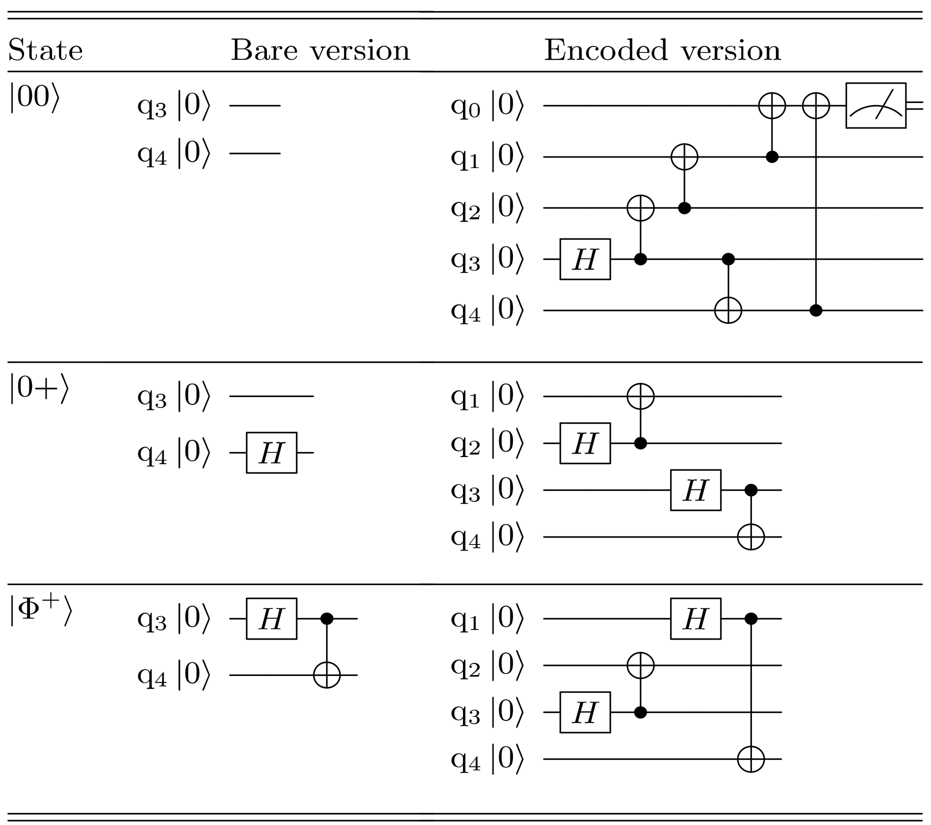
Fault-tolerant quantum computation

In fault-tolerant (FT) protocols, qubit states and gates are logically encoded in a larger space. We consider a code in which 2 logical qubits are encoded in 4+1 physical qubits.

$$\overline{|00\rangle} = (|0000\rangle + |1111\rangle)/\sqrt{2}$$
$$\overline{|01\rangle} = (|1100\rangle + |0011\rangle)/\sqrt{2}$$
$$\overline{|10\rangle} = (|1010\rangle + |0101\rangle)/\sqrt{2}$$
$$\overline{|11\rangle} = (|0110\rangle + |1001\rangle)/\sqrt{2}$$

$$\overline{X1} = X_1X_3$$
$$\overline{X2} = X_1X_2$$
$$\overline{Z1} = Z_1Z_2$$
$$\overline{Z2} = Z_1Z_3$$
$$\overline{HHS} = H_1H_2H_3H_4$$
$$\overline{CZ} = S_1S_2S_3S_4Z_2Z_3$$



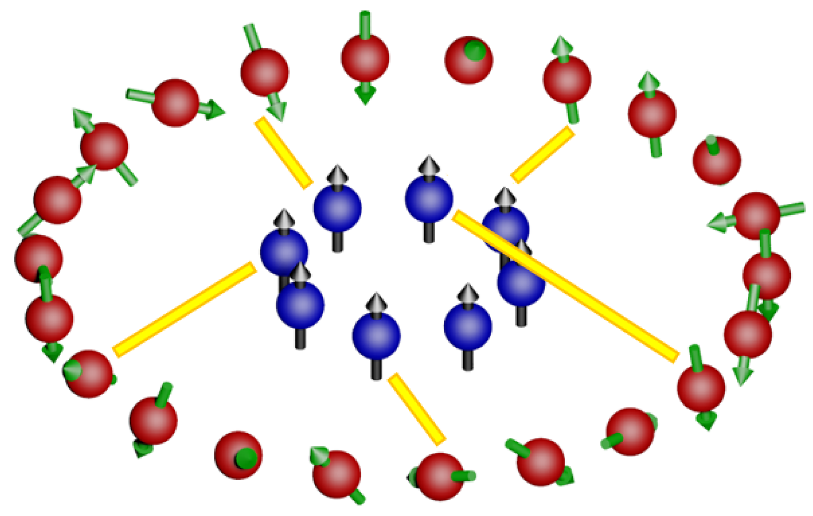
Criterion for success: [1] $D_{\mathrm{enc}} < D_{\mathrm{bare}}$
All encoded circuits of some representative set perform better than the corresponding bare, unencoded circuits

$$\mathrm{D}_{\mathrm{bare/enc}} = \frac{1}{2}\sum_{q_0q_1}\left|p^{\mathrm{bare/enc}}_{q_0q_1} - p^{\mathrm{theory}}_{q_0q_1}\right|$$

System 1: Spin qubits in an environment

The first system consists of 5 spin qubits coupled with strength λ to an environment with N_E two-level systems prepared at inverse temperature β.

$$H = H_Q + H_E + \lambda H_{QE}$$
$$H_Q = -\sum_{n=0}^{4}\sum_{\alpha=x,z} h_n^\alpha \sigma_n^\alpha - \sum_{n,m=0}^{4} J_{nm}^x \sigma_n^x \sigma_m^x$$
$$H_E = -\sum_{n=5}^{N_E+4}\sum_{\alpha=x,y,z} J_n^\alpha \sigma_n^\alpha \sigma_{n+1}^\alpha$$
$$H_{QE} = -\sum_{n=0}^{4}\sum_{\alpha=x,y,z} J_{nj_n}^\alpha \sigma_n^\alpha \sigma_{j_n}^\alpha$$



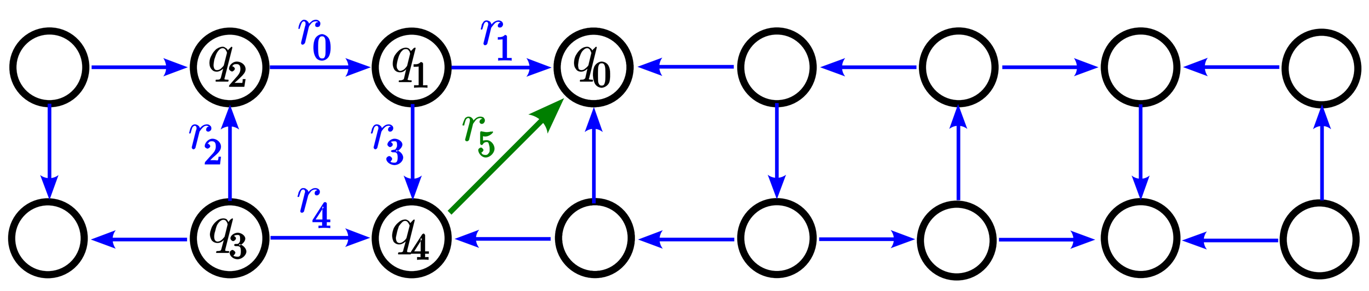
The system is simulated by numerically solving the time-dependent Schrödinger equation (TDSE) to machine precision using the Chebyshev algorithm [2]. Quantum gates are implemented by choosing suitable parameters for H_Q.

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)\,|\Psi(t)\rangle$$

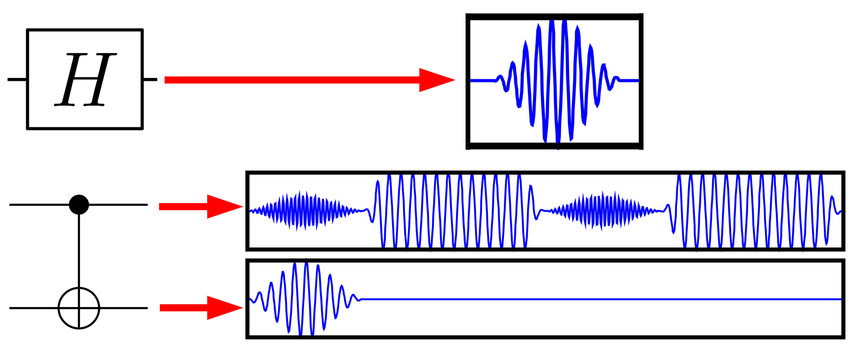
The model is designed to test the performance of the FT protocol exclusively in the presence of decoherence errors due to interaction with the environment. By construction, there are no control or measurement errors in this system.

System 2: Transmon qubits

The second system consists of 5 transmon qubits and 6 coupling resonators. They are modeled in terms of their full circuit Hamiltonian [3].



$$H = H_{\mathrm{tr}} + H_{\mathrm{res}}$$
$$H_{\mathrm{tr}} = \sum_i \left[4E_{Ci}(\hat{n}_i - n_{gi}(t))^2 - E_{Ji}\cos\hat{\varphi}_i\right]$$
$$H_{\mathrm{res}} = \sum_r \Omega_r \hat{a}_r^\dagger \hat{a}_r + \sum_{r,i} G_{ri}\hat{n}_i(\hat{a}_r + \hat{a}_r^\dagger)$$



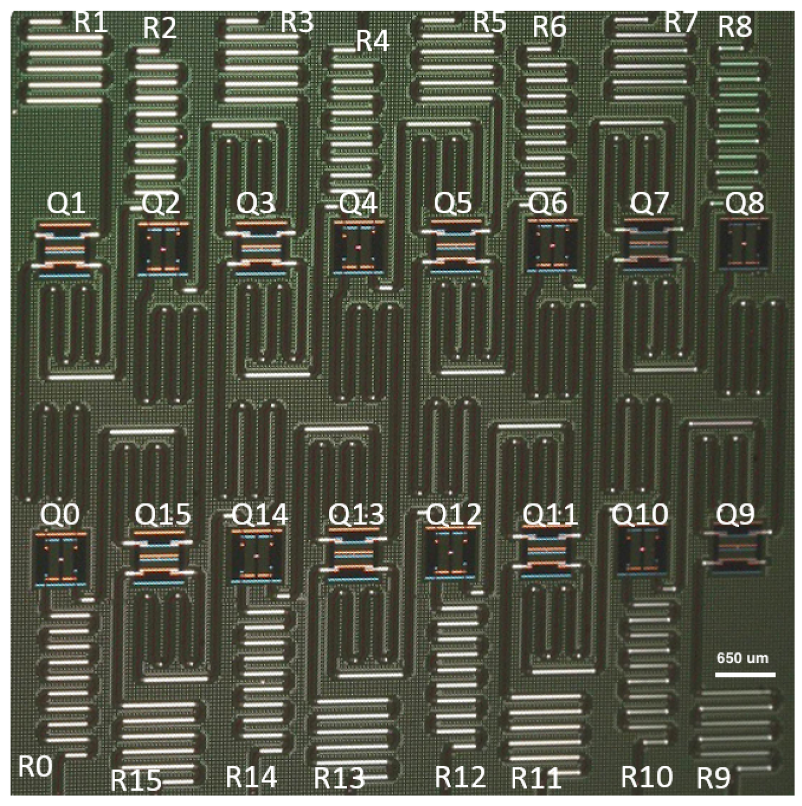
Quantum gates are implemented by the optimized microwave pulses used in the IBM Q [4]. Single-qubit gates use Gaussian envelopes, and two-qubit gates use the echoed cross-resonance scheme [5]. The model is dominated by inherent control errors from the pulses [6]

$$n_{gi}(t) = \Omega_{\mathrm{G}}(t)\cos(2\pi f t - \gamma) + \beta_X \dot{\Omega}_{\mathrm{G}}(t)\cos(2\pi f t - \gamma - \frac{\pi}{2})$$

The system is simulated by solving the TDSE with the Suzuki-Trotter algorithm [7].

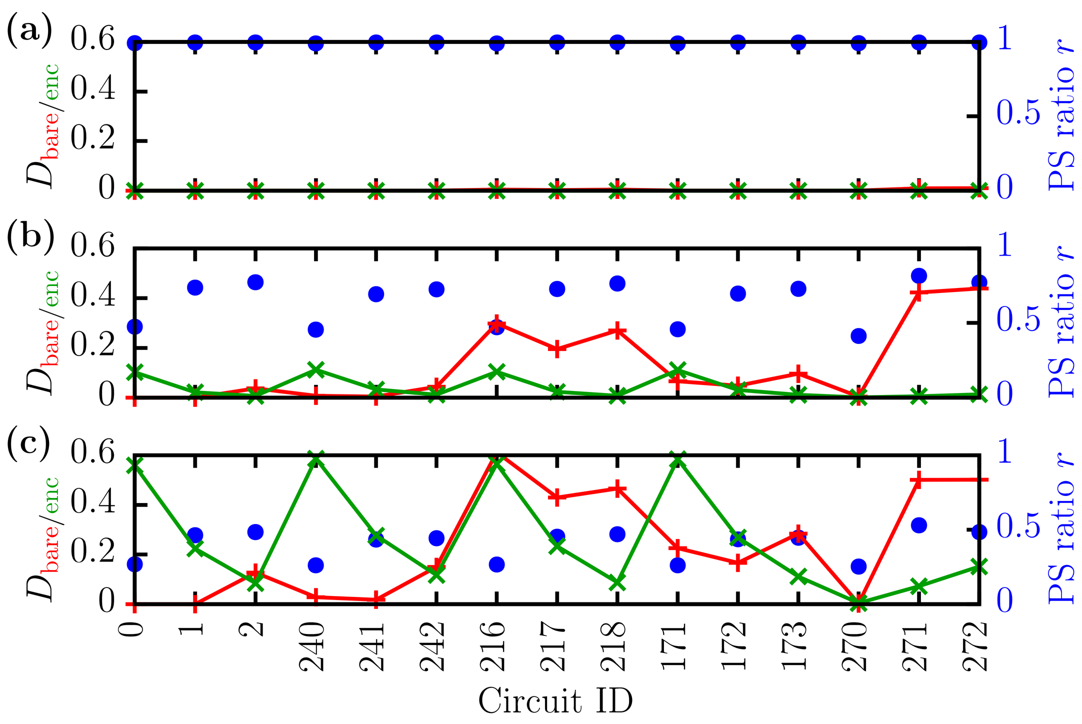
System 3: IBM 16-qubit device

The third system is a subset of the physical 16-qubit device **ibmqx5** [4].

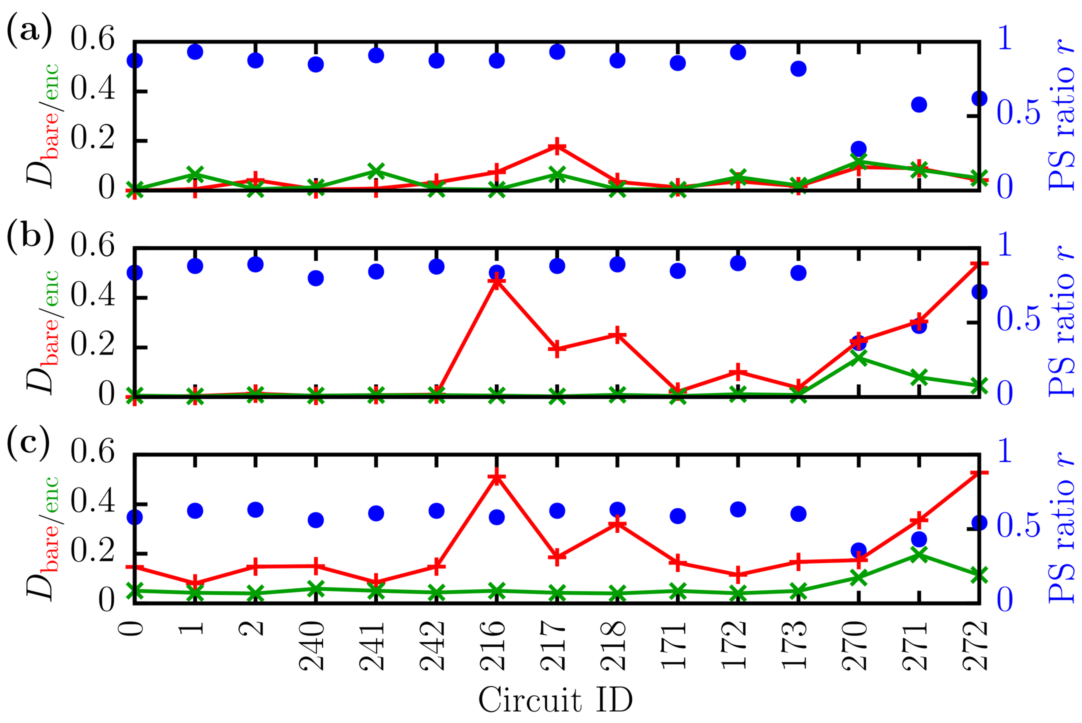


The FT protocol is implemented using the qubits (Q4,Q3,Q2,Q15,Q14) and its performance is tested against the inherent control and measurement errors in the processor [6,8].

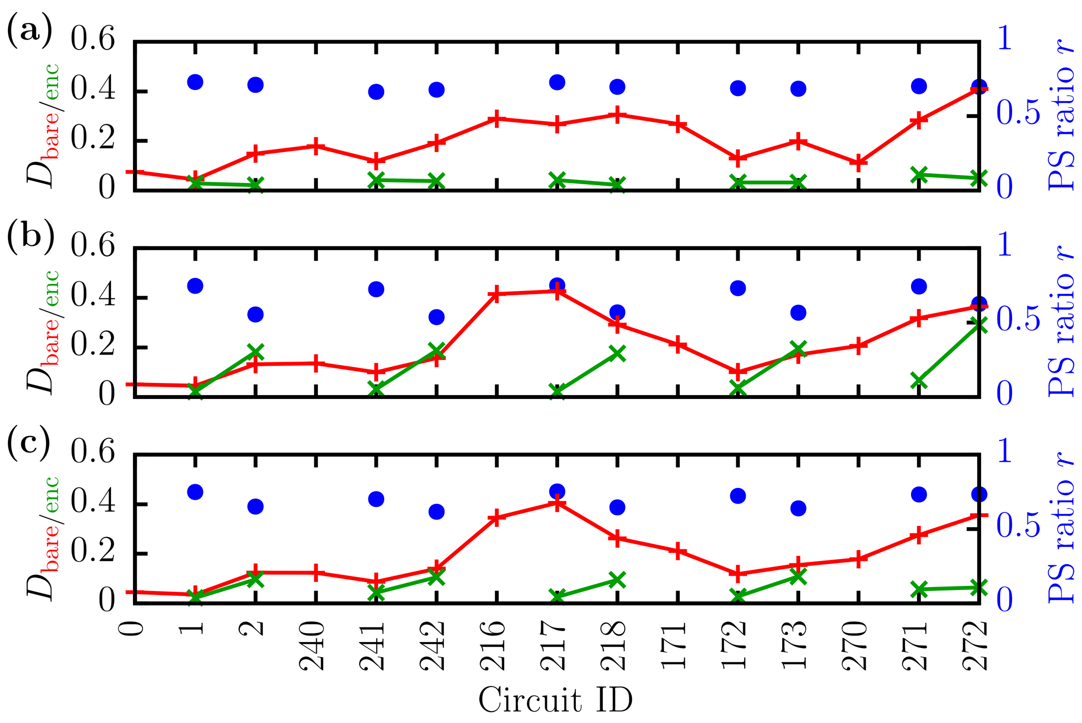
System 1: (a) $\lambda = 0.01$, (b) $\lambda = 0.1$, and (c) $\lambda = 0.2$. The results are for $N_E = 20$ and $\beta = 1$.



System 2: (a) optimized pulses, (b) f-optimized pulses, and (c) additional measurement error.



System 3: (a) April 3, 2018 (b) April 9, 2018, and (c) April 19, 2018. Encoded $\overline{|00\rangle}$ cannot be tested.



ID	Circuit
0-2	$\|i\rangle$
240-242	X1 X1 X1 X1 X1 $\|i\rangle$
216-218	CZ CZ CZ CZ CZ $\|i\rangle$
171-173	CZ X1 X2 Z1 Z1 X1 X1 Z1 Z1 Z2 $\|i\rangle$
270-272	HHS CZ HHS CZ HHS CZ HHS CZ HHS CZ $\|i\rangle$

Conclusion: The FT protocol does not satisfy the criterion for fault tolerance when decoherence errors dominate the system. However, it systematically improves the results for inherent control and measurement errors present in transmon processors [9].

References:

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