Using the fidelity and other metrics to measure gate performance

05.04.2018 I DENNIS WILLSCH

Willsch *et al.*, Phys. Rev. A **96**, 062302 (2017)

JÜLICH Forschungszentrum

What is the fidelity?

- **Measure for gate performance**

- **Idea: Average performance over random states**

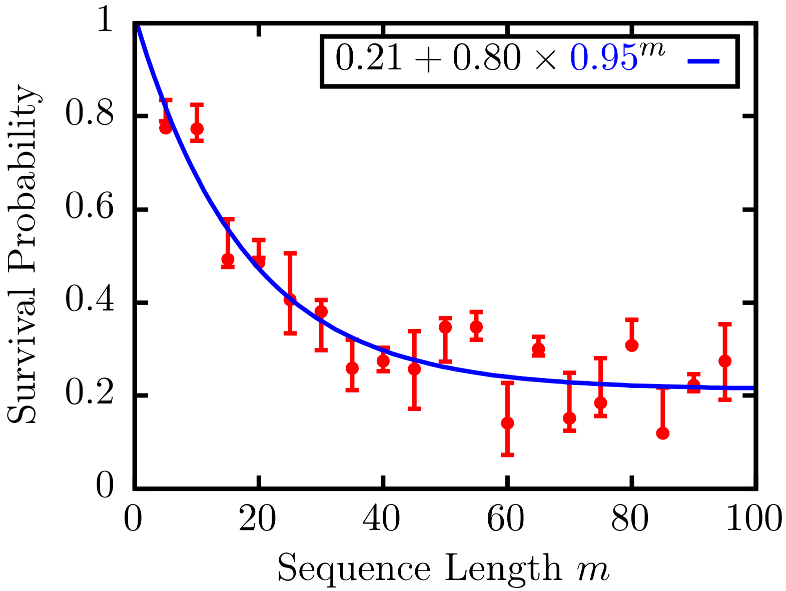
$$F_{\text{avg}} = \int \mathrm{d}|\psi\rangle \ \langle\psi|\, \mathcal{G}_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\, |\psi\rangle$$

Nielsen, Phys. Lett. A **303**, 249 (2002)

- **Experiment: Randomized Benchmarking**

  Emerson *et al.*, J. Opt. B **7**, S347 (2005)
  Magesan *et al.*, Phys. Rev. A **85**, 042311 (2012)

  - Fit exponential decay
  - Produces a single number
  - Scales well to many qubits



- **What is the problem?**

Problems with RB and the fidelity

- **RB does <u>not</u> measure the fidelity**
  - Tiny error in frequency: $F_{\text{avg}} = 0.999$ ⇔ RB: 0.99999
    Proctor *et al.*, Phys. Rev. Lett. **119**, 130502 (2017)
- **Fidelity does <u>not</u> give the error rate for fault-tolerance**
  - ✘ IBM: “fidelity above 0.995 suffices” Chow *et al.*, Phys. Rev. Lett. **109**, 060501 (2012)
  - ✘ Google: “fault-tolerance threshold passed” Barends *et al.*, Nature **508**, 500 (2014)
  - ➔ Required for two-qubit gate: $F_{\text{avg}} > 0.999995$
    Sanders *et al.*, New J. Phys. **1**, 012002 (2016)
- **Are other metrics better?**
- **Can they actually predict gate performance?**

Other gate metrics

$$\mathcal{G}_{ac}(|\psi\rangle\langle\psi|) = M\,|\psi\rangle\langle\psi|\,M^\dagger$$
$$\mathcal{G}_{id}(|\psi\rangle\langle\psi|) = U\,|\psi\rangle\langle\psi|\,U^\dagger$$

Average Gate Fidelity "statistical average"

$$F_{\text{avg}} = \int \mathrm{d}|\psi\rangle\ \langle\psi|\,\mathcal{G}_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\,|\psi\rangle$$

Nielsen, Phys. Lett. A **303**, 249 (2002)

Diamond Norm "worst-case error"

$$\eta_\diamond = \frac{1}{2}\left\|\mathcal{G}_{ac}\circ\mathcal{G}_{id}^{-1} - \mathbb{1}\right\|_\diamond$$

Kitaev, Russian Mathematical Surveys **52**, 1191 (1997)

Unitarity "quantifies leakage"

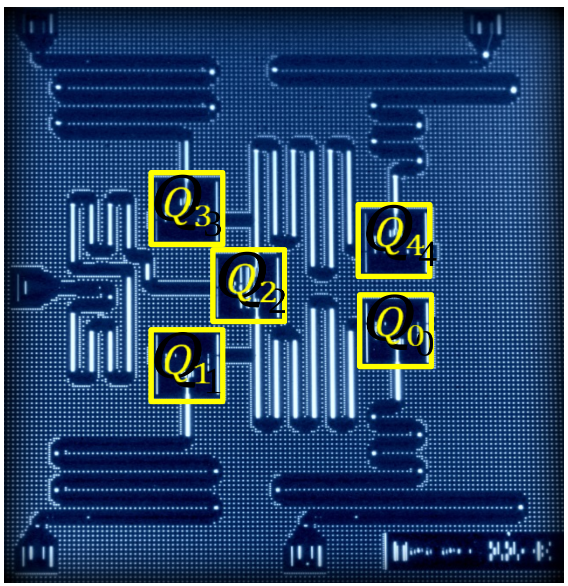
$$u = \frac{d}{d-1}\int \mathrm{d}|\psi\rangle\ \mathrm{Tr}\left[\mathcal{G}'_{ac}(|\psi\rangle\langle\psi|)^\dagger\mathcal{G}'_{ac}(|\psi\rangle\langle\psi|)\right]$$

Wallman et al., New J. Phys. **17**, 113020 (2015)

Simulation of the hardware

Hardware: IBMQX

IBM Quantum Experience



Simulation: TDSE

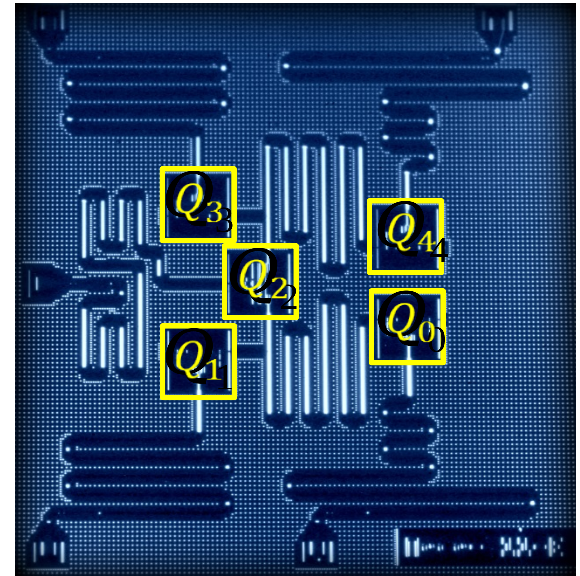
Time-dependent

Schrödinger Equation

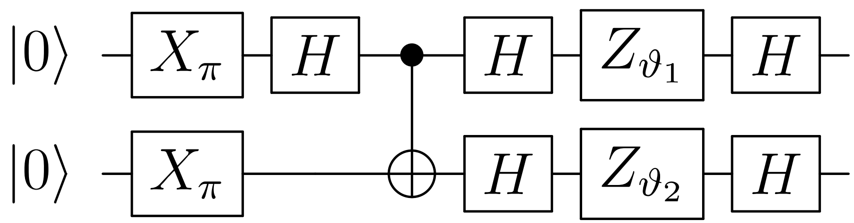
$$\boxed{i\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle = H(t)\left|\Psi(t)\right\rangle}$$

State of the qubits

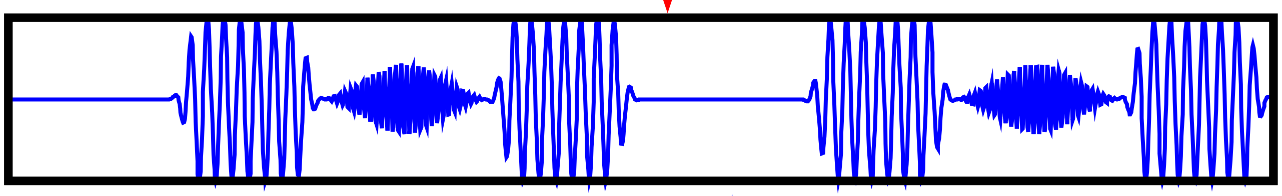
Simulation of the hardware



How to run a quantum circuit?



Each gate → Voltage pulse



$$H_{\mathrm{CPB}} = \sum_{i=1}^{N_{\mathrm{Tr}}} \left[E_{Ci} (\hat{n}_i - n_{gi}(t))^2 - E_{Ji} \cos \hat{\varphi}_i \right]$$

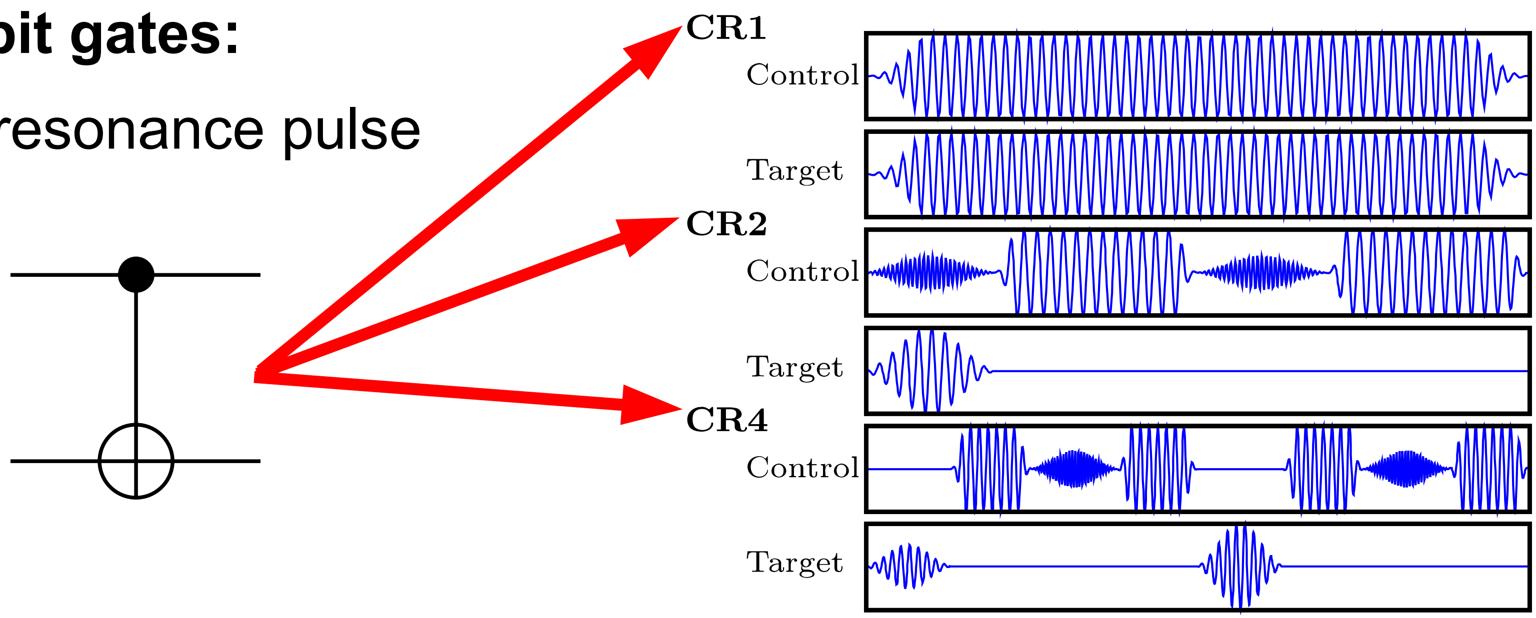
Simulation of the hardware

Single-qubit gates:

Gaussian pulse

Two-qubit gates:

Cross-resonance pulse



Pulses are optimized to be as good as theoretically possible!

Results: Fidelity

Gate	F_{avg}
$X^1_{\pi/2}$	0.9946
$X^2_{\pi/2}$	0.9942
X^1_{π}	0.9949
X^2_{π}	0.9943
$\mathbf{CR2}_{12}$	0.9943
$\mathbf{CR2}_{21}$	0.9947

Fidelity is always the same

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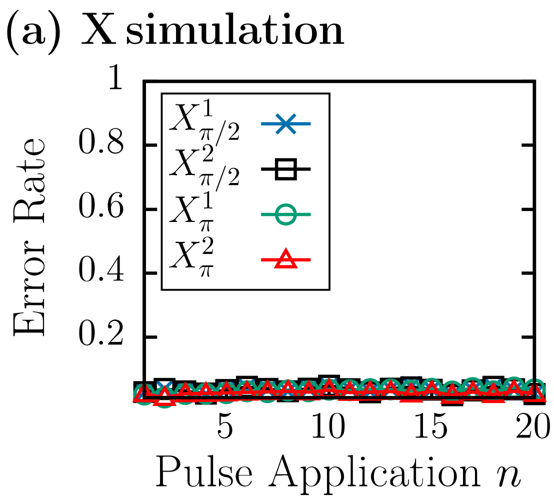

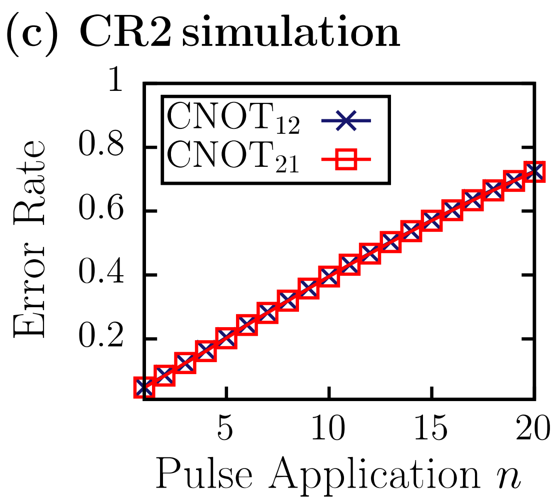

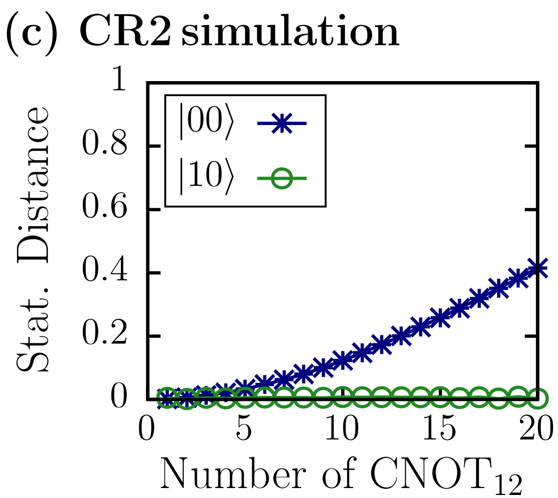

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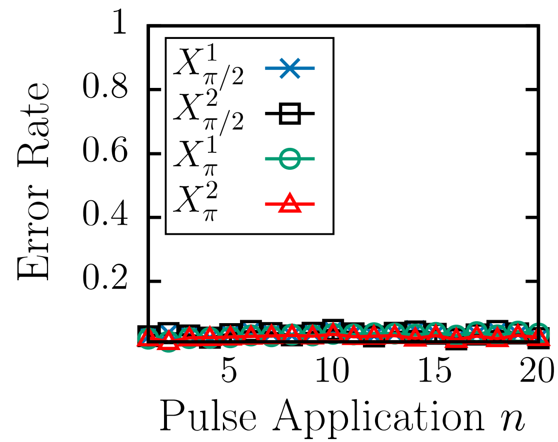
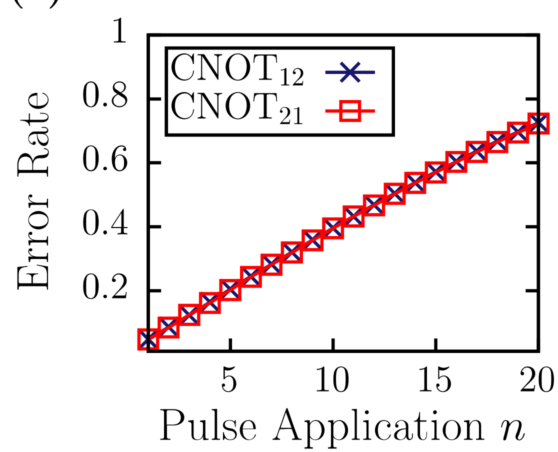
Fidelity is always the same

(a) X simulation

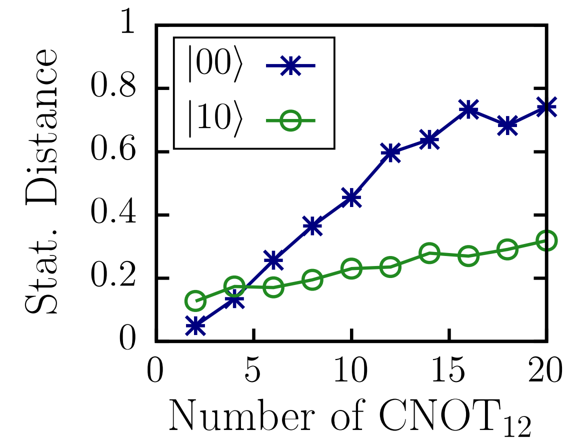


(c) CR2 simulation

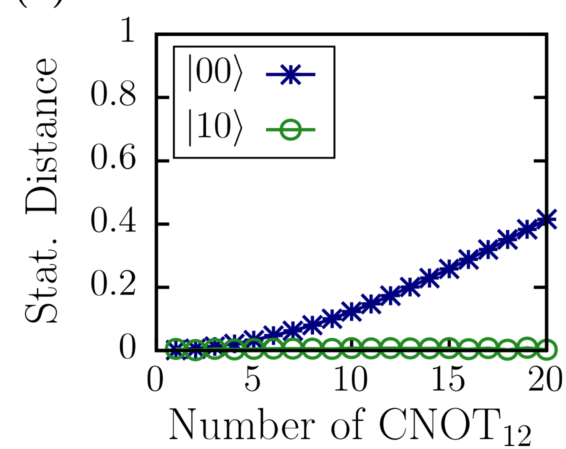


Experiment

(a) IBMQX (using CR2)



(c) CR2 simulation



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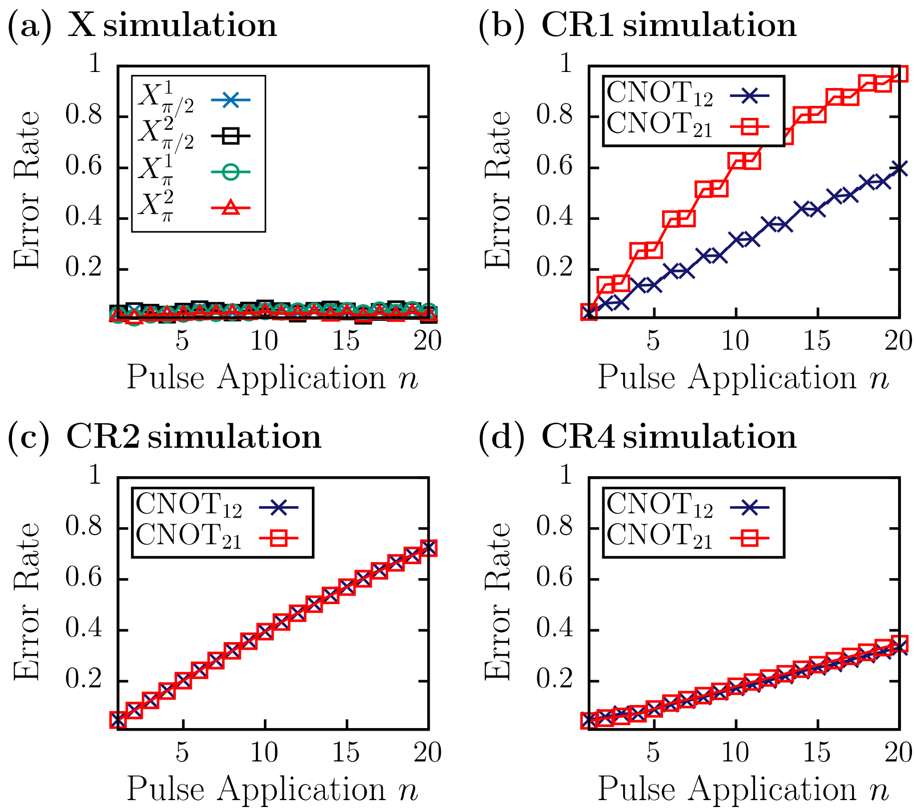
Results: Other gates and metrics

Gate	F_{avg}	$\eta_\diamond$	u
$X^1_{\pi/2}$	0.9946	0.027	0.990
$X^2_{\pi/2}$	0.9942	0.028	0.989
X^1_{π}	0.9949	0.020	0.990
X^2_{π}	0.9943	0.023	0.989
3⟶ $\mathbf{CR1}_{12}$	0.9842	0.029	0.969
1⟶ $\mathbf{CR1}_{21}$	0.9951	0.033	0.991
$\mathbf{CR2}_{12}$	0.9943	0.048	0.991
$\mathbf{CR2}_{21}$	0.9947	0.048	0.992
2⟶ $\mathbf{CR4}_{12}$	0.9934	0.049	0.989
$\mathbf{CR4}_{21}$	0.9946	0.044	0.991



1) Best fidelity **BUT** worst performance

2) Worst diamond norm **BUT** better performance

3) Unitarity: helps theoretically **BUT** not practically

Conclusions

✔ **RB is a stable, scalable procedure for an initial assessment**

✘ **RB does <u>not</u> measure the fidelity** Proctor *et al.*, Phys. Rev. Lett. **119**, 130502 (2017)

✘ **RB conceals systematic errors (→randomization)**

➔ **Don't trust a single, <u>statistical</u> number**

✘ **Fidelity is <u>not</u> sufficient for fault-tolerance**

Sanders *et al.*, New J. Phys. **1**, 012002 (2016)

✘ **<u>None</u> of the gate metrics can reliably <u>predict</u> gate performance**

Willsch *et al.*, Phys. Rev. A **96**, 062302 (2017)

- **for repeated applications**
- **for use in a quantum algorithm**

➔ **Repeat gate pulses**

➔ **Run specific (identity) circuits for benchmarking**

Michielsen *et al.*, Comp. Phys. Comm. **220**, 44 (2017)

THANK YOU
FOR YOUR ATTENTION

Why can't RB measure the fidelity?

Proctor *et al.*, Phys. Rev. Lett. **119**, 130502 (2017)

Experiment:

Physical QIP

Theory:

Mathematical Model

RB $\longrightarrow r$

$$F_{\rm avg} = \int \mathrm{d}|\psi\rangle \; \langle\psi|\, \mathcal{G}_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\, |\psi\rangle$$

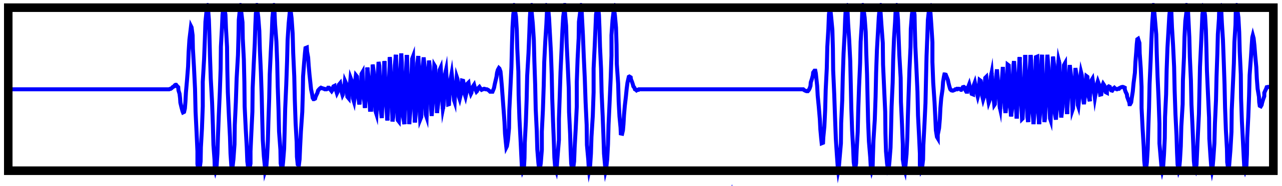
$$F'_{\rm avg} = \int \mathrm{d}|\psi\rangle \; \langle\psi|\, \mathcal{G}'_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\, |\psi\rangle$$

$$F''_{\rm avg} = \int \mathrm{d}|\psi\rangle \; \langle\psi|\, \mathcal{G}''_{ac}(\mathcal{G}_{id}^{-1}(|\psi\rangle\langle\psi|))\, |\psi\rangle$$

Optimization of gate pulses

Willsch *et al.*, Phys. Rev. A **96**, 062302 (2017)

- Many parameters → multidimensional optimization



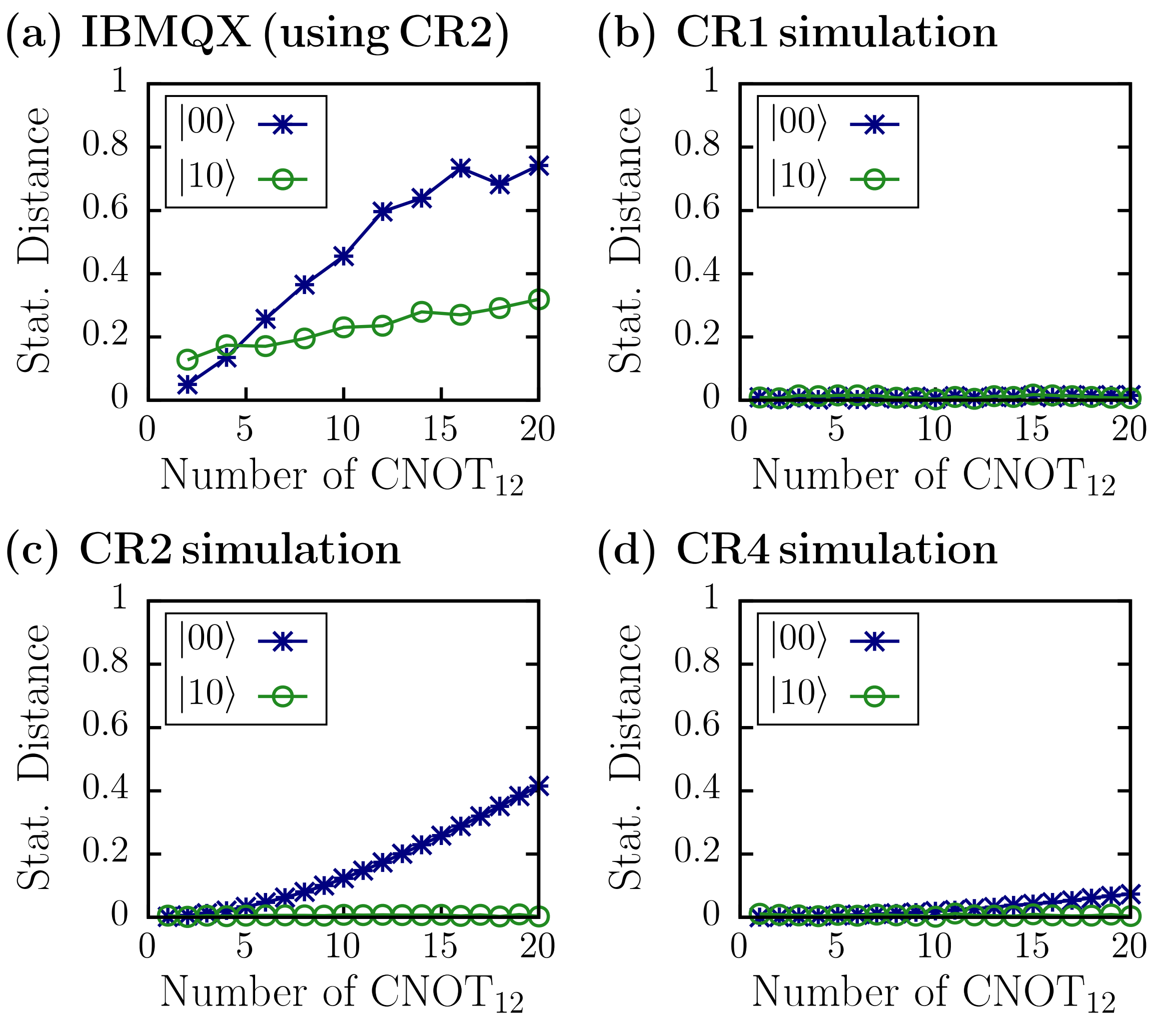
- With gradients: FR, PR, DFP / BFGS (quasi-Newton)
- Gradient-free: Nelder-Mead (downhill simplex), Powell's
- Machine learning → neural networks
- … many other complex ideas

✔ Can improve the fidelity a little bit

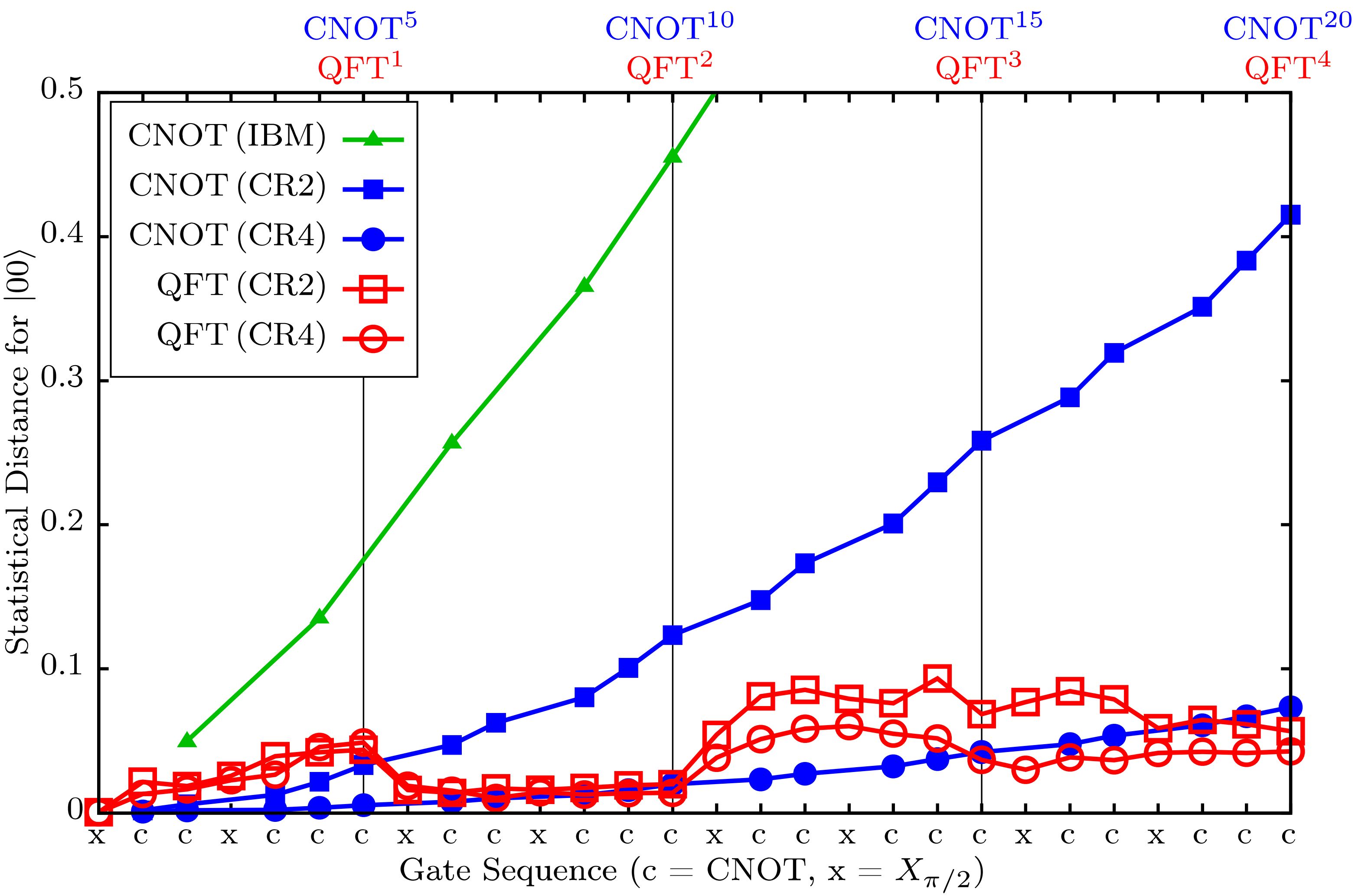
✘ Does <u>not</u> necessarily improve the gates in real applications!

➔ Our observation: Best performance obtained by sticking to the model and fine-tune parameters with Nelder-Mead

Repeated CNOT Gates

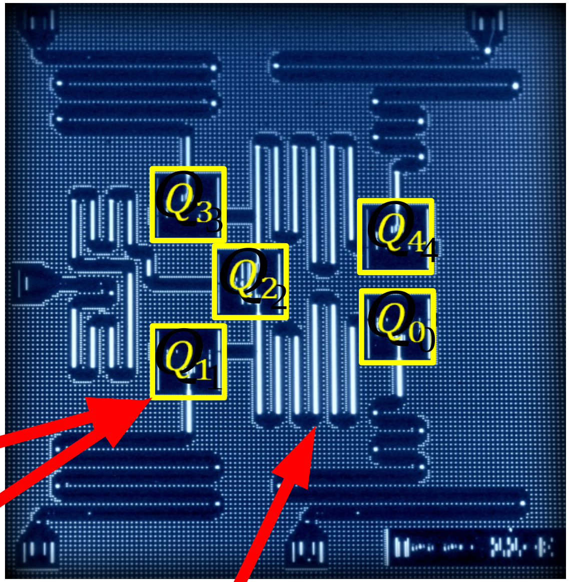


CNOT in QFT



CNOT^5 QFT^1
CNOT^{10} QFT^2
CNOT^{15} QFT^3
CNOT^{20} QFT^4
CNOT (IBM)
CNOT (CR2)
CNOT (CR4)
QFT (CR2)
QFT (CR4)
Statistical Distance for $|00\rangle$
0
0.1
0.2
0.3
0.4
0.5
x c c x c c c x c c x c c c x c c x c c c x c c x c c c
Gate Sequence (c = CNOT, x = $X_{\pi/2}$)

Simulation of the Hardware



Hardware → Hamiltonian

$$H = H_{\mathrm{CPB}} + H_{\mathrm{Res}} + H_{\mathrm{CC}}$$

Qubits: Transmons

$$H_{\mathrm{CPB}} = \sum_{i=1}^{N_{\mathrm{Tr}}} \left[E_{Ci}(\hat{n}_i - n_{gi}(t))^2 - E_{Ji} \cos \hat{\varphi}_i \right]$$

Coupling: Resonators

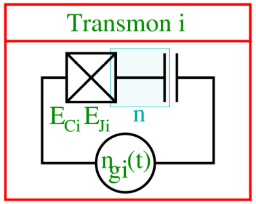
$$H_{\mathrm{Res}} = \omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^{N_{\mathrm{Tr}}} g_i \hat{n}_i (\hat{a} + \hat{a}^\dagger)$$

IBMQX Transmon System

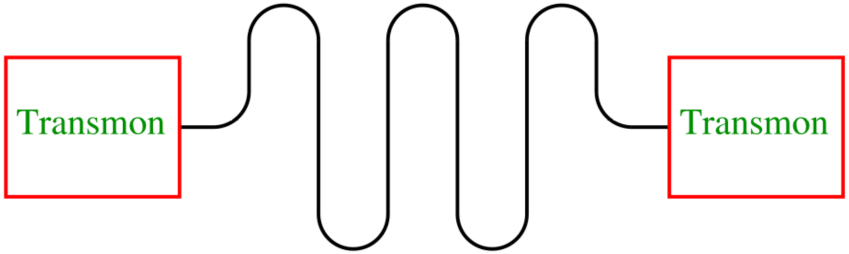
$$H = H_{\mathrm{CPB}} + H_{\mathrm{Res}} + H_{\mathrm{CC}}$$

Transmon	$E_{Ci}/2\pi$	$E_{Ji}/2\pi$	$\omega_i/2\pi$	$\omega_r/2\pi$	$g_i/2\pi$
1	1.204	13.349	5.350	7	0.07
2	1.204	12.292	5.120	7	0.07

$$H_{\mathrm{CPB}} = \sum_{i=1}^{N_{\mathrm{Tr}}} \left[E_{Ci}(\hat{n}_i - n_{gi}(t))^2 - E_{Ji}\cos\hat{\varphi}_i\right]$$



$$H_{\mathrm{Res}} = \omega_r \hat{a}^\dagger \hat{a} + \sum_{i=1}^{N_{\mathrm{Tr}}} g_i \hat{n}_i (\hat{a} + \hat{a}^\dagger)$$



$$H_{\mathrm{CC}} = \sum_{1 \le i < j \le N_{\mathrm{Tr}}} E_{Ci,Cj} \hat{n}_i \hat{n}_j$$