Superconducting flux qubits compared to ideal two-level systems as building blocks for quantum annealers

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Quantum annealing

- Preparation in known ground state of initial Hamiltonian H_{initial}
- Adiabatic transformation to the problem Hamiltonian H_{final}

$$H(s) = A(s)H_{\text{initial}} + B(s)H_{\text{final}}.$$

- Functions $A(s)$ and $B(s)$, with $s = t/T_{\max}$ and $T_{\max}$ annealing time, determine the annealing scheme and satisfy

$$\begin{aligned} A(0) > 0 \quad & A(1) \approx 0 \\ B(0) \approx 0 \quad & B(1) > 0. \end{aligned}$$

- During the annealing process, the system stays in its ground state (if $T_{\max} \to \infty$; adiabatic theorem)
- Final state gives solution (ground state) of problem Hamiltonian
- Hamiltonian of quantum annealer built by D-Wave Systems Inc.:

$$H(s) = -A(s)\sum_k \sigma_k^x - B(s)\left(\sum_k h_k\sigma_k^z + \sum_{l<k} J_{lk}\sigma_k^z\sigma_j^z\right),$$

where h_k, $J_{lk} \in [-1, 1]$ have to be chosen according to the problem

Superconducting flux qubits (rf-SQUID)

- Hamiltonian of a single rf-SQUID:

Harris *et al.*, Phys. Rev. B 81, 134510, 2010
Johnson *et al.*, Nature 473, 194, 2011
Boixo *et al.*, Nat. Comm. 7, 10327, 2016

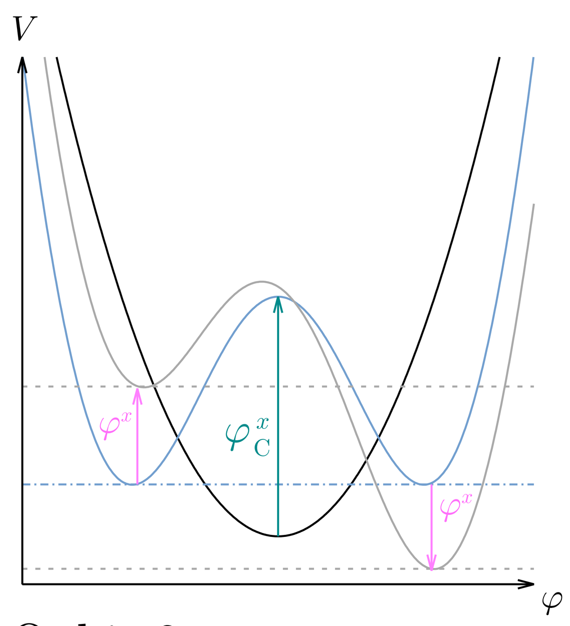
$$\begin{aligned} H_i(s) = &-E_{C_{\text{C}i}}\partial^2_{\varphi_{\text{C}i}} + E_{L_{\text{C}i}}(\varphi_{\text{C}i} - \varphi^x_{\text{C}i}(s))^2/2 \\ &-E_C\partial^2_{\varphi_i} + E_L(\varphi_i - \varphi_i^x(s))^2/2 - E_J\cos(\varphi_i)\cos(\varphi_{\text{C}i}/2), \end{aligned}$$

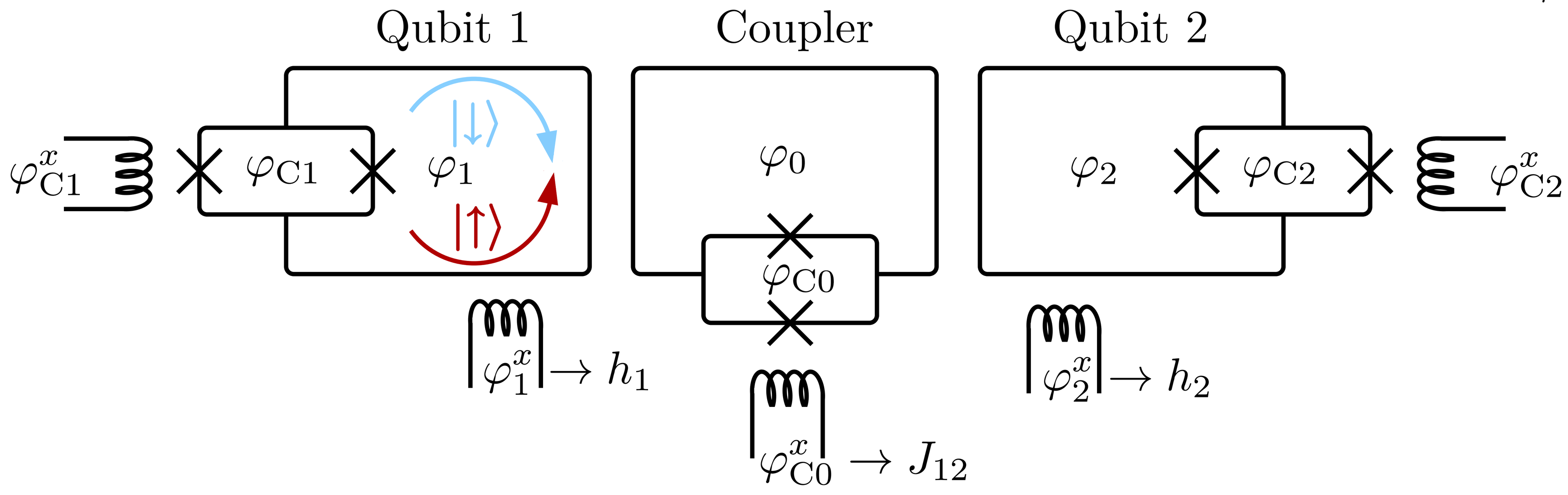
- and interaction terms:

Maassen van den Brink *et al.*, New J. Phys. 7, 230, 2005
Harris *et al.*, Phys. Rev. B 80, 052506, 2009

$$\begin{aligned} H_{\text{int}} = &(M/L_{\text{eff}})E_L(\varphi_1 - \varphi_1^x)(\varphi_0 - \varphi_0^x) + (M/L_{\text{eff}})E_L(\varphi_2 - \varphi_2^x)(\varphi_0 - \varphi_0^x) \\ &+(M^2/L_qL_{\text{eff}})E_L(\varphi_1 - \varphi_1^x)(\varphi_2 - \varphi_2^x) \end{aligned}$$

- external fluxes $\varphi^x_{\text{C}i}(s)$ and $\varphi^x_i(s)$ determine $A(s)$ and $B(s)$ for the qubits
- $\varphi^x_i(s)$ depend on the parameters of the problem Hamiltonian
- $\varphi^x_{\text{C}i}$ gives a tunable Josephson-Junction
  - Qubit: changes potential for φ_i(which defines qubit states) from monostable to bistable
  - Coupler: leads to tunable coupling constant J





Suzuki-Trotter product-formula algorithm

De Raedt, Comp. Phys. Rep. 7, 1, 1987

- Algorithm to solve the time-dependent Schrödinger equation

$$i\partial_t|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

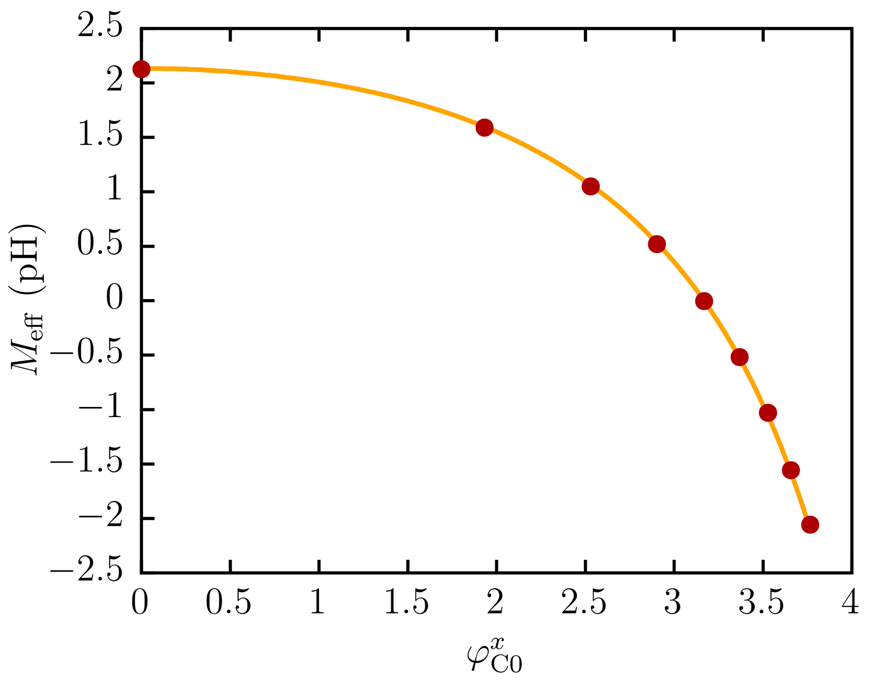
- Hamiltonian is discretized in time
- State vector $|\psi(t)\rangle$ is updated for each time step τ

$$|\psi(t+\tau)\rangle = e^{-iH(t+\tau/2)\tau}|\psi(t)\rangle$$

- If τ sufficiently small, time-evolution operator $e^{-iH(t)\tau}$ is well approximated by $e^{-iH(t)\tau} = e^{-i\sum_k A_k(t)\tau} \approx \prod_k e^{-iA_k(t)\tau}$
- Decomposition $H(t) = \sum_k A_k(t)$ ideally chosen such that exponentials are performed in two-component updates of $|\psi(t)\rangle$

Simulation results

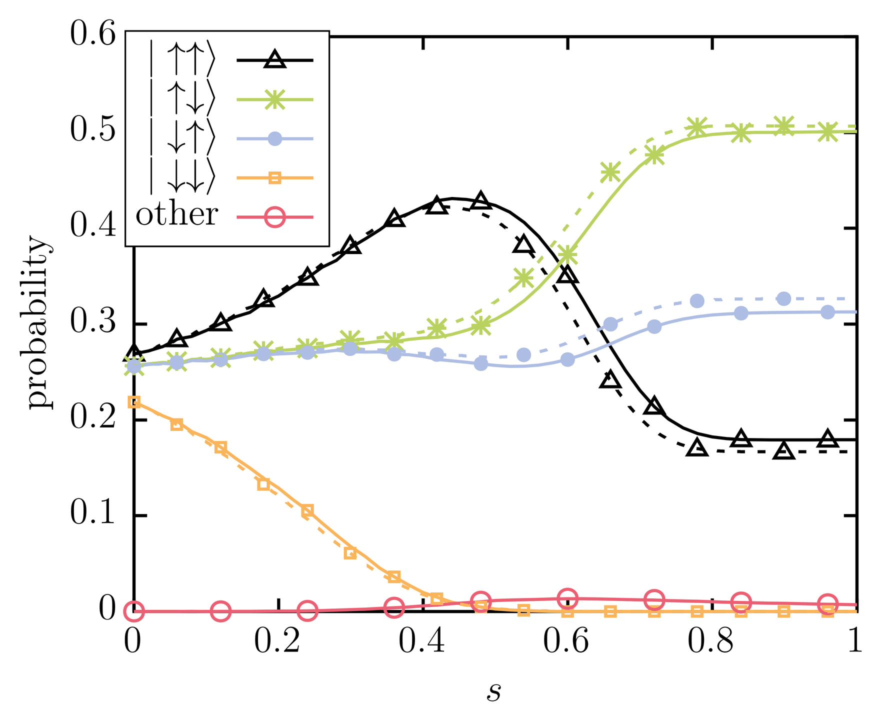
Tunable effective coupling



- Change in $\varphi^x_{\text{C}0}$ leads to different effective mutual inductance M_{eff}
- Analytical calculation includes approximations and basis transformation
  - Leads to $M_{\text{eff}}(\varphi^x_{\text{C}0}) \propto J(\varphi^x_{\text{C}0})$
  - Simulation agrees with theory
- Depending on the qubit states, the coupler is in a coherent state

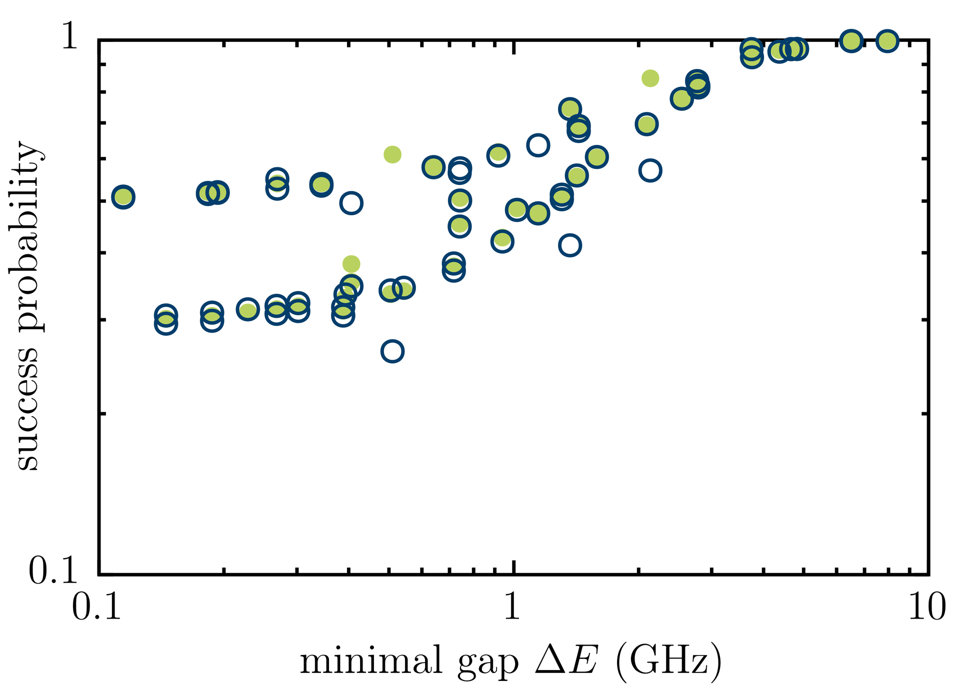
Fig. 1: Effective mutual inductance between the qubits depending on $\varphi^x_{\text{C}0}$ from the simulation (bullets ●) and the theory (line —).

Comparison to the 2-level system



- Evolution during the annealing process
- Small deviations in the probabilities
- Some amount of leakage out of the computational subspace
- All in all, nice agreement

Fig. 2: Probabilities of the basis states during the annealing process for the flux qubits (solid), the ideal 2-level system (dashed) with $T_{\max} = 5\,\text{ns}$. Parameters: $h_1 = 0.96$, $h_2 = 0.94$, $J = -1$



- Final success probability
- Case-dependent deviations in the probabilities (in both directions)
- Possible reasons:
  - Computation of $\varphi^x_{\text{C}0}(J)$ includes approximations
  - Higher order terms that effectively change h_i
- General features are in good agreement

Fig. 3: Success probability depending on the minimal energy gap between the ground state and the first excited state during the annealing process for the ideal 2-level system (bullets ●) and flux qubits (circles ○).

Conclusion

- By using the Suzuki-Trotter product-formula algorithm, we can simulate the dynamics of the full system and compare it to the 2-level system as well as to the analytical calculation including approximations.
- For the investigated case, the simulation results of the effective coupling agree with the theory and the experiment. Thus, the analytical approximations can be justified, and the experimental setup can be described by this Hamiltonian.
- We find deviations during the evolution and the final probabilities between the flux qubits and the 2-level system. However, these are rather small and not surprising due to the approximations made.