Derivation of spontaneously broken gauge symmetry from the consistency of effective field theory II: Scalar field self-interactions and the electromagnetic interaction

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Abstract

We extend our study of deriving the local gauge invariance with spontaneous symmetry breaking in the context of an effective field theory by considering self-interactions of the scalar field and inclusion of the electromagnetic interaction. By analyzing renormalizability and the scale separation conditions of three-, four- and five-point vertex functions of the scalar field, we fix the two couplings of the scalar field self-interactions of the leading order Lagrangian. Next we add the electromagnetic interaction and derive conditions relating the magnetic moment of the charged vector boson to its charge and the masses of the charged and neutral massive vector bosons to each other and the two independent couplings of the theory. We obtain the bosonic part of the Lagrangian of the electroweak Standard Model as a unique solution to the conditions imposed by the self-consistency conditions of the considered effective field theory.

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I. INTRODUCTION

Local gauge invariance is taken as an input in the construction of the Standard Model [1]. On the other hand, a gauge-invariant theory with the spontaneous symmetry breaking can be derived by demanding tree-order unitarity of the S-matrix [2–5]. The modern point of view considers the Standard Model as an effective field theory (EFT) [1], in which tree-order unitarity is in any case violated at sufficiently high energies. This motivated us to address the issue of deriving the Lagrangian of the electroweak interaction from the conditions of self-consistency of EFT. In Ref. [6] we started with constructing the most general Lorentz-invariant EFT Lagrangian of three interacting massive vector bosons and a scalar. Non-trivial relations between the coupling constants of the interaction terms of the most general Lorentz-invariant Lagrangian of a scalar and vector bosons are imposed by the conditions of consistency with the second class constraints which must be satisfied by the systems with spin-one particles [7]. Further restrictions on the interaction terms are imposed by the condition of perturbative renormalizability in the sense of an EFT and scale separation. The last condition requires that contributions of higher order terms of the effective Lagrangian in physical quantities are suppressed by some large scale(s) (for more details see Ref. [6]). To achieve this scale separation we have to demand that the divergences of loop diagrams contributing in *physical* scattering amplitudes generated by the leading order Lagrangian should be removable by renormalizing the parameters of the leading order Lagrangian alone.

In Ref. [6] we considered three- and four-point vertex functions and required perturbative renormalizability and scale separation. This led to conditions imposed on the interaction terms such that we obtained the Lagrangian of spontaneously broken gauge symmetry in the unitary gauge except that the coupling constants of the self-interactions of the scalar field remained unfixed. In the current work we analyse one-loop diagrams contributing to three-, four- and five-point functions of the scalar field and constrain the two free couplings of the self-interactions.

Next, in close analogy to Ref. [8] we "switch on" the electromagnetic interaction. By demanding perturbative renormalizability of the obtained effective Lagrangian, we relate the magnetic moment of the charged vector boson and the mixing of the neutral vector bosons to other parameters of the effective Lagrangian. This completes the derivation of the bosonic part of the electroweak Standard Model in the framework of EFT.

II. FIXING THE COUPLINGS OF THE SCALAR SELF-INTERACTIONS

Here, we continue the study of the most general Lorentz-invariant effective Lagrangian of a scalar and three massive vector boson fields respecting electromagnetic charge conservation, started in Ref. [6]. Two charged spin-one particles are represented by the vector fields $V^\pm_\mu = (V^1_\mu \mp i V^2_\mu)/\sqrt{2}$, while the third component, V^3_μ, and the scalar field Φ are charge-neutral. The effective Lagrangian contains an infinite number of interaction terms and hence depends on an infinite number of parameters. We assume that coupling constants with negative mass dimensions are independent from those of positive and zero mass dimensions. In Ref. [6] we analyzed the Lagrangian containing only interaction terms with coupling constants of non-negative dimensions. By demanding conservation of the second class constraints, perturbative renormalizability in the sense of EFT and scale separation,

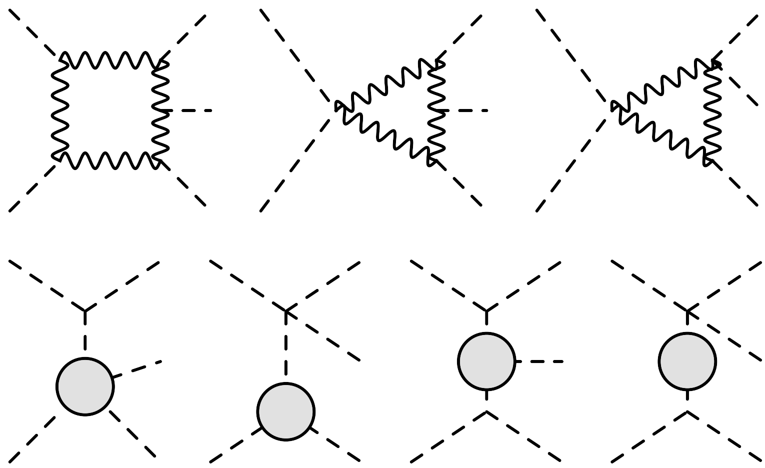

FIG. 1: One-loop contributions to the five-point vertex function of the scalar field. The dashed and wiggly lines correspond to the scalar and the vector bosons, respectively. Diagrams that are generated via permutations of external legs are not shown. Blobs indicate the corresponding one-loop two-, three- and four-point vertex functions. In the last four diagrams only the one-particle-irreducible parts are taken into account.

we showed that the effective Lagrangian can be written in a compact form

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}\, G^a_{\mu\nu} G^{a\mu\nu} + \frac{1}{2}\, V^a_\mu V^{a\mu} \left(M - \frac{g}{2}\,\Phi \right)^2 - g_{A1} \epsilon^{abc}\, \epsilon^{\mu\nu\alpha\beta} V^a_\mu V^b_\nu \partial_\alpha V^c_\beta\,, \\ + \; & \frac{1}{2}\,\partial_\mu \Phi\, \partial^\mu \Phi - \frac{m^2}{2}\,\Phi^2 - a\,\Phi - \frac{b}{3!}\,\Phi^3 - \frac{\lambda}{4!}\,\Phi^4\,, \end{aligned} \tag{1}$$

where

$$G^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g\,\epsilon^{abc}\, V^b_\mu V^c_\nu\,. \tag{2}$$

The value of the parameter a can be changed by shifting the field Φ with a constant value. For convenience we fix the scalar field such that $a \equiv 0$, i.e. the vacuum expectation value of the scalar field is non-vanishing starting at one-loop order. The Lagrangian of Eq. (2) coincides with the SU(2) locally gauge invariant Lagrangian of scalars and vector bosons with spontaneous symmetry breaking in the unitary gauge except for the self-interaction terms of the scalars. As reported in Ref. [6] we checked by explicit calculations that no further constraints on couplings are generated by the condition of perturbative renormalizability and scale separation of the three- and four-point functions of scalar and vector bosons alone. To impose conditions to the two scalar self-interaction couplings we continue our study by a simultaneous analysis of the three-, four- and five-point functions of the scalar field.

We impose the on-mass-shell renormalization condition, i.e. require that all divergences in physical quantities should be removable by redefining the parameters of the effective Lagrangian. As there is no interaction term with five scalar fields in the LO Lagrangian, the sum of divergences of the one-loop diagrams contributing to the five-point function should cancel when all external momenta are put on-mass-shell.

The one-loop diagrams which contribute to the five point function of the scalar field are shown in Fig. 1. We apply dimensional regularization (see, e.g., Ref. [9]) and for calculating the loop diagrams, we independently use the programs FeynCalc [10, 11] and Form [12]. The divergent parts of the one-loop integrals have been checked with the expressions obtained in Ref. [13]. We have checked the generation of all diagrams using FeynArts [14].

Calculating the irreducible one-loop diagrams shown in Fig. 1 (plus permutations), we obtain for the coefficient of the divergent part:

$$\frac{45i\pi^2 g^5 m^4}{2M^5}. \tag{3}$$

The coefficient of the divergent part of the irreducible parts of the reducible diagrams shown in the second line of Fig. 1 (plus permutations) has the form

$$\frac{15i\pi^2 g^2 \left(3b^2 gM + 9bg^2m^2 + 2b\lambda M^2 + 4g\lambda m^2 M\right)}{4M^4}\ . \tag{4}$$

Demanding that the sum of Eqs. (3) and (4) vanishes, we obtain

$$\frac{15i\pi^2 g^2 \left(bM + 2gm^2\right)\left(3bgM + 3g^2m^2 + 2\lambda M^2\right)}{4M^5} = 0, \tag{5}$$

which has two solutions

$$b = -\frac{2gm^2}{M}, \tag{6}$$

and

$$\lambda = -\frac{3g\left(bM + gm^2\right)}{2M^2}\ . \tag{7}$$

In the following we will show that only the latter solution leads to a self-consistent theory. To that end we substitute the bare parameters of the Lagrangian with the renormalized ones and the corresponding counterterms ($p = p_R + \sum_{i=1}^{\infty} \hbar^i \delta p_i$, where p is any of the bare parameters) in Eqs. (6) and (7) and expand in powers of $\hbar$. This generates the following conditions:

$$b_R = -\frac{2g_R m_R^2}{M_R}, \tag{8}$$

$$\delta \mathrm{b}_1 = -\frac{2m_R M_R \left(2\delta \mathrm{m}_1 g_R + \delta \mathrm{g}_1 m_R\right) - 2\delta \mathrm{M}_1 g_R m_R^2}{M_R^2}, \tag{9}$$

$$\cdots,$$

and

$$\lambda_R = -\frac{3g_R(g_R m_R^2 + b_R M_R)}{2M_R^2}, \tag{10}$$

$$\delta\lambda_1 = -\frac{1}{2M_R^3}\Big[3g_R M_R \left(-\delta \mathrm{M}_1 b_R + 2\delta \mathrm{g}_1 m_R^2 + \delta \mathrm{b}_1 M_R\right) + 3M_R^2 \delta \mathrm{g}_1 b_R + 6g_R^2 m_R \left(\delta \mathrm{m}_1 M_R - \delta \mathrm{M}_1 m_R\right)\Big], \tag{11}$$

$$\cdots,$$

respectively. Equations (9) and (11) impose conditions on the divergent parts of one-loop vertex functions, the divergent parts of which are cancelled by the corresponding one-loop counterterms (δb_1, $\delta\lambda_1$, $\delta \mathrm{g}_1$, $\delta \mathrm{M}_1$, $\delta \mathrm{m}_1$).

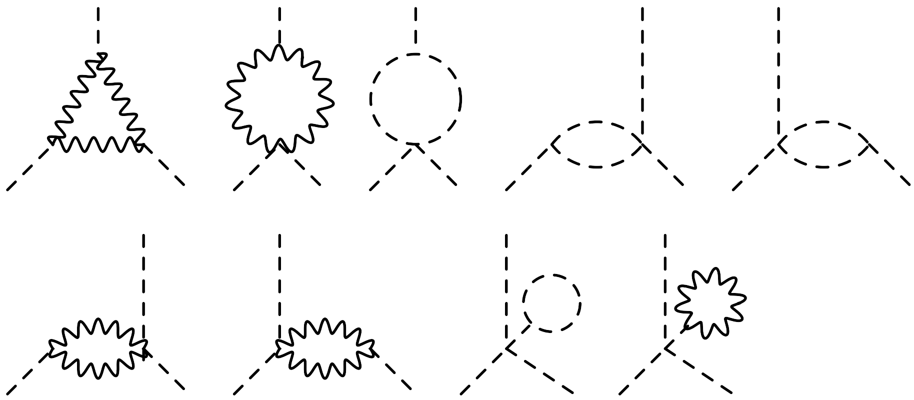

FIG. 2: One-loop contributions to the three-scalar vertex function. The dashed and the wiggly lines correspond to the scalar and the vector bosons, respectively.

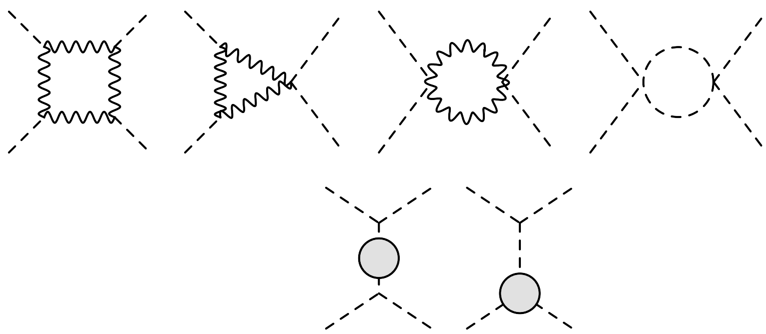

FIG. 3: One-loop contributions to the four-scalar vertex function. The dashed and wiggly lines correspond to the scalar and the vector bosons, respectively. Diagrams that can be generated via permutations of external legs are not shown. Blobs indicate the corresponding one-loop two- and three-point vertex functions. In last two diagrams only the one-particle-irreducible parts are taken into account.

From the calculations of Ref. [6] we have

$$
\begin{aligned}
\delta \mathrm{g}_1 &= -\frac{43}{6}\pi^2 g_R^3\,, \\
\delta \mathrm{M}_1 &= \frac{\pi^2 g_R \left(6 b_R m_R^2 - g_R M_R \left(59 m_R^2 + 54 M_R^2\right)\right)}{12 m_R^2}\,, \\
\delta \mathrm{m}_1 &= \frac{\pi^2 \left(36 b_R g_R M_R^5 + 3 g_R^2 \left(-6 m_R^4 M_R^2 + 18 m_R^2 M_R^4 + m_R^6\right) + 4 m_R^4 M_R^2 \lambda_R\right)}{8 m_R^3 M_R^2}\,.
\end{aligned}
\tag{12}
$$

The one-loop counterterm δg_1 was obtained by demanding that the three-point vertex function of the renormalized vector fields for all three external momenta taken on mass-shell is finite. The same expression takes also care of the cancellation of the divergences in the four-point vertex function of the vector fields when all external momenta are taken on mass shell. The counterterms δM_1 and δm_1 were determined from the condition that the pole masses of the vector bosons and the scalar particle are finite at one-loop order.

The counterterms $\delta \mathrm{b}_1$ and $\delta\lambda_1$ are obtained by calculating the divergent parts of the one-loop diagrams shown in Figs. 2 and 3, respectively:

$$\begin{aligned}\delta\lambda_1 &= \frac{3\pi^2}{4M_R^4}\Big(g_R^2 M_R^2\left(3b_R^2+4\lambda_R\left(m_R^2-3M_R^2\right)\right)+12b_R g_R^3 m_R^2 M_R \\ &+ 9g_R^4\left(m_R^4+M_R^4\right)+4M_R^4\lambda_R^2\Big)\,, \qquad (13)\\ \delta \mathrm{b}_1 &= \frac{\pi^2}{4m_R^2 M_R^3}\Big(9b_R g_R^2 m_R^2 M_R\left(m_R^2-3M_R^2\right)+8b_R m_R^2 M_R^3\lambda_R \\ &+ 9g_R^3\left(m_R^6-6m_R^2M_R^4\right)+36g_R M_R^6\lambda_R\Big)\,. \qquad (14)\end{aligned}$$

By taking into account Eqs. (12) and (14) in Eq. (9) and using Eq. (8), we obtain the following condition which has to be satisfied by renormalized parameters λ_R, g_R, m_R and M_R:

$$\frac{g_R\left(g_R^2\left(5m_R^6-54m_R^2M_R^4\right)-8m_R^4M_R^2\lambda_R+36M_R^6\lambda_R\right)}{m_R^2M_R^3}=0. \qquad (15)$$

The solution to Eq. (15)

$$\lambda_R=\frac{g_R^2\left(54m_R^2M_R^4-5m_R^6\right)}{36M_R^6-8m_R^4M_R^2}, \qquad (16)$$

does not lead to a self-consistent condition. This is because a relation like Eq. (16) can be satisfied for an arbitrary renormalization scheme only if the corresponding bare couplings satisfy the same condition. This, however, imposes the following condition on the counterterms

$$\begin{aligned}\delta\lambda_1 &= \frac{g_R m_R}{2M_R^3\left(2m_R^4-9M_R^4\right)^2}\Big[10m_R^9\left(\delta \mathrm{g}_1 M_R-\delta \mathrm{M}_1 g_R\right)-9m_R^5M_R^4\left(17\delta \mathrm{g}_1 M_R-3\delta \mathrm{M}_1 g_R\right) \\ &+ 486m_R M_R^8\left(\delta \mathrm{g}_1 M_R-\delta \mathrm{M}_1 g_R\right)+10\delta \mathrm{m}_1 g_R m_R^8 M_R-27\delta \mathrm{m}_1 g_R m_R^4 M_R^5+486\delta \mathrm{m}_1 g_R M_R^9\Big],\end{aligned}$$

which is not satisfied by the expressions specified in Eqs. (12) and (13). That is, the first solution to the condition of Eq. (5), given in Eq. (6), does not lead to self-consistent renormalization.

Next, by taking into account Eqs. (12,14) in Eq. (11), we obtain the following condition which has to be satisfied by renormalized parameters b_R, g_R, m_R and M_R:

$$\frac{g_R^2\left(7b_R g_R m_R^2 M_R+2b_R^2M_R^2+6g_R^2m_R^4\right)}{M_R^4}=0\,. \qquad (17)$$

Eq. (17) has two solutions

$$b_R=-\frac{2g_R m_R^2}{M_R},\quad b_R=-\frac{3g_R m_R^2}{2M_R}\,. \qquad (18)$$

However, analogously to the above case, only one, namely

$$b_R=-\frac{3g_R m_R^2}{2M_R} \qquad (19)$$

leads to a self-consistent condition. Substituting Eq. (19) in Eq. (16) leads to

$$\lambda_R = \frac{3 g_R^2 m_R^2}{4 M_R^2}\,. \tag{20}$$

As mentioned above, Eqs. (19) and (20) can be satisfied only if analogous relations hold for corresponding bare quantities, i.e. we have

$$b = -\frac{3 g m^2}{2M}\,, \quad \lambda = \frac{3 g^2 m^2}{4 M^2}\,. \tag{21}$$

This fixes uniquely the coupling constants of the scalar self-interactions to the values corresponding to spontaneously broken gauge symmetry in the unitary gauge.

III. INCLUSION OF THE ELECTROMAGNETIC INTERACTION

To have massless spin-one particles, the photons, in the spectrum of the theory it is necessary that the Lagrangian is invariant under local gauge $U(1)$ transformations [1]. Therefore, we introduce an Abelian gauge field B_μ and its coupling to the charged vector fields and also a gauge-invariant mixing term of the neutral vector fields. The resulting Lagrangian reads (with $a = 0$)

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}\, B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}\, G^a_{\mu\nu} G^{a\mu\nu} + \frac{1}{2}\, V^a_\mu V^{a\mu} \left(M - \frac{g}{2}\,\Phi \right)^2 - g_{A1} \epsilon^{abc}\, \epsilon^{\mu\nu\alpha\beta} V^a_\mu V^b_\nu \partial_\alpha V^c_\beta\,,\\ &+ \frac{c}{2}\, B^{\mu\nu}\, V^3_{\mu\nu} + \frac{\kappa}{2}\, \epsilon^{3ab}\, B^{\mu\nu} V^a_\mu V^b_\nu + \frac{1}{2}\, \partial_\mu \Phi\, \partial^\mu \Phi - \frac{m^2}{2}\, \Phi^2 \left(1 - \frac{g}{4M}\, \Phi \right)^2\,,\end{aligned} \tag{22}$$

where

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\,, \quad V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu\,,\\ G^a_{\mu\nu} &= V^a_{\mu\nu} - g\,\epsilon^{abc}\, V^b_\mu V^c_\nu + e\,\epsilon^{3ab} \left(B_\mu V^b_\nu - B_\nu V^b_\mu \right), \end{aligned} \tag{23}$$

and we have substituted the expressions of Eq. (21). To diagonalize the Lagrangian, Eq. (22), we introduce new vector fields A_μ, Z_μ and $W^\pm_\mu$ as follows:

$$\begin{aligned} B_\mu &= A_\mu + \frac{c}{\sqrt{1-c^2}}\, Z_\mu\,,\\ V^\pm_\mu &= (V^1_\mu \mp i V^2_\mu)/\sqrt{2} = W^\pm_\mu,\ V^3_\mu = \frac{Z_\mu}{\sqrt{1-c^2}}\,. \end{aligned} \tag{24}$$

Next, we analyze the conditions of perturbative renormalizability. We again split the bare parameters as $p = p_R + \sum_{i=1}^\infty \hbar^i \delta p_i$ and fix the counterterms such that they cancel divergences in physical quantities.

The undressed propagators of the Z and $W^\pm$ vector bosons read

$$i\, S^{Z,W}_{0,\mu\nu}(p) = -i\, \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2_{Z,W}}}{p^2 - M^2_{Z,W} + i\,0^+}\,, \tag{25}$$

where $M_W = M$ and $M_Z = M/\sqrt{1-c^2}$. We parameterize the sum of all one-particle-irreducible diagrams contributing to the two-point functions as

$$i\,\Pi^{Z,W}_{\mu\nu}(p) = i\,\left[g_{\mu\nu}\Pi^{Z,W}_1(p^2) + p_\mu p_\nu\,\Pi^{Z,W}_2(p^2)\right]\,. \tag{26}$$

The corresponding dressed propagators have the form

$$i\,S^{Z,W}_{\mu\nu}(p) = -i\,\frac{g_{\mu\nu} - p_\mu p_\nu \frac{1+\Pi^{Z,W}_2(p^2)}{M^2_{Z,W}+\Pi^{Z,W}_1(p^2)+p^2\Pi^{Z,W}_2(p^2)}}{p^2 - M^2_{Z,W} - \Pi^{Z,W}_1(p^2) + i\,0^+}\,. \tag{27}$$

The pole masses are obtained from the solutions to the following equations:

$$z_{Z,W} - M^2_{Z,W} - \Pi_1(z_{Z,W}) = 0\,. \tag{28}$$

In the vicinity of the pole the dressed propagators can be expanded as

$$i\,S^{Z,W}_{\mu\nu}(p) = -i\left[\frac{Z^r_{Z,W}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{z_{Z,W}}\right)}{p^2 - z_{Z,W} + i\,0^+} + R\right]\,, \tag{29}$$

where

$$Z^r_{Z,W} = \frac{1}{1 - \Pi^{Z,W\,\prime}_1(z_{Z,W})}$$

is the wave-function renormalization constant and R denotes the non-pole part.

From Eq. (28) at one-loop order we have

$$z_{Z,W} = M^2_{Z,W} + \Pi_1(M^2_{Z,W})\,. \tag{30}$$

Substituting in Eq. (30)

$$M_W = M = M_R + \hbar\,\delta M_1$$

and

$$M_Z = \frac{M}{\sqrt{1-c^2}} = \frac{M_R}{\sqrt{1-c_R^2}} - \frac{\hbar\left(\delta M_1\left(c_R^2-1\right) - \delta c_1 c_R M_R\right)}{\left(1-c_R^2\right)^{3/2}}$$

and demanding that the pole masses of both the Z and the W bosons must be finite quantities, we obtain

$$\begin{aligned}
\delta M_1 &= \frac{\pi^2}{12\left(c_R^2-1\right)^3 m_R^2}\Big\{M_R\Big[4\left(c_R^2-1\right)e_R m_R^2\left(28c_R^3 g_R - 25c_R g_R - 3c_R^2\kappa_R - 6\kappa_R\right)\\
&+ 4\left(17c_R^4 - 28c_R^2 + 11\right)e_R^2 m_R^2 + g_R^2\Big(\left(37c_R^6 - 32c_R^4 - 73c_R^2 + 59\right)m_R^2\\
&- 18\left(2c_R^6 - 6c_R^4 + 7c_R^2 - 3\right)M_R^2\Big) + 2c_R\left(23c_R^4 - 78c_R^2 + 46\right)g_R m_R^2\kappa_R\\
&+ \left(-3c_R^4 - 17c_R^2 + 11\right)m_R^2\kappa_R^2\Big] - \frac{9\left(c_R^2-1\right)^3 g_R^2 m_R^4}{M_R}\Big\}\,,\\
\delta c_1 &= -\frac{\pi^2}{12c_R\left(c_R^2-1\right)^2}\Big[8\left(c_R^2-1\right)e_R\left(-2c_R^3 g_R + 3c_R g_R + 9c_R^4\kappa_R - 6c_R^2\kappa_R + 3\kappa_R\right)\\
&+ 4\left(3c_R^6 - 22c_R^4 + 30c_R^2 - 11\right)e_R^2 + 26c_R^5 g_R\kappa_R + c_R^4\left(106g_R^2 + \kappa_R^2\right)\\
&+ 20c_R^3 g_R\kappa_R + c_R^2\left(18\kappa_R^2 - 61g_R^2\right) - 30c_R g_R\kappa_R - 37c_R^6 g_R^2 - 11\kappa_R^2\Big]\,.
\end{aligned} \tag{31}$$

Next, we calculate the one-loop contributions to the ΦZZ and ΦW^+W^- vertex functions and demand that the divergences are cancelled in both quantities when taken on mass-shell. Using Eq. (31), we obtain two expressions of δg_1 resulting from the conditions of the finiteness of the ΦZZ and ΦW^+W^- vertex functions:

$$\begin{aligned}
\delta g_1^{(Z)} &= \frac{\pi^2 g_R}{12\left(c_R^2-1\right)^3 M_R^4}\Big[c_R^6\left(8e_R m_R^2 M_R^2\kappa_R + e_R^2\left(8m_R^2M_R^2-18m_R^4\right)+g_R^2\left(6m_R^2M_R^2\right.\right.\\
&+ \left.\left.19M_R^4\right)+2m_R^2\kappa_R^2\left(9m_R^2-8M_R^2\right)\right)+2c_R^5 g_R\left(e_R\left(12m_R^2M_R^2-18m_R^4+56M_R^4\right)\right.\\
&+ \left.\kappa_R\left(-12m_R^2M_R^2+18m_R^4+23M_R^4\right)\right)+c_R^4\left(-4e_RM_R^2\kappa_R\left(4m_R^2+3M_R^2\right)\right.\\
&+ 2e_R^2\left(-8m_R^2M_R^2+9m_R^4+34M_R^4\right)+g_R^2\left(31M_R^4-18m_R^2M_R^2\right)+\kappa_R^2\left(26m_R^2M_R^2\right.\\
&- \left.\left.18m_R^4-3M_R^4\right)\right)+4c_R^3g_R\left(e_R\left(-12m_R^2M_R^2+9m_R^4-53M_R^4\right)-3\kappa_R\left(-3m_R^2M_R^2\right.\right.\\
&+ \left.\left.3m_R^4+13M_R^4\right)\right)-c_R^2\left(4e_RM_R^2\kappa_R\left(3M_R^2-2m_R^2\right)+2e_R^2\left(-4m_R^2M_R^2+3m_R^4+56M_R^4\right)\right.\\
&+ \left.g_R^2\left(145M_R^4-12m_R^2M_R^2\right)+\kappa_R^2\left(10m_R^2M_R^2-6m_R^4+17M_R^4\right)\right)\\
&- 4c_Rg_R\left(e_R\left(-6m_R^2M_R^2+3m_R^4-25M_R^4\right)+\kappa_R\left(3m_R^2M_R^2-3m_R^4-23M_R^4\right)\right)\\
&+ 12c_R^7g_Rm_R^4\left(e_R-\kappa_R\right)+6c_R^8m_R^4\left(e_R^2-\kappa_R^2\right)+M_R^4\left(24e_R\kappa_R+44e_R^2+86g_R^2+11\kappa_R^2\right)\Big]\,,\\
\delta g_1^{(W)} &= \frac{\pi^2 g_R}{12\left(c_R^2-1\right)^3 M_R^4}\Big[2\left(c_R^2-1\right)e_R\left(c_R^3g_R\left(3m_R^4+56M_R^4\right)-c_Rg_R\left(3m_R^4+50M_R^4\right)\right.\\
&- \left.2c_R^2M_R^2\kappa_R\left(2m_R^2+3M_R^2\right)+4M_R^2\kappa_R\left(m_R^2-3M_R^2\right)\right)+\left(c_R^2-1\right)e_R^2\left(4\left(c_R^2-1\right)m_R^2M_R^2\right.\\
&+ \left.3\left(c_R^2-1\right)m_R^4+4\left(17c_R^2-11\right)M_R^4\right)+2c_R^5g_R\kappa_R\left(-20m_R^2M_R^2+2m_R^4+23M_R^4\right)\\
&+ c_R^4\left(g_R^2\left(58m_R^2M_R^2-10m_R^4+31M_R^4\right)+\kappa_R^2\left(-10m_R^2M_R^2+2m_R^4-3M_R^4\right)\right)\\
&- 4c_R^3g_R\kappa_R\left(-23m_R^2M_R^2+2m_R^4+39M_R^4\right)+c_R^2\left(g_R^2\left(-32m_R^2M_R^2+5m_R^4-145M_R^4\right)\right.\\
&+ \left.\kappa_R^2\left(26m_R^2M_R^2-4m_R^4-17M_R^4\right)\right)+4c_Rg_R\kappa_R\left(-13m_R^2M_R^2+m_R^4+23M_R^4\right)\\
&+ c_R^6g_R^2\left(-26m_R^2M_R^2+5m_R^4+19M_R^4\right)+86g_R^2M_R^4-16m_R^2M_R^2\kappa_R^2\\
&+ 2m_R^4\kappa_R^2+11M_R^4\kappa_R^2\Big]\,. \qquad (32)
\end{aligned}$$

The two expressions for the same counterterm have to coincide, leading to the following condition:

$$\begin{aligned}
&\frac{\pi^2 g_R m_R^2}{12\left(c_R^2-1\right)^2 M_R^4}\Big[2\left(c_R^2-1\right)e_R\left(3c_Rg_R\left(4M_R^2-3m_R^2\right)+6c_R^3g_Rm_R^2+4c_R^2M_R^2\kappa_R+4M_R^2\kappa_R\right)\\
&+\left(c_R^2-1\right)e_R^2\left(\left(6c_R^4-6c_R^2-3\right)m_R^2+4\left(2c_R^2-1\right)M_R^2\right)+c_R^4\left(g_R^2\left(32M_R^2-5m_R^2\right)\right.\\
&\left.+4\kappa_R^2\left(3m_R^2-4M_R^2\right)\right)+4c_R^3g_R\kappa_R\left(5m_R^2+4M_R^2\right)+c_R^2\left(g_R^2\left(5m_R^2-44M_R^2\right)\right.\\
&\left.+4\kappa_R^2\left(5M_R^2-2m_R^2\right)\right)-8c_Rg_R\kappa_R\left(m_R^2+5M_R^2\right)-12c_R^5g_Rm_R^2\kappa_R\\
&-6c_R^6m_R^2\kappa_R^2+2\kappa_R^2\left(m_R^2-8M_R^2\right)\Big]=0\,. \qquad (33)
\end{aligned}$$

By demanding that the same counterterm δg_1 in combination with δM_1 and

$$\delta m_1 = \frac{3\pi^2 g_R^2 m_R\left(\left(c_R^2-1\right)m_R^2+\left(3-2c_R^2\right)M_R^2\right)}{4\left(c_R^2-1\right)M_R^2}\,, \qquad (34)$$

removes the divergences from the $\Phi\Phi\Phi$ vertex function, we obtain another condition:

$$\frac{\pi^2 c_R g_R m_R^4}{4\left(c_R^2-1\right)^2 M_R^5}\Big[c_R^3\left(e_R^2\left(4M_R^2-6m_R^2\right)+4e_RM_R^2\kappa_R+3g_R^2M_R^2+2\kappa_R^2\left(3m_R^2-4M_R^2\right)\right)$$

$$
\begin{aligned}
&-12c_R^2 g_R\left(e_R-\kappa_R\right)\left(m_R^2-M_R^2\right)+c_R\left(e_R^2\left(3m_R^2-4M_R^2\right)-4e_R M_R^2\kappa_R-6g_R^2 M_R^2\right.\\
&\left.+\kappa_R^2\left(5M_R^2-3m_R^2\right)\right)+6c_R^4 g_R m_R^2\left(e_R-\kappa_R\right)+3c_R^5 m_R^2\left(e_R^2-\kappa_R^2\right)\\
&\left.+6g_R\left(e_R\left(m_R^2-2M_R^2\right)+\kappa_R\left(M_R^2-m_R^2\right)\right)\right]=0\,. \qquad (35)
\end{aligned}
$$

One more condition is obtained by calculating the one-loop contributions to the ZW^+W^- vertex function and demanding that its divergent part proportional to the Lorentz-structure with the product of three momenta (i.e. containing no metric tensor) vanishes. The resulting expression has the form:

$$
\frac{c_R(e_R-\kappa_R)^2(4c_R g_R+3\,e_R+\kappa_R)}{(1-c_R^2)^{3/2}\,M_R^2}=0\,. \qquad (36)
$$

Eqs. (33), (35) and (36) fix κ_R and c_R to the following unique expressions:

$$
\kappa_R=e_R,\quad c_R=-\frac{e_R}{g_R}\,. \qquad (37)
$$

As argued above, Eq. (37) leads to analogous relations for corresponding bare parameters of the effective Lagrangian.

Thus all parameters of our LO effective Lagrangian of interacting photons, a scalar and massive neutral and two charged vector bosons are uniquely fixed by the self-consitsency conditions such that the Lagrangian corresponding to the electroweak sector of the Standard Model in unitary gauge is obtained.

IV. SUMMARY

In the current work we extended the study of Ref. [6] where following the modern point of view of the Standard Model as the leading order approximation of an effective field theory we analyzed the most general Lorentz-invariant leading order effective Lagrangian of massive vector bosons interacting with a massive scalar field. Here leading order means interaction terms with couplings of non-negative mass dimensions.

In Ref. [6] we analyzed the conditions of perturbative renormalizability and scale separation applied to all three- and four-point functions at one-loop order. These conditions in combination with the second class constraints imposed on systems with spin-one particles led to severe restrictions on the interaction terms of the leading order Lagrangian. However, two coupling constants of the self-interactions of the scalar field remained unfixed. In the current work, using the condition of perturbative renormalizability and scale separation for three-, four- and five-point functions of the scalar field at one-loop order, we were able to fix the remaining two free couplings. Next, we included the coupling to the electromagnetic interaction and again demanded self-consistency in the sense of EFT. Analyzing the renormalizability conditions of the various three-point vertex functions we fixed the two additional free parameters appearing in the most general effective Lagrangian. As the result of this analysis the Lagrangian with spontaneously broken SU(2)×U(1) gauge symmetry taken in unitary gauge naturally appears as the unique leading-order Lagrangian of a self-consistent EFT of a massive scalar interacting with neutral and charged massive vector bosons and the electromagnetic field. It is well-known that such a Lagrangian leads to a well-defined finite S-matrix [15].

The inclusion of the fermionic degrees of freedom is the last step for completing this program of deriving the leading order EFT Lagrangian of the electroweak interaction and will be considered in a forthcoming publication.

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