Towards buoyancy driven flows with FEM

December 08, 2017 | Marc Fehling | Civil Security & Traffic (CST)

Introduction

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- born 1990
- M. Sc. Physics, RUB, 2015
- since November 2015:
  PhD student at JSC,
  Division Civil Security & Traffic

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Motivation: Fire safety in modern architecture

- Modern architecture and large–scale projects (like BER) may not fit in 'building code'.
- Individual considerations necessary:
  - Model experiments, CFD investigations, ...

Figure: Interior of BMW World, Munich.

Modeling fires

Processes to model:

- Fluid dynamics.
  - Navier–Stokes equations.
  - Turbulence modeling.
- Radiation.
  - Discrete Transfer Radiation.
  - Discrete Ordinates.
  - Monte–Carlo.
- Combustion
  - Mixture fraction.
  - Finite rate kinetics.
- Pyrolysis.

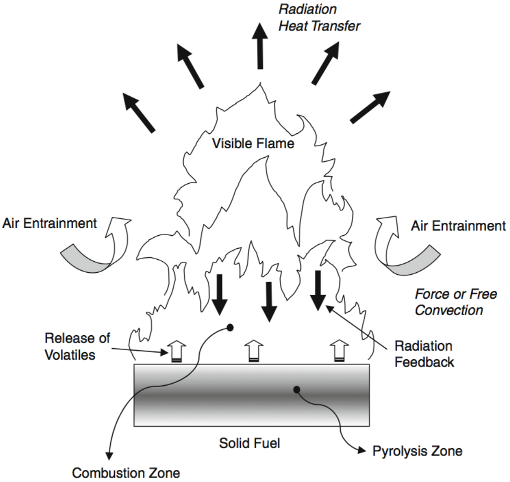


Figure: Burning solid fuel in air with physical processes involved. [1]

Modeling: Equations for smoke propagation

- Smoke propagation with incompressible Navier–Stokes (INS) equations:

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho_0 \left[\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right] + \nabla p - \nabla(2\,\mu\,\epsilon_{ij}(\mathbf{u})) = \mathbf{f}(T)$$

$$\rho_0 \left[\partial_t T + (\mathbf{u} \cdot \nabla) T\right] - 2\,\frac{\mu}{c_\mathrm{p}}\,\epsilon_{ij}(\mathbf{u}) : \nabla \mathbf{u} - \nabla \cdot \left(\frac{\mu}{\mathrm{Pr}}\,\nabla T\right) = \gamma$$

  with strain rate tensor $\epsilon_{ij}(\mathbf{u}) = \frac{1}{2}\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right]$.

- Turbulence model: Smagorinsky–Lilly LES [2]:

$$\nu = \nu_{mol} + \nu_{turb} \qquad \text{with} \qquad \nu_{turb} = (C_s h)^2 \left|\left|\epsilon_{ij}(\mathbf{u})\right|\right|_2 \ .$$

JuFire

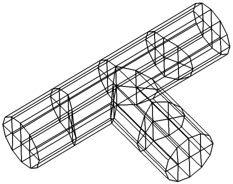
- Investigate applicability of Finite Element Methods (FEM) for fire simulation.
- Use open–source library deal.II [3].
  - 'Toolbox' for the creation of FEM codes.

Differential **E**quations **A**nalysis **L**ibrary

JuFire: Features

Implemented:

- Unstructured grids.
- Continuous Galerkin Methods.
- Adaptive mesh refinement (AMR).
- MPI parallelization.
- Utilization of `CAD` models as manifolds for mesh refinement.

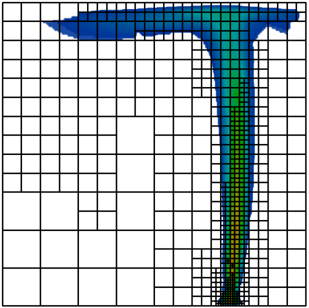


Current field of activity:

- Buoyancy driven flows.

Future work:

- Discontinuous Galerkin methods.
- p–adaptivity.



Adaptive mesh refinement (AMR)

- Solution of differential equations with *FEM* requires:
  - Discretization of space in cells of length h.
  - Shape functions with polynomial degree p.
- But: The more accurate the solution by choosing h and p, the longer the computation.

Adaptive mesh refinement (AMR)

- Solution of differential equations with *FEM* requires:
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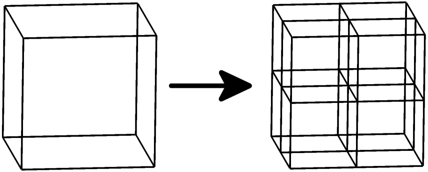
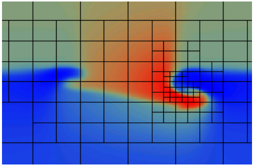
  - Shape functions with polynomial degree *p*.
- But: The more accurate the solution by choosing *h* and *p*, the longer the computation.

→ **Adaptive refinement** as a 'compromise'.
  - Adjustment of parameters locally where necessary.
  - Variable mesh precision at runtime.





J.Dreher, RUB

AMR: Algorithm

How to determine refinement criteria?

Set up decision criterion for refinement/coarsening.
Our choice:

- Estimate error on each cell. Common: $||\nabla u||$
- Normalize values with respect to all cells.

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- Flag top/lower part for refinement/coarsening at each step.
- Define max/min refinement levels as upper/lower bounds.

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- Define max/min refinement levels as upper/lower bounds.
- Cells are not allowed to differ by more than one level of refinement.

AMR: Example

- Demonstration of adaptive mesh refinement via moving vortex test case as a shape–preserving potential stream.

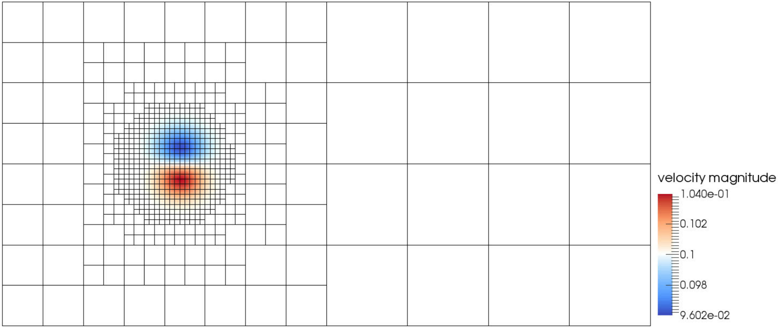


Figure: Video of velocity magnitude of moving vortex, overlayed with corresponding mesh.

AMR: Benefits

- Comparison of runtime and accuracy between uniform and adaptively refined meshes with the moving vortex example.

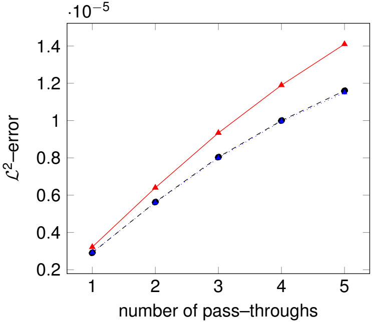


Figure: Global $\mathcal{L}^2$–errors at each periodic pass–through of the vortex.

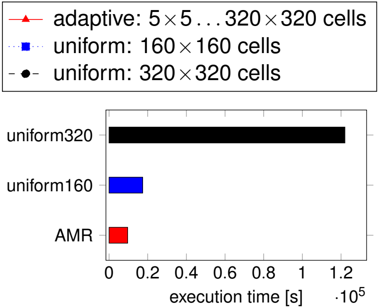


Figure: Execution time of the vortex simulation, run in serial on a common desktop computer.

AMR: Errors

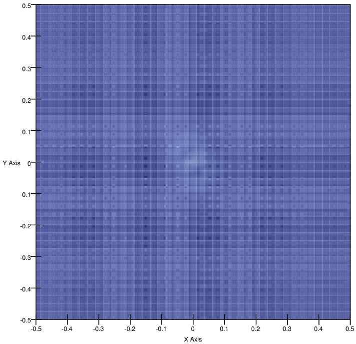


Figure: Static mesh.

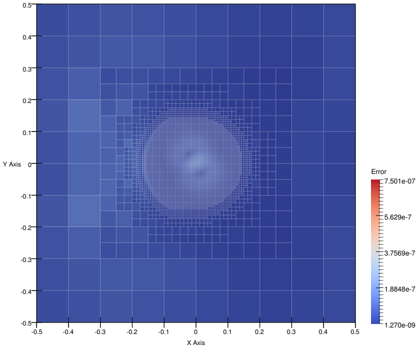
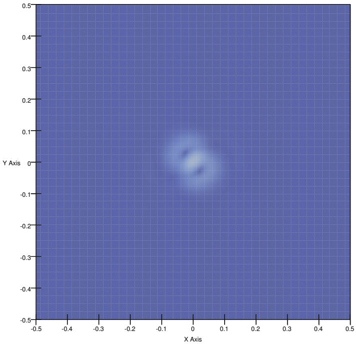


Figure: Adaptive mesh.

1 pass–through.

AMR: Errors



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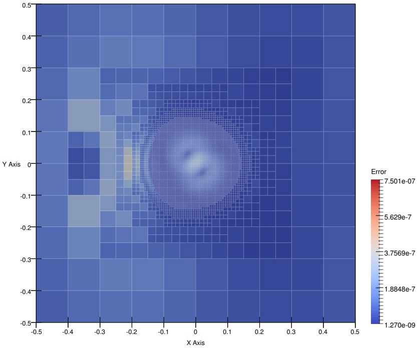

Figure: Adaptive mesh.

2 pass–throughs.

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AMR: Errors

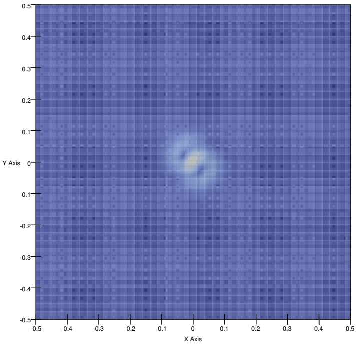

Figure: Static mesh.

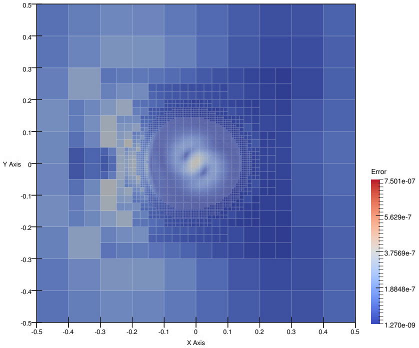

Figure: Adaptive mesh.

3 pass–throughs.

AMR: Errors

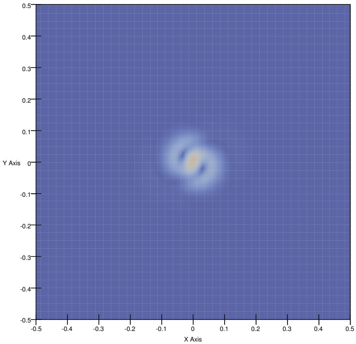

Figure: Static mesh.

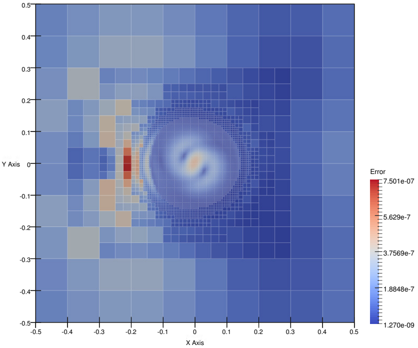

Figure: Adaptive mesh.

4 pass–throughs.

AMR: Errors

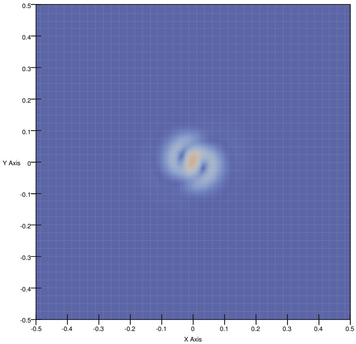

Figure: Static mesh.

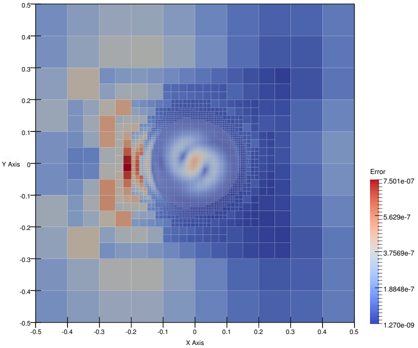

Figure: Adaptive mesh.

5 pass–throughs.

Buoyancy driven flows – Work in progress

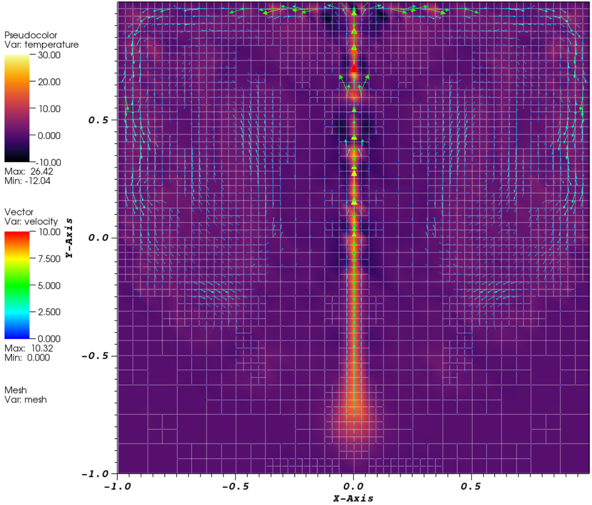

Figure: Video of velocity & temperature with constant heat source.

Buoyancy driven flows – Possible error source

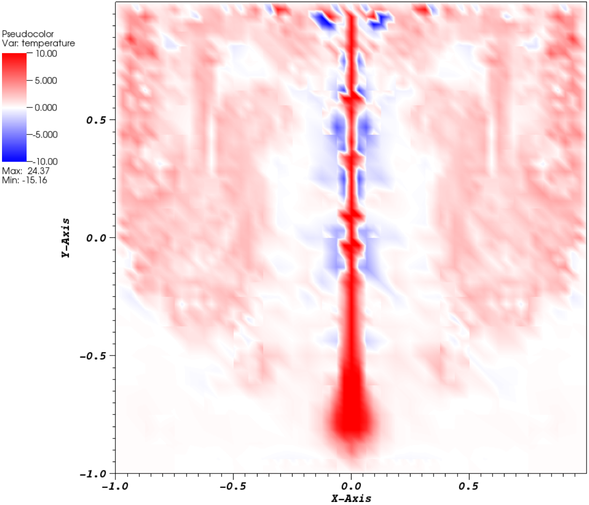

Figure: Temperature with constant heat source.

Heat equation with FEM – Possible error source

$$\partial_t u - \nu \nabla^2 u = 0$$

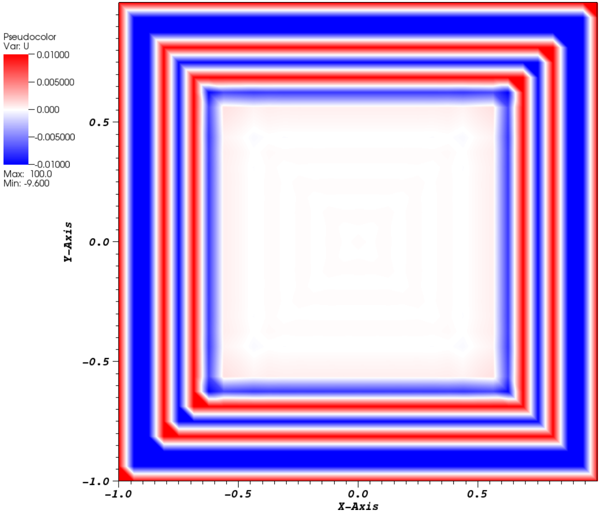

Figure: Solution of heat equation at advanced time.

Flow in exemplary underground – Work in progress

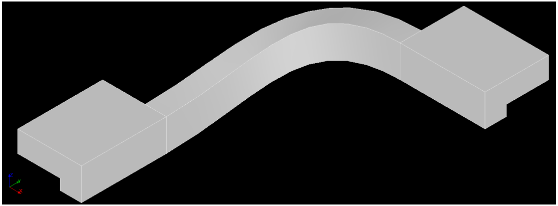

Figure: CAD model of exemplary underground station.

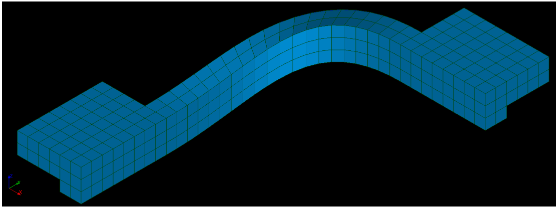

Figure: Initial mesh for simulation.

Summary & Outlook

Summary:

- Adaptive Mesh Refinement works satisfactorily (for now).
- Flaws in solving heat equation need to be cleared.

Future work:

- Implementation of models.
  - Radiation, combustion, pyrolysis, ...
- Extension of numerical methods.
  - DG methods, p-adaptivity, ...
- Comparison with other fire solvers.
- Validation using experiments.

Thank you for your kind attention!

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Towards buoyancy driven flows with FEM: Addendum

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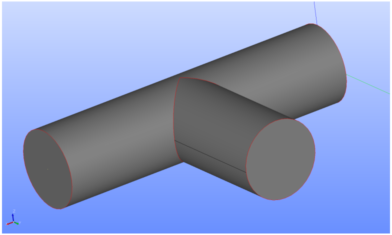
JuFire: Algorithm

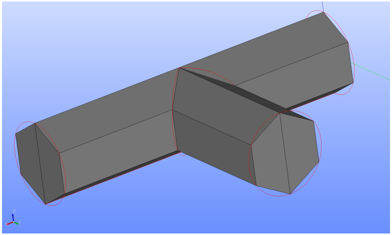
- Time evolution with backward differentiation formula (BDF)
- Taylor–Hood elements [5]: $(\mathbf{u}, p) \in Q_{k+1}^{\dim} \times Q_k$
- Decoupling of $(\mathbf{u}, p)$ by projection scheme
  - Leads to the pressure–Poisson equation
- Stabilization of momentum equation
  - Taylor–Galerkin stabilization [6] for Taylor–Hood elements
    - Additional diffusion in flow direction
  - Grad–div stabilization [7] to enforce $\nabla \cdot \mathbf{u} = 0$
- Neumann series for fast matrix assembly
- Boussinesq approximation for buoyancy force density

Unstructured Grids: Example: T–pipe

- Why T–pipe?
  - Compound bodies.
  - Crooked areas (→ cylinder casing area).
  - Discontinuous edges (→ 'welding seam').
- Develop procedure for the creation of appropriate initial meshes.

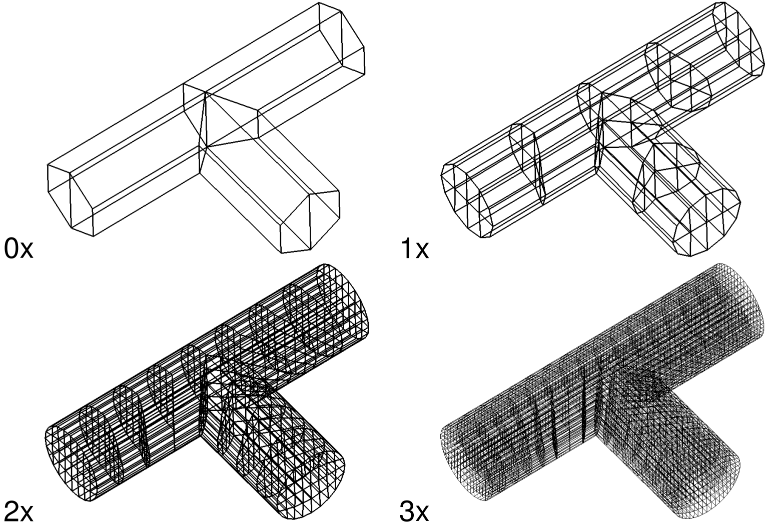


CAD model.



Initial mesh.

Iterations of global refinement:



Numerical methods

Spatial discretization methods for computational fluid dynamics and software packages using it:

- Finite Difference Method (FDM).
  - Open source: NIST Fire Dynamics Simulator (FDS), ...
- Finite Volume Method (FVM).
  - Open source: FireFOAM (OpenFOAM), ...
- Finite Element Method (FEM).
- Lattice–Boltzmann Method (LBM).

FEM: Features

Why use FEM?

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- Unstructured grids.
  - Allow for better domain representation without aliasing.

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FEM: Features

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- Unstructured grids.
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- hp–adaptivity.
  - Dynamic resolution of numerical grid.
  - Adaptive polynomial degree of basis functions.

    $\rightarrow$ Increase accuracy where the action is happening!

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- Unstructured grids.
  - Allow for better domain representation without aliasing.
- hp–adaptivity.
  - Dynamic resolution of numerical grid.
  - Adaptive polynomial degree of basis functions.

    $\rightarrow$ Increase accuracy where the action is happening!
- Discontinuous Galerkin (DG) methods.
  - Allow discontinuities across cell borders.
  - Continuous Galerkin (CG) methods unstable for advection like problems, thus require stabilization.

FEM: Formulation

- Solve variational equation from 'weak' formulation of the differential equation with bilinear form $a(u, v)$:

$$\exists u \in V : \forall v \in V : a(u, v) = f(v)$$

- Choose subspace V_h with basis w_i, out of which the approximate solution $u_\mathrm{h} = \sum u_i w_i \in V_\mathrm{h}$ will be constructed:

$$a(u_\mathrm{h}, w_\mathrm{j}) = \sum a(w_i, w_\mathrm{j})\ u_i = f(w_\mathrm{j}) \quad \rightarrow \quad AU = F$$

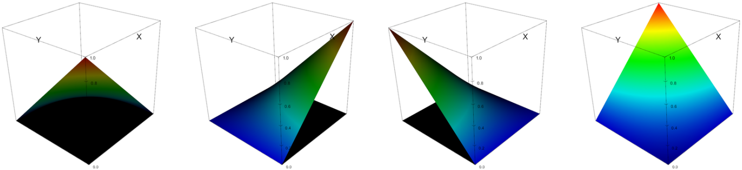

Figure: Q_1 elements in 2D (source: deal.II)

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Verification: Richardson Extrapolation

- How to get convergence rates of time **and** space dependent problems?
- Cancel out redundant error with Richardson extrapolation. Constant ratio r for successive refinement (here: space).

$$\left.\begin{aligned} f_1 - f_0 &= c_t\,\Delta t^{p_t} + c_h\,\Delta h^{p_h} \\ f_2 - f_0 &= c_t\,\Delta t^{p_t} + c_h\,(r\Delta h)^{p_h} \\ f_3 - f_0 &= c_t\,\Delta t^{p_t} + c_h\,(r^2\Delta h)^{p_h} \end{aligned}\right\} \quad \ln\left(\frac{f_3 - f_2}{f_2 - f_1}\right) = p_h\,\ln(r)$$

	Space	Time
FDS	1.9244	2.0721
JuFire	3.0894	0.9715

Table: Examplary convergence rates for a McDermott testcase [4].

HPC on JURECA: Parallelization

- MPI parallelization with Trilinos and p4est through deal.II backends.

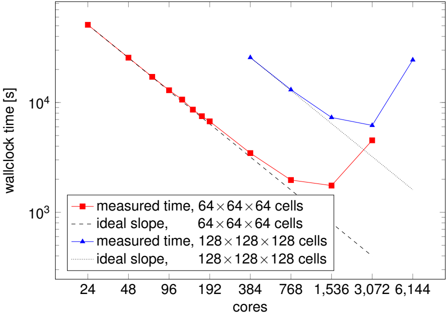

Figure: Strong scaling on JURECA for fixed 3D problems with 262,144 and 2,097,152 cells, respectively.

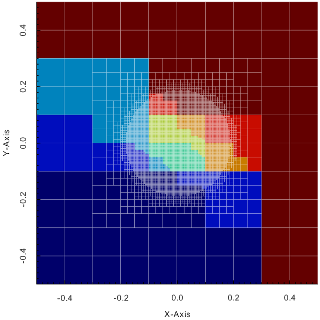

Figure: Examplary domain decomposition with p4est in an AMR case.