Quantum Vavilov-Cherenkov radiation from shearing two transparent dielectric plates

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Using a fully relativistic theory we study the quantum Vavilov-Cherenkov radiation and quantum friction occurring during relative sliding of two transparent dielectric plates with the refractive index n. These phenomena occur above the threshold velocity $v_c = 2nc/(n^2+1)$. Close to the threshold velocity they are dominated by the contribution from s-polarized electromagnetic waves, which agrees with the approximate (relativistic) theory by Pendry [J. B. Pendry, J. Mod. Opt. **45**, 2389 (1998)]. However, in the ultrarelativistic case ($v \to c$), the contributions from both polarizations are strongly enhanced in comparison with the approximate theory, and a new contribution occurs from the mixing of the electromagnetic waves with different polarizations. The numerical results are supplemented by an analytical treatment close to the threshold velocity and the light velocity.

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I. INTRODUCTION

Quantum fluctuations of the electromagnetic field manifest themselves in a wide variety of fields of physics. For example, the Lamb shift of the atomic spectrum and anomalous magnetic moment of the electron were explained with the help of this idea. Most directly, these fluctuations are manifested through the Casimir effect. In the late 1940s Casimir predicted [1] that two macroscopic nonmagnetic bodies with no net electric charge (or charge moments) can experience an attractive force much stronger than gravity. The existence of this force is one of the direct macroscopic manifestations of quantum mechanics. Casimir based his prediction on a simplified model involving two parallel perfectly conducting plates separated by a vacuum. A unified theory of both the van der Waals and Casimir forces between plane parallel material plates, in thermal equilibrium and separated by a vacuum gap, was developed by Lifshitz [2]. To calculate the interaction force Lifshitz used Rytov's theory of the fluctuating electromagnetic field [3]. At present the interest in Casimir forces is increasing because they dominate the interaction between nanostructures and are often responsible for the stiction between moving parts in small devices such as micro- and nanoelectromechanical systems, and can be considered as practical mechanisms for the actuation of such devices. Due to this practical interest, and the fast progress in force detection techniques, experimental and theoretical investigations of Casimir forces have experienced an extraordinary renaissance in the past few years (see [4] for a review and the references therein).

Another manifestation of quantum fluctuations of the electromagnetic field is the noncontact quantum friction between bodies in relative motion. Friction is usually a very complicated process. The simplest case consists of two flat surfaces, separated by a vacuum gap, sliding relative to each other at $T = 0$ K, where the friction is generated by the relative movement of quantum fluctuations [5–12]. The thermal and quantum fluctuation of the current density in one body induces a current density in the other body; the interaction between these current densities is the origin of the Casimir interaction. When two bodies are in relative motion, the induced current will lag slightly behind the fluctuating current inducing it, and this is the origin of the Casimir friction. At present the Casimir friction, with its limiting case, quantum friction, is actively discussed as one of the possible mechanisms of noncontact friction between bodies in the absence of direct mechanical contact between them [12,13]. The Casimir friction was studied in the configuration plate-plate [5–12], neutral particle-plate [12,14–25], and neutral particle-blackbody radiation [10,12,24,26–30]. While the predictions of the theory for the Casimir forces were verified in many experiments [4], the detection of the Casimir friction is still a challenging problem for experimentalists. However, the frictional drag between quantum wells [31,32] and graphene sheets [33,34], and the current-voltage dependence of nonsuspended graphene on the surface of the polar dielectric SiO_2 [35], were accurately described using the theory of the Casimir friction [11,36,37]. At present the frictional drag experiments [31–35] have been performed for a weak electric field when the induced drift motion of the free carriers is smaller than the threshold velocity for quantum friction. Thus in these experiments the frictional drag is dominated by the contributions from thermal fluctuations. However, the measurements of the current-voltage dependence [35] were performed for a high electric field, where the drift velocity is above the threshold velocity, and where the frictional drag is dominated by quantum fluctuations [11,37].

Quantum friction is associated with creation of excitations of a different kind. For transparent dielectrics such excitations are photons and quantum friction is associated with quantum Vavilov-Cherenkov radiation. Thus there is a close connection between quantum friction and the quantum Vavilov-Cherenkov radiation—both of these phenomena are related to the anomalous Doppler effect [5,11,12,38–41]. Quantum Vavilov-Cherenkov radiation was first described by Frank [39] and Ginzburg and Frank [40] (see also [41,42] for reviews of these works). If an object has no internal degrees of freedom (e.g., a point charge), then the energy of the radiation is determined by the change of the kinetic energy of the object. However, if an object has internal degrees of freedom (say, an

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atom), then two types of radiation are possible. If the frequency of the radiation in the *lab* reference frame $\omega > 0$, then in the rest frame of an object, due to the Doppler effect, the frequency of the radiation $\omega' = \gamma(\omega - k_x v)$, where $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$. In the normal Doppler effect region, when $\omega' > 0$, the radiated energy is determined by the decrease of the internal energy. For example, for an atom the state may change from the excited state $|1\rangle$ to the ground state $|0\rangle$. The region of the *anomalous* Doppler effect corresponds to $\omega' < 0$ in which case an object becomes excited when it radiates. For example, an atom could experience the transition from the ground state $|0\rangle$ to the excited state $|1\rangle$ when it radiates. In such a case energy conservation requires that the energies of the radiation and of the excitation result from a decrease of the kinetic energy of the object. That is, the self-excitation of a system is accompanied by a slowing down of the motion of the object as a whole. For a neutral object the interaction of the object with the matter is determined by the fluctuating electromagnetic field due to the quantum fluctuations inside the object.

While a constant translational motion requires for the emission of the radiation at least two bodies in relative motion (otherwise it is not possible due to Lorenz invariance), a single accelerated object can radiate and experience friction. Quantum fluctuations of the electromagnetic field are determined by virtual photons that are continuously created and annihilated in the vacuum. Using a metal mirror in accelerated motion, with velocities near the light velocity, virtual photons can be converted into real photons, leading to radiation emitted by the mirror. This is the dynamic Casimir effect [4,43,44], which recently was observed in a superconducting waveguide [45]. Radiation can be also emitted by a rotating object [46–50]. In fact this phenomenon is closely related to the prediction by Zel'dovich [51] of an amplification of certain waves during scattering from a rotating body. Rotational quantum friction is strongly enhanced close to a dielectric substrate [52]

Pendry was the first to studied quantum friction in detail [5] in the nonrelativistic and nonretarded limit. In Ref. [6] Pendry applied a new formalism developed for quantum friction to estimate the emission of light occurring during relative sliding of two transparent dielectric plates. To take into account some relativistic effects the reflection amplitude for the moving surface was taken as the reflection amplitude in the rest reference frame for this surface. The relation between the frequency and the wave vector, which determines the arguments of these reflection amplitudes, was determined in the different reference frame by the Lorenz transformation. In Ref. [10] we developed a fully relativistic theory of quantum friction from which follows that the theory by Pendry [6] is accurate only to order $(v/c)^2$. Using a toy model the link between quantum Vavilov-Cherenkov radiation and quantum friction was also discussed within the framework of a nonrelativistic theory in Ref. [38].

In the present paper we use a fully relativistic theory [10] to study quantum Vavilov-Cherenkov and quantum friction during shearing of two transparent dielectric plates. Numerical calculations are supplemented by an analytical study close to the threshold $v_c = 2nc/(n^2+1)$ and light velocities. Close to the threshold velocity v_c our results agree with the results obtained by Pendry. In the ultrarelativistic case $(v \to c)$ there is a dramatic enhancement of the contributions to quantum friction related to radiation of both polarizations, and a new contribution occurs from polarization mixing.

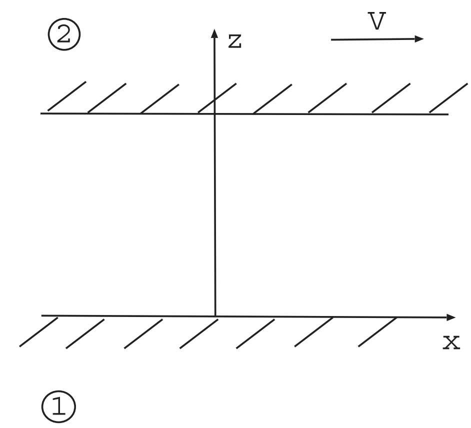

FIG. 1. Two semi-infinite bodies with plane parallel surfaces separated by a distance d. The upper solids moves parallel to other with the velocity v.

II. PHOTON EMISSION AND ANOMALOUS DOPPLER EFFECT

We consider two semi-infinite solids having flat parallel surfaces separated by a distance d and moving with the velocity v relative to each other (see Fig. 1). We introduce the two reference frames K and K' with coordinate axes xyz and $x'y'z'$. In the K frame, body 1 is at rest while body 2 is moving with the velocity v along the x axis (the xy and $x'y'$ planes are in the surface of body 1, x and x' axes have the same direction, and the z and z' axes point toward body **2**). In the K' frame, body 2 is at rest while body 1 is moving with velocity $-v$ along the x axis. If in body 2, in the K' frame, excitation occurs with frequency $\omega'_{\alpha 2}(q')$ and wave vector $-\mathbf{q}' = (-q'_x, -q_y, 0)$, then in the laboratory reference frame K this excitation will have the frequency $\omega_{\alpha 2}(-q_x, -q_y) = \gamma[\omega'_{\alpha 2}(q') - q'_x v] = \omega'_{\alpha 2}(q')/\gamma - q_x v$, where $q'_x = (q_x + \beta k)\gamma$, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$, $k = \omega_{\alpha 1}(q)/c$. The anomalous Doppler effect corresponds to the situation when $\omega_{\alpha 2}(q) < 0$. In this case an excitation in the rest frame K' of body 2 corresponds to a gain of energy in the *lab* frame K. Thus, body 2 can radiate photons while it is excited. Resonance arises when a gain of energy resulting from excitation of body 2 with frequency $\omega_{\text{gain}} = -\omega_{\alpha 2}(\mathbf{q})$ will create excitation in body 1 with frequency $\omega_{\alpha 1}(\mathbf{q}) = -\omega_{\alpha 2}$. Thus, the resonance condition has the form

$$\omega_{\alpha 1}(q) = q_x v - \omega'_{\alpha 2}(q')/\gamma. \quad (1)$$

Equation (1) determines the condition for excitation of pair excitations. For transparent dielectrics such excitations can be photons. For this case Eq. (1) determines the occurrence of Vavilov-Cherenkov radiation. As result of such radiation a pair of photons (a photon with momentum $-\mathbf{q}$ in one body and a photon with momentum $\mathbf{q}$ in the other body) is created that gives rise to the change of the momentum of each body and friction which we denote as quantum friction due to the quantum origin of involved processes. For materials with losses also off-resonant processes are possible when an excitation is created in one body and the resulting photon is absorbed by another body. The condition for such an off-resonant process in the nonrelativistic case has the form

$$\omega_2(q) - q_x v < 0. \tag{2}$$

Equation (2) was used by Landau for obtaining the critical velocity of a superfluid flowing past a wall [53].

For two transparent identical media $\omega_{\alpha 1}(q) = v_0 q$ and $\omega'_{\alpha 2}(q') = v_0 q'$, where $v_0 = c/n$, n is the refractive index. In this case the condition (1) is reduced to the form

$$q_x v = \omega_{\alpha 1}(q) + \frac{\omega'_{\alpha 2}(q')}{\gamma} > v_0 q_x \left(2 - \frac{v v_0}{c^2}\right) \tag{3}$$

or

$$v > v_c = \frac{2v_0}{1 + (v_0/c)^2} = \frac{2nc}{n^2+1}. \tag{4}$$

The condition (4) was already obtained by Pendry [6] using an approximate relativistic theory. For $n \gg 1$ the condition (4) reduces to the nonrelativistic result $v > v_c = 2c/n$ obtained in Ref. [38]. Thus, the condition for the validity of the nonrelativistic theory is $v_0/c \ll 1$ or $n \gg 1$. However, for transparent dielectrics, at the high frequencies typically involved, $n \sim 1$ and a relativistic theory should be used.

Multiplying Eq. (1) with the Plank constant and with the photon emission rate per unit area, and taking into account that this rate is invariant under the Lorentz transformation, we get

$$Fv = P_1 + \frac{P'_2}{\gamma}, \tag{5}$$

where F is the friction force, P_1 is the power of photon emission energy which is equal to the power of excitation energy in body 1 in the K system, and P'_2 is the power of excitation energy in body 2 in the K' system. For the lossy materials the energy absorbed by the bodies is converted into heat. In this case P_1 and P'_2 are equal to the heat power in the corresponding reference frames.

III. A FULLY RELATIVISTIC THEORY FOR THE CASIMIR FRICTION BETWEEN TWO PLATES SLIDING RELATIVE TO EACH OTHER

According to a fully relativistic theory [10] the contributions of the evanescent waves (which dominate at large velocities and low temperatures) to the friction force F_{1x}, and the radiation power P_1 absorbed by plate 1 in the K frame, are determined by formulas

$$\begin{pmatrix} F_{1x} \\ P_1 \end{pmatrix} = \int \frac{d^2q}{(2\pi)^2} \int_0^{cq} \frac{d\omega}{2\pi} \begin{pmatrix} \hbar q_x \\ \hbar\omega \end{pmatrix} \times \Gamma(\omega,q)\mathrm{sgn}(\omega')[n_2(\omega') - n_1(\omega)] \tag{6}$$

where the positive quantity

$$\Gamma_{12}(\omega,\mathbf{q}) = \frac{4\mathrm{sgn}(\omega')}{|\Delta|^2}\big[(q^2-\beta k q_x)^2 - \beta^2 k_z^2 q_y^2\big]\big\{\mathrm{Im}R_{1p}\big[(q^2-\beta k q_x)^2 \mathrm{Im}R'_{2p}|\Delta_{ss}|^2 + \beta^2 k_z^2 q_y^2 \mathrm{Im}R'_{2s}|\Delta_{sp}|^2\big] + (p \leftrightarrow s)\big\}e^{-2k_z d} \tag{7}$$

can be identified as a spectrally resolved photon emission rate,

$$\Delta = (q^2 - \beta k q_x)^2 \Delta_{ss}\Delta_{pp} - \beta^2 k_z^2 q_y^2 \Delta_{ps}\Delta_{sp},$$
$$\Delta_{pp} = 1 - e^{-2k_z d} R_{1p} R'_{2p}, \quad \Delta_{sp} = 1 + e^{-2k_z d} R_{1s} R'_{2p},$$

$n_i(\omega) = [\exp(\hbar\omega/k_B T_i) - 1]^{-1}$, $k_z = \sqrt{q^2 - (\omega/c)^2}$, $R_{1p(s)}$ is the reflection amplitude for surface 1 in the K frame for a $p(s)$-polarized electromagnetic wave, $R'_{2p(s)} = R_{2p(s)}(\omega',q')$ is the reflection amplitude for surface 2 in the K' frame for a $p(s)$-polarized electromagnetic wave, $\omega' = \gamma(\omega - q_x v)$, $q'_x = \gamma(q_x - \beta k)$, $\Delta_{ps} = \Delta_{sp}(p \leftrightarrow s)$. The symbol $(p \leftrightarrow s)$ denotes the terms that are obtained from the preceding terms by permutation of indexes p and s. In the domains of the *normal* Doppler effect ($\omega - q_x v > 0$) the last factor in the integrand in Eq. (6) can be written in the form

$$\mathrm{sgn}(\omega')[n_2(\omega') - n_1(\omega)] = n_2(\omega')[1 + n_1(\omega)] - n_1(\omega)[1 + n_2(\omega')].$$

Thus, in this domain the energy and momentum transfer are related as when the excitations are annihilated in one body and created in another body. Such processes are only possible at $T \neq 0$ K, i.e., they are associated with thermal radiation. On the other hand, in the case of the *anomalous* Doppler effect ($\omega - q_x v < 0$)

$$\begin{aligned}\mathrm{sgn}(\omega')[n_2(\omega') - n_1(\omega)] &= 1 + n_2(|\omega'|) + n_1(\omega)\\ &= [1 + n_2(|\omega'|)][1 + n_1(\omega)]\\ &\quad - n_2(|\omega'|)n_1(\omega).\end{aligned}$$

In this case the excitations are created and annihilated simultaneously in both bodies. Such processes are possible even at $T = 0$ K and are associated with quantum friction.

If in Eq. (6) one neglects terms of order β^2 then the contributions from waves with p and s polarization will be separated. In this case Eq. (6) is reduced to the formula obtained in Ref. [7]:

$$\begin{pmatrix} F_{1x} \\ P_1 \end{pmatrix} = \frac{1}{2\pi^3} \int d^2q \int_0^{cq} d\omega \begin{pmatrix} \hbar q_x \\ \hbar\omega \end{pmatrix} \times e^{-2k_z d}\left(\frac{\mathrm{Im}R_{1p}\mathrm{Im}R'_{2p}}{|\Delta_{pp}|^2} + \frac{\mathrm{Im}R_{1s}\mathrm{Im}R'_{2s}}{|\Delta_{ss}|^2}\right) \times [n_2(\omega') - n_1(\omega)]. \tag{8}$$

Thus, to order β^2 the mixing of the waves with different polarizations can be neglected, which agrees with the results obtained in Ref. [10]. At $T_1 = T_2 = 0$ K the propagating waves do not contribute to the friction and the radiative heat transfer. However, the contribution from the evanescent waves does not vanish. Taking into account that $n_1(\omega) = 0$ at $T_1 = T_2 = 0$ K and $\omega > 0$,

$$n_2(\omega') = \begin{cases} -1 & \text{for} \quad 0 < \omega < q_x v \\ 0 & \text{for} \quad \omega > q_x v \end{cases},$$

from Eq. (6) we get the friction mediated by the evanescent waves at zero temperature (denoted as quantum friction [5]) and the radiative heat transfer

$$\begin{pmatrix} F_{1x} \\ P_1 \end{pmatrix} = \int_{-\infty}^{\infty} \frac{dq_y}{2\pi} \int_0^{\infty} \frac{dq_x}{2\pi} \int_0^{q_x v} \frac{d\omega}{2\pi} \begin{pmatrix} \hbar q_x \\ \hbar\omega \end{pmatrix} \Gamma_{12}(\omega,\mathbf{q}). \quad (9)$$

If in Eq. (9) one neglects the terms of order β^2, then the contributions from the waves with p and s polarization will be separated. In this case Eq. (9) is reduced to the approximate (relativistic) formula used by Pendry in Ref. [6]:

$$\begin{pmatrix} F_x \\ P_1 \end{pmatrix} = -\frac{\hbar}{\pi^3} \int_0^{\infty} dq_y \int_0^{\infty} dq_x \int_0^{q_x v} d\omega \begin{pmatrix} \hbar q_x \\ \hbar\omega \end{pmatrix} \times \left(\frac{\mathrm{Im}R_{1p}\mathrm{Im}R'_{2p}}{|D_{pp}|^2} + \frac{\mathrm{Im}R_{1s}\mathrm{Im}R'_{2s}}{|D_{ss}|^2} \right) e^{-2k_z d}. \quad (10)$$

In the nonrelativistic and nonretarded limit, which can be formally obtained in the limit $c \to \infty$, Eq. (10) is reduced to the formula obtained by Pendry in Ref. [5].

IV. QUANTUM FRICTION BETWEEN TWO TRANSPARENT PLATES

For the transparent dielectrics the reflection amplitudes are given by Fresnel's formulas:

$$R_p = \frac{in^2k_z - \sqrt{n^2(\omega/c)^2 - q^2}}{in^2k_z + \sqrt{n^2(\omega/c)^2 - q^2}}, \qquad R_s = \frac{ik_z - \sqrt{n^2(\omega/c)^2 - q^2}}{ik_z + \sqrt{n^2(\omega/c)^2 - q^2}}. \quad (11)$$

In this case the friction force can be written in the form

$$F_{1x} = \frac{\hbar v_0}{d^4}\tilde{g}\left(\frac{v}{v_0},\frac{v}{c},n\right), \quad (12)$$

Where $\tilde{g}$ is a function of two dimensionless velocity ratios and the refractive index n. In the nonrelativistic limit ($\beta^2 \ll 1$), the dependence on vacuum velocity c drops out and

$$F_{1x}^{\mathrm{nrel}} = \frac{\hbar v_0}{d^4}\left[g_s\left(\frac{v}{v_0}\right) + g_p\left(\frac{v}{v_0},n\right)\right], \quad (13)$$

where the s-wave contribution g_s depends only on the ratio of the velocity v to the light speed in the medium v_0. This result was already noted in Ref. [38]. However, the p-wave contribution depends also on the refractive index n.

The imaginary part of the reflection amplitude $R_{1p(s)}$, given by Eq. (11), is only nonzero when $\omega > v_0 q > v_o q_x$. Similarly, $\mathrm{Im}R_{2p(s)}$ is nonzero only when $q_x v - \omega > v_0 q'/\gamma > v_0(q_x - \beta\omega/c)$. Both these conditions limit the range of integration to

$$v_0 q_x < \omega < \frac{(v - v_0)q_x}{1 - v v_0/c^2}. \quad (14)$$

From this condition follows that the minimal velocity v_c, at which friction occurs, is determined by Eq. (4).

For transparent dielectrics, where there are no resonances in the reflection amplitudes, in the frequency range where the quantum friction is nonvanishing $|R_{s(p)}| \leqslant 1$. Thus a good estimation of the friction force and the radiative heat transfer can be obtained by neglecting the multiple scattering of the electromagnetic waves by the dielectric surfaces, in the vacuum gap between them. In this approximation $D_{pp} \approx D_{ss} \approx D_{sp} \approx D_{sp} \approx 1$,

$$\Delta \approx (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 = \frac{(qq')^2}{\gamma^2},$$

$$(q^2 - \beta k q_x)^2 \mathrm{Im}R'_{2p}|\Delta_{ss}|^2 + \beta^2 k_z^2 q_y^2 \mathrm{Im}R'_{2s}|\Delta_{sp}|^2 \approx \frac{(qq')^2}{\gamma^2}\mathrm{Im}R'_{2p} + \beta^2 k_z^2 q_y^2 \mathrm{Im}(R'_{2p} + R'_{2s}),$$

and

$$\Gamma_{12} = 4\mathrm{sgn}(\omega')\Bigg[(\mathrm{Im}R_{1p}\mathrm{Im}R'_{2p} + \mathrm{Im}R_{1s}\mathrm{Im}R'_{2s}) \times \left(1 + \gamma^2\beta^2\frac{k_z^2 q_y^2}{q^2 q'^2}\right) + \gamma^2\beta^2\frac{k_z^2 q_y^2}{q^2 q'^2} \times (\mathrm{Im}R_{1p}\mathrm{Im}R'_{2s} + \mathrm{Im}R_{1s}\mathrm{Im}R'_{2p})\Bigg]. \quad (15)$$

Thus the relativistic effects not only produce a mixing of the waves with different polarizations but also modify the contributions from the different polarizations. These effects were not taken into account in the approximate relativistic theory used by Pendry [6].

Close to the threshold velocity $v \approx v_c$, when $\xi_{\min} \approx \xi_{\max}$ and

$$y_{\max} = \frac{n^2+1}{n\sqrt{n^2-1}}\sqrt{\frac{v - v_c}{v_0}} \ll 1,$$

the integration over q_y in Eq. (9) is restricted by the range $0 < |q_y| < y_{\max} q_x \ll q_x$. In this case, to lowest order in $y_{\max}$, the mixing of the waves with different polarizations can be neglected and the friction force is determined by the formula (see Appendix A)

$$F_{1x} \approx \frac{\hbar v_0}{d^4}\left[\tilde{g_s}\left(\frac{v}{v_0},n\right) + \tilde{g_p}\left(\frac{v}{v_0},n\right)\right] \quad (16)$$

and the radiative heat transfer $P_1 = v_0 F_{1x}$, where

$$\tilde{g_s}\left(\frac{v}{v_0},n\right) = \frac{\zeta(3)}{5\pi^2}\frac{n(n^2+1)^5}{(n^2-1)^5\sqrt{n^2-1}}\left(\frac{v - v_c}{v_0}\right)^{5/2},$$

and $\tilde{g}_p = \tilde{g}_s/n^4$. In the nonrelativistic limit ($n \gg 1$)

$$g_s\left(\frac{v}{v_0}\right) = \frac{\zeta(3)}{5\pi^2}\left(\frac{v - v_c}{v_0}\right)^{5/2},$$

and $g_p(v/v_0,n) = g_s(v/v_0)/n^4$.

Close to the light velocity ($\gamma \gg 1$) (see Appendix B) the s-wave contribution to the friction force, given by the approximate formula (10), is finite at $v \to c$,

$$F_{xs}^{\mathrm{approx}} \approx \frac{3\hbar v}{4\pi^2 d^4}\frac{\sqrt{2}}{n^2-1}\ln(n + \sqrt{n^2-1}), \quad (17)$$

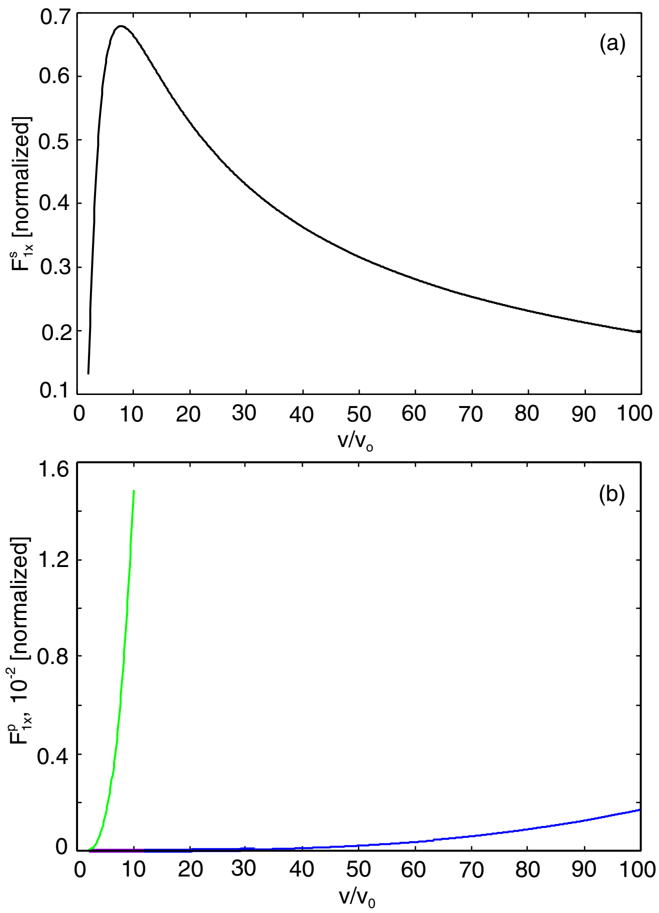

FIG. 2. The dependence of friction force between two transparent dielectric plates on the relative sliding velocity. Normalization factor for the forces $\hbar v_0/\pi^3 d^4$, $v_0 = c/n$. (a) and (b) Results of a nonrelativistic theory for the contributions from s- and p-polarized electromagnetic waves, respectively. The s-wave contribution in a nonrelativistic theory depends only on the ratio v/v_0. The p-wave contributions are shown for $n = 10$ (green line) and $n = 100$ (blue line).

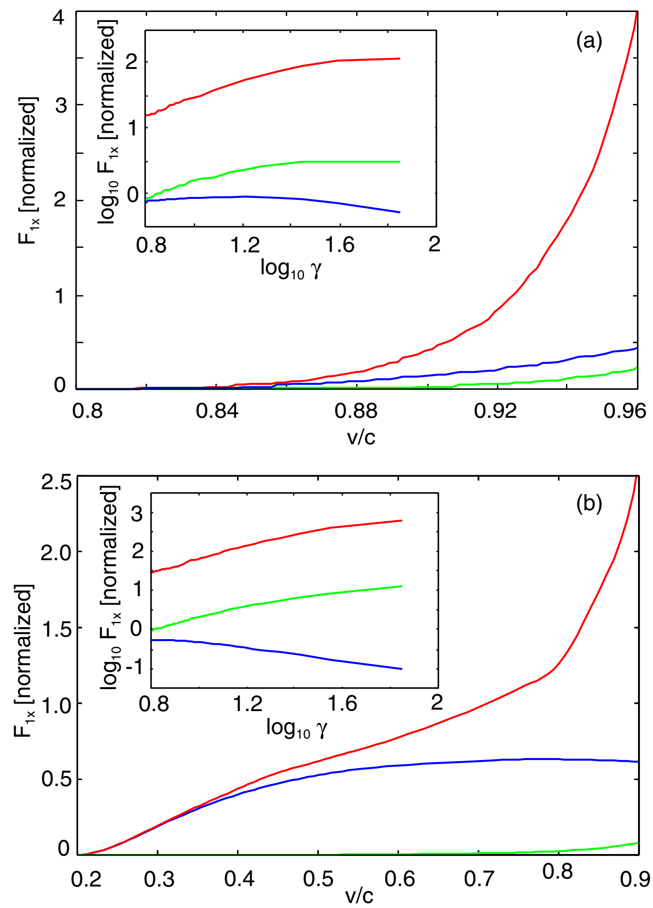

FIG. 3. The same as in Fig. 2 but for a fully relativistic theory. Figures (a) and (b) show the results of a fully relativistic theory (red line) for $n = 2$ and 10, respectively. The blue and green lines show the separate contributions from the s- and p-polarized electromagnetic waves, respectively, obtained using the approximate formula (10). Insets show the friction forces in the ultrarelativistic case ($1 - \beta \ll 1$).

and diverges as $\sim \gamma$ in a fully relativistic theory given by Eq. (B1):

$$F_{xs} \approx \frac{3\hbar v}{4\pi^2 d^4} \frac{\sqrt{n-1}}{2(n+1)^{3/2}} \gamma. \tag{18}$$

Other contributions can be estimated in a similar way.

Figures 2 and 3 show the dependence of friction force between two transparent dielectric plates on the relative sliding velocity in a nonrelativistic theory [Figs. 2(a) and 2(b)] and a fully relativistic theory for $n = 2$ [Fig. 3(a)] and $n = 10$ [Fig. 3(b)]. In a nonrelativistic theory the contributions to the friction force from s- and p-polarized waves are separated. The threshold velocity v_c for appearance of the Vavilov-Cherenkov radiation in the nonrelativistic theory is equal to $2v_0$. The friction in this theory is dominated by the s-wave contribution which depends only on the velocity ratio v/v_0. In a fully relativistic theory friction and radiation only exist for $v > v_c = 2nc/(n^2 + 1)$, which is equal to $0.8c$ for $n = 2$ [Fig. 3(a)] and $0.2c$ for $n = 10$ [Fig. 3(b)]. Figures 3(a) and 3(b) also show results of an approximate relativistic theory for the contributions to the friction force from s-polarized waves (blue line) and p-polarized waves (green line) given by Eq. (10). In such an approximate theory the reflection amplitude from the moving surface is approximated by the reflection amplitude in the comoving reference frame at the frequencies and wave vectors determined by the Lorenz transformation. The polarization mixing is not taken into account in this approximated theory, where the coupling between waves with different polarizations is neglected. Close to the threshold velocity the mixing of the waves with different polarizations is unimportant and the friction is dominated by the contribution from the s-polarized electromagnetic waves, which can be accurately described using an approximate theory. However, in the ultrarelativistic case ($\gamma \gg 1$) both contributions from the different polarizations are strongly enhanced in comparison with the approximate theory, and a new contribution occurs connected with the polarization mixing.

V. CONCLUSION

Two transparent dielectric plates during relative sliding emit quantum Valivov-Cherenkov radiation when the sliding velocity exceeds a threshold velocity $v_c = 2nc/(n^2+1)$. This radiation is responsible for quantum friction which we have studied using a fully relativistic theory. Close to the threshold velocity the friction force $\sim (v-v_c)^{5/2}$ and is dominated by the contribution from the s-polarized electromagnetic waves. However, close to the light velocity the contributions from both polarizations are strongly enhanced and a new contribution occurs connected with the mixing of the waves with different polarizations. As was shown by Pendry [6], surface roughness also can strongly enhance the Vavilov-Cherenkov radiation. In the relativistic case this problem requires further investigation.

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APPENDIX A: CLOSE TO THE THRESHOLD VELOCITY

Close to the threshold $[(v-v_c)/v_0 \ll 1]$ the range in ω becomes narrow. For small $\omega - q_x v$,

$$k_{zn}^2 = n^2\left(\frac{\omega}{c}\right)^2 - q^2 = \left[\frac{(\omega - q_x v_0)(\omega + q_x v_0)}{v_0^2} - q_y^2\right] \approx \left(\frac{\omega - q_x v_0}{v_0}\right) 2q_x - q_y^2, \tag{A1}$$

$$\begin{aligned} k_{nz}'^2 &= \left(\frac{\omega'}{v_0}\right)^2 - q'^2 = -\gamma^2 \frac{[q_x(v-v_0) - \omega(1 - vv_0/c^2)][q_x(v+v_0) - \omega(1+vv_0/c^2)]}{v_0^2} - q_y^2 \\ &= \frac{1}{v_0}\left[\frac{(n^2+1)^2}{n^2(n^2-1)}(v-v_c)q_x - (\omega - q_x v_0)\right] 2q_x - q_y^2. \end{aligned} \tag{A2}$$

From Eqs. (A1) and (A2) follow the ranges in ω and q_y: $\omega_- < \omega < \omega_+$ where

$$\omega_- = q_x v_0 + \frac{v_0 q_y^2}{2q_x}, \quad \omega_+ = q_x v_0 + \frac{(n^2+1)^2}{n^2(n^2-1)}(v-v_c)q_x - \frac{v_0 q_y^2}{2q_x},$$

and $0 \leqslant q_y \leqslant q_{\max}$ where

$$q_{\max}^2 = \frac{(n^2+1)^2}{n^2(n^2-1)} \frac{v-v_c}{v_0} q_x^2 \ll q_x^2.$$

After the changing of the variables $q_y \to q_x y_{\max} y$,

$$\omega \to q_x v_0 \left(1 + \frac{y_{\max}^2}{2} + z y_{\max}^2 \frac{1-y^2}{2}\right),$$

where

$$y_{\max}^2 = \frac{(n^2+1)^2}{n^2(n^2-1)} \frac{v-v_c}{v_0},$$

the imaginary parts for the reflection amplitudes can be written in the form

$$\mathrm{Im} R_s = \frac{2k_z k_{zn}}{k_z^2 + k_{zn}^2} \approx \frac{2k_{zn}}{k_z} \approx \frac{2n}{q_x\sqrt{n^2-1}} \sqrt{\left(\frac{\omega - q_x v_0}{v_0}\right) 2q_x - q_y^2} = \frac{2n y_{\max}}{\sqrt{n^2-1}} \sqrt{1-y^2}\sqrt{1+z} \sim \sqrt{\frac{v-v_c}{v_0}}, \tag{A3}$$

$$\begin{aligned} \mathrm{Im} R_s' &= \frac{2k_z k_{zn}'}{k_z^2 + k_{zn}'^2} \approx \frac{2k_{zn}'}{k_z} \approx \frac{2n}{q_x\sqrt{n^2-1}} \sqrt{\frac{1}{v_0}\left[\frac{(n^2+1)^2}{n^2-1}(v-v_c)q_x - (\omega - q_x v_0)\right] 2q_x - q_y^2} \\ &= \frac{2n y_{\max}}{\sqrt{n^2-1}} \sqrt{1-y^2}\sqrt{1-z} \sim \sqrt{\frac{v-v_c}{v_0}}, \end{aligned} \tag{A4}$$

$\mathrm{Im} R_p \approx \mathrm{Im} R_s/n^2$, $\mathrm{Im} R_p' \approx \mathrm{Im} R_s'/n^2$. Because the integrand in Eq. (9) is proportional to the product of the imaginary parts of the reflection amplitudes, which are of the order $(v-v_c)/v_0$, to lowest order in $(v-v_c)/v_0$, all other terms in the integrand should be taken at $v = v_c$. In this approximation the mixing of the waves with different polarizations can be neglected because they are of order $q_y^2 \sim (v-v_c)/v_0$ and the reflection amplitudes $R_s = R_p = 1$ and the integral for the contribution to the friction force from s-polarized waves is reduced to

$$F_{1x}^s \approx \frac{\hbar v_0}{\pi^3}\int_0^\infty dq_x q_x^3 \frac{e^{-2q_x d\sqrt{n^2-1}/n}}{\left(1-e^{-2q_x d\sqrt{n^2-1}/n}\right)^2}\left(\frac{2n}{\sqrt{n^2-1}}\right)^2 y_{\max}^5 \int_0^1 dy(1-y^2)^2\int_{-1}^1 dz\sqrt{1-z^2}, \tag{A5}$$

which produces Eq. (16).

APPENDIX B: CLOSE TO THE LIGHT VELOCITY: $v \to c$

Introducing new variables $\omega = q_x v\xi$ and $q_y = q_x y$ the integration in Eq. (15) over q_x can be performed analytically, giving

$$\begin{pmatrix} F_x \\ P_1 \end{pmatrix} = -\frac{3\hbar v}{8\pi^3 d^4}\int_{\xi_{\min}}^{\xi_{\max}} d\xi \int_0^{y_{\max}} dy \begin{pmatrix} 1 \\ v\xi \end{pmatrix}\frac{1}{\kappa_z^4}\bigg[(\mathrm{Im}R_{1p}\mathrm{Im}R'_{2p} + \mathrm{Im}R_{1s}\mathrm{Im}R'_{2s})\left(1+\gamma^2\beta^2\frac{\kappa_z^2 y^2}{w^2 w'^2}\right) + \gamma^2\beta^2\frac{\kappa_z^2 y^2}{w^2 w'^2}(\mathrm{Im}R_{1p}\mathrm{Im}R'_{2s} + \mathrm{Im}R_{1s}\mathrm{Im}R'_{2p})\bigg], \tag{B1}$$

where $\kappa_z^2 = 1-\beta^2\xi^2 + y^2$, $w^2 = 1+y^2$, $w'^2 = \gamma^2(1-\beta^2\xi)^2 + y^2$,

$$\xi_{\min} = \frac{1}{n\beta}, \quad \xi_{\max} = 1 - \frac{1}{\gamma^2\beta(n-\beta)} = \frac{n\beta - 1}{\beta(n-\beta)}.$$

In new variables $k_z = q_x\kappa_z$,

$$k_{nz}^2 = (n^2-1)\left(\frac{\omega}{c}\right)^2 - k_z^2 = q_x^2\left(y_1^2 - y^2\right),$$

$$k_{nz}'^2 = (n^2-1)\left(\frac{\omega'}{c}\right)^2 - k_z^2 = q_x^2\left(y_0^2 - y^2\right),$$

where $y_1 = n^2\beta^2\xi^2 - 1$,

$$y_0^2 = \gamma^2[n^2\beta^2(1-\xi)^2 - (1-\beta^2\xi)^2],$$

$$\mathrm{Im}R_s = \frac{2k_z k_{nz}}{k_z^2 + k_{nz}^2} = \frac{2\sqrt{1-\beta^2 o^2 + y^2}\sqrt{y_1^2 - y^2}}{(n^2-1)\beta^2 o^2},$$

$$\mathrm{Im}R'_s = \frac{2k_z k'_{nz}}{k_z^2 + k_{nz}'^2} = \frac{2\sqrt{1-\beta^2 o^2 + y^2}\sqrt{y_0^2 - y^2}}{(n^2-1)\gamma^2\beta^2(1-o)^2},$$

$$\mathrm{Im}R_p = \frac{2n^2 k_z k_{nz}}{n^4 k_z^2 + k_{nz}^2} = \frac{2n^2\sqrt{1-\beta^2 o^2 + y^2}\sqrt{n^2\beta^2 o^2 - 1 - y^2}}{(n^2-1)\beta^2 o^2 + (n^4-1)(1-\beta^2 o^2 + y^2)},$$

$$\mathrm{Im}R'_p = \frac{2n^2 k_z k_{nz}}{n^4 k_z^2 + k_{nz}^2} = \frac{2n^2\sqrt{1-\beta^2 o^2 + y^2}\sqrt{y_0^2 - y^2}}{y_0^2 + (n^4-1)(1-\beta^2 o^2 + y^2) + 1 - \beta^2 o^2},$$

$$y_{\max} = \begin{cases} y_0 & \text{for} \quad o_c < o < o_{\max} \\ 0 & \text{for} \quad o_{\min} < o < o_c \end{cases},$$

where $o_c = \gamma/(1+\gamma) \approx 1 - 1/\gamma$. At $\gamma \gg 1$ the main contribution during integration gives the region $o_c < o < o_{\max}$. The s-wave contribution is given by

$$\begin{pmatrix} F_{xs} \\ P_{1s} \end{pmatrix} \approx \frac{3\hbar v}{8\pi^3 d^4}\int_{o_c}^{o_{\max}} do \int_0^1 dy \begin{pmatrix} 1 \\ v \end{pmatrix}\frac{4y_0^2\sqrt{1-y^2}}{(n^2-1)^{3/2}\gamma^2(1-o)^2} \times \left\{\frac{1}{1-\beta^2 o^2 + y_0^2 y^2} + \gamma^2\frac{y_0^2 y^2}{(1+y_0^2 y^2)\left[\gamma^2(1-\beta^2 o)^2 + y_0^2 y^2\right]}\right.$$

$$\approx \frac{3\hbar v}{4\pi^2 d^4}\begin{pmatrix} 1 \\ v \end{pmatrix}\left\{\left[\frac{\sqrt{2}}{n^2-1}\ln(n+\sqrt{n^2-1}) - \frac{1}{\sqrt{n^2-1}(n+1)} - \frac{2}{(n^2-1)\sqrt{\gamma}} + \frac{1}{(n^2-1)^{3/2}\gamma}\right] + \left[\frac{\sqrt{n-1}}{2(n+1)^{3/2}}\gamma - \frac{1}{\sqrt{n^2-1}(n+1)}\ln(n-1)\gamma - \frac{1}{2(n^2-1)^{3/2}\gamma}\right]\right\}. \tag{B2}$$

Other contributions can be estimated in a similar way.

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