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Towards the equation of state at finite density from the lattice

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Abstract

A new precision lattice simulation set is analyzed for the equation of state to sixth order. We used lattice results at imaginary chemical potentials to calculate the Taylor coefficients, from which the pressure, trace anomaly, energy and entropy density as well as the baryon number can be derived. We discuss an alternative extrapolation strategy and show first results for zero strangeness chemical potential.

Keywords: Equation of state, lattice QCD

1. Introduction

The equation of state of the strongly interacting matter has been calculated from first principles by lattice QCD [1, 2]. The dynamical effect of the charm quark was calculated in [3]. While at LHC energies these results are satisfactory, for RHIC physics one has to extend the range of the computations to finite densities. For lattice QCD this is a very difficult task, since the simulated action turns complex when a baryo-chemical potential is introduced. A remedy for this situation is the calculation of the Taylor expansion coefficients of e.g. the pressure that represents a small parameter expansion in μ_B/T [4, 5]. Similar expansion μ_B/T is also possible for the transition temperature [6, 7]. In the Beam Energy Scan II program the expansion parameter μ_B/T can be as large as 4 at chemical fereze-out, but individual fluid cells can have even higher chemical potentials [8].

Apart from the zero chemical potential, imaginary values are equally unproblematic to simulate [9, 10, 11]. The transition line [12, 13, 6], fluctuations [14] and the equation of state [15] have been calculated by studying the chemical potential dependence of thermodynamic observables, such as the chiral condensate, or, the (imaginary) baryon and other charge densities. The observed dependence on the (imaginary) chemical potential can be used to extrapolate to real chemical potentials using the analytic nature of the transition [16].

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2. Taylor coefficients from imaginary μ_B

Recently we have calculated high order baryon fluctuations on an $N_t = 12$ lattice using imaginary-μ_B chemical potentials. Most importantly, we determined the

$$\chi_n^B(T,\hat{\mu}_B) = \left.\frac{\partial^n(p/T^4)}{(\partial\hat{\mu}_B)^n}\right|_T \tag{1}$$

with $\hat{\mu}_B = \mu_B/T$ for several temperatures as well as imaginary values of the chemical potential $0 \le \mathrm{Im}\,\hat{\mu}_B < \pi$. All these simulations can be described by the global pressure function:

$$\chi_0^B(\hat{\mu}_B) = c_0 + c_2\hat{\mu}_B^2 + c_4\hat{\mu}_B^4 + c_6\hat{\mu}_B^6 + c_8\hat{\mu}_B^8 + c_{10}\hat{\mu}_B^{10} + \ldots, \tag{2}$$

where c_n are the (temperature dependent) Taylor coefficients for the equation of state: $n!c_n = \chi_n^B(T,\hat{\mu}_B = 0)$. We keep the truncated series up to c_{10} order. We extract the $\chi_1^B(T,\hat{\mu}_B),\ldots,\chi_4^B(T,\hat{\mu}_B)$ quantities from each simulation and fit these as a function of $\hat{\mu}_B$ with the corresponding derivative of Eq. (2). For c_8 and c_{10} we introduce priors to stabilize the fit. For the details of the correlated fit over all our ensembles at a given temperature T see Ref. [17]. Results at different temperatures are not correlated. The fit coefficients $\chi_{2n}^B(T,\hat{\mu}_B = 0)$ we plot in Fig. 1.

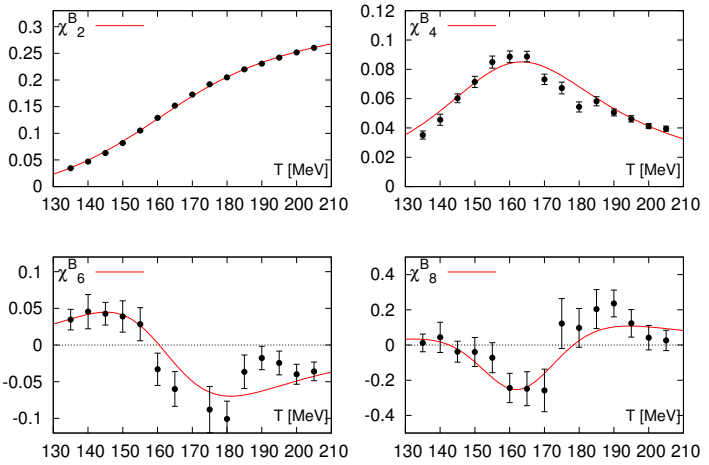

Fig. 1. Results for $\chi_2^B, \chi_4^B, \chi_6^B$ and an estimate for χ_8^B as functions of the temperature, obtained from the single-temperature analysis. The χ_8^B determination was guided by a prior (see [17]). The red curve in each panel corresponds to a simple analytical estimate.

3. Analytical estimate

The pattern in Fig. 1 can be understood as the consequence of a μ_B dependent transition temperature. To illustrate this we plotted the baryon density, normalized to $\hat{\mu}_B$ for zero and an imaginary chemical potential in the left panel of Fig. 2. The shifting of T_c is numerically given by the κ curvature of the phase diagram, which we take from Ref. [18]. Using this κ coefficient to rescale the temperature axis by $(1 - \kappa\hat{\mu}_B^2)$ we find that the rescaled $\mu_B = 0$ curve well reproduces our direct lattice result at imaginary μ_B.

Turning this observation into an ansatz, we can now model $\chi_B/\hat{\mu}_B(T,\hat{\mu}_B)$ using the given κ coefficient and our χ_B^2 result from the first panel of Fig. 1. These assumed functions can be analyzed for their $\hat{\mu}_B$

dependence at fixed T and the Taylor coefficients can be calculated. The resulting curves are shown and compared to the lattice results in red in Fig. 1. For such a simple ansatz the agreement is remarkable.

What do we learn from the approximate agreement between the analytic estimate and the lattice result? If this simple ansatz was exact, the baryon density could also be extended to real finite density. Since the strength of the transition is μ_B-independent in this construction this ansatz assumes the absence of a critical end point. As long as a lattice simulation is consistent with this simple ansatz, it is also consistent with the scenario without a critical end point.

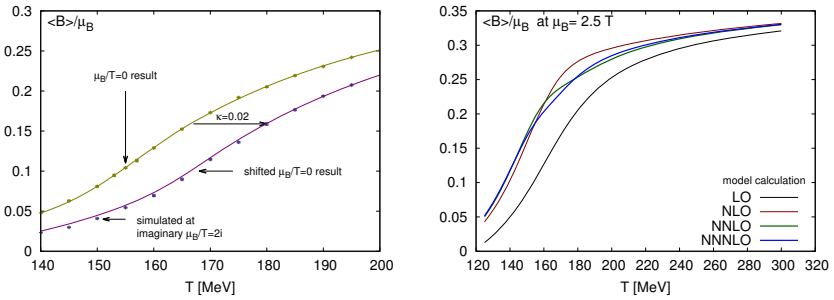

Fig. 2. *Left:* $\chi_1^B(T,\hat{\mu}_B)$ as a function of the temperature for zero and a typical fixed imaginary $\hat{\mu}_B$ value. The inflection point marks the transition, which is at a higher temperature for imaginary $\hat{\mu}_B$. The imaginary $\hat{\mu}_B$ result (blue error bars) is very close to the shifted $\mu_B = 0$ result (violet curve). *Right:* Using the analytical estimates for χ_n^B as Taylor coefficients we extrapolated to $\hat{\mu}_B = 2.5$ and plotted the baryon density. The truncation of the Taylor series introduces artificial inflection points.

4. Alternative extrapolation

Having a high order Taylor expansion of the pressure does not necessarily gives an accurate description of the equation of state. In the right panel of Fig. 2 we show four truncations of the Taylor series. Keeping only χ_2^B (LO) or χ_4^B (NLO) the results are plausible. However, higher orders introduce "wiggles" that might destabilize the hydrodynamic application where it is used. The reason for this behaviour is the oscillatory pattern of high χ_n^B coefficients, which in turn, is caused by the μ_B dependent transition temperature. In the transition region these coefficients must extrapolate from the deconfined phase to the confined phase, or the other way around, depending on the used sign of μ_B^2.

A less problematic prescription could be obtained if the extrapolation did not cross phase boundaries. Fig. 3 shows an example for such an extrapolation. We connect the points on the QCD phase diagram with equal $\chi_1^B(T,\hat{\mu}_B)/\hat{\mu}_B$ values and extrapolate these to $\hat{\mu}_B > 0$ using a low order polynomial or a rational function. The contours (once continuum extrapolated) can be used to determine $\chi_1^B(T,\mu_B)$ at $\mu_B > 0$. For this we also need the already continuum extrapolated χ_2^B (see Ref. [19]) that associates values to the contour lines. Here we show results for three lattice spacings, that are lying on top of each other. The extrapolation, however, introduces significant systematical errors beyond $\hat{\mu}_B > 2$, which requires further studies.

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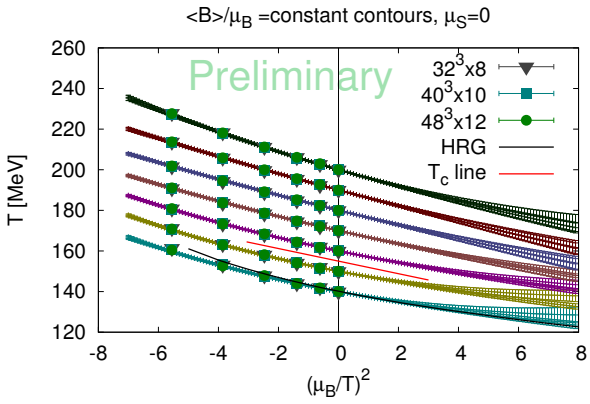

Fig. 3. Contours of fixed baryon-density-over-chemical-potential ratios on the QCD phase diagram. On the left hand side lattice simulations at imaginary chemical potentials constrain the contours, which are extrapolated to real chemical potentials on the right hand side. The contour at T_c is consistent with the curvature of the transition line.

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