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Toroidal Effects on Island Formation in a Helically Perturbed Tokamak Equilibrium

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Abstract

The characteristics of the ergodic magnetic field at the edge of a tokamak plasma produced by an additional helical field are described usually in terms of the Fourier components of the perturbing field to lowest order in the aspect ratio $\varepsilon$ [1]. Fieldline tracing calculations for the planned Dynamic Ergodic Divertor (DED) at TEXTOR-94 [2], which has an aspect ratio of $\varepsilon = 0.25$ lead to about 30% smaller values for the island width and the related Chirikov parameter and to quite larger deviations in the field line diffusion coefficient $D_{FL}$. This effect was found to have a significant dependence on the poloidal location of the perturbation coils.

Starting from the expression for the island width in terms of the vector potential, given in [1], which is correct to any order in the aspect ratio, we derive an expression, which is in full agreement with the numerical results. This is achieved by taking into account the full toroidal geometry when replacing the perturbation of the vector potential by the perturbation of the magnetic field.

1 Introduction

Artificial destruction of nested toroidal surfaces at the plasma edge by means of the magnetic field distortion through special coils is one of the established approaches to the problem of plasma-wall interaction in thermonuclear devices [3]. This allows to increase the transport of particles and heat in the peripheral plasma region and to reduce the energy of particles impinging the wall elements and enhance their living time significantly. Additionally the concentration of impurities in the plasma core can be diminished dramatically due to the so called “screening effect” [3, 4].

Unfortunately even with a high level of stochastization of field lines they intersect the wall surface at certain preferential positions and this leads to local overheating of the wall elements. To avoid this disadvantage a concept of dynamic ergodic divertor (DED) has been proposed [5, 6]. It implies a time variation of the electric current in the perturbation coils with a frequency up to 10 kHz which should lead to poloidal displacement of the plasma-wall contact and smooth down the heat load.

It is planned now to realize the DED concept on TEXTOR-94 tokamak. Different to Tore-Supra the perturbation coils are located at the inner edge of the tokamak. Field line tracing calculations done for this configuration revealed a significant deviation from the standard analytical formula [1] by comparing the island widths in the perturbed configuration. A strong dependence on the location of the perturbation
coils was found out. Here we derive an expression which is correct to any order in
the aspect ratio and we apply it to an equilibrium with circular cross section to get
a first order result which is in full agreement with the numerical calculations.

2 Basic formula for island width

Following [1, 7] we introduce intrinsic coordinates $\Phi_{\text{cor}}, \theta^*, \varphi^*$, being the toroidal
flux, the poloidal and toroidal coordinate, respectively. In this reference system
with the basic vectors
\[ \vec{\nabla} \Phi_{\text{cor}}, \vec{\nabla} \theta^*, \vec{\nabla} \varphi^*, \]
field lines are straight and the magnetic field of a tokamak equilibrium can be
written in the form [7]
\[ \vec{B}_{\text{eq}} = \vec{\nabla} \times \left[ \Phi_{\text{tor}} \cdot \vec{\nabla} \theta^* - \Phi_{\text{pol}}^\text{eq} \cdot \vec{\nabla} \varphi^* \right], \]
where $\Phi_{\text{pol}}^\text{eq}$ is the poloidal flux. A helical perturbation can be described as
\[ \delta \Phi_{\text{pol}}(\Phi_{\text{tor}}) = \sum_{m,n = -\infty}^{\infty} \delta \Phi_{m,n}(\Phi_{\text{tor}}) \cdot e^{i(m\theta^* + n\varphi^*)}. \]

With this notation and using the Hamiltonian formalism one finds that the resonant
magnetic surface with the safety factor $q = m/n$ splits into a chain of islands. The
half width of the islands in terms of the toroidal flux is given by the expression [1]
\[ \delta \Phi_{\text{tor}} = \left[ \frac{8 \cdot \delta \Phi_{m,n}}{d \left( \frac{1}{q} \right)} \right]^{1/2}, \]
where all variables are surface quantities.

3 Poloidal variation of the radial island width

In metric units the radial island diameter is
\[ d = \frac{2 \delta \Phi_{\text{tor}}}{|\vec{\nabla} \Phi_{\text{tor}}|}. \]

Since
\[ \vec{\nabla} \Phi_{\text{tor}} = \frac{d \Phi_{\text{cor}}}{d \Phi_{\text{pol}}} \cdot \vec{\nabla} \Phi_{\text{pol}} = q \cdot \vec{\nabla} \Phi_{\text{pol}} \]
and
\[ \vec{\nabla} \Phi_{\text{pol}} = R B_{\text{pol}} \cdot \vec{e}_\perp \]
where $\vec{e}_\perp$ is the unit vector perpendicular to the unperturbed magnetic surface, one
can write
\[ |\vec{\nabla} \Phi_{\text{tor}}| = q R B_{\text{pol}}. \]

Due to the Shafranov shift and a possibly noncircular cross section $|\vec{\nabla} \Phi_{\text{tor}}|$ is a
function of the poloidal coordinate and the radial diameters of the islands depend
on their poloidal location $\theta_i$ ($i = 1, \ldots, m$).
4 Replacing the vector potential $\delta \Phi$ by the magnetic field $\delta B$

Analogously to equation (2) we write the perturbation field as

$$\delta \vec{B} = -\vec{\nabla} \times \left( \delta \Phi_{\text{pol}} \cdot \vec{\nabla} \phi^* \right) = -\vec{\nabla} \delta \Phi_{\text{pol}} \times \vec{\nabla} \phi^*. \tag{9}$$

Scalar multiplication by $\frac{R}{|B_{\text{tor}}|} \vec{\nabla} \Phi_{\text{tor}}$ leads to

$$\frac{R \cdot |\nabla \Phi_{\text{tor}}|}{|B_{\text{tor}}|} \cdot \delta B_L = -\frac{R \cdot J}{|B_{\text{tor}}|} \cdot \frac{\partial}{\partial \theta^*} \delta \Phi_{\text{pol}} = -\frac{\partial}{\partial \theta^*} \delta \Phi_{\text{pol}}. \tag{10}$$

$J$ is the Jacobian of the basis (1) and in a coordinate system where $\phi^*$ is the geometrical toroidal angle (which is appropriate for Fourier expansions) we have $R \cdot J / B_{\text{tor}} = 1$, whereas the factor associated to $\delta B_L$ is a function of $\theta^*$.

The Fourier decomposition of equation (10) leads to

$$\delta \Phi_{m,n} = \left[ \frac{S}{m} \cdot \delta B_L \right]_{m,n} = \frac{S}{m} \cdot h_{m,n}, \tag{11}$$

where

$$S = \frac{|\nabla \Phi_{\text{tor}}|}{J} = \frac{qRB_{\text{pol}}}{J}. \tag{12}$$

and the $h_{m,n}$ are the coefficients of the expansion of the function

$$h(\Phi_{\text{tor}}, \theta^*, \phi) = \frac{S}{\bar{S}} \cdot \delta B_L.$$ 

with $\bar{S}$ is the average value of $S$ with respect to the geometrical poloidal angle $\theta$.

In general, the $h_{m,n}$ are different from the coefficients of the expansion of $\delta B_L$.

5 Circular cross section

For circular cross sections to first order in the aspect ratio $\varepsilon = r / \bar{R}$ one can write

$$B_{\text{pol}} = B_{\text{pol}} \cdot (1 + \Lambda \varepsilon \cos \theta), \tag{14}$$

where $\Lambda = \beta_{\text{pol}} + i / 2 - 1$, is the average over the poloidal cross section of the magnetic surface. $li$ is the internal inductance. Thus

$$|\nabla \Phi_{\text{tor}}| = r B_{\text{tor}} \cdot (1 - \Delta' \cos \theta) \tag{15}$$

with $\Delta'$ being the derivative of the Shafranov shift,

$$\Delta' = \frac{d\Delta}{dr} = -\varepsilon \cdot (1 + \Lambda), \tag{16}$$

and

$$S = r \bar{R} \cdot \left( \frac{R}{\bar{R}} \right)^3 \cdot \left( \frac{B_{\text{pol}}}{B_{\text{pol}}} \right) = r \bar{R} \cdot (1 + \varepsilon \cos \theta)^3 \cdot (1 + \Lambda \varepsilon \cos \theta) = r \bar{R} \cdot g(\theta). \tag{17}$$
Taking into account that
\[ \frac{d}{d\Phi_{\text{tor}}} \left( \frac{1}{q} \right) = -\frac{1}{q^2} \frac{dq}{dr} \cdot (r B_{\text{tor}})^{-1}, \] (18)
the island diameter in metric units can be written in the form
\[ d_i = \left( 1 - \Delta' \cos \theta_i \right)^{-1} \left[ \frac{32 q r R}{m s} \cdot \frac{h_{m,n}}{B_{\text{tor}}} \right]^{1/2}, \] (19)
with the shear parameter \( s = r/q \cdot dq/dr \). The first factor here reproduces the observation (see fig.1) that the islands inside are broader than at the outer side.

Different to [1] the Fourier components \( h_{m,n} \) now appear in eq. (19) instead of \( \delta B_{m,n} \). In case of localized coils the local value of \( g(\theta) \) affects the \( h_{m,n} \) and the island diameter, as it was observed by field line tracing calculations.

6 Comparison with the Numerical Results

For the DED at TEXTOR-94 the coils are located at \( \pi - \Delta \theta \leq \theta \leq \pi + \Delta \theta \) (see fig. 2a) with \( \Delta \theta \approx \pi/5 \). Assuming the amplitude of \( \delta B \) being constant in this domain one gets approximately
\[ h_{m,n} = \delta B_{m,n} \cdot \frac{1}{2\Delta \theta} \int_{\pi - \Delta \theta}^{\pi + \Delta \theta} (1 + \varepsilon \cos \theta)^3 (1 + \varepsilon \Lambda \cos \theta) \, d\theta \approx 0.45 \cdot \delta B_{m,n} \] (20)
for \( \varepsilon = 0.244 \) and \( \Lambda = 0.5 \).

For coils located at the low field side \( -\Delta \theta \leq \theta \leq 0 \) and \( \Delta \theta \approx \pi/2 \) (see fig. 2b), we get
\[ h_{m,n} \approx 1.6 \cdot \delta B_{m,n}. \] (21)

For the case of fig. 2c where the coils are located at top and bottom the lowest order formula [1] should be nearly correct, since \( g(\theta) \approx 1 \).

The following table contains the mean radial island diameters measured from the Poincaré plots for the different locations of the perturbation coils in fig. 2. They are compared with the results of the expression (19). The calculations were done for the aspect ratio \( \varepsilon = 0.244 \) and \( \Lambda = 0.5 \). The Fourier components and the island widths were measured at the \( q = 3 \) surface. The units are arbitrary. Also the results of the lowest order expression are reported.

It is seen that the presented theoretical results practically coincide with the values measured from the Poincaré plots, \( (dp_{\text{pinc}})^1 \) whereas the results from the formula of [1], are larger by a factor up to 1.7 for configuration a) and smaller by 0.74 for configuration b). They coincide for configuration c) since \( h \approx \delta B_{\perp} \) in this case.

Comparison of \( \delta B_{m,n} \) with \( h_{m,n} \) confirms the approximations of the expressions (20), (21).

\[ ^1 \text{Here the average part is taken.} \]
<table>
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<th>config</th>
<th>$\beta_{pol}$</th>
<th>$\delta B_{m,n}$</th>
<th>$h_{m,n}$</th>
<th>$d_{B_{m,n}}$</th>
<th>$d_{h_{m,n}}$</th>
<th>$d_{Poinc}$</th>
<th>$d_B/d_{Poinc}$</th>
<th>$d_h/d_{Poinc}$</th>
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<tr>
<td>a</td>
<td>0.5</td>
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<td>0.34</td>
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<td>0.45</td>
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<td>0.38</td>
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<tr>
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<td>0.62</td>
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<tr>
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<td>0.50</td>
<td>0.53</td>
<td>0.51</td>
<td>0.98</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 1: For different poloidal locations of the perturbation coils (see fig. 2) and for different $\beta_{pol}$, the Fourier components of the perturbation field $\delta B_\perp$ and of the function $h$ are listed. $d_{B_{m,n}}$, $d_{h_{m,n}}$ are the radial island diameters calculated by the formula of [1] and by eq. (19) respectively. The island diameters are compared to the mean values $d_{Poinc}$ deduced from the Poincaré plots.

7 Conclusion

An analytical expression of higher order in the aspect ratio for the island width in perturbed magnetic tokamak equilibria has been derived. It correctly accounts for the poloidal position of the perturbation coils and the toroidal geometry, and it reproduces the results from numerical calculations.

References

[1] Ph. Ghendrih, H. Capes, F. Nguyen, A. Samain
Control of the Edge Transport with the Ergodic Divertor

Magnetic Field Line Properties
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Figure 1: Poincaré plot of a moderately perturbed tokamak equilibrium. The abscissa represents the poloidal angle, the ordinate is the small radius $r$. The high field side is in the center. The poloidal dependence of the island chains is due to the Shafranov shift.

Figure 2: Sketch of the three different coil systems located at a) the high field side, b) the low field side and c) top and bottom.