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Cross-correlations of conserved charges from the lattice

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Abstract

We present a lattice calculation on the cross-correlations of conserved charges (baryon number, electric charge and strangeness) near the transition temperature. We extrapolate to small baryo-chemical potentials, and thus we cover typical STAR energies. We confront our finding to the latest STAR data set on cross-correlations. In this work we present continuum lattice results with resolution up to $N_t = 16$.

Keywords: lattice QCD, cross-correlations, phase diagram, finite density

1. Introduction

Correlations of conserved charges are important observables for the finite-density investigations. In this work we focus on the off-diagonal combinations. One possible way to extend lattice results to finite density is to perform Taylor expansions of the thermodynamic observables around chemical potential $\mu_B = 0$ [1, 2, 3, 4, 5]: fluctuations of conserved charges are directly related to the Taylor expansion coefficients of such observables. They allow for a comparison between theoretical and experimental results to extract the chemical freeze-out temperature T_f and chemical potential μ_{Bf} as functions of the collision energy [6, 7, 8, 9]. The higher order fluctuations are also an important signature for the critical endpoint, as they give access to the correlation length [3, 10, 11].

In this work we use analytical continuation from imaginary chemical potential [12, 13, 14, 15, 16]. It agrees well with the results of the Taylor expansion as shown for the transition temperature [17].

We simulate the lower-order fluctuations at imaginary chemical potential and extract the higher order fluctuations as derivatives of the lower order ones at $\mu_B = 0$. This method has been successfully used in the past and proved to lead to a more precise determination of the higher order fluctuations, compared to their direct calculation [18, 19, 17].

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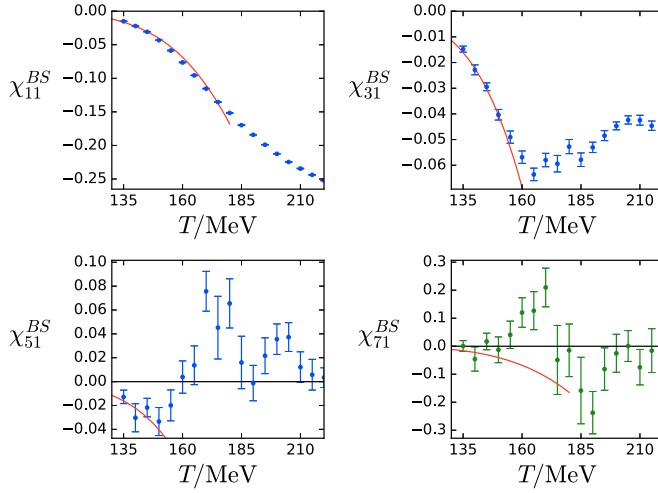


Fig. 1. Results for χ_{11}^{BS} , χ_{31}^{BS} , χ_{51}^{BS} and an estimate for χ_{71}^{BS} on a $N_t = 12$ lattice as functions of the temperature, obtained from the single-temperature analysis. We plot χ_{71}^{BS} in green to point out that its determination is guided by a prior, which is linked to χ_{31}^{BS} . The red curve in each panel corresponds to the Hadron Resonance Gas (HRG) model result. [20]

2. Cross-correlations on an $N_t = 12$ -lattice

We first present an analysis with high precision on an $N_t = 12$ lattice. A more detailed description as well as precise information on the lattice set-up can be found in ref. [20, 21]. In the following we use the notation $\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k} \langle p/T^4 \rangle}{(\partial \hat{\mu}_B)^i (\partial \hat{\mu}_Q)^j (\partial \hat{\mu}_S)^k}$, with $\hat{\mu} = \mu/T$. We make the ansatz

$$\chi_{01}^{BS}(\hat{\mu}_B) = \chi_{11}^{BS} \hat{\mu}_B + \frac{1}{3!} \chi_{31}^{BS} \hat{\mu}_B^3 + \frac{1}{5!} \chi_{51}^{BS} \hat{\mu}_B^5 + \frac{1}{7!} \chi_{71}^{BS} \hat{\mu}_B^7 + \frac{1}{9!} \chi_{91}^{BS} \hat{\mu}_B^9 \quad (1)$$

where $\frac{\chi_{31}^{BS}}{\chi_{11}^{BS}}$ and $\frac{\chi_{51}^{BS}}{\chi_{31}^{BS}}$ are constrained by a prior, normally distributed with $\mu = -1.25$ and $\sigma = 2.75$ and the independent fit parameters are χ_{11}^{BS} , χ_{31}^{BS} and χ_{51}^{BS} . The results which we obtain from a fully correlated fit to this ansatz and its first three derivatives $\chi_{11}^{BS}(\hat{\mu}_B)$, $\chi_{21}^{BS}(\hat{\mu}_B)$ and $\chi_{31}^{BS}(\hat{\mu}_B)$ are presented in fig. 1. To connect to experimental results, we calculate the ratio of the cumulants of the net-baryon number distribution as functions of temperature and chemical potential by means of their Taylor expansion in powers of μ_B/T . This is possible by combining different diagonal and non-diagonal fluctuations to obtain a result at the strangeness neutral point and with $\langle n_Q \rangle = 0.4 \langle n_B \rangle$. Our results for $S_B \sigma_B^3 / M_B$ and κ_B / σ_B^2 are shown in fig. 2. Here M_B is the mean, σ_B^2 is the variance, S_B is the skewness and κ_B the kurtosis of the the baryon number distribution.

3. Cross-correlations in the continuum

Now we will present our preliminary results on the cross-correlations in the continuum. The curves shown in fig. 3 are not final, as they do not yet include a full analysis of the systematic error. To extrapolate to the continuum we need to incorporate the temperature and the $1/N_t^2$ dependence of our data in this fit ansatz. We expect our results for $\chi(T)$ to lie on a smooth curve. We implement this information by fitting the results with a spline in the temperature direction. For the $1/N_t^2$ we fit a linear function through the data from lattices with the sizes $32^3 \times 10$, $40^3 \times 10$, $40^3 \times 12$, $48^3 \times 12$, $48^3 \times 16$ and $64^3 \times 16$. Our whole analysis is done in one combined fit. Therefore now the fit parameter in ansatz similar to eq. (1) become functions of T and N_t .

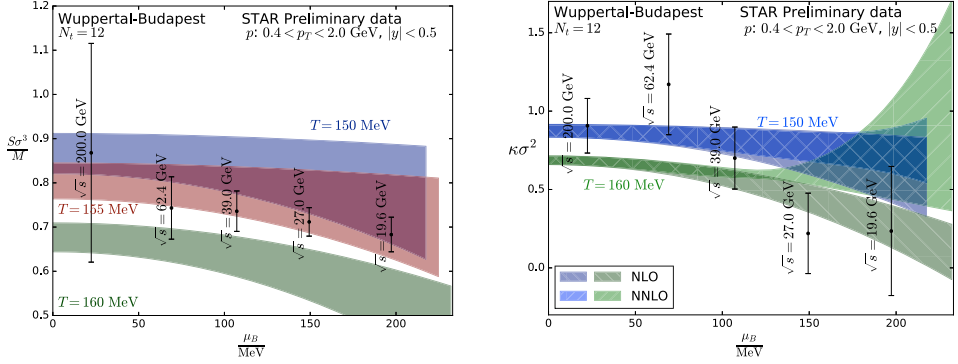


Fig. 2. $S_B \sigma_B^3 / M_B$ (left panel) and $\kappa_B \sigma_B^2$ (right panel) extrapolated to finite chemical potential. The left panel is extrapolated up to $\mathcal{O}(\mu_B^2)$. In the right panel, the darker bands correspond to the extrapolation up to $\mathcal{O}(\mu_B^2)$, whereas the lighter bands also include the $\mathcal{O}(\mu_B^3)$ term. [20]

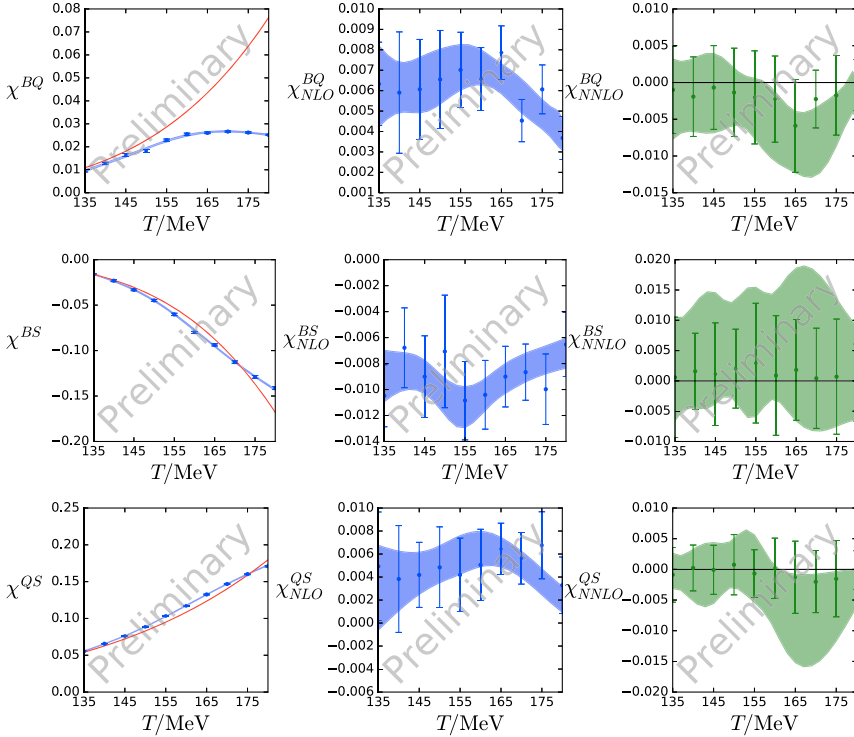


Fig. 3. Preliminary results for the cross-correlations in the continuum. The NNLO contribution is plotted in green, as it is again constrained by a prior. The red curves correspond to the HRG model results.

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