

# Parallel & Scalable Machine Learning

Introduction to Machine Learning Algorithms

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**LECTURE 6** 

# **Validation & Regularization**

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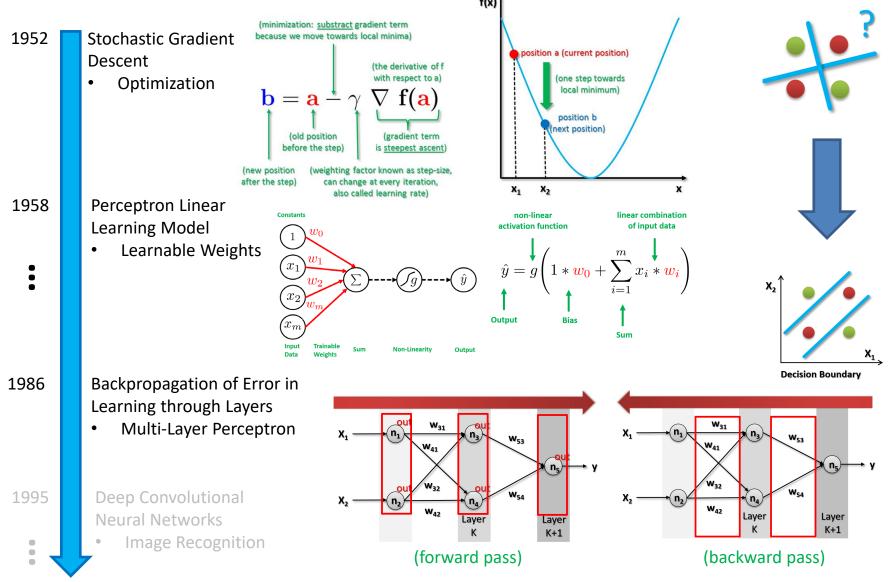








#### **Review of Lecture 5 – Feed Forward Neural Networks**



#### **Outline of the Course**

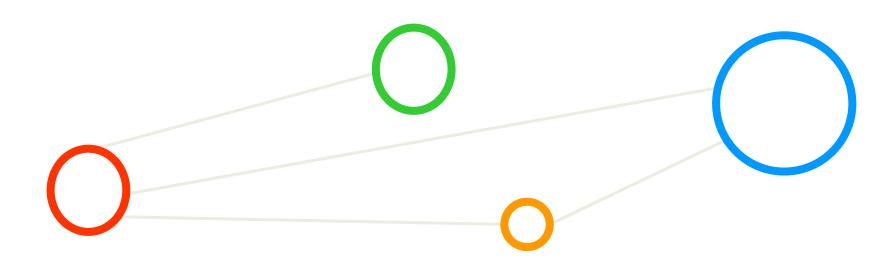
- 1. Parallel & Scalable Machine Learning driven by HPC
- 2. Introduction to Machine Learning Fundamentals
- 3. Introduction to Machine Learning Fundamentals
- 4. Feed Forward Neural Networks
- 5. Feed Forward Neural Networks
- 6. Validation and Regularization
- 7. Validation and Regularization
- 8. Data Preparation and Performance Evaluation
- 9. Data Preparation and Performance Evaluation
- 10. Theory of Generalization
- 11. Unsupervised Clustering and Applications
- 12. Unsupervised Clustering and Applications
- 13. Deep Learning Introduction

Theoretical Lectures

**Practical Lectures** 



# **Outline**



#### **Outline**

#### Validation for Model Selection

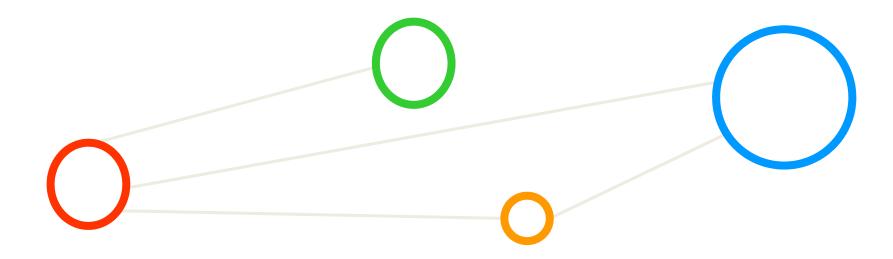
- Problem of Overfitting
- Overfitting Reasoning & Validation
- Creating ANN Network Topologies
- Many Parameters & Hidden Layers
- Validation Datasets & Splits

#### Regularization

- Overfitting Reasoning
- Regularization & Validation Counter Approach
- Regularization Techniques
- Dropout Regularizer
- Optimizers RMSprop & Adam

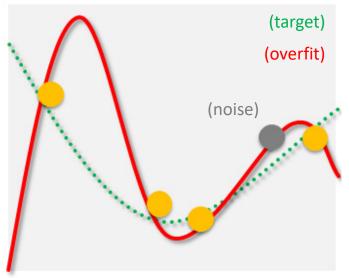


# **Validation for Model Selection**



## **Machine Learning Challenges – Problem of Overfitting**

- Overfitting refers to fit the data too well more than is warranted thus may misguide the learning
- Overfitting is not just 'bad generalization' e.g. the VC dimension covers noiseless & noise targets
- Theory of Regularization are approaches against overfitting and prevent it using different methods
  - Key problem: noise in the target function leads to overfitting
    - Effect: 'noisy target function' and its noise misguides the fit in learning
    - There is always 'some noise' in the data
    - Consequence: poor target function ('distribution') approximation
  - Example: Target functions is second order polynomial (i.e. parabola)
    - Using a higher-order polynomial fit
    - lacksquare Perfect fit: low  $E_{in}(g)$  , but large  $E_{out}(g)$



(but simple polynomial works good enough)
('over': here meant as 4th order,

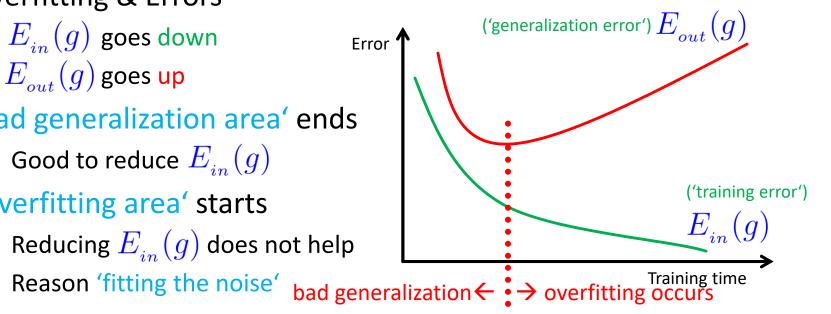
a 3<sup>rd</sup> order would be better, 2<sup>nd</sup> best)

# **Problem of Overfitting – Clarifying Terms**

- A good model must have low training error (E<sub>in</sub>) and low generalization error (E<sub>out</sub>)
- Model overfitting is if a model fits the data too well (Ein) with a poorer generalization error (Eout) than another model with a higher training error (E<sub>in</sub>)

[1] Introduction to Data Mining

- Overfitting & Errors
  - $\blacksquare E_{in}(g)$  goes down
  - $\blacksquare E_{out}(g)$  goes up
- 'Bad generalization area' ends
  - Good to reduce  $E_{in}(g)$
- 'Overfitting area' starts
  - Reducing  $E_{in}(g)$  does not help



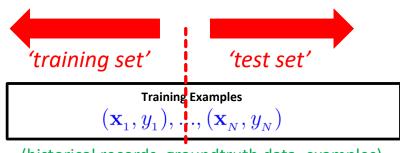
The two general approaches to prevent overfitting are (1) regularization and (2) validation

#### **Terminologies & Different Dataset Elements**

- Target Function  $f: X \to Y$ 
  - Ideal function that 'explains' the data we want to learn
- Labelled Dataset (samples)
  - 'in-sample' data given to us:  $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_N, y_N)$
- Learning vs. Memorizing
  - The goal is to create a system that works well 'out of sample'
  - In other words we want to classify 'future data' (ouf of sample) correct
- Dataset Part One: Training set
  - Used for training a machine learning algorithms
  - Result after using a training set: a trained system
- Dataset Part Two: Test set
  - Used for testing whether the trained system might work well
  - Result after using a test set: accuracy of the trained model

### **Model Evaluation – Training and Testing Phases**

- Different Phases in Learning (cf. day one remote sensing)
  - Training phase is a hypothesis search
  - Testing phase checks if we are on right track (once the hypothesis clear)
- Work on 'training examples'
  - Create two disjoint datasets
  - One used for training only (aka training set)
  - Another used for testing only (aka test set)

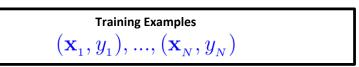


(historical records, groundtruth data, examples)

- Exact seperation is rule of thumb per use case (e.g. 10 % training, 90% test)
- Practice: If you get a dataset take immediately test data away ('throw it into the corner and forget about it during modelling')
- Reasoning: Once we learned from training data it has an 'optimistic bias'

# **Learning Approaches – Supervised Learning – Formalization**

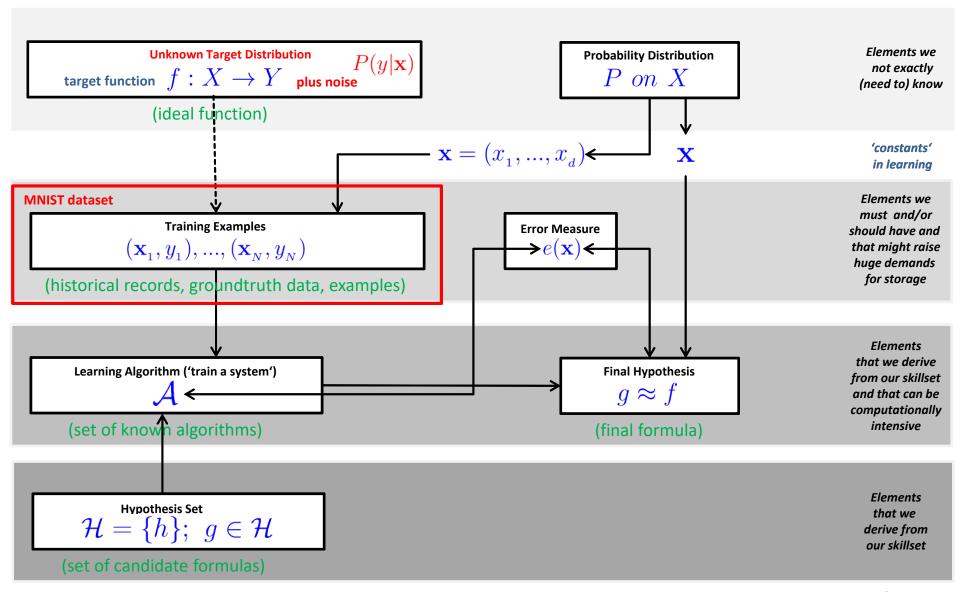
- Each observation of the predictor measurement(s) has an associated response measurement:
  - Input  $\mathbf{x} = x_1, ..., x_d$
  - Output  $y_i, i = 1, ..., n$
  - Data  $(\mathbf{x}_{_{1}},y_{_{1}}),...,(\mathbf{x}_{_{N}},y_{_{N}})$



(historical records, groundtruth data, examples)

- Goal: Fit a model that relates the response to the predictors
  - Prediction: Aims of accurately predicting the response for future observations
  - Inference: Aims to better understanding the relationship between the response and the predictors
- Supervised learning approaches fits a model that related the response to the predictors
- Supervised learning approaches are used in classification algorithms such as SVMs
- Supervised learning works with data = [input, correct output]

## **Supervised Learning – Training Examples**



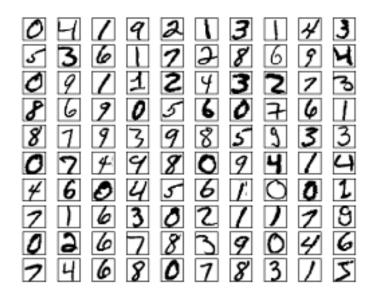
### Handwritten Character Recognition MNIST Dataset

#### Metadata

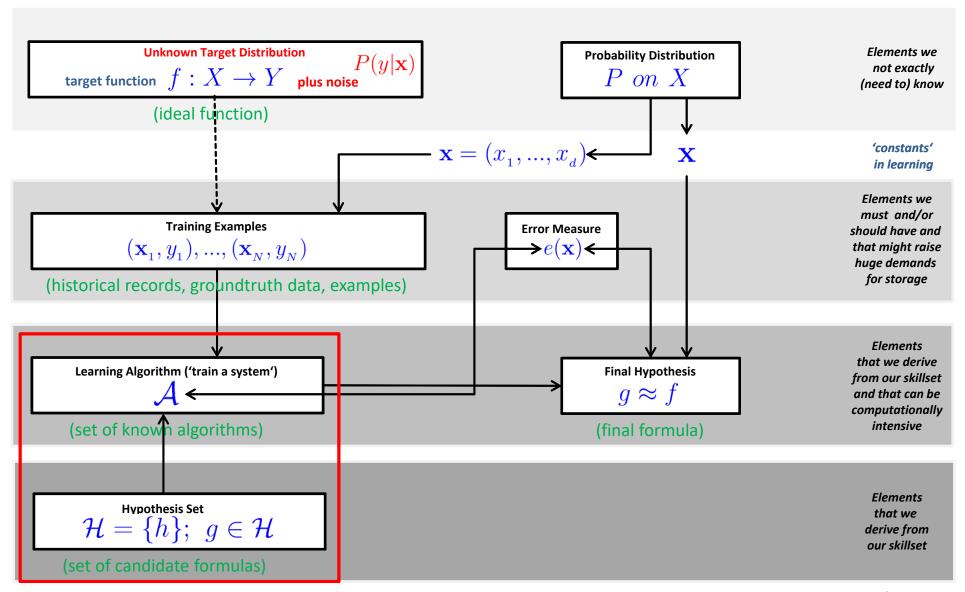
- Subset of a larger dataset from US National Institute of Standards (NIST)
- Handwritten digits including corresponding labels with values 0 to 9
- All digits have been size-normalized to 28 \* 28 pixels and are centered in a fixed-size image for direct processing
- Not very challenging dataset, but good for experiments / tutorials

#### Dataset Samples

- Labelled data (10 classes)
- Two separate files for training and test
- 60000 training samples (~47 MB)
- 10000 test samples (~7.8 MB)



## **Supervised Learning – Many Hypothesis to Choose**



## **Different Models – Understanding the Hypothesis Set**

$$\mathcal{H}=\{h\};\;g\in\mathcal{H}$$

$$\mathcal{H} = \{h_1, ..., h_m\};$$

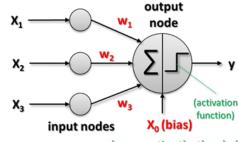
(all candidate functions derived from models and their parameters)

- Choosing from various model approaches h<sub>1</sub>, ...,
   h<sub>m</sub> is a different hypothesis
- Additionally a change in model parameters of h<sub>1</sub>, ..., h<sub>m</sub> means a different hypothesis too

 $h_1$ 

(e.g. support vector machine model)

 $h_2$ 



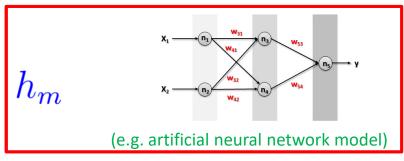
(representing the threshold)

(e.g. linear perceptron model)

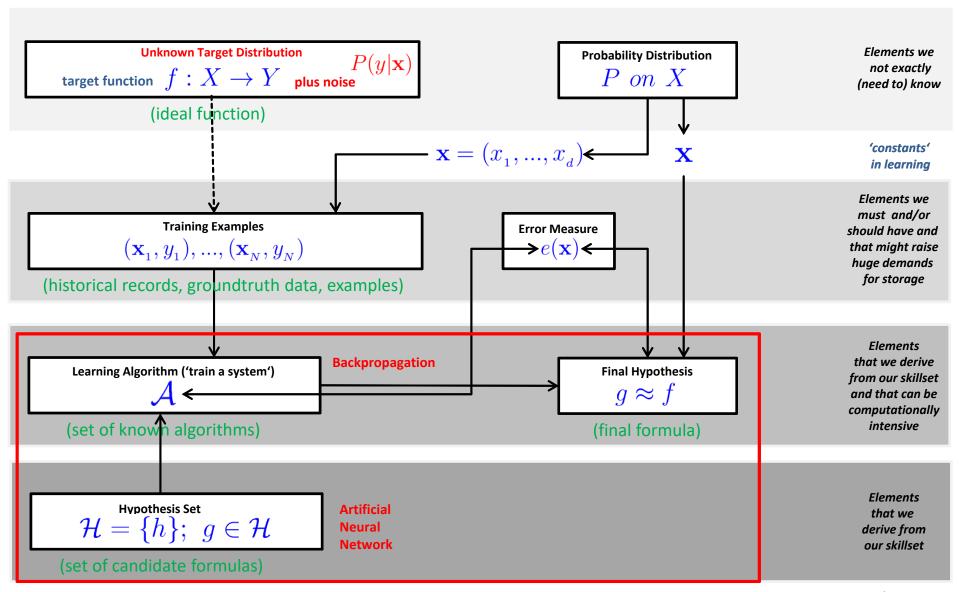
'select one function' that best approximates

Final Hypothesis 
$$q \approx f$$



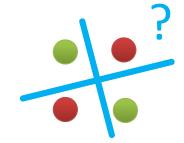


#### **Supervised Learning – Training Examples**



### **Artificial Neural Network (ANN)**

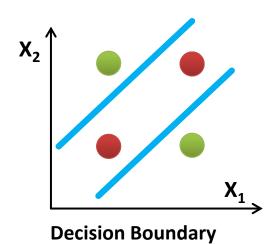
Simple perceptrons fail: 'not linearly seperable'

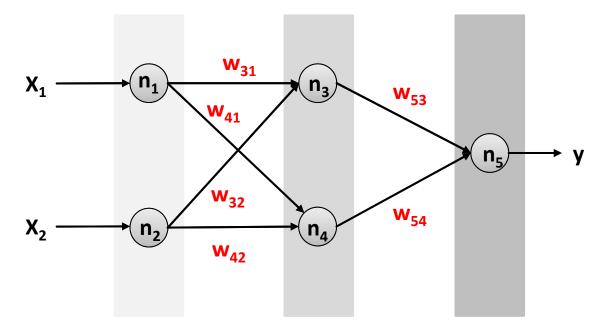


X <sub>1</sub>	X <sub>2</sub>	Y
0	0	-1
1	0	1
0	1	1
1	1	-1

(Idea: instances can be classified using two lines at once to model XOR)

**Labelled Data Table** 





Two-Layer, feed-forward Artificial Neural Network topology

#### **High-level Tools – Keras**

- Keras is a high-level deep learning library implemented in Python that works on top of existing other rather low-level deep learning frameworks like Tensorflow, CNTK, or Theano
- The key idea behind the Keras tool is to enable faster experimentation with deep networks
- Created deep learning models run seamlessly on CPU and GPU via low-level frameworks

```
keras.layers.Dense(units,
                   activation=None,
                   use bias=True,
                   kernel initializer='glorot uniform',
                   bias initializer='zeros',
                   kernel regularizer=None,
                   bias regularizer=None,
                   activity regularizer=None,
                   kernel constraint=None,
                   bias constraint=None)
keras.optimizers.SGD(lr=0.01,
                     momentum=0.0,
                     decay=0.0,
```

nesterov=False)

- **Tool Keras supports inherently** the creation of artificial neural networks using Dense layers and optimizers (e.g. SGD)
- **Includes regularization (e.g.** weight decay) or momentum



#### **ANN – MNIST Dataset – Create ANN Blueprint**

- ✓ Data Preprocessing done (i.e. data normalization, reshape, etc.)
- 1. Define a neural network topology
  - Which layers are required?
  - Think about input layer need to match the data what data we had?
  - Maybe hidden layers?
  - Think Dense layer Keras?
  - Think about final Activation as Softmay (cf. Day One)  $\rightarrow$  output probability
- 2. Compile the model  $\rightarrow$  model representation for Tensorflow et al.
  - Think about what loss function you want to use in your problem?
  - What is your optimizer strategy, e.g. SGD (cf. Day One)
- 3. Fit the model  $\rightarrow$  the model learning takes place
  - How long you want to train (e.g. NB\_EPOCHS)
  - How much samples are involved (e.g. BATCH\_SIZE)

# **Exercises – Create a Simple ANN Model – One Dense**



#### **MNIST Dataset – Model Parameters & Data Normalization**

```
import numpy as np
from keras.datasets import mnist
from keras.models import Sequential
from keras.layers.core import Dense, Activation
from keras.optimizers import SGD
from keras.utils import np_utils
```

```
# parameter setup
NB_EPOCH = 20
BATCH_SIZE = 128
NB_CLASSES = 10 # number of outputs = number of digits
OPTIMIZER = SGD() # optimization technique
VERBOSE = 1
```

```
# download and shuffled as training and testing set
(X_train, y_train), (X_test, y_test) = mnist.load_data()

# X_train is 60000 rows of 28x28 values --> reshaped in 60000 x 784
RESHAPED = 784
X_train = X_train.reshape(60000, RESHAPED)
X_test = X_test.reshape(10000, RESHAPED)
X_train = X_train.astype('float32')
X_test = X_test.astype('float32')

# normalize
X_train /= 255
X_test /= 255
```

```
NB_CLASSES: 10 Class Problem
```

- NB\_EPOCH: number of times the model is exposed to the overall training set – at each iteration the optimizer adjusts the weights so that the objective function is minimized
- BATCH\_SIZE: number of training instances taken into account before the optimizer performs a weight update to the model
- OPTIMIZER: Stochastic Gradient Descent ('SGD') – only one training sample/iteration
  - Data load shuffled between training and testing set in files
  - Data preparation, e.g. X\_train is 60000 samples / rows of 28 x 28 pixel values that are reshaped in 60000 x 784 including type specification (i.e. float32)
  - Data normalization: divide by
     255 the max intensity value
     to obtain values in range [0,1]

print(X\_train.shape[0], 'train samples')
print(X test.shape[0], 'test samples')

# output number of samples

### MNIST Dataset – A Multi Output Perceptron Model

- The Sequential() Keras model is a linear pipeline (aka 'a stack') of various neural network layers including Activation functions of different types (e.g. softmax)
- Dense() represents a fully connected layer used in ANNs that means that each neuron in a layer is connected to all neurons located in the previous layer

```
function 'softmax' is a generalization of the sigmoid function – it squashes an n-dimensional vector of arbitrary real values into a n-dimenensional vector of real values in the range of 0 and 1 – here it aggregates 10 answers provided by the Dense layer with 10 neurons
```

The non-linear activation

```
# convert class label vectors using one hot encoding
Y_train = np_utils.to_categorical(y_train, NB_CLASSES)
Y_test = np_utils.to_categorical(y_test, NB_CLASSES)
```

```
# model Keras sequential
model = Sequential()

# add fully connected layer - input with output
model.add(Dense(NB_CLASSES, input_shape=(RESHAPED,)))

# add activation function layer to get class probabilities
model.add(Activation('softmax'))
```

```
\# printout a summary of the model to understand model complexity {\tt model.summary}()
```

```
# specify loss, optimizer and metric
model.compile(loss='categorical_crossentropy', optimizer=OPTIMIZER, metrics=['accuracy'])
```

history = model.fit(X\_train, Y\_train, batch\_size=BATCH\_SIZE, epochs=NB\_EPOCH, verbose=VERBOSE)

```
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
```

Loss function is a multi-class logarithmic loss: target is ti,j and prediction is pi,j

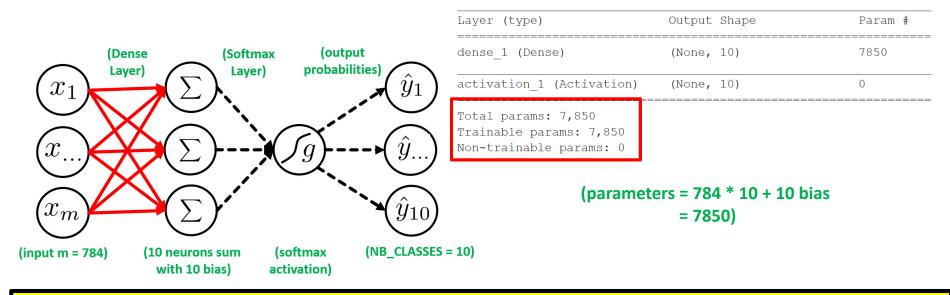
$$L_i = -\sum_j t_{i,j} \log(p_{i,j})$$

Train the model ('fit')

# model training

#### **MNIST Dataset & Model Summary & Parameters**

- Activation Function Softmax
  - Softmax enables probabilities for 10 classes



 Relevant for validation: Choosing a model with different layers is a model selection that directly also influences the number of parameters (e.g. add Dense layer from Keras means new weights)



# printout a summary of the model to understand model complexity
model.summary()

### **Model Evaluation – Testing Phase & Confusion Matrix**

- Model is fixed
  - Model is just used with the testset
  - Parameters are set.
- Evaluation of model performance
  - Counts of test records that are incorrectly predicted
  - Counts of test records that are correctly predicted
  - E.g. create confusion matrix for a two class problem

Counting per sample		Predicted Class	
		Class = 1	Class = 0
Actual Class	Class = 1	f <sub>11</sub>	f <sub>10</sub>
	Class = 0	f <sub>01</sub>	f <sub>00</sub>

(serves as a basis for further performance metrics usually used)

## **Model Evaluation – Testing Phase & Performance Metrics**

Counting per sample		Predicted Class		
		Class = 1	Class = 0	
Actual Class	Class = 1	f <sub>11</sub>	f <sub>10</sub>	(100% accuracy in learning often points to problems using machine learning methos in practice)
	Class = 0	f <sub>01</sub>	$f_{00}$	

Accuracy (usually in %)

$$egin{aligned} Accuracy = rac{number\ of\ correct\ predictions}{total\ number\ of\ predictions} \end{aligned}$$

Error rate

$$egin{aligned} Error \ rate = rac{number \ of \ wrong \ predictions}{total \ number \ of \ predictions} \end{aligned}$$

# **Exercises – Evaluate Multi Output Perceptron Model**



#### MNIST Dataset – A Multi Output Perceptron Model – Output

```
Epoch 7/20
60000/60000
     Epoch 8/20
60000/60000 [============] - 2s 26us/step - loss: 0.4271 - acc: 0.8866
Epoch 9/20
60000/60000 [===
       Epoch 10/20
60000/60000 [============] - 2s 26us/step - loss: 0.4052 - acc: 0.8910
Epoch 11/20
Epoch 12/20
Epoch 13/20
Epoch 14/20
Epoch 15/20
60000/60000 [===========] - 2s 25us/step - loss: 0.3727 - acc: 0.8982
Epoch 16/20
Epoch 17/20
60000/60000 [============] - 1s 25us/step - loss: 0.3641 - acc: 0.9001
Epoch 18/20
Epoch 19/20
Epoch 20/20
# model evaluation
score = model.evaluate(X test, Y test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
                              Multi Output Perceptron:
10000/10000 [============ ] - 0s 41us/step
Test score: 0.33423959468007086
                              ~91,01% (20 Epochs)
```

Test accuracy: 0.9101

#### **ANN – MNIST Dataset – Extend ANN Blueprint**

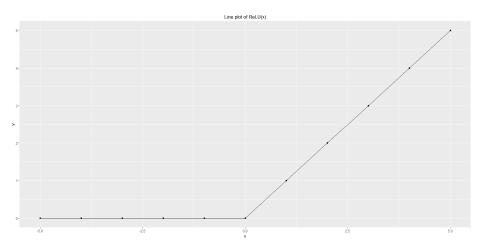
- ✓ Data Preprocessing done (i.e. data normalization, reshape, etc.)
- ✓ Initial ANN topology existing
- ✓ Initial setup of model works (create, compile, fit)

#### Extend the neural network topology

- Which layers are required?
- Think about input layer need to match the data what data we had?
- Maybe hidden layers?
- How many hidden layers?
- What activation function for which layer (e.g. maybe ReLU)?
- Think Dense layer Keras?
- Think about final Activation as Softmay (cf. Day One) → output probability

#### **Selected Activation Functions**

Rectified Linear Unit

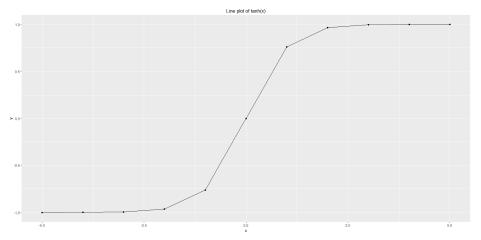


1 -5 0

2 -4 0 3 -3 0 4 -2 0

6 0 0

Tanh

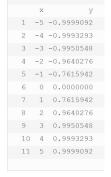


[5] big-data.tips,

'Relu Neural Network'



[6] big-data.tips, 'tanh'

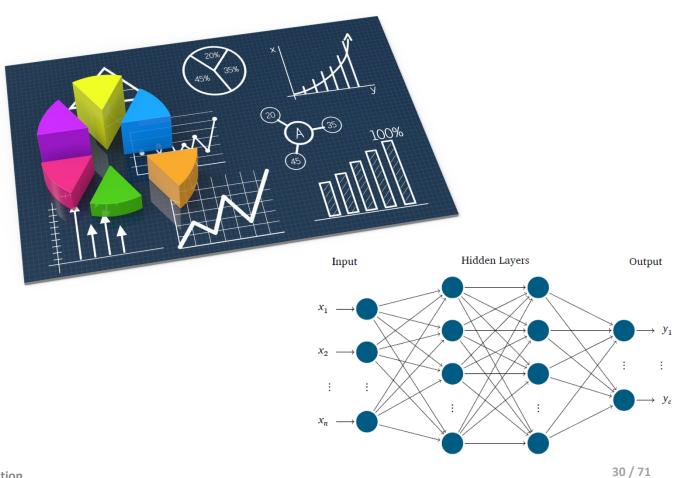




model.add(Dense(N\_HIDDEN))
model.add(Activation('tanh'))

# **Exercises – Add Two Hidden Layers**

Multi Output Perceptron: ~91,01% (20 Epochs)



### **ANN – MNIST Dataset – Add Two Hidden Layers**

- All parameter value remain the same as before
- We add N\_HIDDEN as parameter in order to set 128 neurons in one hidden layer – this number is a hyperparameter that is not directly defined and needs to be find with parameter search

```
# parameter setup
NB_EPOCH = 20
BATCH_SIZE = 128
NB_CLASSES = 10 # number of outputs = number of digits
OPTIMIZER = SGD() # optimization technique
VERBOSE = 1
N_HIDDEN = 128 # number of neurons in one hidden layer
```

```
# model Keras sequential
model = Sequential()

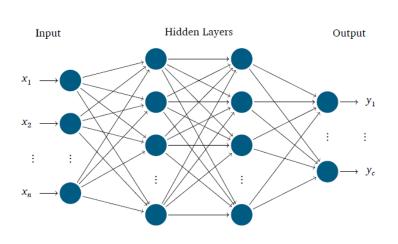
# modeling step
# 2 hidden layers each N_HIDDEN neurons
model.add(Dense(N_HIDDEN, input_shape=(RESHAPED,)))
model.add(Activation('relu'))
model.add(Dense(N_HIDDEN))
model.add(Activation('relu'))
model.add(Dense(NB_CLASSES))
```

# add activation function layer to get class probabilities
model.add(Activation('softmax'))

- The non-linear Activation function 'relu' represents a so-called Rectified Linear Unit (ReLU) that only recently became very popular because it generates good experimental results in ANNs and more recent deep learning models it just returns 0 for negative values and grows linearly for only positive values
- A hidden layer in an ANN can be represented by a fully connected Dense layer in Keras by just specifying the number of hidden neurons in the hidden layer

### **MNIST Dataset & Model Summary & Parameters**

- Added two Hidden Layers
  - Each hidden layers has 128 neurons



Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 128)	100480
activation_1 (Activation)	(None, 128)	0
dense_2 (Dense)	(None, 128)	16512
activation_2 (Activation)	(None, 128)	0
dense 3 (Dense)	(None, 10)	1290
_	, , ,	
activation_3 (Activation)	(None. 10)	0
=======================================	=======================================	:========
Total params: 118,282		

Total params: 118,282 Trainable params: 118,282 Non-trainable params: 0

Relevant for validation: Choosing a model with different layers is a model selection that directly also influences the number of parameters (e.g. add Dense layer from Keras means new weights)



# printout a summary of the model to understand model complexity
model.summary()

#### **ANN 2 Hidden – MNIST Dataset – Output**

```
Epoch 7/20
Epoch 8/20
Epoch 9/20
Epoch 10/20
Epoch 11/20
Epoch 12/20
Epoch 13/20
Epoch 14/20
Epoch 15/20
Epoch 16/20
Epoch 17/20
Epoch 18/20
Epoch 19/20
Epoch 20/20
```

```
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
```

- ✓ Multi Output Perceptron: ~91,01% (20 Epochs)
- ✓ ANN 2 Hidden Layers: ~95,14 % (20 Epochs)

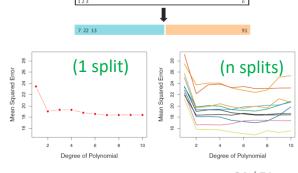
#### **Validation & Model Selection – Terminology**

- The 'Validation technique' should be used in all machine learning or data mining approaches
- Model assessment is the process of evaluating a models performance
- Model selection is the process of selecting the proper level of flexibility for a model

modified from [4] 'An Introduction to Statistical Learning'

- 'Training error'
  - Calculated when learning from data (i.e. dedicated training set)
- 'Test error'
  - Average error resulting from using the model with 'new/unseen data'
  - 'new/unseen data' was not used in training (i.e. dedicated test set)
  - In many practical situations, a dedicated test set is not really available
- 'Validation Set'
  - Split data into training & validation set
- 'Variance' & 'Variability'
  - Result in different random splits (right)

(split creates a two subsets of comparable size)



#### Validation Technique – Formalization & Goal

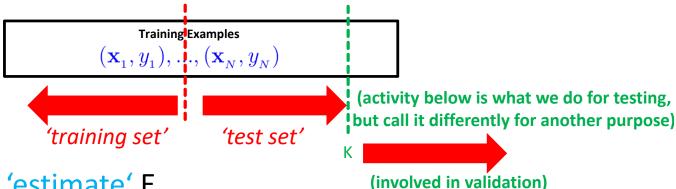
- Validation is a very important technique to estimate the out-of-sample performance of a model
- Main utility of regularization & validation is to control or avoid overfitting via model selection
  - Regularization & Validation
    - Approach: introduce a 'overfit penalty' that relates to model complexity
    - Problem: Not accurate values: 'better smooth functions'

(regularization uses a term that captures the overfit penalty)

$$E_{out}(h) = E_{in}(h) + \textbf{overfit penalty} \quad \text{(minimize both to be better proxy for E}_{out})$$
 
$$\uparrow \qquad \qquad \uparrow \qquad \qquad \text{(validation estimates} \qquad \qquad \text{(regularization estimates} \qquad \qquad \text{this quantity)}$$

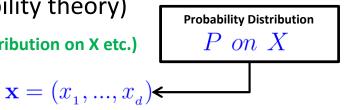
- Validation (measuring E<sub>out</sub> is not possible as this is an unknown quantity, another quantity is needed that is measurable that at least estimates it)
  - Goal 'estimate the out-of-sample error' (establish a quantity known as validation error)
  - Distinct activity from training and testing (testing also tries to estimate the E<sub>out</sub>)

# Validation Technique – Pick one point & Estimate E<sub>out</sub>



- Understanding 'estimate' E<sub>out</sub>
  - On one out-of-sample point  $(\mathbf{x}, y)$  the error is  $e(h(\mathbf{x}), y)$
  - E.g. use squared error:  $e(h(\mathbf{x}), f(\mathbf{x})) = (h(\mathbf{x}) f(\mathbf{x}))^2$  $e(h(\mathbf{x}), y) = (h(\mathbf{x}) - y)^2$
  - Use this quantity as estimate for E<sub>out</sub> (poor estimate)
  - Term 'expected value' to formalize (probability theory)

(Taking into account the theory of Lecture 1 with probability distribution on X etc.)



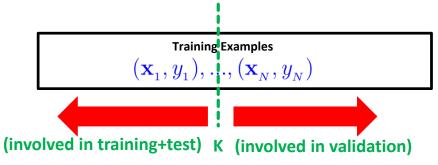
(aka 'random variable')

$$\mathbb{E}[e(h(\mathbf{x}),y)] = E_{out}(h)$$
 (aka the long-run average value of repetitions of the experiment)

(one point as unbiased estimate of E<sub>out</sub> that can have a high variance leads to bad generalization)

### **Validation Technique – Validation Set**

- Validation set consists of data that has been not used in training to estimate true out-of-sample
- Rule of thumb from practice is to take 20% (1/5) for validation of the learning model
- Solution for high variance in expected values  $\mathbb{E}[e(h(\mathbf{x}), y)] = E_{out}(h)$ 
  - Take a 'whole set' instead of just one point (x, y) for validation



(we need points not used in training to estimate the out-of-sample performance)

(we do the same approach with the testing set, but here different purpose)

Idea: K data points for validation

$$(\mathbf{x}_{\!\scriptscriptstyle 1},y_{\!\scriptscriptstyle 1}),...,(\mathbf{x}_{\!\scriptscriptstyle K},y_{\!\scriptscriptstyle K})$$
 (validation set)

$$E_{val}(h) = rac{1}{K} \sum_{k=1}^{K} e(h(\mathbf{x})_k, y_k)$$
 (validation error)

 Expected value to 'measure' the out-of-sample error (expected values averaged over set)

$$\mathbb{E}[E_{val}(h)] = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[e(h(\mathbf{x})_k, y_k)] = E_{out}$$

'Reliable estimate' if K is large

(on rarely used validation set, otherwise data gets contaminated)

(this gives a much better (lower) variance than on a single point given K is large)

### **Validation Technique – Model Selection Process**

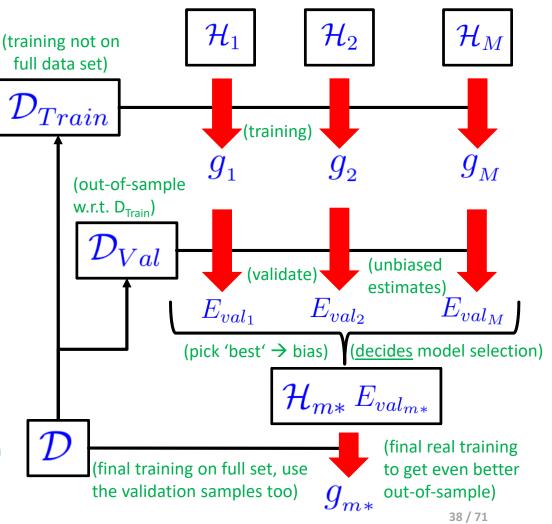
- Model selection is choosing (a) different types of models or (b) parameter values inside models
- Model selection takes advantage of the validation error in order to decide → 'pick the best'

$$\mathcal{H}=\{h\};\;g\in\mathcal{H}$$

(set of candidate formulas across models)

- Many different models
   Use validation error to
   perform select decisions
- Careful consideration:
  - Picked means decided' hypothesis has already bias (→ contamination)
  - Using  $\mathcal{D}_{Val}$  M times

(test this on unseen data good, but depends on availability in practice)



**Final Hypothesis** 

 $g_{m*} \approx f$ 

## **Exercises – Add Validation – Table & Groups**

- ✓ Multi Output Perceptron: ~91,01% (20 Epochs)
- ✓ ANN 2 Hidden Layers: ~95,14% (20 Epochs) overfit?



VAL_SPLIT	Accuracy Groups	
0.0	97,79%	
0.1	97,83%	
0.2	97,64%	
0.3	97,52 %	
0.4		
0.5	97,13 %	39 / 71

### **ANN 2 Hidden 1/5 Validation – MNIST Dataset**

- If there is enough data available one rule of thumb is to take 1/5 (0.2) 20% of the datasets for validation only
- Validation data is used to perform model selection (i.e. parameter / topology decisions)

```
# parameter setup
NB_EPOCH = 20
BATCH_SIZE = 128
NB_CLASSES = 10 # number of outputs = number of digits
OPTIMIZER = SGD() # optimization technique
VERBOSE = 1
N_HIDDEN = 128 # number of neurons in one hidden layer
VAL_SPLIT = 0.2 # 1/5 for validation rule of thumb
```

- The validation split parameter enables an easy validation approach during the model training (aka fit)
- Expectations should be a higher accuracy for unseen data since training data is less biased when using validation for model decisions (check statistical learning theory)
- VALIDATION\_SPLIT: Float between 0 and 1
- Fraction of the training data to be used as validation data
- The model fit process will set apart this fraction of the training data and will not train on it
- Intead it will evaluate the loss and any model metrics on the validation data at the end of each epoch.

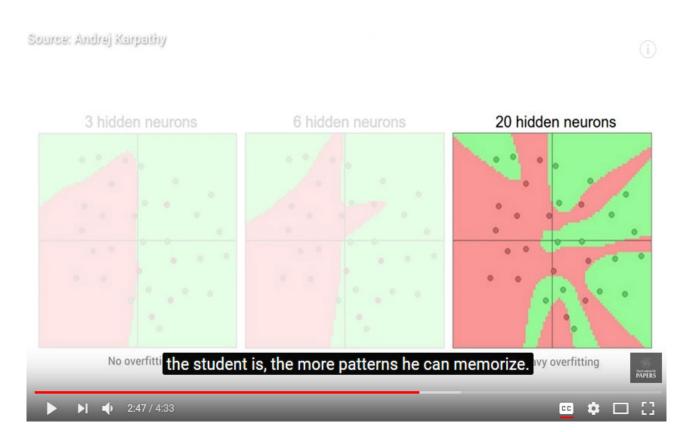
```
# model training
history = model.fit(X_train, Y_train, batch_size=BATCH_SIZE, epochs=NB_EPOCH, verbose=VERBOSE, validation_split = VAL_SPLIT)
```

Train on 48000 samples, validate on 12000 samples

## ANN 2 Hidden – 1/5 Validation – MNIST Dataset – Output

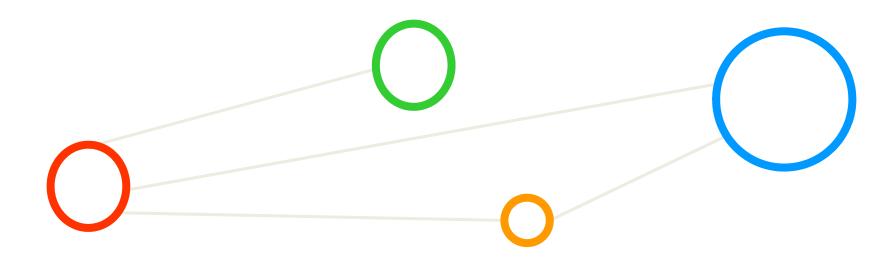
```
Epoch 7/20
48000/48000 [============] - 1s 18us/step - loss: 0.2967 - acc: 0.9148 - val_loss: 0.2759 - val_acc: 0.9212
48000/48000
        [===========] - 1s 18us/step - loss: 0.2825 - acc: 0.9187 - val_loss: 0.2636 - val_acc: 0.9248
Epoch 9/20
Epoch 10/20
48000/48000 [==============] - 1s 17us/step - loss: 0.2593 - acc: 0.9259 - val_loss: 0.2461 - val_acc: 0.9311
Epoch 11/20
48000/48000 [=============] - 1s 18us/step - loss: 0.2494 - acc: 0.9283 - val_loss: 0.2367 - val_acc: 0.9388
Epoch 12/20
Epoch 13/20
48000/48000 [============= ] - 1s 18us/step - loss: 0.2319 - acc: 0.9334 - val loss: 0.2228 - val acc: 0.9392
Epoch 14/20
48000/48000 [============] - 1s 18us/step - loss: 0.2242 - acc: 0.9358 - val_loss: 0.2172 - val_acc: 0.9397
Epoch 15/20
48000/48000 [=============] - 1s 17us/step - loss: 0.2172 - acc: 0.9381 - val_loss: 0.2105 - val_acc: 0.9418
Epoch 16/20
Epoch 17/20
48000/48000 [============] - 1s 18us/step - loss: 0.2040 - acc: 0.9417 - val_loss: 0.2007 - val_acc: 0.9447
Epoch 18/20
48000/48000 [============] - 1s 18us/step - loss: 0.1982 - acc: 0.9432 - val_loss: 0.1949 - val_acc: 0.9473
Epoch 19/20
Epoch 20/20
48000/48000 [============] - 1s 17us/step - loss: 0.1876 - acc: 0.9464 - val_loss: 0.1866 - val_acc: 0.9499
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
10000/10000 [=========== ] - 0s 21us/step
Test score: 0.18584023508876563
Test accuracy: 0.9462
```

## [Video] Overfitting in Deep Neural Networks



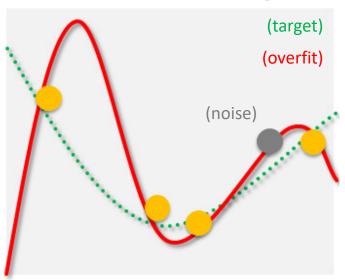
[4] Overfitting and Regularization For Deep Learning, YouTube

# Regularization



### **Machine Learning Challenges – Problem of Overfitting**

- Overfitting refers to fit the data too well more than is warranted thus may misguide the learning
- Overfitting is not just 'bad generalization' e.g. the VC dimension covers noiseless & noise targets
- Theory of Regularization are approaches against overfitting and prevent it using different methods
  - Key problem: noise in the target function leads to overfitting
    - Effect: 'noisy target function' and its noise misguides the fit in learning
    - There is always 'some noise' in the data
    - Consequence: poor target function ('distribution') approximation
  - Example: Target functions is second order polynomial (i.e. parabola)
    - Using a higher-order polynomial fit
    - lacksquare Perfect fit: low  $E_{in}(g)$  , but large  $E_{out}(g)$



(but simple polynomial works good enough)
('over': here meant as 4th order,

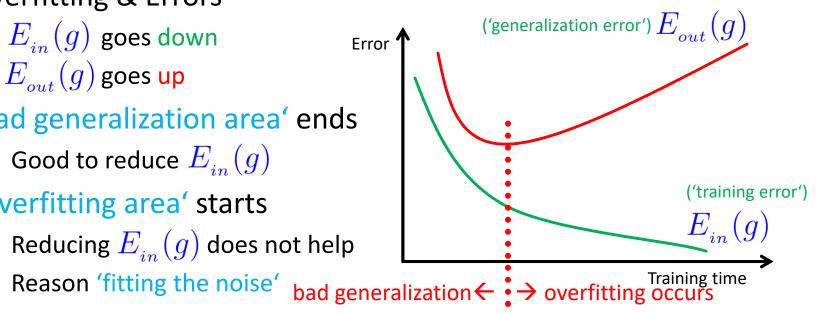
a 3<sup>rd</sup> order would be better, 2<sup>nd</sup> best)

## **Problem of Overfitting – Clarifying Terms**

- A good model must have low training error (E<sub>in</sub>) and low generalization error (E<sub>out</sub>)
- Model overfitting is if a model fits the data too well (Ein) with a poorer generalization error (Eout) than another model with a higher training error (E<sub>in</sub>)

[1] Introduction to Data Mining

- Overfitting & Errors
  - $\blacksquare E_{in}(g)$  goes down
  - $\blacksquare E_{out}(g)$  goes up
- 'Bad generalization area' ends
  - Good to reduce  $E_{in}(g)$
- 'Overfitting area' starts
  - Reducing  $E_{in}(g)$  does not help

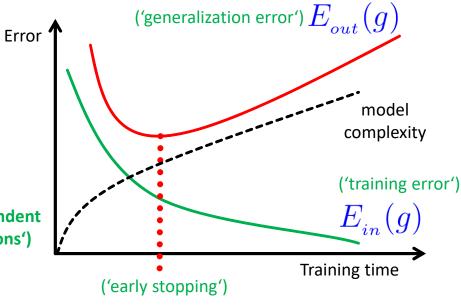


The two general approaches to prevent overfitting are (1) regularization and (2) validation

### **Problem of Overfitting – Model Relationships**

- Review 'overfitting situations'
  - When comparing 'various models' and related to 'model complexity'
  - Different models are used, e.g. 2<sup>nd</sup> and 4<sup>th</sup> order polynomial
  - Same model is used with e.g. two different instances (e.g. two neural networks but with different parameters)
- Intuitive solution
  - Detect when it happens
  - 'Early stopping regularization term' to stop the training
  - Early stopping method (later)

('model complexity measure: the VC analysis was independent of a specific target function – bound for all target functions')



'Early stopping' approach is part of the theory of regularization, but based on validation methods

### **Problem of Overfitting – ANN Model Example**

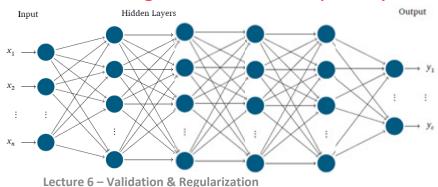
Error

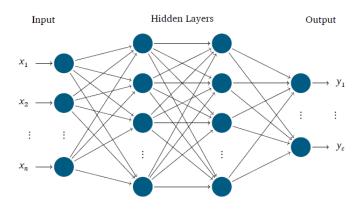
#### Two Hidden Layers

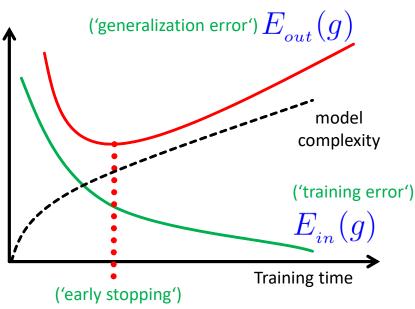
- Good accuracy and works well
- Model complexity seem to match the application & data

#### Four Hidden Layers

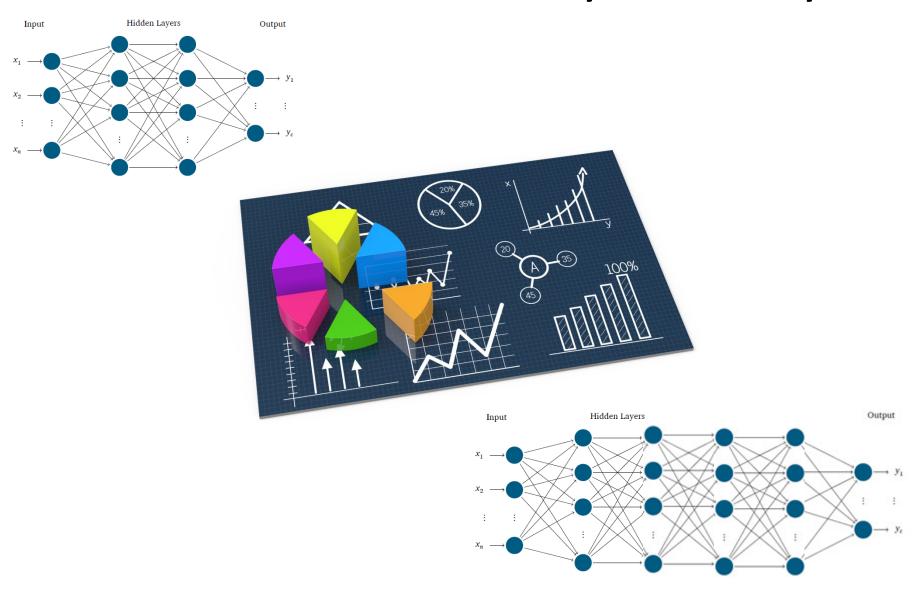
- Accuracy goes down
- $E_{in}(g)$  goes down
- $E_{out}(g)$  goes up
- Significantly more weights to train
- Higher model complexity





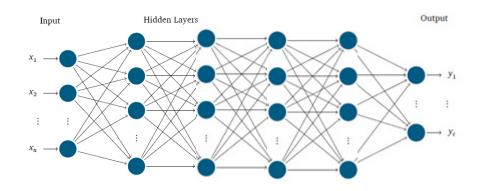


# **Exercises - Add more Hidden Layers - Accuracy?**



### **MNIST Dataset & Model Summary & Parameters**

- Four Hidden Layers
  - Each hidden layers has 128 neurons



Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 128)	100480
activation_1 (Activation)	(None, 128)	0
dense_2 (Dense)	(None, 128)	16512
activation_2 (Activation)	(None, 128)	0
dense_3 (Dense)	(None, 128)	16512
activation_3 (Activation)	(None, 128)	0
dense_4 (Dense)	(None, 128)	16512
activation_4 (Activation)	(None, 128)	0
dense_5 (Dense)	(None, 10)	1290
activation_5 (Activation)	(None, 10)	0 =======
Total params: 151,306		

Total params: 151,306 Trainable params: 151,306 Non-trainable params: 0



# printout a summary of the model to understand model complexity
model.summary()

#### Exercises - Add more Hidden Layers - 4 Hidden Layers

```
Epoch 7/20
Epoch 8/20
Epoch 9/20
Epoch 10/20
Epoch 11/20
Epoch 12/20
Epoch 13/20
Epoch 14/20
Epoch 15/20
Epoch 16/20
Epoch 17/20
Epoch 18/20
Epoch 19/20
Epoch 20/20
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
           Training accuracy should still be
print('Test accuracy:', score[1])
```

Test accuracy: 0.9571

Training accuracy should still be above the test accuracy – otherwise overfitting starts!

#### Exercises - Add more Hidden Layers - 6 Hidden Layers

```
Epoch 7/20
Epoch 8/20
Epoch 9/20
Epoch 10/20
48000/48000 [============== ] - 1s 28us/step - loss: 0.1963 - acc: 0.9415 - val_loss: 0.1860 - val_acc: 0.9461
Epoch 11/20
48000/48000 [============== ] - 1s 28us/step - loss: 0.1812 - acc: 0.9470 - val_loss: 0.1779 - val_acc: 0.9487
Epoch 12/20
Epoch 13/20
48000/48000 [============= ] - 1s 28us/step - loss: 0.1580 - acc: 0.9540 - val_loss: 0.1651 - val_acc: 0.9543
Epoch 14/20
Epoch 15/20
Epoch 16/20
Epoch 17/20
48000/48000 [============= ] - 1s 28us/step - loss: 0.1240 - acc: 0.9630 - val_loss: 0.1495 - val_acc: 0.9573
Epoch 18/20
48000/48000 [============== ] - 1s 27us/step - loss: 0.1170 - acc: 0.9663 - val_loss: 0.1447 - val_acc: 0.9563
Epoch 19/20
48000/48000 [============== ] - 1s 27us/step - loss: 0.1114 - acc: 0.9674 - val_loss: 0.1391 - val_acc: 0.9587
Epoch 20/20
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
                                  Training accuracy should still be
print('Test accuracy:', score[1])
                                  above the test accuracy – otherwise
```

overfitting starts!

51 / 71

Test accuracy: 0.9614

Test score: 0.13102742895036937

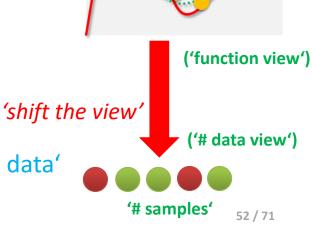
### **Problem of Overfitting – Noise Term Revisited**

- '(Noisy) Target function' is not a (deterministic) function
  - Getting with 'same x in' the 'same y out' is not always given in practice
  - Idea: Use a 'target distribution' instead of 'target function'
  - Fitting some noise in the data is the basic reason for overfitting and harms the learning process
  - Big datasets tend to have more noise in the data so the overfitting problem might occur even more intense

- Unknown Target Distribution  $P(y|\mathbf{x})$  target function  $f:X \to Y$  plus noise (ideal function)
  - (noise)

(target)

- 'Different types of <u>some</u> noise' in data
  - Key to understand overfitting & preventing it
  - 'Shift of view': refinement of noise term
  - Learning from data: 'matching properties of # data'

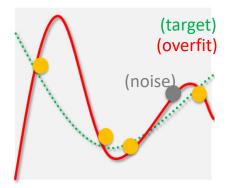


### **Problem of Overfitting – Stochastic Noise**

- Stoachastic noise is a part 'on top of' each learnable function
  - Noise in the data that can not be captured and thus not modelled by f
  - Random noise : aka 'non-deterministic noise'
  - Conventional understanding established early in this course
  - Finding a 'non-existing pattern in noise not feasible in learning'
- Practice Example
  - Random fluctuations and/or measurement errors in data
  - Fitting a pattern that not exists 'out-of-sample'
  - Puts learning progress 'off-track' and 'away from f'





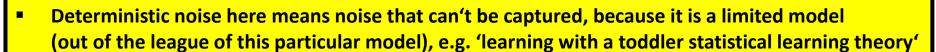


Stochastic noise here means noise that can't be captured, because it's just pure 'noise as is'
(nothing to look for) – aka no pattern in the data to understand or to learn from

### **Problem of Overfitting – Deterministic Noise**

- Part of target function f that H can not capture:  $f(\mathbf{x}) h^*(\mathbf{x})$ 
  - Hypothesis set H is limited so best h\* can not fully approximate f
  - h\* approximates f, but fails to pick certain parts of the target f
  - Behaves like noise', existing even if data is 'stochastic noiseless'
- Different 'type of noise' than stochastic noise
  - lacktriangledown Deterministic noise depends on  ${\mathcal H}$  (determines how much more can be captured by
  - E.g. same f, and more sophisticated  $\mathcal{H}$ : noise is smaller (stochastic noise remains the same, nothing can capture it)
  - Fixed for a given x, clearly measurable
     (stochastic noise may vary for values of x)

(learning deterministic noise is outside the ability to learn for a given  $h^*$ )



(f)

(h\*)

#### **Problem of Overfitting – Impacts on Learning**

- The higher the degree of the polynomial (cf. model complexity), the more degrees of freedom are existing and thus the more capacity exists to overfit the training data
- Understanding deterministic noise & target complexity
  - Increasing target complexity increases deterministic noise (at some level)
  - Increasing the number of data N decreases the deterministic noise
- Finite N case:  $\mathcal{H}$  tries to fit the noise
  - Fitting the noise straightforward (e.g. Perceptron Learning Algorithm)
  - Stochastic (in data) and deterministic (simple model) noise will be part of it
- Two 'solution methods' for avoiding overfitting
  - Regularization: 'Putting the brakes in learning', e.g. early stopping (more theoretical, hence 'theory of regularization')
  - Validation: 'Checking the bottom line', e.g. other hints for out-of-sample (more practical, methods on data that provides 'hints')

### **High-level Tools – Keras – Regularization Techniques**

- Keras is a high-level deep learning library implemented in Python that works on top of existing other rather low-level deep learning frameworks like Tensorflow, CNTK, or Theano
- The key idea behind the Keras tool is to enable faster experimentation with deep networks
- Created deep learning models run seamlessly on CPU and GPU via low-level frameworks

 Dropout is randomly setting a fraction of input units to 0 at each update during training time, which helps prevent overfitting (using parameter rate)

```
from keras import regularizers
model.add(Dense(64, input_dim=64,
kernel_regularizer=regularizers.12(0.01),
activity_regularizer=regularizers.11(0.01)))
```

 L2 regularizers allow to apply penalties on layer parameter or layer activity during optimization itself – therefore the penalties are incorporated in the loss function during optimization



[2] Keras Python Deep Learning Library

### **Exercises – Underfitting & Add Dropout Regularizer**

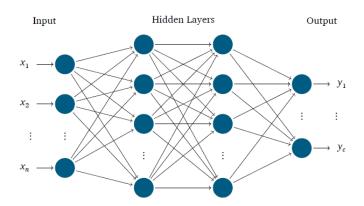
- Run with 20 Epochs first (not trained enough); then 200 Epochs
  - Training accuracy should be above the test accuracy otherwise 'underfitting'



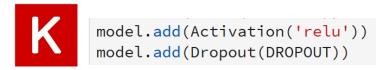
VAL_SPLIT	Dropout	Accuracy Groups
0.0	0.10	98,0%
0.1	0.20	97,8%
0.2	0.25	97,78%
0.3	0.30	97,47 %
0.4	0.40	97,23%

### **ANN – MNIST Dataset – Add Weight Dropout Regularizer**

```
# parameter setup
NB_EPOCH = 20
BATCH_SIZE = 128
NB CLASSES = 10 # number of outputs = number of digits
OPTIMIZER = SGD() # optimization technique
VERBOSE = 1
N_HIDDEN = 128 # number of neurons in one hidden layer
VAL_SPLIT = 0.2 # 1/5 for validation rule of thumb
DROPOUT = 0.3 # regularization
# modeling step
# 2 hidden layers each N HIDDEN neurons
model.add(Dense(N_HIDDEN, input_shape=(RESHAPED
model.add(Activation('relu'))
model.add(Dropout(DROPOUT))
model.add(Dense(N HIDDEN))
model.add(Activation('relu')
model.add(Dropout(DROPOUT))
model.add(Dense(NB CLASSES))
```

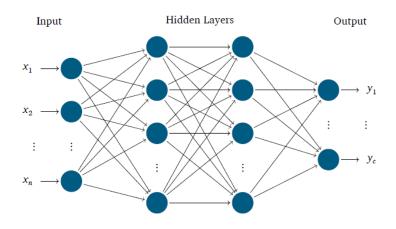


- A Dropout() regularizer randomly drops with ist dropout probability some of the values propagated inside the Dense network hidden layers improving accuracy again
- Our standard model is already modified in the python script but needs to set the DROPOUT rate
- A Dropout() regularizer randomly drops with ist dropout probability some of the values propagated inside the Dense network hidden layers improving accuracy again



### **MNIST Dataset & Model Summary & Parameters**

- Only two Hidden Layers but with Dropout
  - Each hidden layers has 128 neurons



Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 128)	100480
activation_1 (Activation)	(None, 128)	0
dropout_1 (Dropout)	(None, 128)	0
dense_2 (Dense)	(None, 128)	16512
activation_2 (Activation)	(None, 128)	0
dropout_2 (Dropout)	(None, 128)	0
dense_3 (Dense)	(None, 10)	1290
activation_3 (Activation)	(None, 10)	0
Total params: 118,282 Trainable params: 118,282 Non-trainable params: 0		



# printout a summary of the model to understand model complexity
model.summary()

### ANN – MNIST – DROPOUT (20 Epochs)

```
Epoch 7/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.4616 - acc: 0.8628 - val loss: 0.3048 - val acc: 0.9127
Epoch 8/20
Epoch 9/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.4181 - acc: 0.8762 - val_loss: 0.2776 - val_acc: 0.9198
Epoch 10/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.3990 - acc: 0.8838 - val_loss: 0.2657 - val_acc: 0.9234
Epoch 11/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.3819 - acc: 0.8876 - val loss: 0.2551 - val acc: 0.9258
Epoch 12/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.3688 - acc: 0.8920 - val_loss: 0.2465 - val_acc: 0.9283
Epoch 13/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.3571 - acc: 0.8943 - val loss: 0.2388 - val acc: 0.9299
Epoch 14/20
Epoch 15/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.3359 - acc: 0.9015 - val_loss: 0.2261 - val_acc: 0.9339
Epoch 16/20
Epoch 17/20
48000/48000 [=============== ] - 1s 22us/step - loss: 0.3142 - acc: 0.9085 - val loss: 0.2122 - val acc: 0.9375
Epoch 18/20
48000/48000 [=============== ] - 1s 21us/step - loss: 0.3103 - acc: 0.9095 - val_loss: 0.2076 - val_acc: 0.9390
Epoch 19/20
48000/48000 [================= ] - 1s 21us/step - loss: 0.3019 - acc: 0.9118 - val_loss: 0.2018 - val_acc: 0.9409
Epoch 20/20
48000/48000 [=============== ] - 1s 21us/step - loss: 0.2931 - acc: 0.9132 - val loss: 0.1974 - val acc: 0.9419
```

```
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
```

10000/10000 [=======] - 0s 29us/step

Test score: 0.19944561417847873 Test accuracy: 0.9404 Regularization effect not yet because too little training time (i.e. other regularlization ,early stopping' here)

### ANN - MNIST - DROPOUT (200 Epochs)

```
Epoch 187/200
Epoch 188/200
48000/48000 [=================== ] - 1s 21us/step - loss: 0.0795 - acc: 0.9753 - val_loss: 0.0799 - val_acc: 0.9765
Epoch 189/200
48000/48000 [========================== ] - 1s 21us/step - loss: 0.0774 - acc: 0.9763 - val_loss: 0.0802 - val_acc: 0.9763
Epoch 190/200
48000/48000 [========================== ] - 1s 21us/step - loss: 0.0773 - acc: 0.9770 - val_loss: 0.0799 - val_acc: 0.9758
Epoch 191/200
48000/48000 [========================== ] - 1s 21us/step - loss: 0.0746 - acc: 0.9771 - val_loss: 0.0804 - val_acc: 0.9762
Epoch 192/200
48000/48000 [========================== ] - 1s 21us/step - loss: 0.0761 - acc: 0.9771 - val loss: 0.0805 - val acc: 0.9762
Epoch 193/200
48000/48000 [========================= ] - 1s 21us/step - loss: 0.0750 - acc: 0.9772 - val_loss: 0.0800 - val_acc: 0.9763
Epoch 194/200
Epoch 195/200
48000/48000 [========================= ] - 1s 21us/step - loss: 0.0748 - acc: 0.9768 - val_loss: 0.0799 - val_acc: 0.9767
Epoch 196/200
Epoch 197/200
48000/48000 [========================= ] - 1s 21us/step - loss: 0.0740 - acc: 0.9771 - val_loss: 0.0799 - val_acc: 0.9767
Epoch 198/200
Epoch 199/200
48000/48000 [========================== ] - 1s 21us/step - loss: 0.0759 - acc: 0.9769 - val_loss: 0.0794 - val_acc: 0.9767
Epoch 200/200
48000/48000 [========================= ] - 1s 21us/step - loss: 0.0730 - acc: 0.9778 - val_loss: 0.0794 - val_acc: 0.9771
```

Test score: 0.07506137332450598

Test accuracy: 0.9775

- Regularization effect visible by long training time using dropouts and achieving highest accuracy
- Note: Convolutional Neural Networks: 99,1 %

## ANN - MNIST - w/o DROPOUT (200 Epochs)

```
Epoch 187/200
Epoch 188/200
Epoch 189/200
Epoch 190/200
Epoch 191/200
Epoch 192/200
Epoch 193/200
Epoch 194/200
Epoch 195/200
Epoch 196/200
Epoch 197/200
Epoch 198/200
Epoch 199/200
Epoch 200/200
```

```
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
```

10000/10000 [=========== ] - 0s 27us/step

Test score: 0.07599342362476745

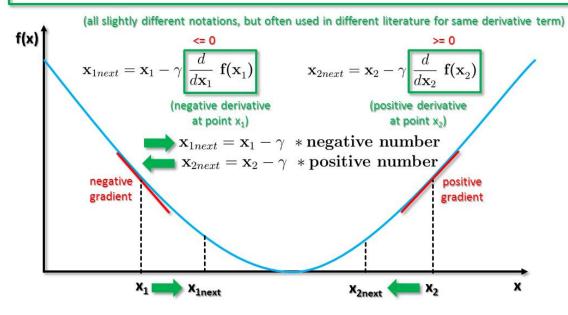
Test accuracy: 0.9764

 No regularization method by long training time for comparison – slight drop in accuracy since simple dataset

#### MNIST Dataset & SGD Method – Revisited

- Gradient Descent (GD) uses all the training samples available for a step within a iteration
- Stochastic Gradient Descent (SGD) converges faster: only one training samples used per iteration

$$\mathbf{b} = \mathbf{a} - \gamma \ \nabla \ \mathbf{f(a)} \quad \mathbf{b} = \mathbf{a} - \gamma \ \frac{\partial}{\partial \mathbf{a}} \ \mathbf{f(a)} \quad \mathbf{b} = \mathbf{a} - \gamma \ \frac{d}{d\mathbf{a}} \ \mathbf{f(a)}$$





[4] Big Data Tips, Gradient Descent

### MNIST Dataset & RMSprop & Adam Optimization Methods

- RMSProp is an advanced optimization technique that in many cases enable earlier convergence
- Adam includes a concept of momentum (i.e. veloctity) in addition to the acceleration of SGD

```
Epoch 7/20
Epoch 8/20
48000/48000 [=========================== ] - 1s 25us/step - loss: 0.1051 - acc: 0.9690 - val_loss: 0.0984 - val_acc: 0.9735
Epoch 9/20
48000/48000 [==================== ] - 1s 25us/step - loss: 0.0970 - acc: 0.9706 - val_loss: 0.0996 - val_acc: 0.9747
Epoch 10/20
Epoch 11/20
48000/48000 [=========================== ] - 1s 25us/step - loss: 0.0880 - acc: 0.9734 - val_loss: 0.0945 - val_acc: 0.9763
Epoch 12/20
48000/48000 [===============] - 1s 25us/step - loss: 0.0873 - acc: 0.9745 - val_loss: 0.0957 - val_acc: 0.9761
Epoch 13/20
Epoch 14/20
48000/48000 [==============] - 1s 25us/step - loss: 0.0804 - acc: 0.9763 - val_loss: 0.1002 - val_acc: 0.9767
Epoch 15/20
48000/48000 [============== ] - 1s 25us/step - loss: 0.0788 - acc: 0.9771 - val loss: 0.0991 - val acc: 0.9772
Epoch 16/20
Epoch 17/20
48000/48000 [===============] - 1s 25us/step - loss: 0.0758 - acc: 0.9776 - val loss: 0.1033 - val acc: 0.9753
Epoch 18/20
48000/48000 [========================== ] - 1s 26us/step - loss: 0.0755 - acc: 0.9781 - val_loss: 0.0996 - val_acc: 0.9773
Epoch 20/20
# model evaluation
score = model.evaluate(X_test, Y_test, verbose=VERBOSE)
print("Test score:", score[0])
print('Test accuracy:', score[1])
10000/10000 [============ ] - 0s 33us/step
Test score: 0.09596708530617616
Test accuracy: 0.9779
```



from keras.optimizers import RMSprop

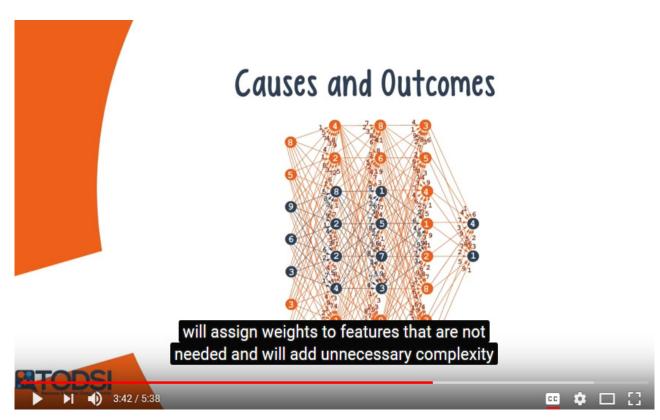
OPTIMIZER = RMSprop() # optimization technique

## **Exercises – Underfitting & Change to Adam**

Run with 20 Epochs With Adam Optimizer

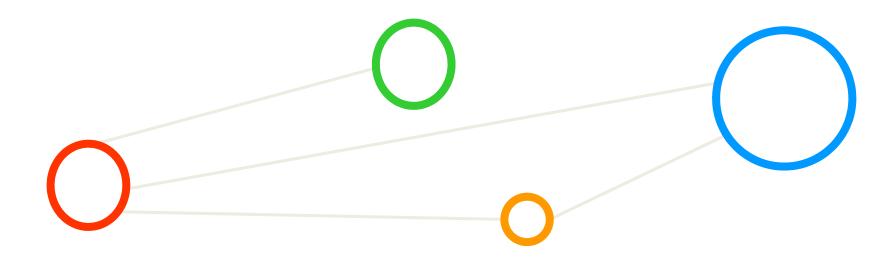


### [Video] Overfitting in Deep Neural Networks



[3] How good is your fit?, YouTube

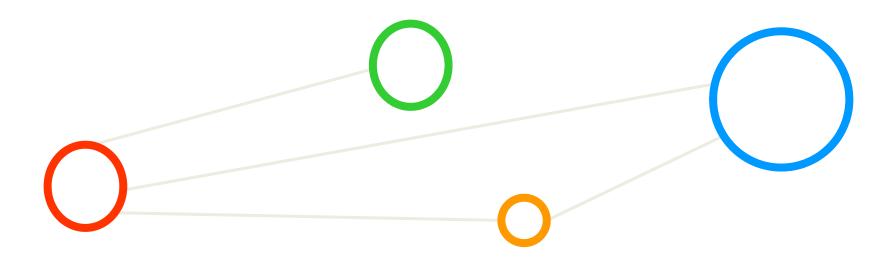
# **Appendix A – SSH Commands JURECA**



### **Appendix A – SSH Commands JURECA**

- salloc --gres=gpu:4 --partition=gpus --nodes=1 -account=training1904 --time=00:30:00 -reservation=prace\_ml\_gpus\_tue
- module --force purge; module use /usr/local/software/jureca/OtherStages module load Stages/Devel-2018b GCCcore/.7.3.0 module load TensorFlow/1.12.0-GPU-Python-3.6.6 module load Keras/2.2.4-GPU-Python-3.6.6
- srun python PYTHONSCRIPTNAME

# **Lecture Bibliography**



#### **Lecture Bibliography**

[1] An Introduction to Statistical Learning with Applications in R,

Online: http://www-bcf.usc.edu/~gareth/ISL/index.html

[2] Keras Python Deep Learning Library,

Online: <a href="https://keras.io/">https://keras.io/</a>

[3] YouTube Video, 'How good is your fit? - Ep. 21 (Deep Learning SIMPLIFIED)', Online: https://www.youtube.com/watch?v=cJA5IHIIL30

• [4] YouTube Video, 'Overfitting and Regularization For Deep Learning | Two Minute Papers #56',

Online: <a href="https://www.youtube.com/watch?v=6aF9sJrzxaM">https://www.youtube.com/watch?v=6aF9sJrzxaM</a>

[5] www.big-data.tips, 'Relu Neural Network'
 Online: http://www.big-data.tips/relu-neural-network

• [6] www.big-data.tips, 'tanh',

Online: <a href="http://www.big-data.tips/tanh">http://www.big-data.tips/tanh</a>

## Slides Available at http://www.morrisriedel.de/teaching

