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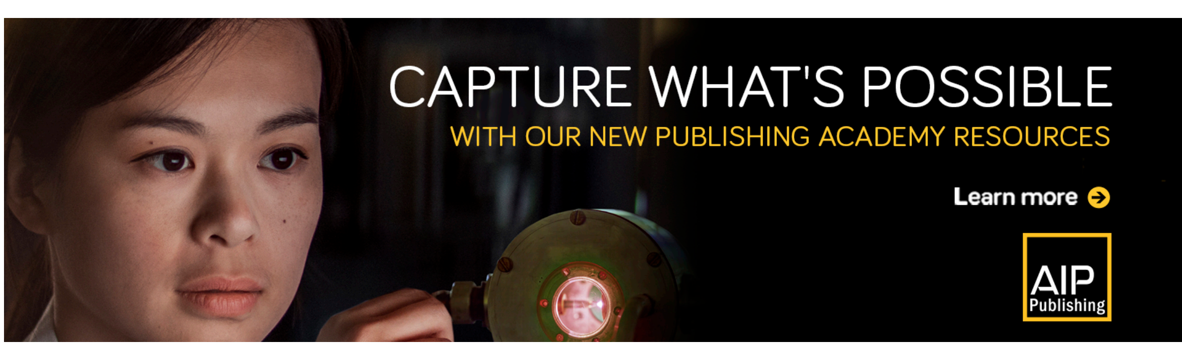
Gerd (Gerhard) H. Wolf



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Dynamic stabilization of the Rayleigh-Taylor instability of miscible liquids and the related "frozen waves"

Gerd (Gerhard) H. Wolf
Institute for Energy and Climate Research (IEK-4), Forschungszentrum Jülich, 52425 Jülich, Germany

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Superimposed miscible liquids, the heavier one on top, when subjected to vibrations vertical to their interface (dynamic stabilization), can only be maintained for a certain period. A mechanism is presented explaining the resulting process of degradation and "anomalous diffusion" through that interface. Superimposed liquids, the lighter one on top, exposed to horizontal vibrations, develop a saw-tooth-like pattern called "frozen waves." These are subject to conditions similar to those of dynamic stabilization and, if miscible, thus can also only be maintained for a certain period. A further analysis of these processes would be desirable, also in view of their relation to analogue phenomena. *Published by AIP Publishing.* https://doi.org/10.1063/1.5017846

Although the effect of vibrations on liquid interfaces had been already investigated in 1831 by Faraday[1] and later also by others (e.g., Ref. 2), it was only several decades ago that the dynamic stabilization of the Rayleigh-Taylor instability of superimposed liquids and their related phenomena like parametric resonances/Faraday instabilities and dynamic equilibria had been considered, demonstrated,[3–5] and extensively studied.[6–10]

While at that time and later on,[11] the issue of dynamic stabilization was the main focus of attention, more recently—partly also motivated in view of experiments under microgravity—dynamic equilibria ("frozen waves") excited by horizontal vibrations, parallel to a statically stable interface of superimposed liquids, gained increased interest.

Several publications (e.g., Refs. 3 and 12–16) deal with the phenomenon of a more or less saw-tooth-like pattern that develops when superimposed **immiscible** liquids of different density—the lighter one on top of the heavier one—become exposed to **horizontal** vibrations. In the more recent literature, they are mostly called "frozen waves." They have first been observed[3] in the context of experiments demonstrating the dynamic stabilization of the Rayleigh-Taylor instability of superimposed liquids (theoretically supported by Ref. 7) and the related dynamic equilibria. There[3] they were identified as the spatial sequence of dynamic equilibria with an alternating angle of inclination, growing out of the now unstable horizontal equilibrium. The inclination of these dynamic equilibria grew as the amplitude of the vibrations was increased, and it could become so steep that instead of looking like saw teeth, the two liquids developed into a sequence of separated nearly vertical layers. Although, concerning the quantitative relations of these frozen waves (mainly their angle of inclination), rather simple and crude assumptions had been made in Ref. 3, these relations could be largely confirmed later by more refined studies and treatments.[16]

To understand this behavior, the underlying physics of dynamic stabilization had been transferred to the corresponding dynamic equilibria,[3] in analogy to the Kapitsa pendulum[17,18] when oscillating horizontally. This implies that the conditions for dynamic stabilization are now transferred to those of the dynamic equilibria[3,16] of the "frozen waves" when accounting for the angle of the different forces. In other words, while in the case of dynamic stabilization of the Rayleigh-Taylor instability, the heavier liquid is on top of the lighter one, i.e., the liquids are **separated vertically**, in the case of dynamic equilibria, specifically in the form of "frozen waves," the two liquids are **separated mainly horizontally** but with a certain (alternating) angle of inclination which can approach a vertical position. The physical conditions and processes to maintain the boundary layers are similar when accounting for the angle of the acting forces.

However, while for the Kapitsa pendulum only one mode (eigenfrequency) needs to be stabilized, in superimposed liquids and in "frozen waves," there exists a whole continuum of modes, out of which, for given vibration parameters, only a certain bandwidth of possible modes can be dynamically stabilized. Approaching the upper limit, the vibrations excite parametric resonances (e.g., Refs. 3, 9, and 19) or "Faraday instabilities." This can even be seen from Fig. 1(b) of Ref. 3, where parametric resonances also occurred. Therefore, for the dynamic equilibria of frozen waves in **immiscible** liquids, both a sufficient viscosity and surface tension[9] on the one hand (to suppress parametric instabilities) and not too large vessel dimensions on the other hand (to avoid too slow-growing modes) are needed to contain the spectrum of possible modes within the given bandwidth.

It is worthwhile to note that analogies of this method concern also other fields of physics, like the "strong focusing" of accelerators[20] or the Paul-trap.[18,21]

More recently, Gaponenko *et al.*[22] and Shevtsova *et al.*[23] dealt with superimposed **miscible** liquids and with the patterns of the now diffuse boundary layer developing under periodic excitations—including experiments under reduced gravity. Also here, the interfaces, when exposed to vibrations, develop saw-tooth-like patterns or even some kind of rectangular structures, which usually end up with various distortions and a turbulent mixture or even the break-down of the interfaces. These processes were analyzed with extensive mathematical and numerical instrumentation. In their emergence, the excitation and growth of the frozen waves were mainly attributed to Kelvin-Helmholtz instabilities and, in their final state, mainly to Faraday instabilities. These Faraday instabilities were attributed to Raleigh-Taylor instabilities being periodically excited by the instantaneous acceleration of each vibration.

In fact, this is the same interpretation as in the very first publication[3] on dynamic stabilization and dynamic equilibria and the very reason why there Faraday instabilities were referred to as "parametric resonances," limiting or exceeding the possible range of parameters at the upper end of the bandwidth under which complete dynamic stabilization and dynamic equilibria are possible. Starting from these early results,[3] an extensive study on parametric resonances had already been made in 1974.[10]

Thus, while in Refs. 22 and 23 the Faraday instabilities were correctly attributed to an induced Raleigh-Taylor instability resulting from oscillations with a major component perpendicular to the boundary layer in analogy to the dynamic stabilization of superimposed liquids, however, what may not have been recognized and therefore appears to be missing in their interpretation is that, in the case of **miscible** liquids, the very Raleigh-Taylor instability as such (meaning those modes which are *not caused* be the vibrations themselves)—averaged over the vibration periods—cannot be completely stabilized by such vibrations. Consequently also no lasting corresponding dynamic equilibria can be maintained because, for diffuse boundary layers, there exist also low-frequency modes lying outside the lower end of the bandwidth which can be stabilized. May be their effect had been camouflaged by the faster growing parametric resonances/Faraday instabilities, which obviously had not been avoided (e.g., by choosing more suitable vibration parameters).

The problem of these too slowly growing modes was already addressed in the very first paper[3] on this subject, where the diffuse boundary layers of miscible liquids were also considered as a means to suppress parametric resonances/ Faraday instabilities without needing sufficient viscosity since modes with a wavelength shorter than the thickness of the layer do not grow any faster when further reducing their wavelength.[24] However, it was known already at that time (see the footnote of Ref. 3) that in a diffuse boundary layer also modes (of short wavelength perpendicular to the boundary layer) can develop that grow so slowly[24] that they fall outside the range of the bandwidth of modes that can be dynamically stabilized or brought into a dynamic equilibrium, although they may be suppressed for other "granular" reasons, such as in magnetized plasmas by Larmor-radius effects.[6]

It was this very fact of incomplete stabilization which, already at that time, had stimulated specific investigations also with miscible liquids.[5] There an aqueous solution of ZnJ (density 1.8 g/ccm) and pure water, separated by a diffuse boundary layer, was subjected to vibration experiments. Their main result was—as expected—that a degradation process occurred and indeed no lasting stabilized state could be maintained. However, compared with the turbulent breakdown time of the un-stabilized state, the time until a complete mixture has been obtained and could be prolonged by two to three orders of magnitude. Figure 1 shows a sequence of photographs (taken from the experiments of Ref. 5) covering—from left to right—a time span of 8 min.

The left-most picture (a) shows the stable state before turning the vessel upside down, and the right-most one (f) shows the process of the final turbulent mixture at the moment (after 8 min) when the vibrations were switched off. Picture (b) shows the initial state after turn-over, and pictures (c)–(e) show the state at equidistant intervals until said 8 min.

In the experiments described,[5] these degradation processes phenomenologically led to some kind of "anomalous diffusion" between the two components exceeding the speed of normal diffusion being several orders of magnitude but equally prolonging the inverted state of the two components by 2–3 orders of magnitude.

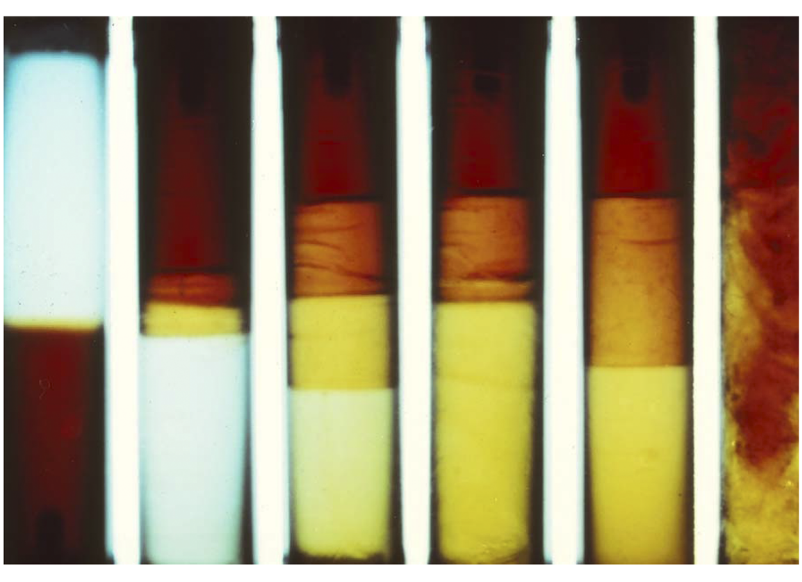

FIG. 1. Photograph on the left (a) showing a cylindrical glass vessel of inner diameter 0.78 cm, filled with an aqueous solution of ZnJ with a density of 1.8 g/ccm upon which pure water has been superimposed. This was exposed to vertical harmonic oscillations with a frequency of 100 Hz and a maximum instantaneous acceleration of 60 g. It was then turned upside down (b). The next three pictures [(c)–(e)] from the left to right show about equidistant snapshots (illuminated by a stroboscope coupled to the phase of the oscillation) over a period of 8 min, beginning immediately after the turn-over and ending after 8 min. The right-side picture (f) shows the turbulent state when the vibration had been stopped after said 8 min. [From the experiments of Ref. 5. Reproduced with permission from G. H. Wolf, "Dynamic stabilization of hydrodynamic interchange instabilities—A model for plasma physics," AIP Conf. Proc. **1**, 293 (1970). Copyright 1970 AIP Publishing LLC.]

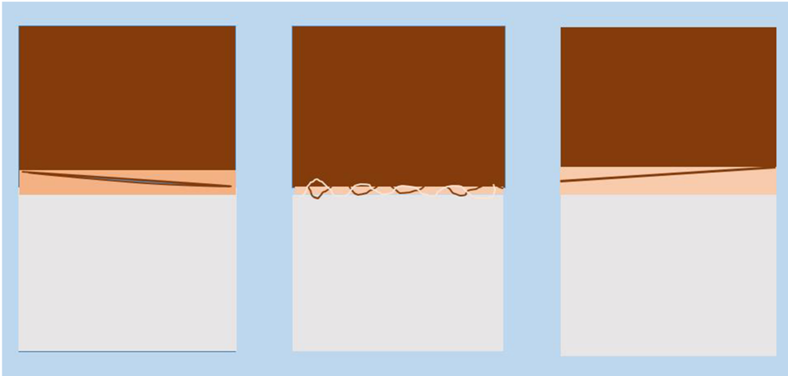

FIG. 2. A possible explanation of the process shown in Fig. 1. From left to right: falling leaves, narrowing the boundary layer; Faraday instabilities, broadening the boundary layer; and again development of falling leaves.

Two interesting further details need to be addressed.

1. As a result of or during the turnover, the boundary layer separating the two liquids initially broke up into three individual layers (fourth picture only two layers).
2. A possible process driving the above-mentioned "anomalous diffusion" could just be seen, which offers the following interpretation.

As shown in Fig. 2, from left to right, there may exist, to begin with (left), a boundary layer which is sufficiently broad to suppress or damp short wavelength parametric instabilities but is the optimal breeding ground for those of the unstabilized modes, which grow fastest under the given vibration conditions. These remnant modes—phenomenologically—defoliate the boundary layer [see, e.g., Figs. 1(c)–1(e)] by taking some part of it away, looking and *moving* like falling leaves.

As a result (middle), the remaining boundary layer became smaller and, thus, less fertile for further falling leaves. However, it therefore allowed now short wavelength parametric resonances/Faraday instabilities to develop, which, by causing local turbulence, broaden the boundary layer (see also Figs. 5 and 6 of Ref. 22) to such an extent that they undermine their very basis of existence but prepare again the breeding ground for the next (right) defoliation process. In other words, in broad boundary layers, slow-growing un-stabilized modes (defoliation) dominate and in thin boundary layers parametric resonances/Faraday instabilities dominate, each of them preparing the breeding ground for the other. This seems to be related to the broad group of phenomena like Edge Localized Modes (ELMs)[25,26] in tokamaks, predator-prey relations in ecology (e.g., Ref. 27), or catalytic reactions on surfaces.[28]

Of course, these mainly qualitative explanations and considerations open a range of interesting questions concerning the interplay of these competing unstable modes, their detailed quantitative description and parametric dependencies (e.g., why do the interfaces split, which, under given vibration parameters, is the dominating mode out of which the "falling leaves" develop, and why do they continue to fall undistorted?), and their role for the speed of the "anomalous diffusion." Regrettably, these questions were neither addressed nor, apparently, recognised in the publications so far known to the author. Therefore, it would be desirable, if these gaps could be closed using the powerful experimental and theoretical instrumentation now available to the teams working in this field.

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