

# Noise-induced Stop-and-Go Dynamics

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**Abstract** Stop-and-go waves are commonly observed in traffic and pedestrian flows. In traffic theory they are described by phase transitions of metastable models. The self-organization phenomenon occurs due to inertia mechanisms but requires fine tuning of the parameters. Here, a novel explanation for stop-and-go waves based on stochastic effects is presented for pedestrian dynamics. We show that the introduction of specific coloured noises in a stable microscopic model allows to describe realistic pedestrian stop-and-go behaviour without requirement of metastability and phase transition. We compare simulation results of the stochastic model to real pedestrian trajectories and discuss plausible values for the model's parameters.

## 1 Introduction

Stop-and-go waves in traffic flow is a fascinating collective phenomenon that attracted the attention of scientists for several decades [15, 19, 25] (see [7, 18] for reviews). Curiously, congested flows self-organise in waves of slow and fast traffic (stop-and-go) instead of streaming homogeneously. Stop-and-go dynamics are observed in road traffic, bicycle and pedestrian movements [39] in reality as well as during experiments, where the disturbance due to the infrastructure cannot explain their presence [28]. Besides its scientific interest, such self-organisation phenomena impact transportation networks in terms of safety, economy, and comfort.

Stop-and-go behaviours are often analysed with microscopic, mesoscopic (kinetic) and macroscopic models based on non-linear differential systems (see for in-

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stance [2, 14, 8]), but also with discrete models like cellular automata. The models based on systems of differential equations have homogeneous equilibrium solutions that can be unstable for certain values of the parameters. Periodic or quasi-periodic solutions in unstable cases describe non-homogeneous dynamics, with potentially stop-and-go waves for fine tuning of the parameters, while the stable cases describe homogeneous dynamics.

Phase transition and metastability in self-driven dynamical systems far from the equilibrium are commonly observed in physics, theoretical biology or social science [3, 35, 5, 4, 16]. In traffic, typical continuous models are inertial second order systems based on relaxation processes. Stop-and-go and matching to Korteweg–de Vries (KdV) and modified KdV soliton equations occur when the inertia of the vehicles exceed critical values [2, 24, 30]. Empirical evidence for phase transitions in traffic, like hysteresis or capacity drop, have been observed in real data as well as during experiments [19, 28]. Yet the number of phases and their characteristics remain actively debated [34].

Some studies describe pedestrian stop-and-go dynamics by means of, as traffic models, instability and phase transitions [26, 23, 20, 21]. However, to our knowledge, empirical evidence for phase transitions and metastability have not been observed for pedestrian flow. Pedestrian dynamics shows no pronounced inertia effect since human capacity nearly allows any speed variation at any time. Furthermore, pedestrian motion does not show mechanical delays. Nevertheless, stop-and-go behaviour is observed at congested density levels [27, 39].

In this work, we propose an novel explanation of stop-and-go phenomena in pedestrian flows as a consequence of stochastic effects. We first present statistical evidence for the existence of Brownian noise in pedestrian speed time-series. Then a microscopic model composed of a minimal deterministic part for the convection and a relaxation process for the noise is proposed and analysed. Simulation results show that the stochastic approach allows to describe realistic pedestrian stop-and-go dynamics without metastability and fine tuning of the parameters.

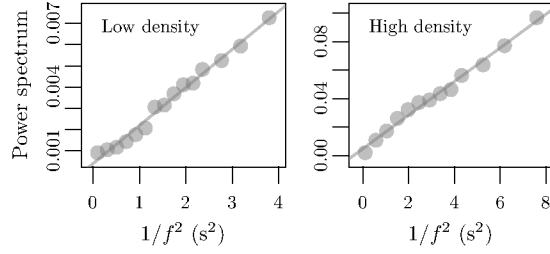
## 2 Definition of the stochastic model

Stochastic effects can have various roles in the dynamics of self-driven systems [11]. Generally speaking, the introduction of white noise in models tends to increase the disorder in the system [35] or to prevent self-organisation [12], while coloured noises can affect the dynamics and generated complex patterns [1, 6]. Coloured noise have been observed in human response [9, 38]. Pedestrian as well as driver behaviours result from complex human cognition. They are intrinsically stochastic in the sense that the deterministic modelling of the driving, i.e. the modelling of the human cognition composed of up to  $10^{11}$  neurons [37], is not possible. Furthermore the behaviour of a pedestrian, as well as a driver, may be influenced by multiple factors, e.g. experience, culture, environment, psychology, etc. Stochastic effects and notion of noise are the main emphasis of many pedestrian or road traffic modelling

approaches (see, e.g., white noises [13, 30], pink-noise [29], action-point [36], or again inaccuracies or risk-taking behavior [33, 10]).

Fig. 1 presents statistical evidence for the existence of a Brownian noise in time series of pedestrian speeds. Such a noise has power spectral density (PSD) proportional to the inverse of the square noise frequency  $1/f^2$ . A characteristic linear tendency is observed independent of the density (see [31] for details on the data). Such a noise with exponentially decreasing time-correlation function can be described by using the Ornstein-Uhlenbeck process (see e.g. [22]).

**Fig. 1** Periodogram power spectrum estimate for the speed time-series of pedestrians at low and high density levels. The power spectrum is roughly proportional to the inverse of square frequency  $1/f^2$ . This is a typical characteristic of a Brownian noise.



We denote in the following  $x_k(t)$  the curvilinear position of the pedestrian  $k$  at time  $t$ . The pedestrian  $k+1$  is the predecessor of  $k$ . The model is the Langevin equation

$$\begin{cases} dx_k(t) = V(x_{k+1}(t) - x_k(t)) dt + \varepsilon_k(t) dt, \\ d\varepsilon_k(t) = -\frac{1}{\beta} \varepsilon_k(t) dt + \alpha dW_k(t). \end{cases} \quad (1)$$

Here  $V : s \mapsto V(s)$  is a differentiable and non-decreasing *optimal velocity* (OV) function for the convection [2], while  $\varepsilon_k(t)$  is a noise described by the Ornstein-Uhlenbeck stochastic process. In the following, an affine function  $V(s) = \frac{1}{T}(s - \ell)$  is used with  $T$  the time gap between the agents and  $\ell$  their size. The quantities  $(\alpha, \beta)$  are positive parameters related to the noise.  $\alpha$  is the volatility while  $\beta$  is the noise relaxation time.  $W_k(t)$  is a Wiener process. Note that alternatively, the model can be defined as the special stochastic form of the Full Velocity Difference model [17]  $\ddot{x}_k = [V(x_{k+1} - x_k) - \dot{x}_k]/\beta + V'(x_{k+1} - x_k)(\dot{x}_{k+1} - \dot{x}_k) + \alpha \xi_k$ , where  $\xi_k$  is a white noise.

Considering a line of  $n$  agents with periodic boundary conditions, the system is

$$\begin{cases} d\eta(t) = (A\eta(t) + a)dt + b dw(t) \\ \text{with } \eta(t) = {}^t(x_1(t), \varepsilon_1(t), \dots, x_n(t), \varepsilon_n(t)) \in \mathbb{R}^{2n} \\ A = \begin{bmatrix} R & S \\ S & R \end{bmatrix} \in \mathbb{R}^{2n \times 2n} \text{ with } R = \begin{bmatrix} -1/T & 1 \\ 0 & -1/\beta \end{bmatrix} \text{ and } S = \begin{bmatrix} 1/T & 0 \\ 0 & 0 \end{bmatrix} \\ a = -\frac{\ell}{T} {}^t(1, 0, \dots, 1, 0) \in \mathbb{R}^{2n} \text{ and } b = \alpha {}^t(0, 1, \dots, 0, 1) \in \mathbb{R}^{2n}. \end{cases} \quad (2)$$

Here  $w(t)$  is a  $2n$ -vector of independent Wiener processes. Such a linear stochastic process is Markovian. It has a normal distribution with expectation  $m(t)$  and variance/covariance matrix  $C(t)$  such that, by using the Fokker-Planck equation,

$$\dot{m}(t) = Am(t) + a \quad \text{and} \quad \dot{C}(t) = AC(t) + (C(t))^T A + \text{diag}(b) \quad (3)$$

with  $m(0) = \eta_0$  and  $C(0) = 0$ . The expected value  $m(t)$  is asymptotically the homogeneous solution for which  $x_{k+1}(t) - x_k(t) = L/n$  and  $\varepsilon_k(t) = 0$  for all  $k$  and  $t$ ,  $L$  being the length of the system. The roots  $(\lambda_1, \lambda_2)$  of the characteristic equation for the homogeneous configuration

$$\left[\lambda + \frac{1}{T}(1 - e^{i\theta})\right] [\lambda + 1/\beta] = 0, \quad (4)$$

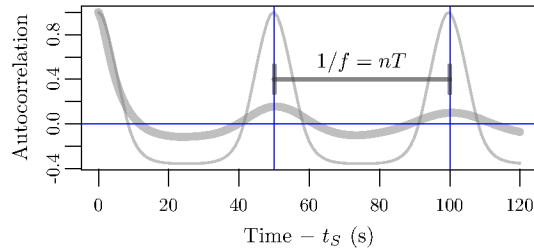
have strictly negative real parts for any  $\theta \in (0, 2\pi)$ . Therefore the homogeneous solution is stable for the system (2) for any values of the parameters. Note that the more unstable configuration is the one with maximal period for which  $\theta \rightarrow 0$ .

### 3 Numerical experiments

The system (2) is simulated using an explicit Euler-Maruyama numerical scheme with time step  $\delta t = 0.01$  s. The parameter values are  $T = 1$  s,  $\ell = 0.3$  m,  $\alpha = 0.1 \text{ ms}^{-3/2}$  and  $\beta = 5$  s. Such values are close to the statistical estimates for pedestrian flow presented in [31]. The length of the system is  $L = 25$  m, corresponding to the experimental situation, and the boundary conditions are periodic.

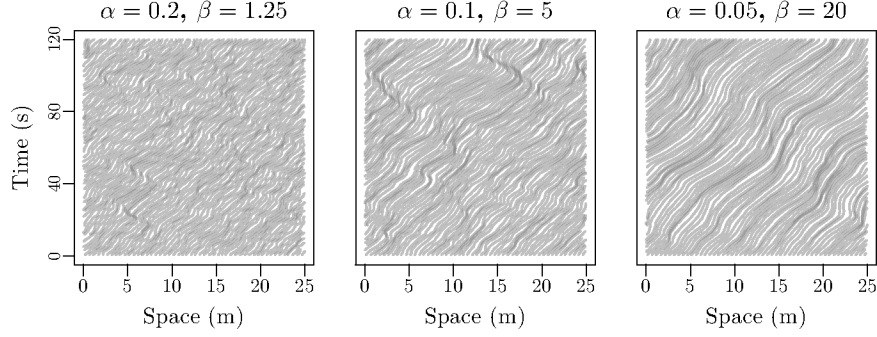
Simulations are carried out for systems with  $n = 25, 50$  and  $75$  agents with the stochastic model Eq. (2) and the unstable deterministic optimal velocity model introduced in [32] from jam initial condition. The mean time-correlation functions for the distance spacing in stationary state (i.e. for large simulation time) are presented in Fig. 2. The peaks of the time-correlations match for both stochastic and deterministic models, i.e. the frequency of the stop-and-go waves are the same. A wave propagate backward in the system at a speed  $c = -\ell/T$  while vehicles travel in average at the speed  $v = (L/n - \ell)/T$ . In adequacy with the theory, the wave period for any agent is  $(v - c)/L = nT$ .

**Fig. 2** Mean time-correlation function of the distance spacing for the stochastic and deterministic models in stationary state. The same period  $1/f = nT$  for the stop-and-go waves is observed. The simulation time to consider the system stationary is  $t_S = 2 \cdot 10^5$  s.



Some experiments are carried out for different values of the noise parameters  $\alpha$  and  $\beta$ . In Fig. 3, the trajectories of 50 agents are presented for  $\alpha = 0.05, 0.1$  and  $0.2 \text{ ms}^{-3/2}$  while  $\beta = 1.25, 5$  and  $20$  s (the values are set in order to keep

the amplitude of the noise  $\sigma = \alpha\sqrt{\beta/2}$  constant). The noise tends to be white when  $\beta$  is low, while the noise autocorrelation is high for large relaxation times  $\beta$ . Unstable waves emerge locally and disappear when  $\beta$  is small (i.e. for a white noise, see Fig. 3, left panel), while stable waves with large amplitude occur for high  $\beta$  (Fig. 3, right panel). The parameters of the noise influence the amplitude of the time-correlation function, but not the frequency that only depends on the parameters  $n$  and  $T$ , see Fig. 4.



**Fig. 3** Simulated trajectories for different values of the noise parameters (units:  $\alpha$  in  $\text{ms}^{-3/2}$ ,  $\beta$  in s). The initial configuration is homogeneous. Here  $n = 50$  agents are considered.

**Fig. 4** Mean time-correlation function of the distance spacing in the stationary state for different values of the noise parameters. The noise parameters do not influence the frequency of the waves (that only depends on  $n$  and  $T$ ). The simulation delay time is  $t_S = 2 \cdot 10^5$  s.

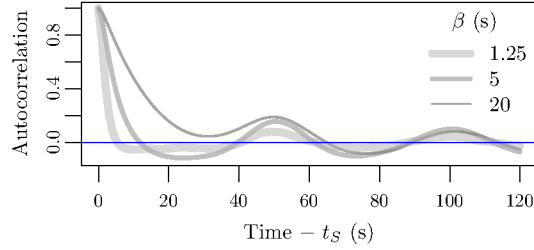
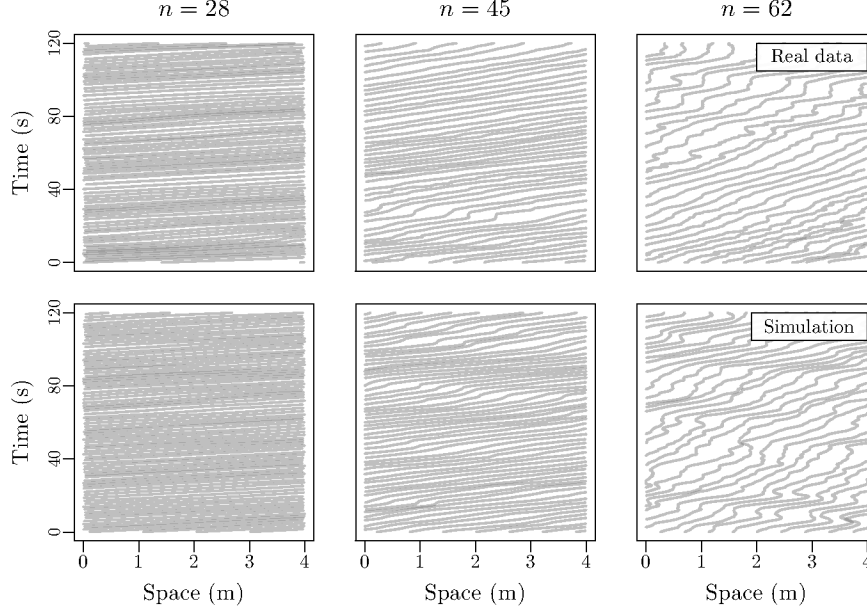


Fig. 5 presents real trajectories for experiments with 28, 45 and 62 participants (see [31]) and simulations with the stochastic model. The simulations are in good agreement with the data. Stop-and-go waves appear for semi-congested ( $n = 45$ ) and congested ( $n = 62$ ) states, while free states ( $n = 28$ ) seem homogeneous in both empirical data and simulation.

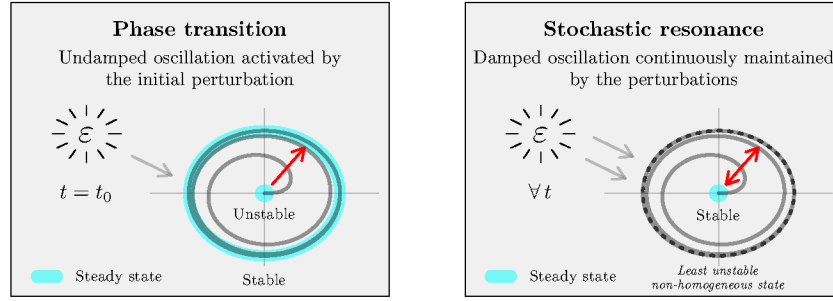


**Fig. 5** Real (top panels) and simulated (bottom panels) trajectories for different densities. The initial configuration is homogeneous.

## 4 Discussion

We have proposed an original explanation for stop-and-go phenomena in pedestrian flows as the consequence of a coloured noise in the dynamics of the speed. In this stochastic approach, oscillations in the system are due to the perturbations provided by the noise. Such a mechanism qualitatively describes stop-and-go waves, especially when the system is poorly damped. The approach differs from classical deterministic traffic models with inertia for which stop-and-go occurs due to metastability and phase transitions to periodical dynamics (see Fig. 6).

Two mechanisms based on relaxation processes are identified for the description of stop-and-go waves. In the novel stochastic approach, the relaxation time is related to the noise and is estimated to approximately 5 s [31]. The parameter corresponds to the mean time period of the stochastic deviations from the phenomenological equilibrium state. Such a time can be high, especially when the deviations are small and the spacings are high. In the classical inertial approaches, the relaxation time is interpreted as the driver/pedestrian reaction time and is estimated by around 0.5 to 1 s. Technically, such a parameter can not exceed the physical time gap between the agents (around 1 to 2 s) without generating unrealistic (colliding) behaviour and has to be set carefully.



**Fig. 6** Illustrative scheme for the modelling of stop-and-go dynamics with phase transition in the periodic solution (left panel) and the noise-induced oscillating behaviour (right panel).

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