

Benchmarking the Quantum Approximate Optimization Algorithm

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Quantum Approximate Optimization Algorithm

E. Farhi et. al.,
arXiv:1411.4028, 2014

- Optimization problem with cost function $C(z) = \langle z | H_C | z \rangle$
where $z = z_1 z_2 \dots z_N$, $z_i \in \{-1, 1\}$ and H_C diagonal w.r.t. $\{|z_i\rangle\}$

- Prepare

$$|\vec{\gamma}, \vec{\beta}\rangle = U_B(\beta_p) U_C(\gamma_p) \dots U_B(\beta_1) U_C(\gamma_1) |+\rangle^{\otimes N}$$

where $\vec{\gamma} = (\gamma_1, \dots, \gamma_p)$, $\vec{\beta} = (\beta_1, \dots, \beta_p)$ are variational parameters and

$$U_C(\gamma) = e^{-i\gamma H_C}, \quad U_B(\beta) = e^{-i\beta \sum_{i=1}^N \sigma_i^x}.$$

- Compute $E_p(\vec{\gamma}, \vec{\beta}) = \langle \vec{\gamma}, \vec{\beta} | H_C | \vec{\gamma}, \vec{\beta} \rangle$ and minimize w.r.t. $\vec{\gamma}$ and $\vec{\beta}$.

Practical Aspects

- $E_p(\vec{\gamma}, \vec{\beta}) = \sum_z |\langle z | \vec{\gamma}, \vec{\beta} \rangle|^2 C(z)$ can be computed on a simulator;
needs to be sampled on a real chip
- Ratio $r = \frac{E_p(\vec{\gamma}, \vec{\beta}) - E_{\max}}{E_{\min} - E_{\max}}$ is related to approximation ratio
- Optimization algorithm used: Nelder-Mead

Quantum Annealing

T. Kadowaki & H. Nishimori,
Phys. Rev. E 58, 5355, 1998

- Preparation of known ground state of initial Hamiltonian H_{initial}
- Adiabatic transformation to the problem Hamiltonian H_{final}

$$H(s) = A(s)H_{\text{initial}} + B(s)H_{\text{final}}.$$

- Functions $A(s)$ and $B(s)$, with $s = t/T_a$ and T_a annealing time, determine the annealing scheme and satisfy

$$A(0) > 0, \quad A(1) \approx 0, \quad B(0) \approx 0, \quad B(1) > 0.$$

- During the annealing process, the system stays in its ground state (if $T_a \rightarrow \infty$; adiabatic theorem)

- Final state gives solution (ground state) of problem Hamiltonian

- Hamiltonian of quantum annealer built by D-Wave Systems Inc.:

$$H(s) = -A(s) \sum_k \sigma_k^x - B(s) \left(\sum_k h_k \sigma_k^z + \sum_{l < k} J_{lk} \sigma_k^z \sigma_l^z \right),$$

where $h_k, J_{lk} \in [-1, 1]$ have to be chosen according to the problem

Results and Conclusions

M. Willsch et. al.,
arXiv:1907.02359, 2019

Test set: 2-SAT problems with unique ground state and highly degenerate first excited state

QAOA on a Simulator and a Real Device

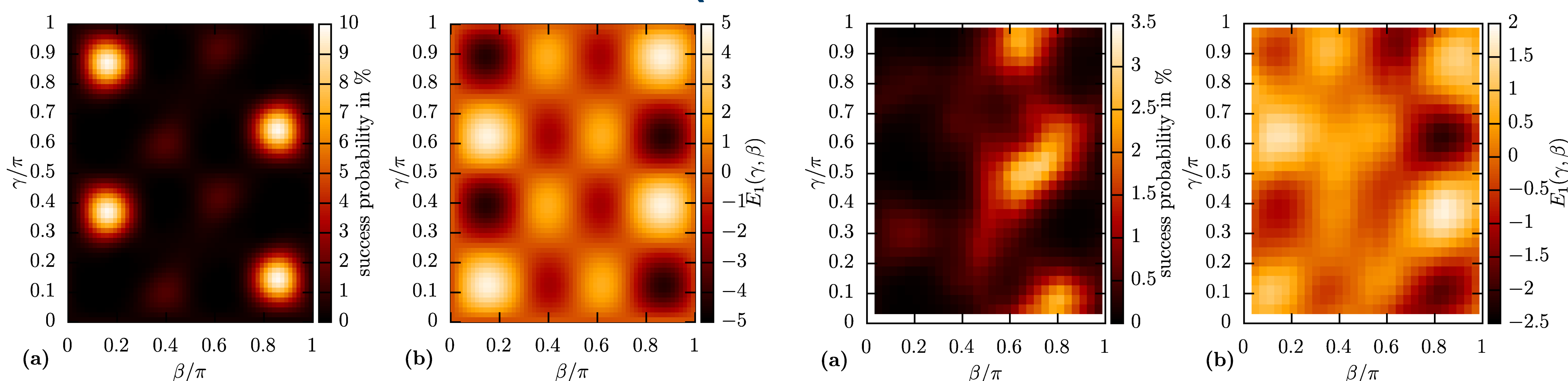


Fig. 1: QAOA ($p = 1$) performed on the IBM simulator. (a) Success probability (b) expectation value $E_1(\gamma, \beta)$.

Fig. 2: QAOA ($p = 1$) performed on the IBM Q Experience (IBM Q 16 Melbourne). (a) Success probability (b) expectation value $E_1(\gamma, \beta)$.

QAOA on JUQCS*: Optimizing the Success Probability or the Expectation Value

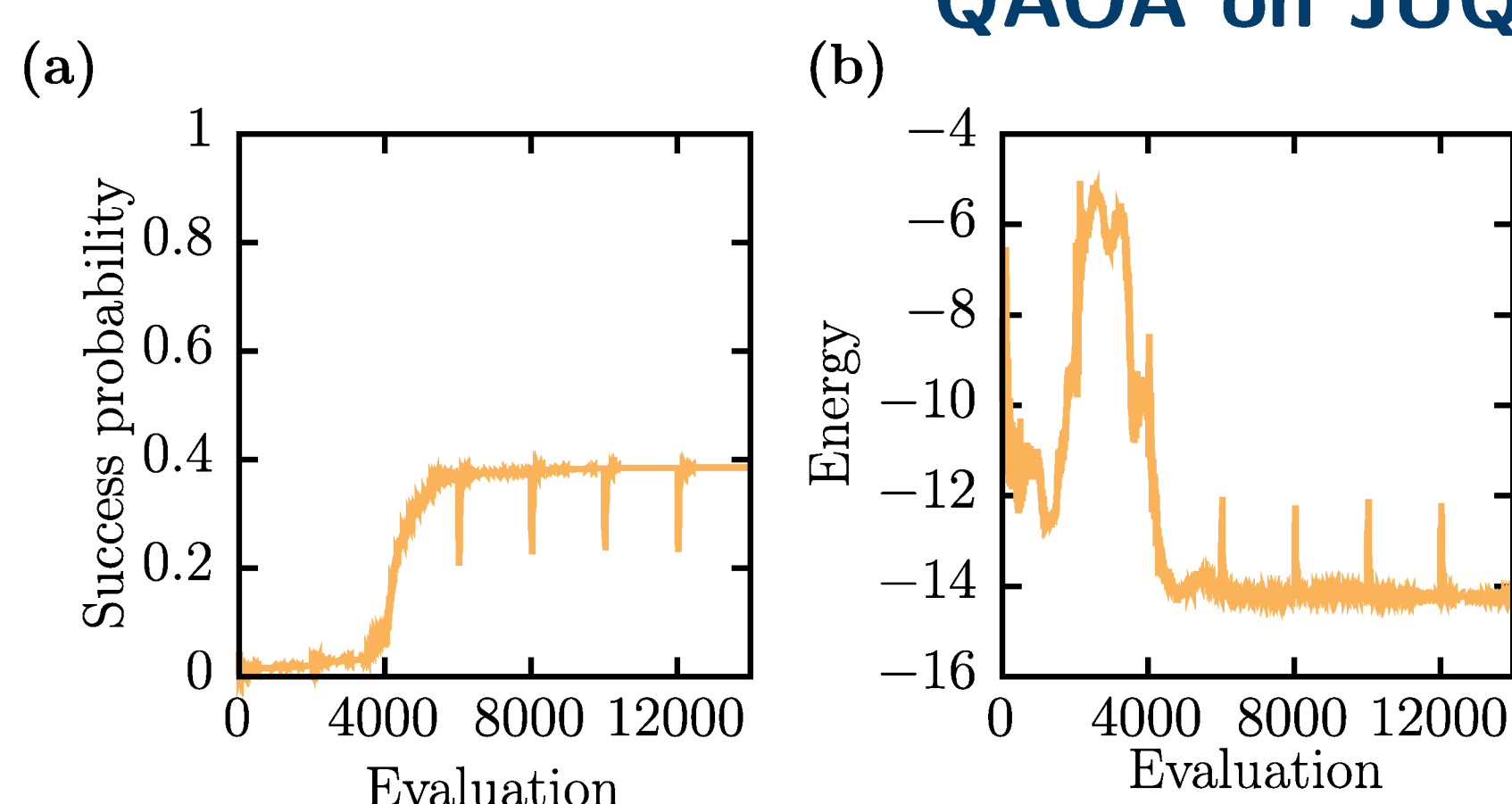


Fig. 3: (a) Success probability and (b) energy expectation value during the optimization w.r.t. success probability for an 18-variable problem instance.

QAOA ($p = 10$)	Optimization w.r.t. (init)	
	E	P
E (init)	-14.36	-8.81
P (init)	0.6%	0.5%
E (after)	-14.97	-12.16
P (after)	0.1%	8.5%

Tab 1: Expectation value E and success probability P before and after the optimization of $(\vec{\gamma}, \vec{\beta})$

- Initialization of variational parameters can be crucial
- Optimization w.r.t. energy expectation value E yields in general different results than optimization w.r.t. success probability P (not applicable in practice; only benchmarking)
→ energy landscape may have more local optima
→ positions of the optima (energy/probability) may not be aligned

*Jülich Universal
Quantum Computer
Simulator

Comparison between QAOA and Quantum Annealing

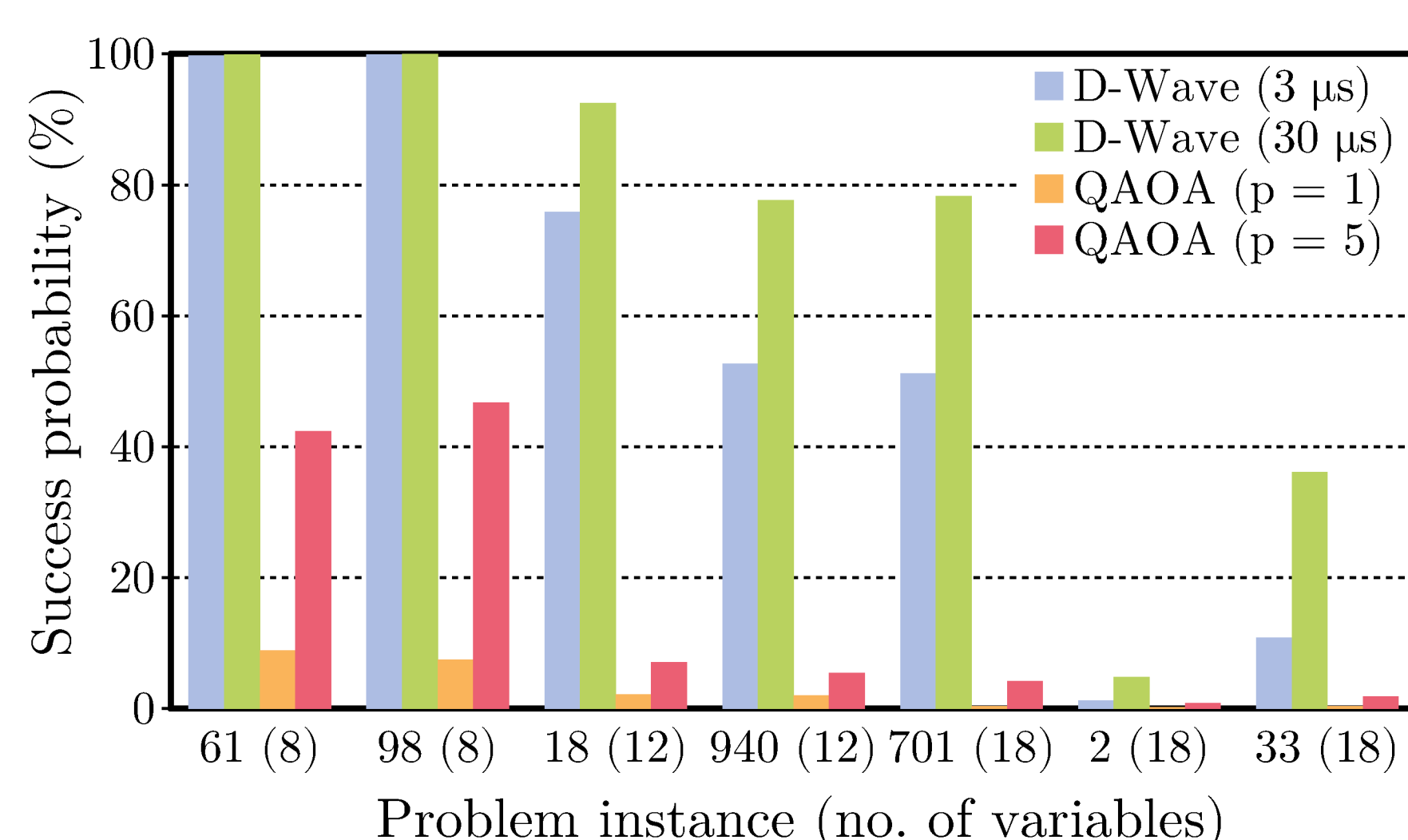


Fig. 3: Success probabilities for QAOA (JUQCS) and quantum annealing (DW_2000Q_2_1 chip).

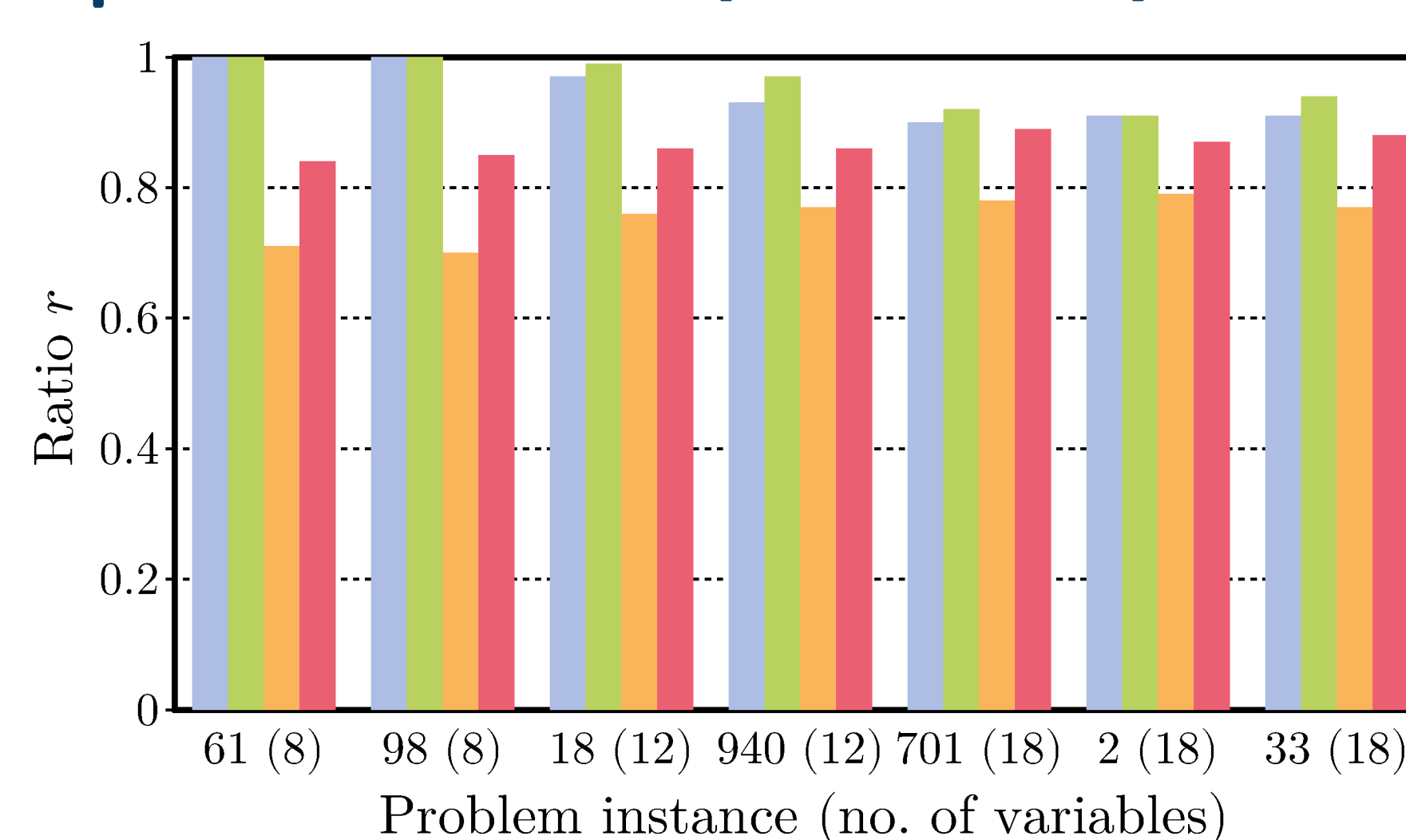


Fig. 4: Ratio r for QAOA (JUQCS) and quantum annealing (DW_2000Q_2_1 chip).

- D-Wave quantum annealer (real chip) outperforms QAOA on a simulator (ideal case)
- For the hardest cases, both do not seem to work well
→ similar trends for success probability
→ success depends on problem instance
- Ratio r
→ for QAOA more stable than for QA
→ for QAOA, tends to increase with the number of variables
→ increase from $p = 1$ to $p = 5$ is larger