Benchmarking the Quantum Approximate Optimization Algorithm

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Quantum Approximate Optimization Algorithm

E. Farhi *et. al.*, arXiv:1411.4028, 2014

- Optimization problem with cost function $C(z) = \langle z|H_C|z\rangle$ where $z = z_1 z_2 \dots z_N$, $z_i \in \{-1, 1\}$ and H_C diagonal w.r.t. $\{|z_i\rangle\}$
- Prepare
$$|\vec{\gamma}, \vec{\beta}\rangle = U_B(\beta_p)U_C(\gamma_p)\cdots U_B(\beta_1)U_C(\gamma_1)|+\rangle^{\otimes N}$$
where $\vec{\gamma} = (\gamma_1, \dots, \gamma_p)$, $\vec{\beta} = (\beta_1, \dots, \beta_p)$ are variational parameters and
$$U_C(\gamma) = e^{-i\gamma H_C}, \quad U_B(\beta) = e^{-i\beta \sum_{i=1}^N \sigma_i^x}.$$
- Compute $E_p(\vec{\gamma}, \vec{\beta}) = \langle \vec{\gamma}, \vec{\beta}|H_C|\vec{\gamma}, \vec{\beta}\rangle$ and minimize w.r.t. $\vec{\gamma}$ and $\vec{\beta}$.

Practical Aspects

- $E_p(\vec{\gamma}, \vec{\beta}) = \sum_z |\langle z|\vec{\gamma}, \vec{\beta}\rangle|^2 C(z)$ can be computed on a simulator; needs to be sampled on a real chip
- Ratio $r = \dfrac{E_p(\vec{\gamma}, \vec{\beta}) - E_{\max}}{E_{\min} - E_{\max}}$ is related to approximation ratio
- Optimization algorithm used: Nelder-Mead

Quantum Annealing

T. Kadowaki & H. Nishimori, Phys. Rev. E 58, 5355, 1998

- Preparation of known ground state of initial Hamiltonian H_{initial}
- Adiabatic transformation to the problem Hamiltonian H_{final}
$$H(s) = A(s)H_{\text{initial}} + B(s)H_{\text{final}}.$$
- Functions $A(s)$ and $B(s)$, with $s = t/T_{\text{a}}$ and T_{a} annealing time, determine the annealing scheme and satisfy
$$A(0) > 0, \quad A(1) \approx 0, \quad B(0) \approx 0, \quad B(1) > 0.$$
- During the annealing process, the system stays in its ground state (if $T_{\text{a}} \to \infty$; adiabatic theorem)
- Final state gives solution (ground state) of problem Hamiltonian
- Hamiltonian of quantum annealer built by D-Wave Systems Inc.:
$$H(s) = -A(s)\sum_k \sigma_k^x - B(s)\left(\sum_k h_k\sigma_k^z + \sum_{l<k} J_{lk}\sigma_k^z\sigma_j^z\right),$$
where h_k, $J_{lk} \in [-1, 1]$ have to be chosen according to the problem

Results and Conclusions

M. Willsch *et. al.*, arXiv:1907.02359, 2019

Test set: 2-SAT problems with unique ground state and highly degenerate first excited state

QAOA on a Simulator and a Real Device

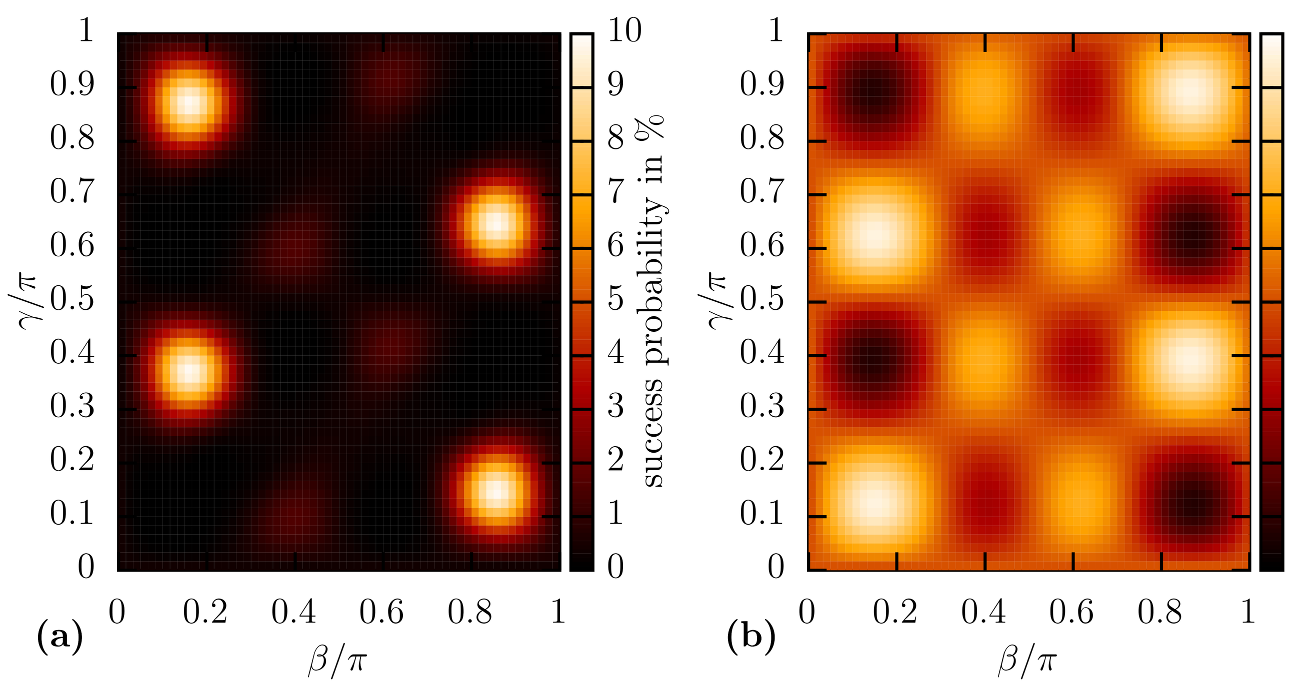

Fig. 1: QAOA ($p = 1$) performed on the IBM simulator. (a) Success probability (b) expectation value $E_1(\gamma,\beta)$.

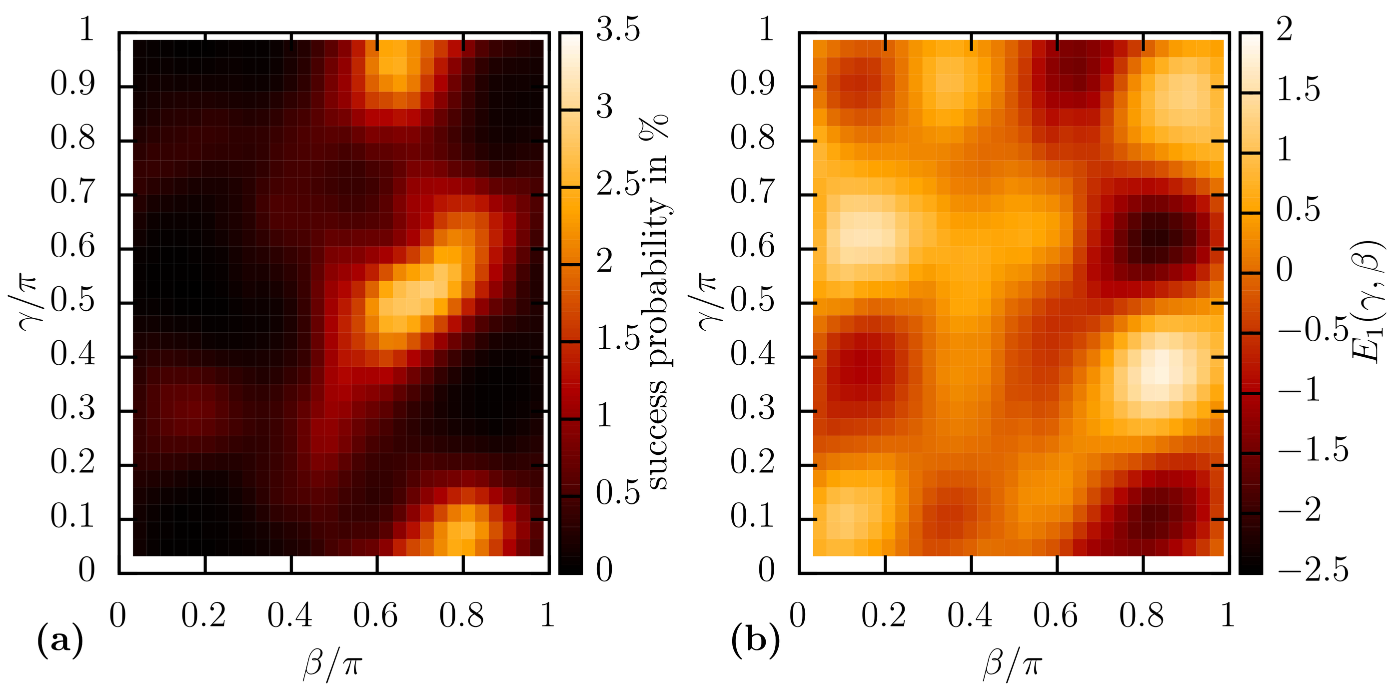

Fig. 2: QAOA ($p = 1$) performed on the IBM Q Experience (IBM Q 16 Melbourne). (a) Success probability (b) expectation value $E_1(\gamma,\beta)$.

- Comparing the values, QAOA on the real chip performs rather poorly
- For the expectation value, the minimum is at the correct position
- Regarding the success probability, the pattern is deformed

QAOA on JUQCS*: Optimizing the Success Probability or the Expectation Value

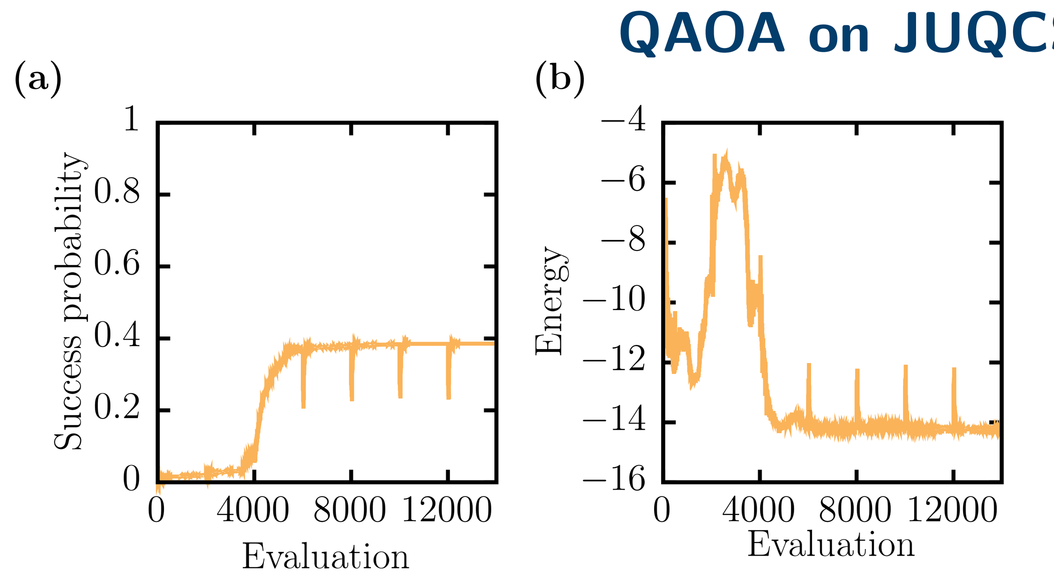

Fig. 3: (a) Success probability and (b) energy expectation value during the optimization w.r.t. success probability for an 18-variable problem instance.

QAOA ($p = 10$)	Optimization w.r.t. (init)		
	E	P (anneal)	P (rand)
E (init)	− 14.36		− 8.81
P (init)	0.6%		0.5%
E (after)	− 14.97	− 12.16	− 14.22
P (after)	0.1%	8.5%	38.6%

Tab 1: Expectation value E and success probability P before and after the optimization of $(\vec{\gamma}, \vec{\beta})$

- Initialization of variational parameters can be crucial
- Optimization w.r.t. energy expectation value E yields in general different results than optimization w.r.t. success probability P (not applicable in practice; only benchmarking)
  - → energy landscape may have more local optima
  - → positions of the optima (energy/probability) may not be aligned

*Jülich Universal Quantum Computer Simulator

Comparison between QAOA and Quantum Annealing

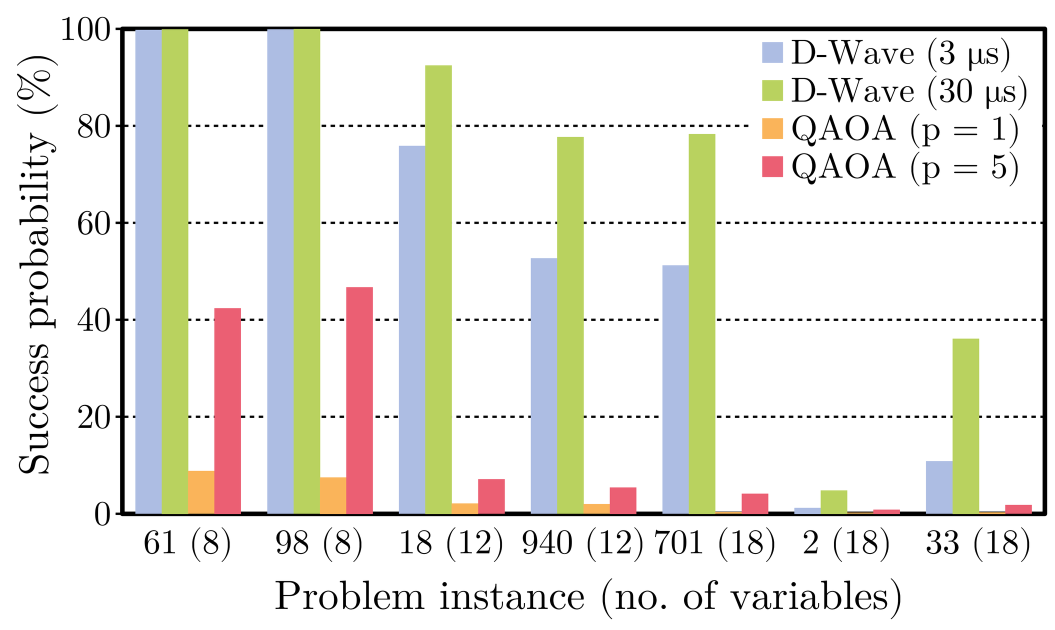

Fig. 3: Success probabilities for QAOA (JUQCS) and quantum annealing (DW_2000Q_2_1 chip).

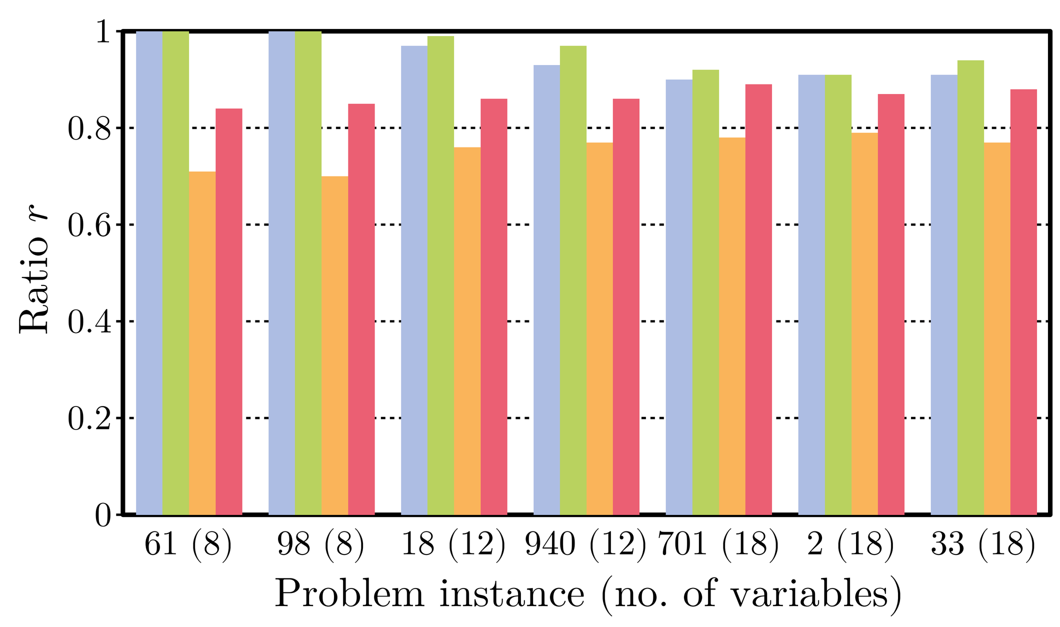

Fig. 4: Ratio r for QAOA (JUQCS) and quantum annealing (DW_2000Q_2_1 chip).

- D-Wave quantum annealer (real chip) outperforms QAOA on a simulator (ideal case)
- For the hardest cases, both do not seem to work well
  - → similar trends for success probability
  - → success depends on problem instance
- Ratio r
  - → for QAOA more stable than for QA
  - → for QAOA, tends to increase with the number of variables
  - → increase from $p = 1$ to $p = 5$ is larger