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# Applications of Seismic Full-Waveform Inversion on Shallow-Seismic and Ultrasonic Data

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Conventional seismic imaging methods utilise a small portion of the information in the seismic data we obtain. Most methods analyse their arrival times or specific signal amplitudes only. In this report we further develop and apply a new seismic inversion and imaging technique that uses the full information content of the seismic recordings. Full waveform inversion (FWI) is an algorithm that accounts for the full seismic waveform. It iteratively retrieves multiparameter physical models of the material by numerically solving the wave equation and optimisation problem. FWI is currently a cutting-edge seismic inversion and imaging technique that enables to exploit the full information contained in the seismic waveforms over a broad range of frequencies and apertures for an improved estimation of physical parameters. It allows for a mapping of structures on spatial scales down to approximately half of the seismic wavelength, hence providing a tremendous improvement of resolution compared to travel-time tomography based on ray-theory. We especially focus on the applications of FWI on two different scales, including near-surface scale which extends from Earth's free surface to a depth around 10 meters and laboratory scale which targets on material from a few millimetre to less than one meter. Numerical tests and real-world applications are used to show the high resolution of FWI in reconstructing physical properties of the earth model and artificial material.

## 1 Introduction

The reconstruction of near-surface elastic-parameter models is of fundamental importance for near-surface geophysical and geotechnical studies. Surface waves dominate the shallow-seismic wavefield and are attractive for determining near-surface structures due to their relatively high signal-to-noise ratio in field recordings. With a rapid development in the theories of surface-wave methods, it has become increasingly popular over the last two decades to use surface waves as a non-invasive way to estimate near-surface structures. Those methods, however, only use part of the information contained in the seismic data and, therefore, lose resolution in the reconstructed models (Pan *et al.*<sup>1</sup>).

With a rapid increase in computational power, it has become feasible to use full-waveform inversion (Tarantola<sup>2</sup>) to resolve a subsurface model by fitting the observed waveform directly. Based on full-wavefield modelling, FWI is able to fully exploit the waveform information and is getting increasingly popular among shallow-seismic methods. Due to the existence of surface waves, the acoustic approximation, which is widely adopted in explorational seismics, is no longer valid in shallow seismics. The inclusion of surface waves in the wavefields also increases the nonlinearity of FWI. Shallow-seismic FWI is an ill-posed problem and could converge toward a local minimum especially when a poor initial model is provided. Therefore, special care should be paid when utilising FWI to resolve near-surface model (Groos *et al.*<sup>3</sup>).

Besides the application of FWI on shallow-seismic data, we also look at FWI in the context of non-destructive testing. The aim is to characterise small anomalies in building

materials with ultrasonic measurements to detect defects and inclusions which can significantly affect the integrity of building materials. Defects and intrusions in building materials can be build due to altering processes or an inadequate production. The advantage of ultrasonic measurements is, that they are non-destructive. Therefore, it can be used for almost every kind of testing of building materials, even for vulnerable structures like historic buildings. Thus, the results are of high practical interest. The project is done in cooperation with the Fraunhofer Institute for Nondestructive Testing (IZFP) in Saarbrücken.

The aim of full waveform inversion is the estimation of elastic material parameters and their composition to a subsurface model out of seismic observations. For most subsurface configurations there do not exist exact analytical solutions, so only approximated solutions can be found using numerical methods. Therefore, we iteratively minimise the misfit between the modelled and the observed data using time-domain staggered-grid finite differences. The model with the smallest misfit then describes the observed data best.

The big advantage of using FWI is, that it uses the whole information content of the observed data. In contrast, most methods only use the arrival time or specific signal amplitudes of the observations. Consequently, we have a large improvement in model resolution by using FWI compared to other methods such as traveltimes tomography.

## 2 Methodology

### 2.1 Forward Modelling

Seismic modelling is the fundamental part of FWI and requires nearly all the computation time. In dependence of the field of application, the wave-propagation physics for an underlying subsurface model has to be described by an appropriate wave equation. On the one hand, this comprises the acoustic wave equation in this work. On the other hand, the problem has to be solved for two-dimensional or three-dimensional subsurface models (corresponding applications are referred to as 2D or 3D FWI). The numerical implementation of the wave equations consists of a time-domain finite-difference (FD) time-stepping method in the Cartesian coordinates. In this report, we use the FD scheme to solve the velocity-stress formulation of visco-elastic wave equation (Bohlen<sup>4</sup>). Due to finite model sizes, the wave equations are expanded by a perfectly matched layer to avoid artificial boundary reflections.

### 2.2 Inversion

Fig. 1 shows the general workflow of FWI. The solution of the inverse problem comprises several steps. The method is initialised by the choice of a 2D or 3D initial parameter model. Seismic velocities and/or mass density are assigned to the model  $m_0$  at the first iteration. The initial model can be estimated from a prior information or computed by conventional methods. For each source of the acquisition geometry, seismic modelling is applied, *i. e.* the wavefield is emitted by the source and forward-propagates across the medium. A time series of this wavefield has to be stored in memory with respect to the whole volume. Synthetic seismic data is recorded at the receivers and the difference between the observed and the synthetic data is calculated, which results in residuals. For each source, the residual wavefield is backpropagated from the receivers to the source position.

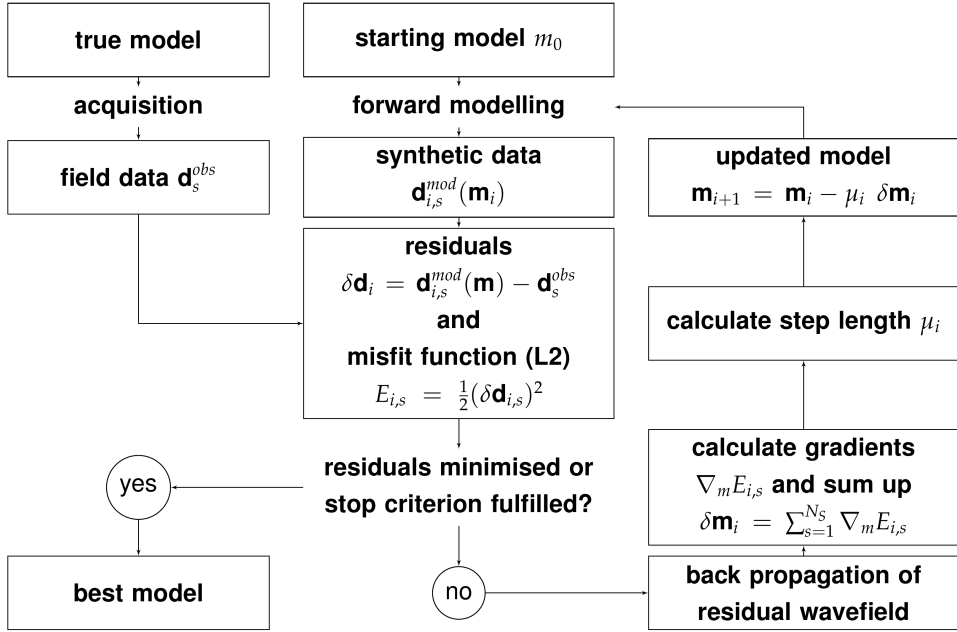


Figure 1. General workflow of FWI.

The cross-correlation of forward- and backpropagated wavefields yields shot-specific gradients. The computation of the global gradient  $\delta m$  for the entire acquisition geometry is given by the summation of all gradients  $\delta m_{i,s}$ . An optimised gradient  $\delta m_i$  is computed by subsequent preconditioning and application of the optimisation algorithm (*e. g.* conjugate gradient, L-BFGS). The update of the model parameter is the final step of an FWI iteration. The gradient  $\delta m_i$  has to be scaled by an optimal step length  $\mu_i$  to get a proper model update at iteration step  $i$ . The estimation of  $\mu_i$  is performed at each iteration with a line search algorithm and requires additional modellings.

### 3 Shallow-Seismic FWI

#### 3.1 Synthetic Test

A checkerboard model is used to test the resolution of FWI. A homogeneous half-space model with an S-wave velocity of 200 m/s, a P-wave velocity of 500 m/s, and a density of 2000 kg/m<sup>3</sup> is used as the background model. The checkerboard is formed by blocks of 1 m × 1 m, and the S-wave velocity of each block is 220 or 180 m/s depending on its position (Fig. 2), *i. e.* ±10 % perturbation in S-wave velocity is added to the background model. Twenty-four two-component geophones are placed along the free surface with an interval of 1 m (red dashed line in Fig. 2a). Six shot gathers are simulated with a vertical-force source spaced every 6 m (red stars in Fig. 2a). The nearest source-receiver offset for the leftmost source is 3 m. A shifted Ricker wavelet with a centre frequency of 40 Hz

is used as the source wavelet. The minimal wavelength of the observed surface wave is around 2 m, which is twice the size of the checkerboard block.

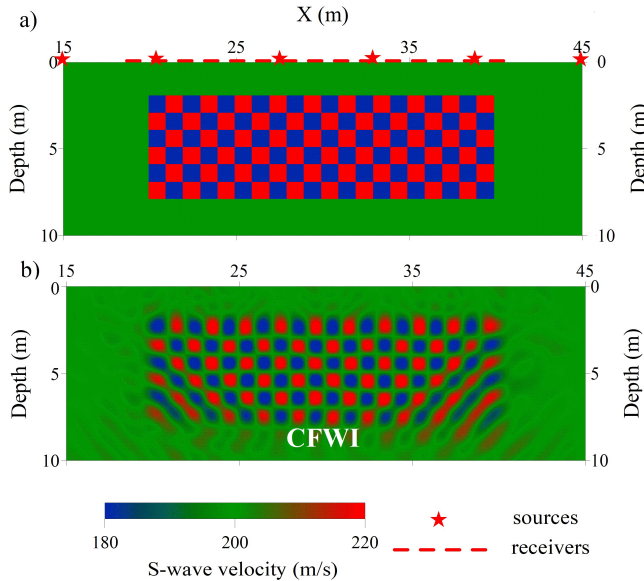


Figure 2. Checkerboard model test. a) True S-wave velocity model. Stars represent the locations of the sources, and the dashed line represents the geophone spread. b) Inversion results of Conventional FWI. The background without the checkerboard is used as the initial model.

We use the background model as the initial model and apply FWI on the observed data. The true source wavelet is used and only the S-wave velocity model is updated during the inversion. The conjugate gradient algorithm is used as the optimisation algorithm, and the gradient is calculated by the adjoint state algorithm. The checkerboard model can be reconstructed using FWI (Fig. 2b), which proves a relatively high resolution (approximately half a wavelength) of FWI. The lower left and right corners in the checkerboard are not reconstructed as well as the other areas, mainly due to a lower illumination in the lower corners.

### 3.2 Field Data Test

The field data set was acquired in Rheinstetten, Germany. The near-surface material at the test site was mainly composed of layered fluvial sediments. A vertical source and forty-eight vertical-component geophones were placed along the survey line with a 1 m spacing (triangles in Fig. 3a). The source was placed every 4 m, and the first source position was located between the first and the second geophones. A total of 12 shot gathers are used in this example (asterisks in Fig. 3a).

We built a 1D depth-dependent S-wave velocity model as the initial model for FWI by using the conventional surface-wave method (Fig. 3a). The initial P-wave velocity was

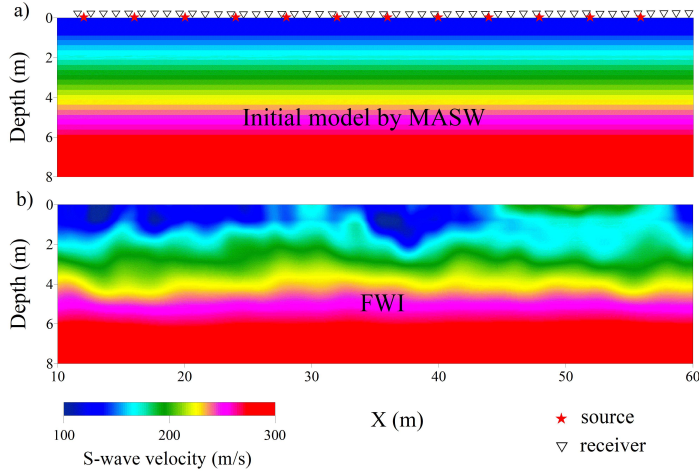


Figure 3. S-wave velocity model for the Rheinstetten data. a), b) and c) represent the initial model estimated by 1D MASW, the inversion result of MFWI after 52 iterations, and the pseudo-2D MASW result obtained by combining 16 1D profiles, respectively. Asterisks and inverted triangles represent the locations of the sources and the receivers, respectively.

built by inverting traveltimes of the first arrivals. We applied a 3D-to-2D transformation to the observed data since we used a 2D forward solver. We delayed the whole shot gathers by 0.03 s and killed the traces with a source-receiver offset shorter than 2 m since the signals in those traces are partly clipped. Since the acquired data contain relatively strong attenuation effects, we used constant  $Q$  models as passive information with a viscoelastic forward solver to mitigate the influence of viscosity. We started FWI by inverting a subset of data up to 10 Hz. The upper frequency limit of the low-pass filter was progressively increased to 60 Hz with a 5 Hz interval, with a minimum of 3 iterations in each stage. The inversion converged after a total of 52 iterations.

The inverted S-wave velocity model shows some 2D lateral variations of the subsurface (Fig. 3b). A "V"-shaped low-velocity body can be identified at the central shallow part of the model, which corresponds to a refilled trench at the test site. The inversion result also shows a velocity inversion (high-velocity layer on top of a low-velocity layer) around 50 m and 0.2 m depth. It is worth mentioning that the low-velocity body is also identified at the same location in the reconstructed P-wave velocity model; however, its "V" shape is less clear in the P-wave velocity model compared to the S-wave velocity model.

## 4 Ultrasonic FWI

### 4.1 Information about the Software

We develop an FWI software which is freely available under the GNU Affero General Public License. To parallelise the inverse problem, it is reformulated to a matrix-vector for-

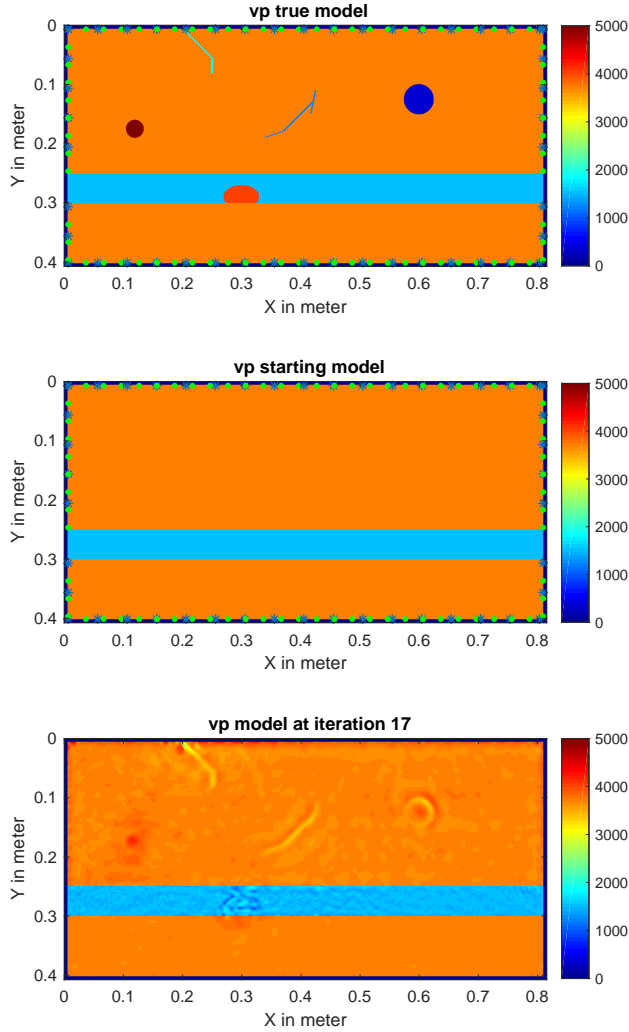


Figure 4. 2D elastic full waveform inversion of synthetic 2D data, exemplary shown for the parameter P-wave velocity ( $vp$ ). Top: True model, middle: starting model, bottom: inversion result.

malism. The different implementations use explicit time domain finite difference forward modelling for the wave field calculations. The different FWI implementations consider 2D and 3D wave propagation and allow the reconstruction of acoustic, elastic, viscoacoustic and viscoelastic medium properties.

## 4.2 Synthetic Test

We use a 2D elastic FWI in the time domain and apply it to measured ultrasonic data at a concrete block, that is specifically build for this case. Inside the concrete block is a bore,

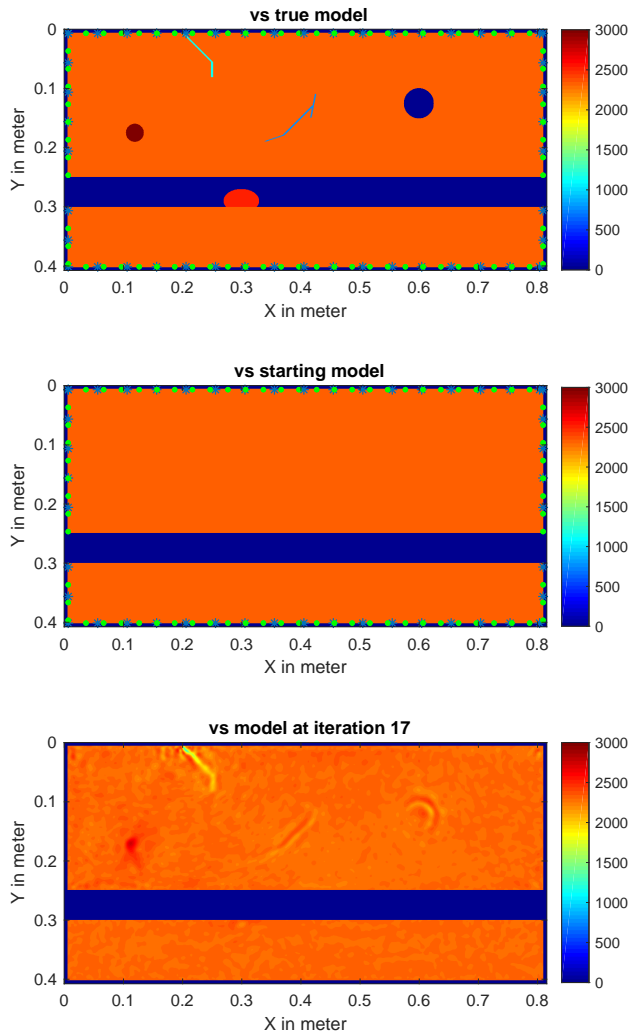


Figure 5. 2D elastic full waveform inversion of synthetic 2D data, exemplary shown for the parameter S-wave velocity ( $v_s$ ). Top: True model, middle: starting model, bottom: inversion result.

that can be filled with different materials with different properties. The block as well as the data acquisition is provided by the Fraunhofer Institute in Saarbrücken. The project mainly focuses on the reconstruction of the shape and properties (such as velocity and density) of anomalies.

The first step is to calculate some 2D synthetic tests to approximate a suitable quantity and positions of sources and receivers. After that we will perform the ultrasonic measurements on the concrete block and invert the field data. The small wavelengths of ultrasound are necessary to dissolve also small-scale anomalies. In our case the ultrasonic transmitter has a peak frequency of 100 kHz. Numerical seismic modelling is the fundamental part of



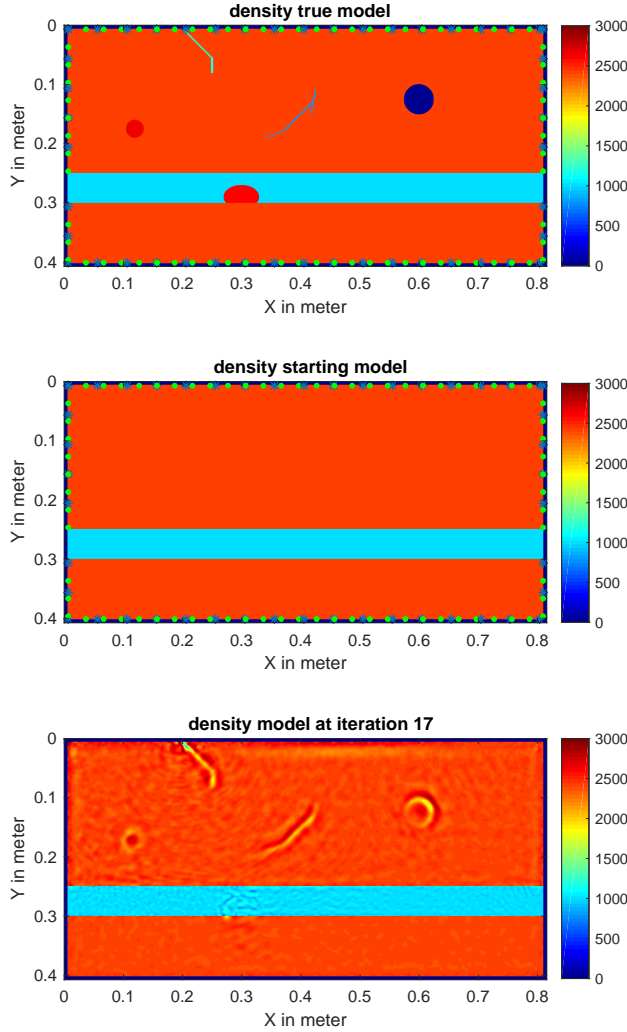


Figure 6. 2D elastic full waveform inversion of synthetic 2D data, exemplary shown for the density. Top: True model, middle: starting model, bottom: inversion result.

FWI, because usually there exist no exact analytical solution for a given subsurface configuration. The numerical modelling requires nearly all the computation time. The physics of wave propagation underlie the theory of continuum mechanics and can be described by an appropriate wave equation. The FD scheme solves the velocity-stress formulation of the elastic wave equation.

Figs. 4, 5 and 6 show the inversion results of 2D synthetic data for the P-wave velocity, S-wave velocity and density, respectively. The 2D synthetic data were obtained by forward modelling the wavefield in the true model (top model in Figs. 4, 5 and 6). The initial model contains a water-filled elongated pipe with a stone, two differently shaped cracks,

one high velocity anomaly and one air-filled low velocity anomaly. The starting model for the inversion is shown in the middle model in the figures. We follow a multi-scale approach to invert the data, that means we use multiple consecutive inversions containing different frequency ranges to prevent cycle skipping. We start with small frequencies creating a smooth model and go up step by step to higher frequencies. The final inversion result after 17 iterations in the last frequency range is shown in the bottom model of figures. As we can see, the shape of very small structures like cracks can be reconstructed quite good in all the three parameters. The inversion of the round high velocity anomaly on the left converges to the true velocity in the initial model. In contrast, the velocity of the air filled round low velocity anomaly on the right cannot be obtained correctly, as elastic waves do not propagate in air. Overall, it shows that FWI has a high resolution in reconstructing multiparameter physical models of artificial material and is promising to be used in ultrasonic testing in the future.

## 5 Concluding Remarks

We have applied full-waveform inversion to both shallow-seismic and ultrasonic data. Synthetic examples prove the high-resolution of FWI in reconstructing physical properties of the earth model and the artificial material. One real-world application proves the applicability of FWI. These results encourage us to use FWI in both shallow-seismic surveys and ultrasonic testings in the future.

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