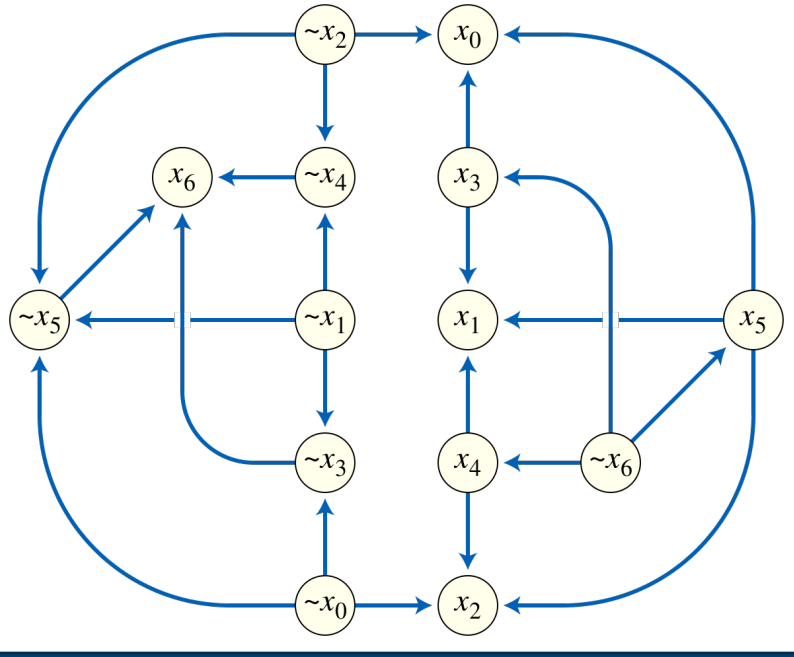

OPTIMIZATION PROBLEMS FOR BENCHMARKING THE HYBRID SOLVER SERVICE VERSION 2 AND THE ADVANTAGE QPU

CADS SEMINAR, OCTOBER 20, 2020 I MANPREET JATTANA

CONTENTS & TEAM

1. Vegetable garden optimization
2. 2-satisfiability
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4. Conclusions

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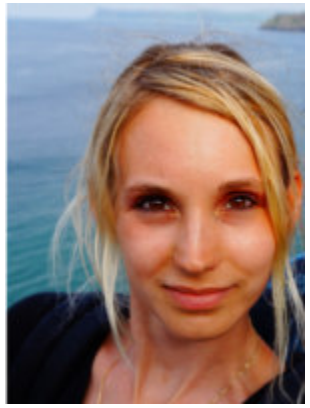

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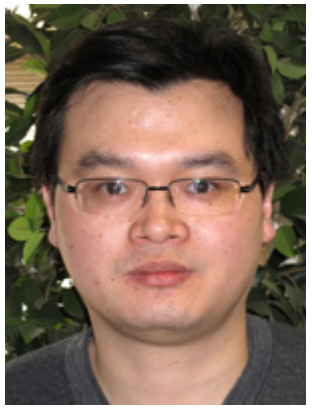


Dr. Fengping Jin

VEGETABLE GARDEN OPTIMIZATION

Overview

	Tomato	Cucumber	Lettuce
Tomato		😢	🙂
Cucumber	😢		🙂
Lettuce	🙂	🙂	

- Application: Companion planting in polyculture vegetable gardens
- Problem: Find an optimal placement of plants in the garden considering the characteristics of their nearest neighbors

VEGETABLE GARDEN OPTIMIZATION

Overview

$$\text{minimize} \quad \sum_{ii'}^{n,n} J_{ii'} \left(1 + \sum_{jj'}^{t,t} x_{ij} C_{jj'} x_{i'j'} \right)$$

$$\text{subject to} \quad \sum_{i}^{n} \left(1 - \sum_{j}^{t} x_{ij} \right)^2 = 0$$

$$\sum_{j}^{t} \left(c_j - \sum_{i}^{n} x_{ij} \right)^2 = 0$$

$$\sum_{i,j}^{n,t} \left(i\%2 - s_j \right)^2 x_{ij} = 0$$

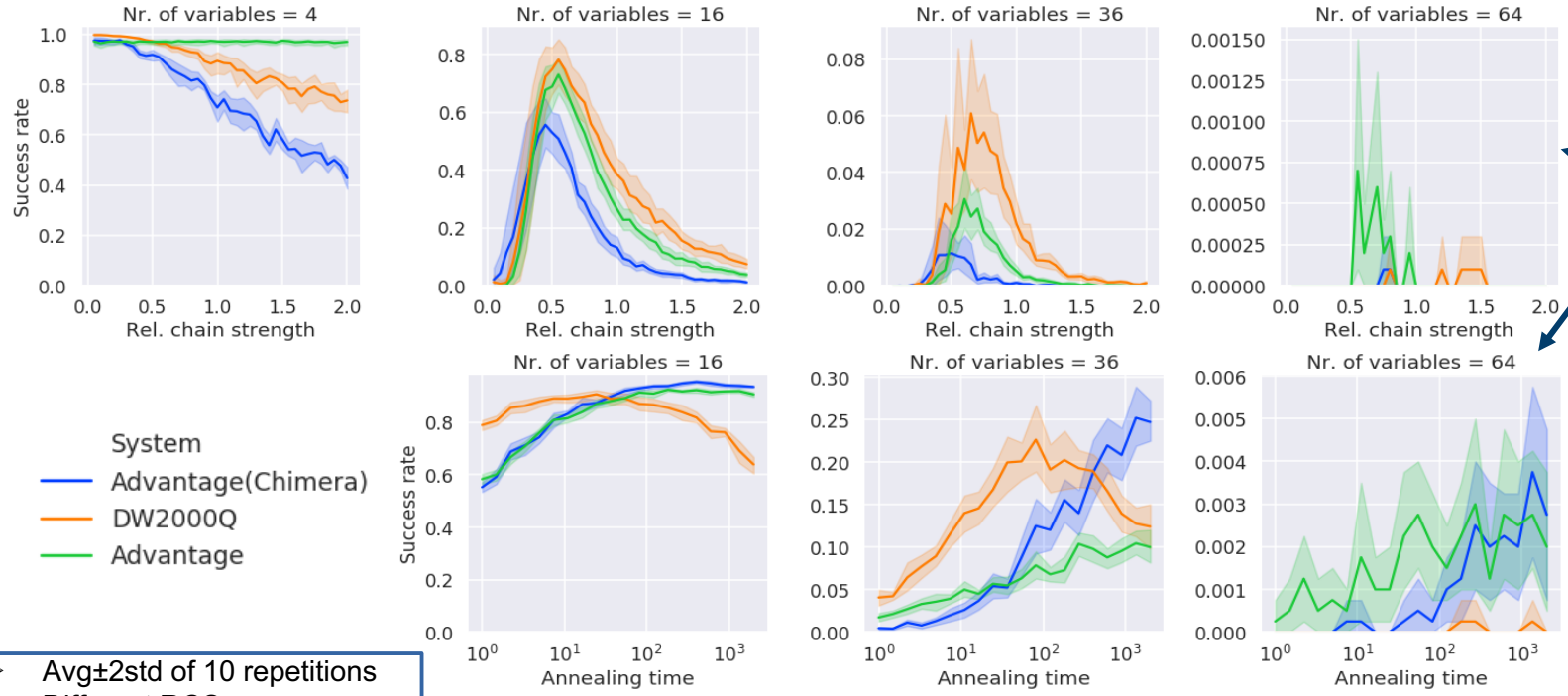
- Quadratic Assignment Problem (QAP) with constraints
- Assign plants to pots in the garden:
  - Qubit $x_{ij} = 1$ means species j is placed in pot i
  - All plants should have a good relationship with their neighbors
  - Use all available plants and pots
  - Big plants shouldn't shadow small ones
- Flexible problem size
- Embedding required

n: nr. of pots in the garden
t: nr. of plant species to place
J: connectivity between pots
C: relationship between species
c_j: nr. of plants of species ***j***
s_j: size of species ***j***

VEGETABLE GARDEN OPTIMIZATION

Advantage vs DW2000Q: smaller problems (4 – 256 variables)

- Success rate (SR) under increasing relative chain strength (RCS) and annealing times (AT)
- Influence of Advantage's extra couplers on success rate:
  - **Chimera embedding on Advantage**
  - **Chimera embedding on DW2000Q**
  - **Pegasus embedding on Advantage**



- Avg±2std of 10 repetitions
- Different RCS

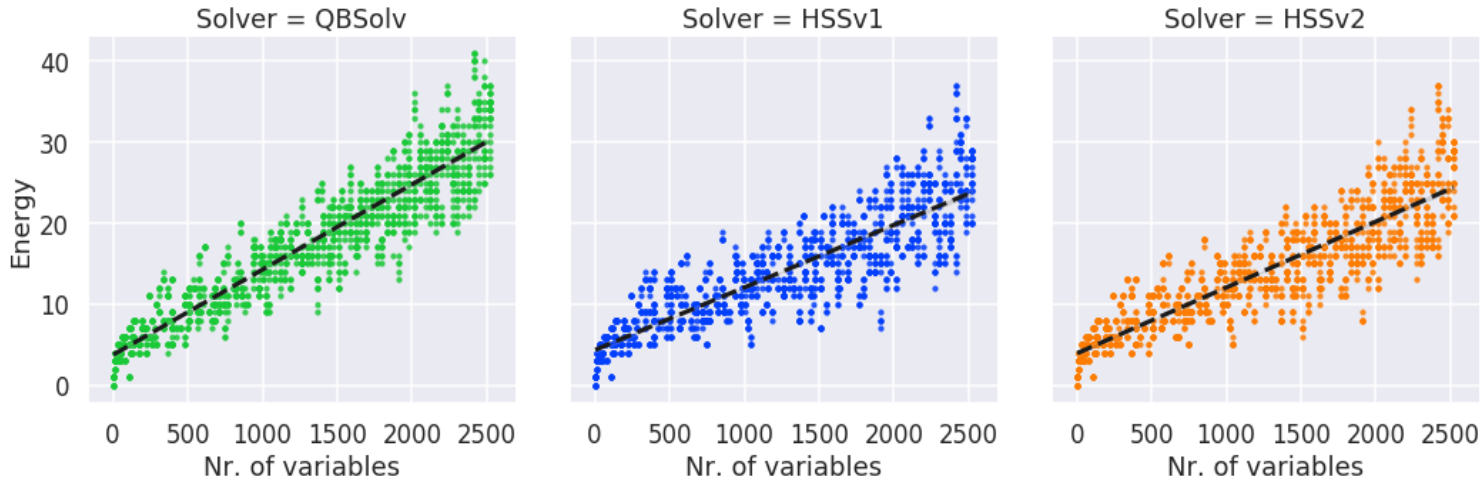
- Avg±2std of 10 embeddings
- Default AT: 20 μs

Observations:

- Embedding up to 196 (64) vars possible on Advantage (DW2000Q)
- Successful up to 64 vars
- Tuning RCS increases SR
- Tuning AT increases SR (esp. on Advantage)
- ≤ 36 vars: similar embeddings in Chimera and Pegasus → DW2000Q better
- 64 vars: shorter embeddings on Pegasus → Advantage better

VEGETABLE GARDEN OPTIMIZATION

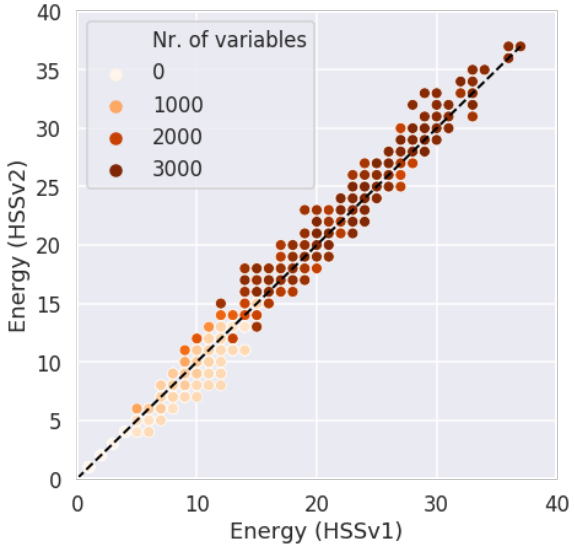
Hybrid solver service (HSS) v1 vs v2: bigger problems (4 – 2520 variables)



Experiment:

1760 problems solved using:

- **QBSolv** (legacy software solver)
- **HSSv1** (current hybrid solver)
- **HSSv2** (new hybrid solver)



Timing for 2520 vars problem with default parameters (n=10)

Solver	Energy	Runtime (s)
HSSv1	28.1 ± 0.568	6.396 ± 0.023
HSSv2	29.0 ± 0.471	6.407 ± 0.003
QBSolv*	35.5 ± 0.972	66.056 ± 20.99

* Intel Core i5-4200U CPU @ 1.60GHz (Notebook)

Observations:

- **HSSv1**, **HSSv2** better & faster than **QBSolv**
- **HSSv2** better at smaller problems
- **HSSv1** better at bigger problems

2-SATISFIABILITY

Overview

- Mathematical formulation:

$F = (L_{1,1} \vee L_{1,2}) \wedge (L_{2,1} \vee L_{2,2}) \wedge \ldots \wedge (L_{M,1} \vee L_{M,2})$,

where $L_{j,k} = x_i$ or $\overline{x_i}$ with $x_i = 0,1$

→ find assignment to x_i that makes *F* true

- Reformulation as Ising Problem:

$H_{2SAT} = \sum_{\alpha=1}^{M} g_\alpha h_{2SAT}(\epsilon_{\alpha,1} s_{i[\alpha,1]}, \epsilon_{\alpha,2} s_{i[\alpha,2]})$, where $g_\alpha > 0$ is a scaling factor, $\epsilon_\alpha = 1$ for x_i , $\epsilon_\alpha = -1$ for $\overline{x_i}$

  - Example: for clause $(x_1 \vee x_2)$, $h_{2SAT} = s_1 s_2 - (s_1 + s_2) + 1$

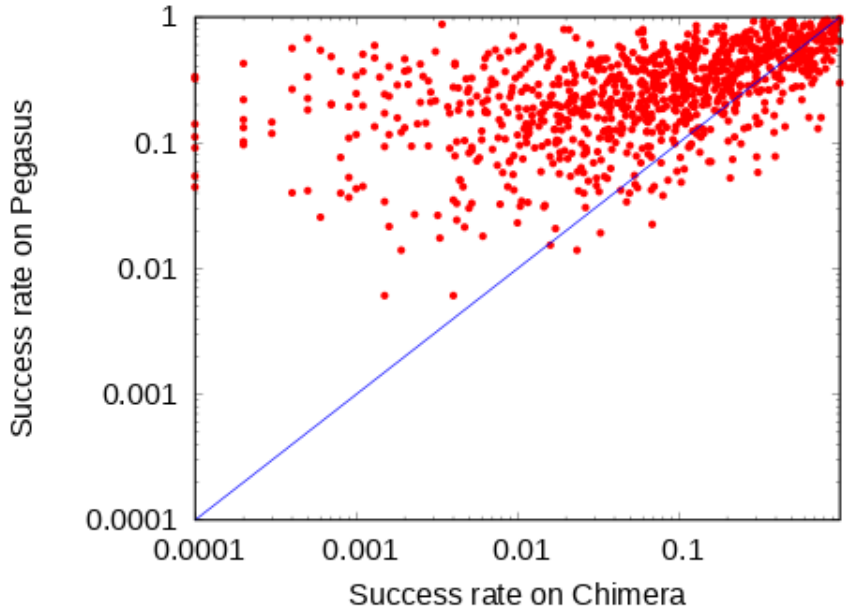
- Chosen problems: 18 variables with 19 clauses

  - Properties: a known unique ground state and a highly degenerate first excited state

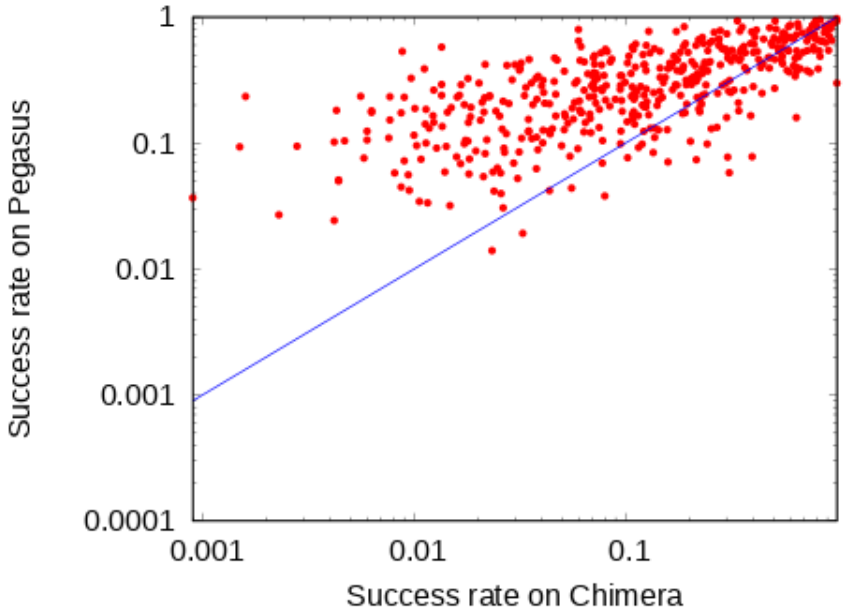
2-SATISFIABILITY: RESULTS

Success rate of the 1000 2-SAT problems for reformulation with g_α=1

All problems



Problems with direct mapping



- Largest enhancement for cases with no direct mapping on Chimera but direct mapping on Pegasus
  ➔ improvement due to increased connectivity

- 476 problems on DW2000Q and 923 problems on Advantage have a direct mapping
- Advantage performs better for a majority of the problems, especially the difficult problems

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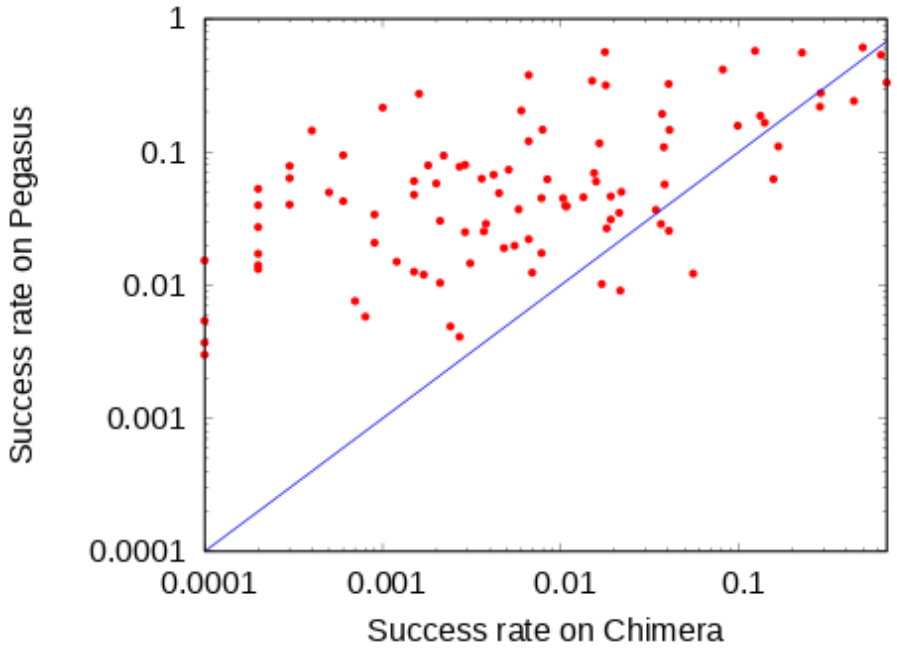
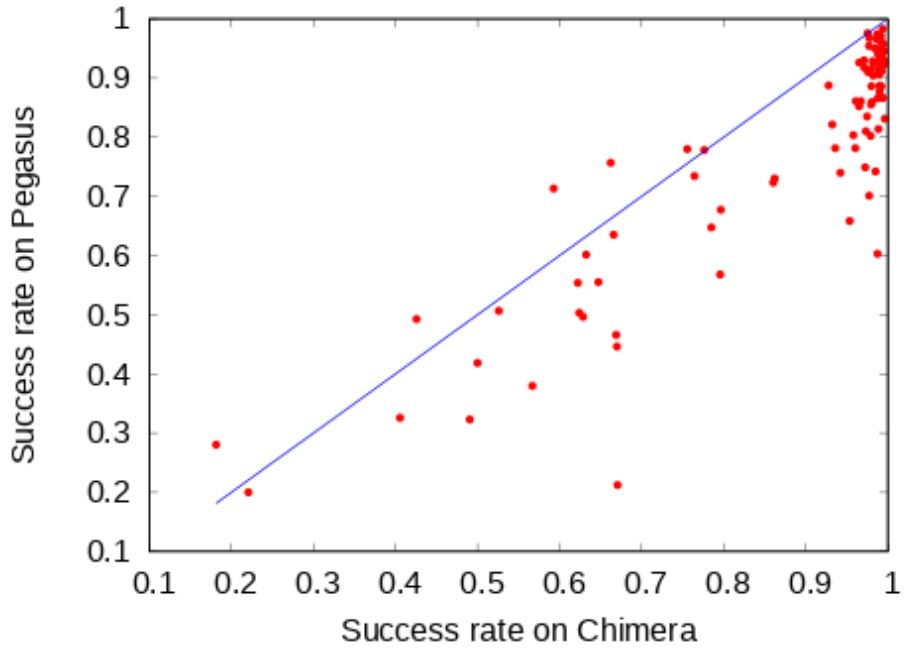
2-SATISFIABILITY: RESULTS

Success rate of 2 chosen 2-SAT problems for reformulation with random g_α

Hard problem



Easy problem



- For a hard problem Advantage performs better
- However, for an easy problem DW2000Q performs better

2-SATISFIABILITY

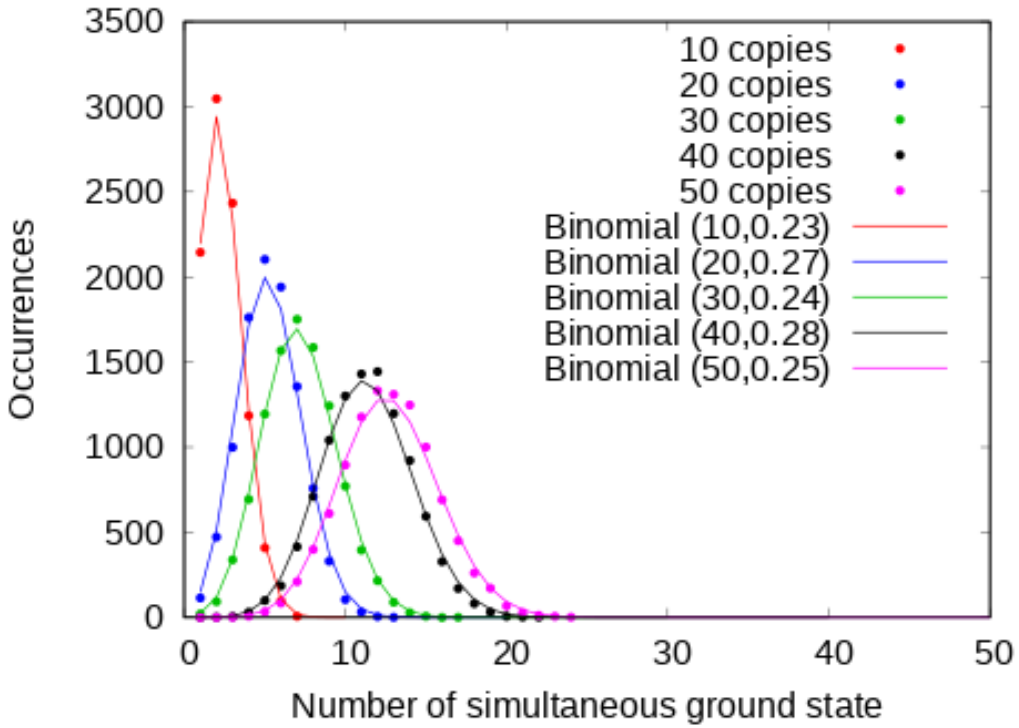
Multiple-copy problem

- To utilize the capacity of Advantage with a large number of qubits, copies of a 2-SAT problem are submitted to the system

- Further, the copies are connected through one bond to form a chain of copies, which becomes the new multiple-copy problem
  - Properties: a unique ground state, which is a combination of the ground states for each copy, for a certain range of bond strengths (< 1.0)

2-SATISFIABILITY: RESULTS

Multiple-copy problem (unconnected)



- Number of (logical) qubits ranges from 180 to 900
- The occurrence of number of simultaneous ground states approximately follows binomial distribution
  - Note that no combined state with simultaneous ground states for all copies could be found
- Parts of the solver corresponding to each copy work almost independently of each other

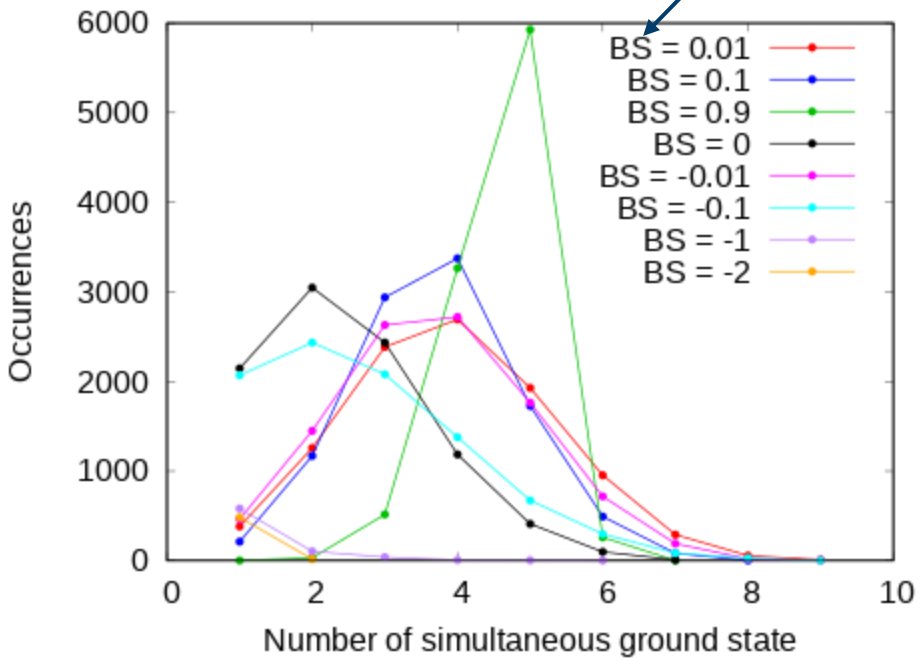
* Total number of samples = 10000

2-SATISFIABILITY: RESULTS

Multiple-copy problem (connected)

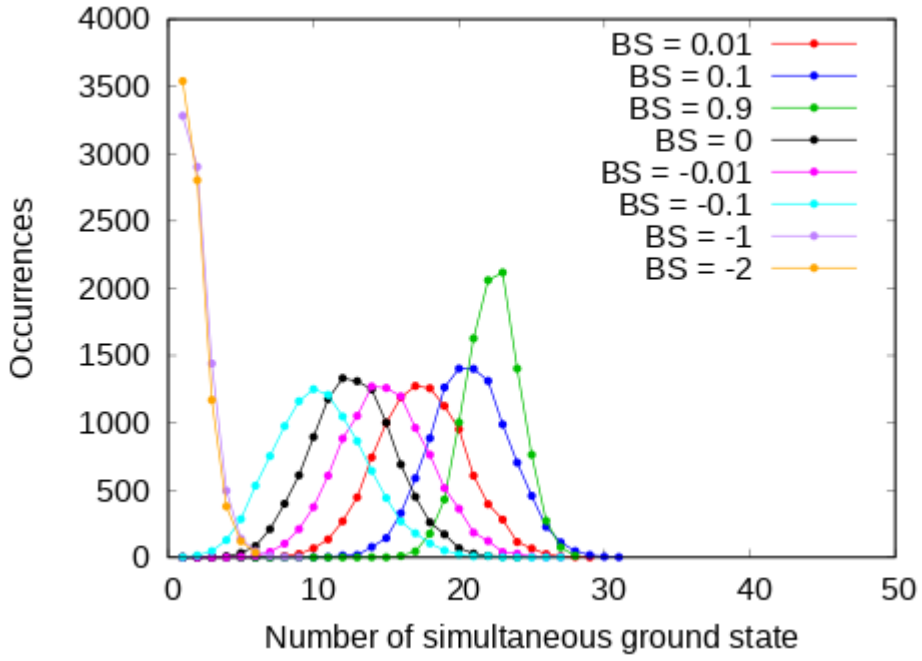
BS = Bond strength

10 copies



180 logical qubits

50 copies



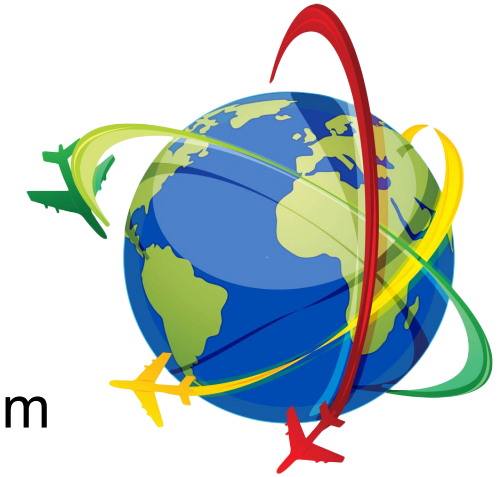
900 logical qubits

* Total number of samples = 10000

- Ground state for the new problems could not be found
- However, Hybrid solver v2 could find the ground state

TAIL ASSIGNMENT

Overview



- Application: Airline planning
- Mathematical formulation: Linear assignment problem

$$\begin{aligned} &\text{minimize} && \vec{f}^T \vec{x} \\ &\text{subject to} && A\vec{x} = \vec{b} \end{aligned}$$

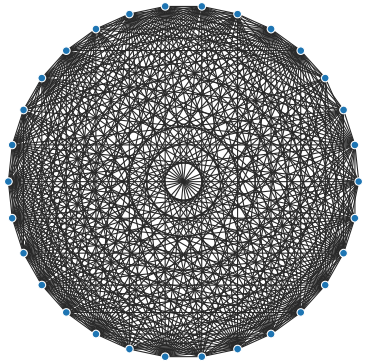
← encodes cost of assigning aircrafts to routes

← encodes constraints: each aircraft ↔ one route

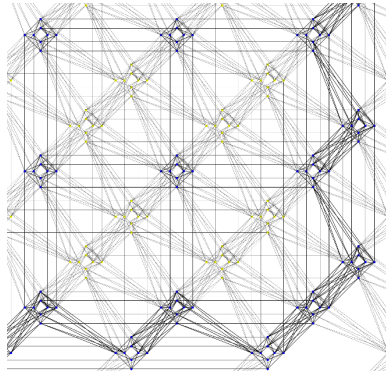
- Reformulation as Ising problem

$$\min_{x_i=0,1} \left(\left(A\vec{x} - \vec{b} \right)^2 + \lambda \vec{f}^T \vec{x} \right) = \min_{s_i=\pm 1} \left(\sum_i s_i + \sum_{i<j} J_{ij} s_i s_j + \text{const} \right)$$

- 25-40 qubits almost fully-connected & 50-120 sparse Ising problems



embedding needed



In collaboration with:
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Jeppesen and
Chalmers University of Technology

BOEING

OpenSuperQ

QUANTUM FLAGSHIP

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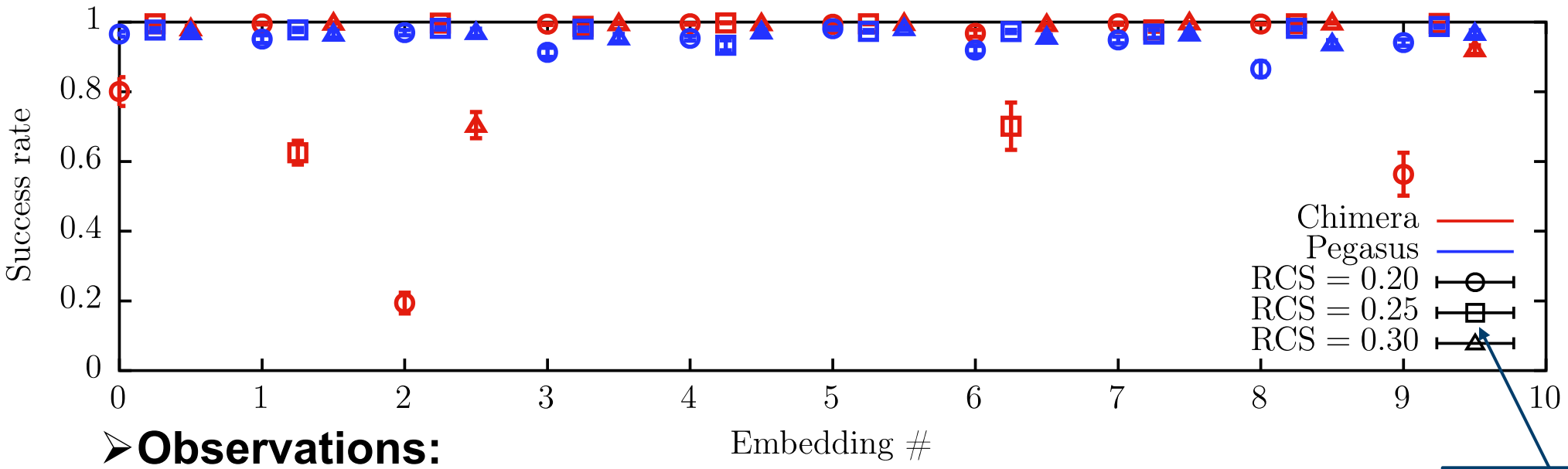
TAIL ASSIGNMENT: RESULTS

25 qubits: “simple” problems



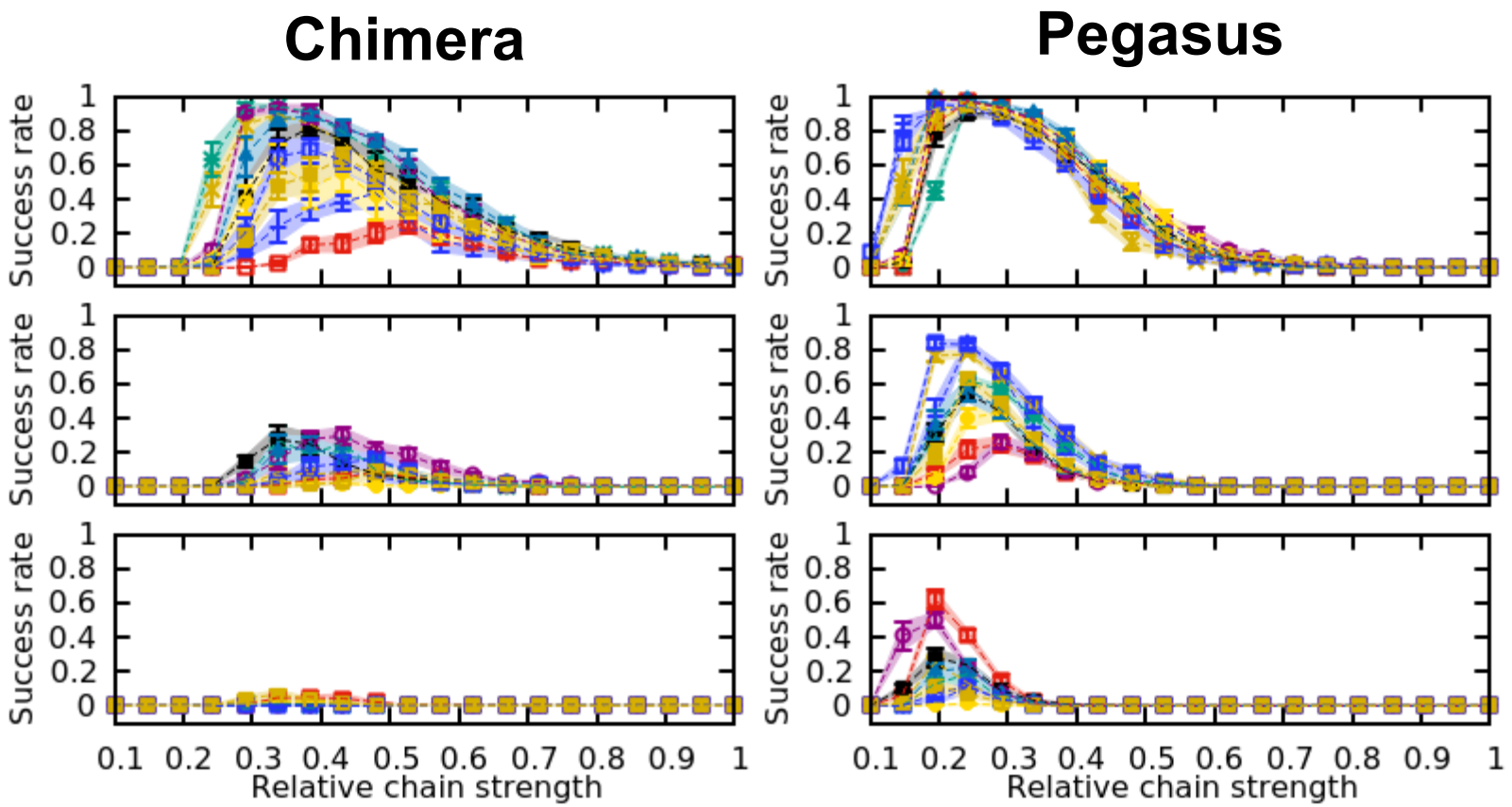
- **Observations:**
  - Embeddings on Chimera sometimes give a low success rate
  - Embeddings on Pegasus have on average a high success rate
  - **However:** if the embedding for Chimera is good, Chimera outperforms Pegasus

    → **Conjecture:** additional unused couplers on Pegasus disturb the annealing process

TAIL ASSIGNMENT: RESULTS

30-40 qubits: “harder” problems (90% nonzero couplers)

Scan of 10 different embeddings and 20 relative chain strengths:



Observations:

- In general, Pegasus performs better than Chimera
- Peak shifted to the left: Optimal chain strength is smaller

TAIL ASSIGNMENT: RESULTS

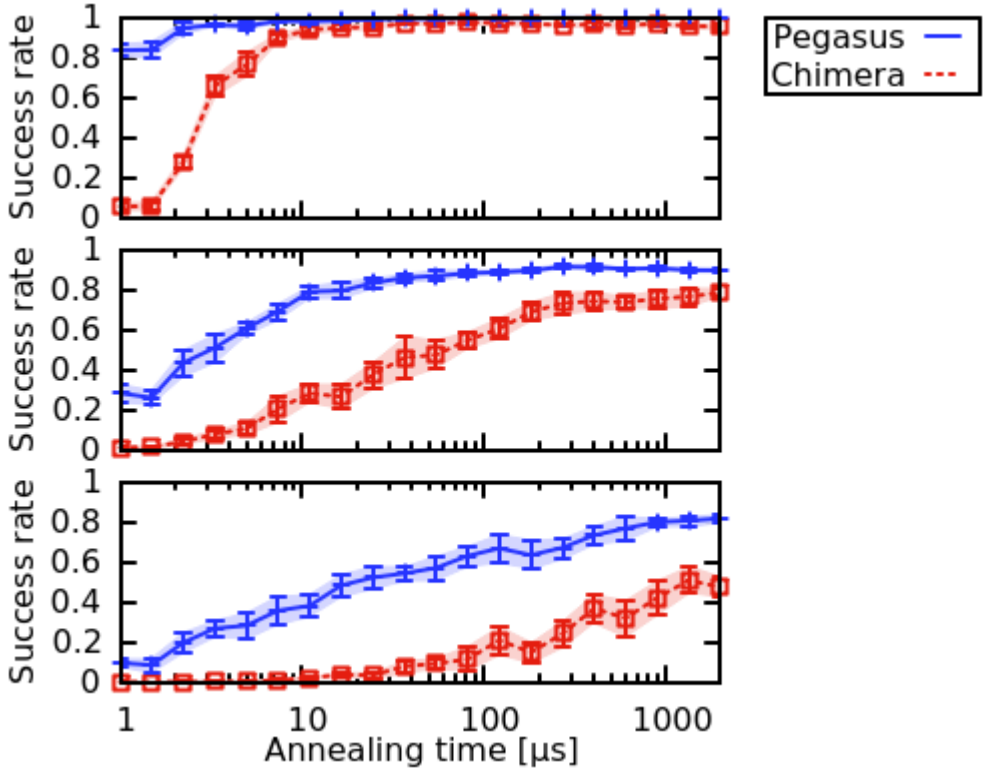
30-40 qubits: “harder” problems (90% nonzero couplers)

For the best embedding and chain strength found: Scan of the annealing time

30 qubits

36 qubits

40 qubits



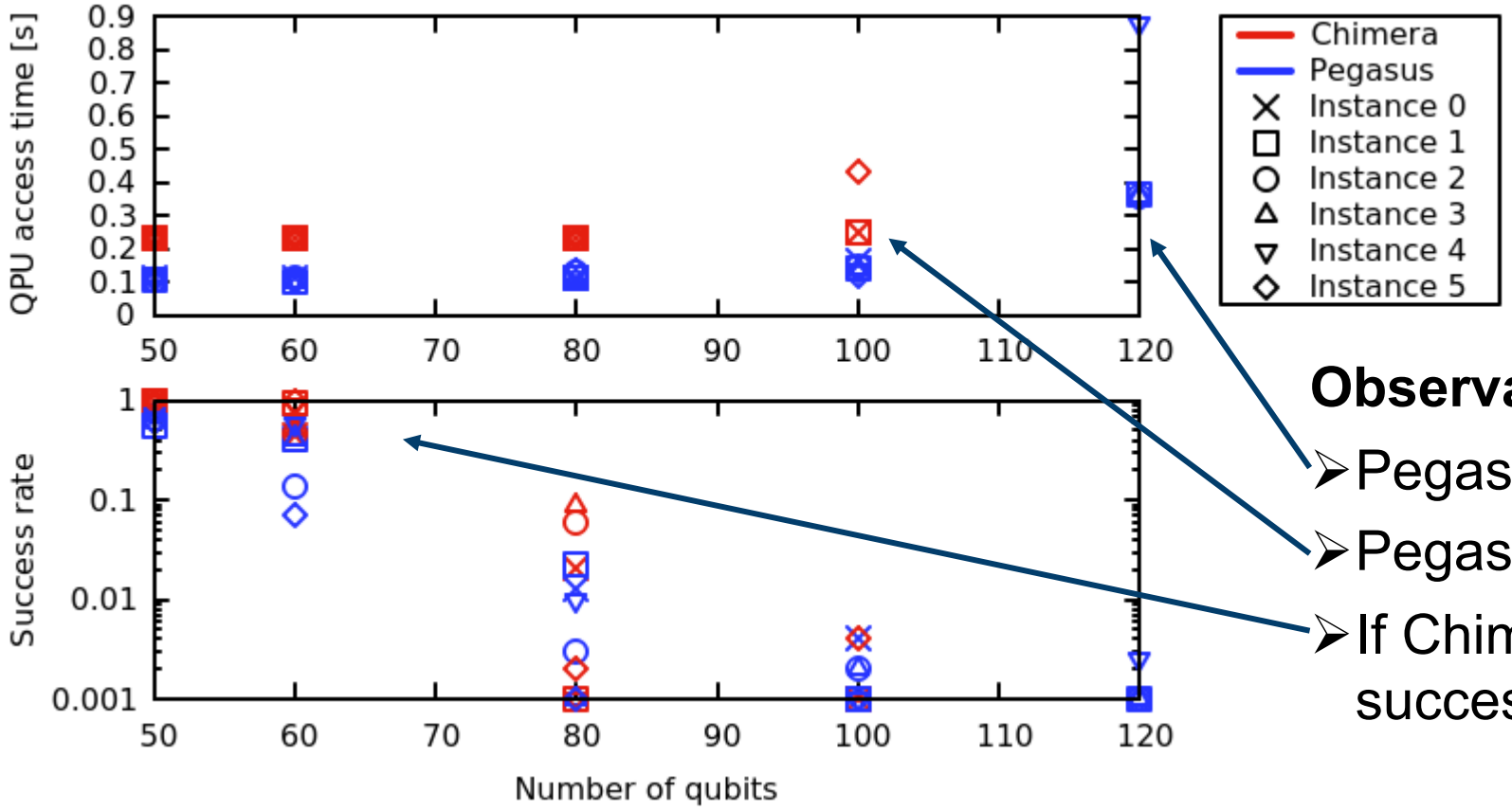
Observations:

- Success rate increases as annealing time is increased
- Larger 30-40 qubits problems have a clear advantage on the D-Wave Advantage

TAIL ASSIGNMENT: RESULTS

50-120 qubits: larger but sparser problems (20% nonzero couplers)

The fastest successful runs that reproducibly gave a solution:



Observations:

- Pegasus solves larger problems
- Pegasus solves problems faster
- If Chimera can solve a problem, the success rate is sometimes higher

CONCLUSIONS

Advantage QPU:

- Problems that are “hard” enough to
  1. make use of the many additional couplers on Pegasus
  2. have embeddings with smaller chain lengths on Pegasus than Chimera

  show a clear advantage on the D-Wave Advantage
- Problems that can be solved on both chips are solved faster on the D-Wave Advantage
- **However:** problems that already fit well on Chimera tend to perform worse on Pegasus

  → Conjecture: additional unused couplers on Pegasus disturb the annealing process

Hybrid solver v2:

- Garden problems: Better and ~10x faster than QBSolv (default parameters); better at smaller problems than HSSv1, but worse at bigger ones
- Hard multiple-copy problems, constructed from the 2-SAT problems, cannot be solved by the D-Wave Advantage, but can be solved on Hybrid solver v2