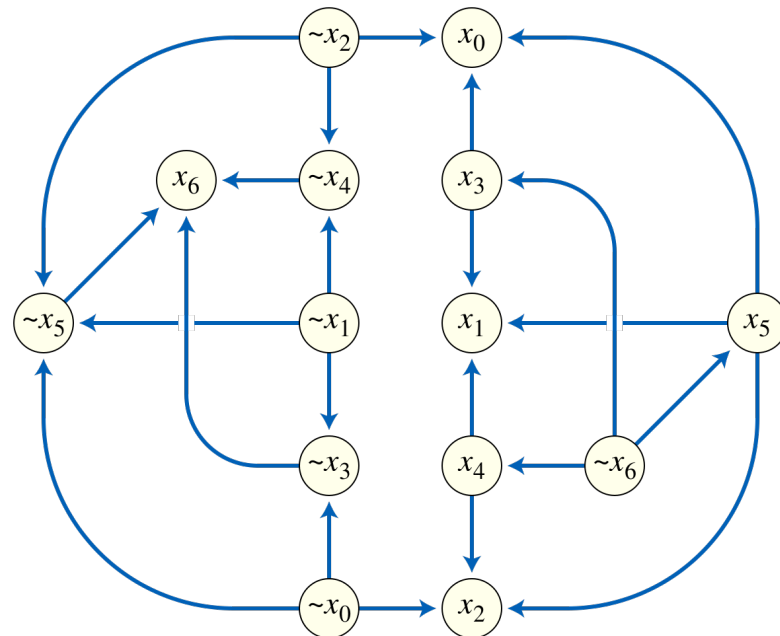




OPTIMIZATION PROBLEMS FOR BENCHMARKING THE HYBRID SOLVER SERVICE VERSION 2 AND THE ADVANTAGE QPU

DWAVE QUBITS 2020, SEPTEMBER 2020 | MANPREET JATTANA

CONTENTS & TEAM

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2. 2-satisfiability
3. Tail assignment
4. Conclusions



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VEGETABLE GARDEN OPTIMIZATION

Overview



	Tomato	Cucumber	Lettuce
Tomato		☹️	😊
Cucumber	☹️		😊
Lettuce	😊	😊	

- Application: Companion planting in polyculture vegetable gardens
- Problem: Find an optimal placement of plants in the garden considering the characteristics of their nearest neighbors

VEGETABLE GARDEN OPTIMIZATION

Overview

➤ Quadratic Assignment

Problem (QAP) with constraints

➤ Assign plants to pots in the garden:

➤ Qubit $x_{ij} = 1$ means species j is placed in pot i

➤ All plants should have a good relationship with their neighbors

➤ Use all available plants and pots

➤ Big plants shouldn't shadow small ones

➤ Flexible problem size

➤ Embedding required

minimize

$$\sum_{i,i'}^{n,n} J_{ii'} \left(1 + \sum_{j,j'}^{t,t} x_{ij} C_{jj'} x_{i'j'} \right)$$

subject to

$$\sum_i^n \left(1 - \sum_j^t x_{ij} \right)^2 = 0$$

$$\sum_j^t \left(c_j - \sum_i^n x_{ij} \right)^2 = 0$$

$$\sum_{i,j}^{n,t} (i \% 2 - s_j)^2 x_{ij} = 0$$



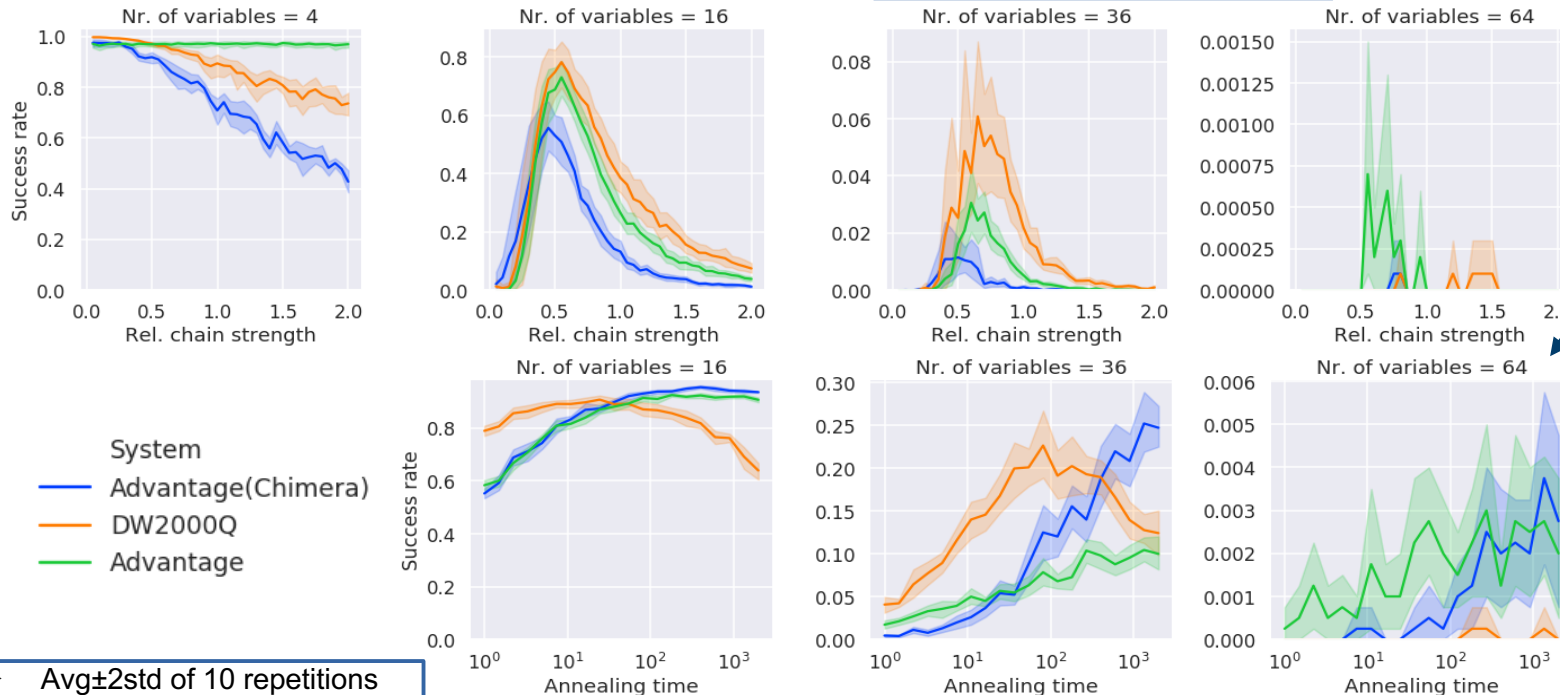
n : nr. of pots in the garden
 t : nr. of plant species to place
 J : connectivity between pots
 C : relationship between species
 c_j : nr. of plants of species j
 s_j : size of species j

VEGETABLE GARDEN OPTIMIZATION

Advantage vs DW2000Q: smaller problems (4 – 256 variables)

- Success rate (SR) under increasing relative chain strength (RCS) and annealing times (AT)
- Influence of Advantage's extra couplers on success rate:

- Chimera embedding on Advantage
- Chimera embedding on DW2000Q
- Pegasus embedding on Advantage

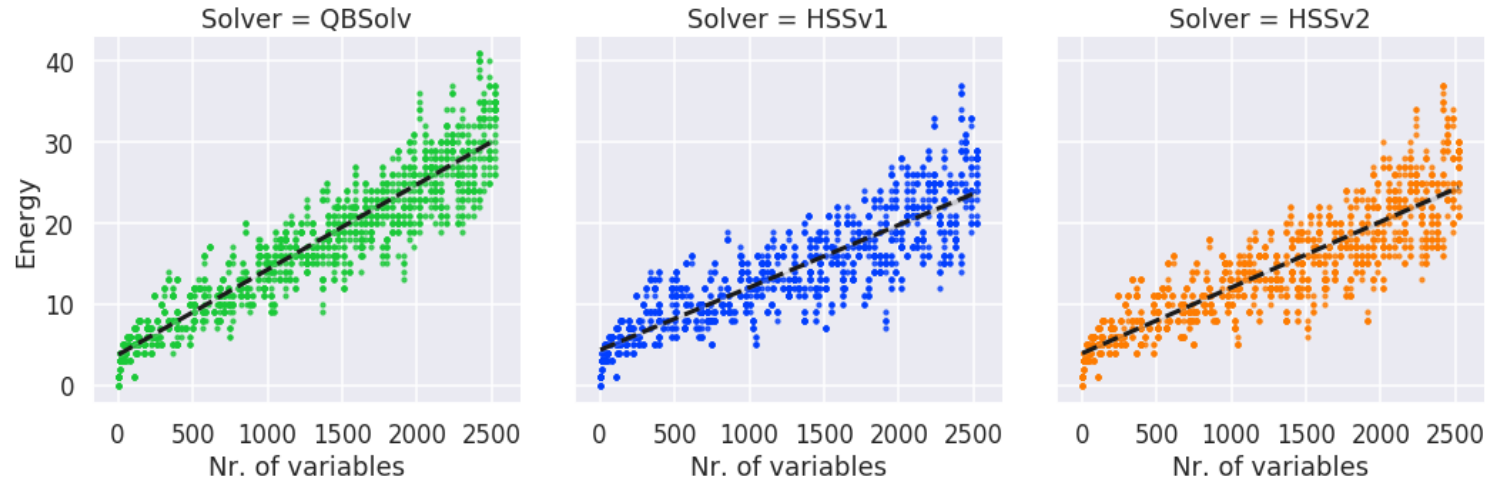


Observations:

- Embedding up to 196 (64) vars possible on Advantage (DW2000Q)
- Successful up to 64 vars
- Tuning RCS increases SR
- Tuning AT increases SR (esp. on Advantage)
- ≤ 36 vars: similar embeddings in Chimera and Pegasus → DW2000Q better
- 64 vars: shorter embeddings on Pegasus → Advantage better

VEGETABLE GARDEN OPTIMIZATION

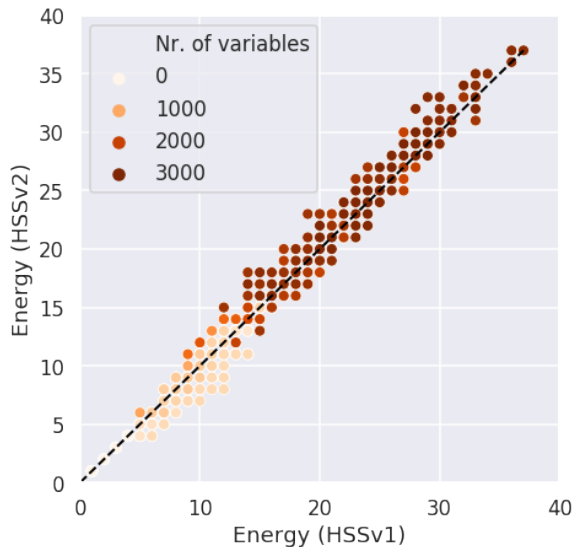
Hybrid solver service (HSS) v1 vs v2: bigger problems (4 – 2520 variables)



Experiment:

1760 problems solved using:

- **QBSolv** (legacy software solver)
- **HSSv1** (current hybrid solver)
- **HSSv2** (new hybrid solver)



Timing for 2520 vars problem
with default parameters (n=10)

	Energy	Runtime (s)
Solver		
HSSv1	28.1 ± 0.568	6.396 ± 0.023
HSSv2	29.0 ± 0.471	6.407 ± 0.003
QBSolv*	35.5 ± 0.972	66.056 ± 20.99

* Intel Core i5-4200U CPU @ 1.60GHz (Notebook)

Observations:

- **HSSv1**, **HSSv2** better & faster than **QBSolv**
- **HSSv2** better at smaller problems
- **HSSv1** better at bigger problems

2-SATISFIABILITY

Overview

➤ Mathematical formulation:

$$F = (L_{1,1} \vee L_{1,2}) \wedge (L_{2,1} \vee L_{2,2}) \wedge \dots \wedge (L_{M,1} \vee L_{M,2}),$$

where $L_{j,k} = x_i$ or $\bar{x}_i$ with $x_i = 0,1$

→ find assignment to x_i that makes F true

➤ Reformulation as Ising Problem:

$H_{2SAT} = \sum_{\alpha=1}^M g_{\alpha} h_{2SAT}(\epsilon_{\alpha,1} s_{i[\alpha,1]}, \epsilon_{\alpha,2} s_{i[\alpha,2]})$, where $g_{\alpha} > 0$ is a scaling factor, $\epsilon_{\alpha} = 1$ for x_i , $\epsilon_{\alpha} = -1$ for $\bar{x}_i$

➤ Example: for clause $(x_1 \vee x_2)$, $h_{2SAT} = s_1 s_2 - (s_1 + s_2) + 1$

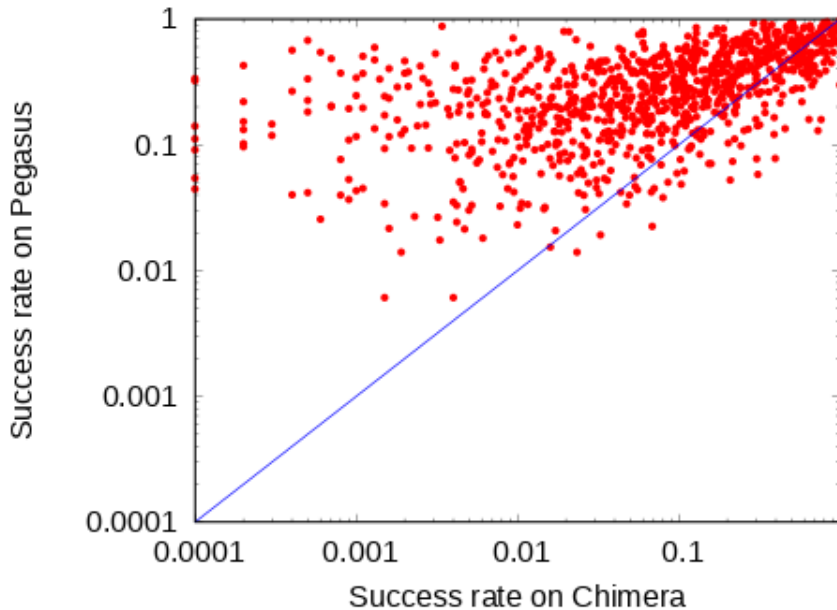
➤ Chosen problems: 18 variables with 19 clauses

➤ Properties: a known unique ground state and a highly degenerate first excited state

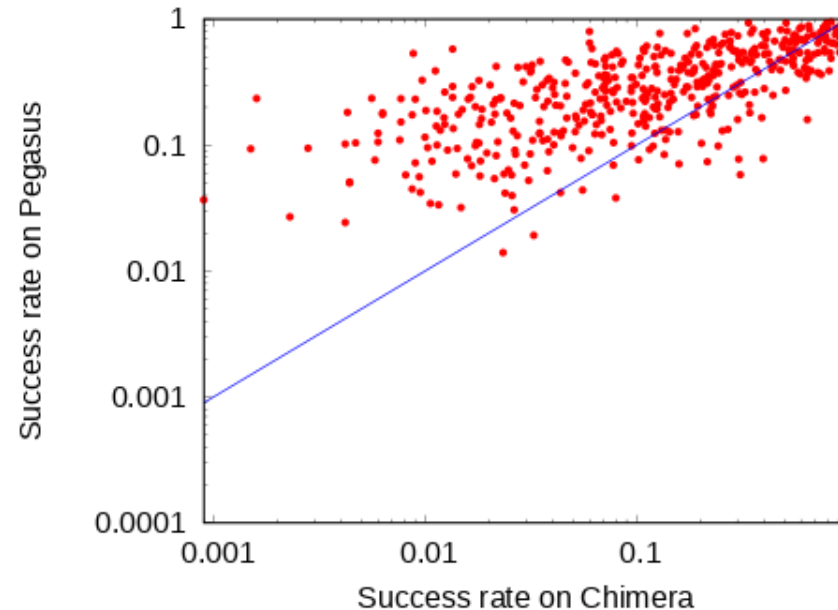
2-SATISFIABILITY: RESULTS

Success rate of the 1000 2-SAT problems for reformulation with $g_\alpha=1$

All problems



Problems with direct mapping



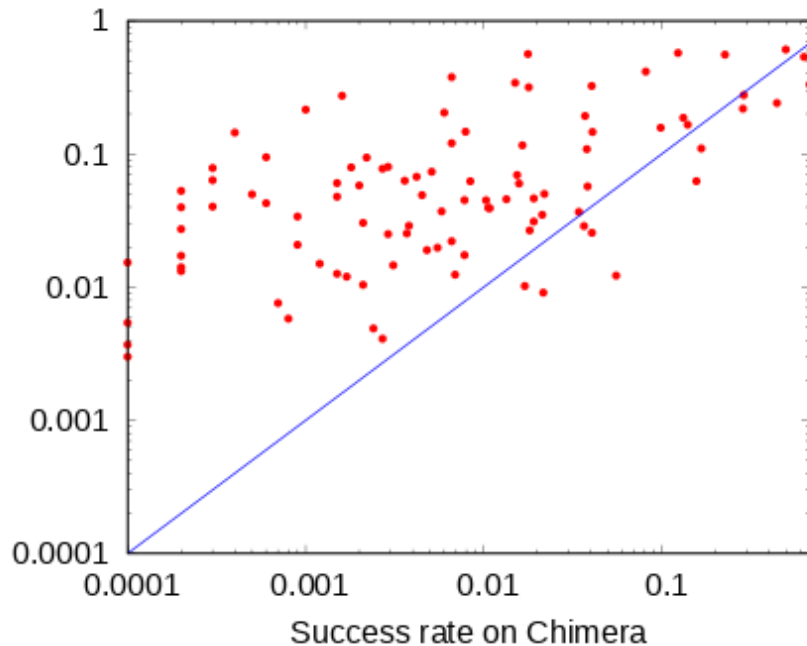
- 476 problems on DW2000Q and 923 problems on Advantage have a direct mapping
- Advantage performs better for a majority of the problems, especially the difficult problems

- Largest enhancement for cases with no direct mapping on Chimera but direct mapping on Pegasus
➔ improvement due to increased connectivity

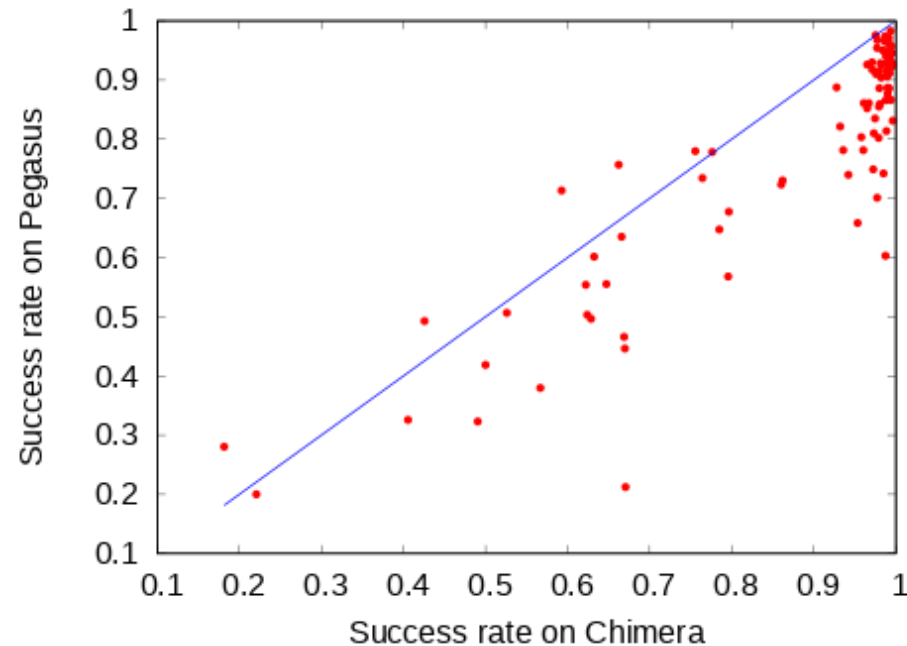
2-SATISFIABILITY: RESULTS

Success rate of 2 chosen 2-SAT problems for reformulation with random g_α

Hard problem



Easy problem



- For a hard problem Advantage performs better
- However, for an easy problem DW2000Q performs better

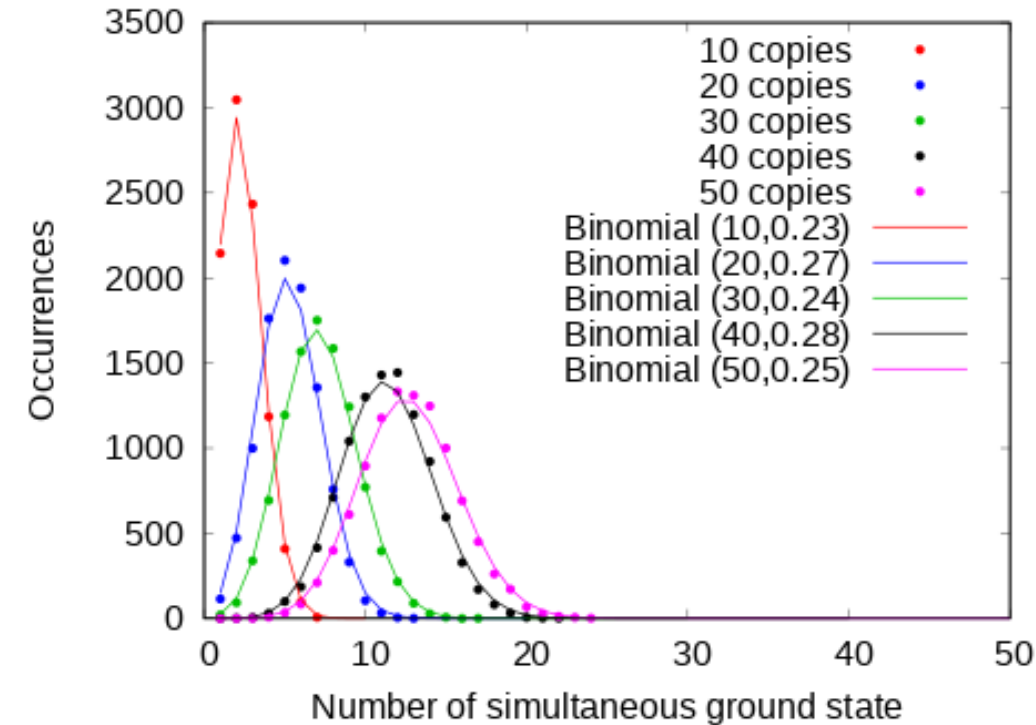
2-SATISFIABILITY

Multiple-copy problem

- To utilize the capacity of Advantage with a large number of qubits, copies of a 2-SAT problem are submitted to the system
- Further, the copies are connected through one bond to form a chain of copies, which becomes the new multiple-copy problem
 - Properties: a unique ground state, which is a combination of the ground states for each copy, for a certain range of bond strengths (< 1.0)

2-SATISFIABILITY: RESULTS

Multiple-copy problem (unconnected)



- Number of (logical) qubits ranges from 180 to 900
- The occurrence of number of simultaneous ground states approximately follows binomial distribution
 - Note that no combined state with simultaneous ground states for all copies could be found
- Parts of the solver corresponding to each copy work almost independently of each other

* Total number of samples = 10000

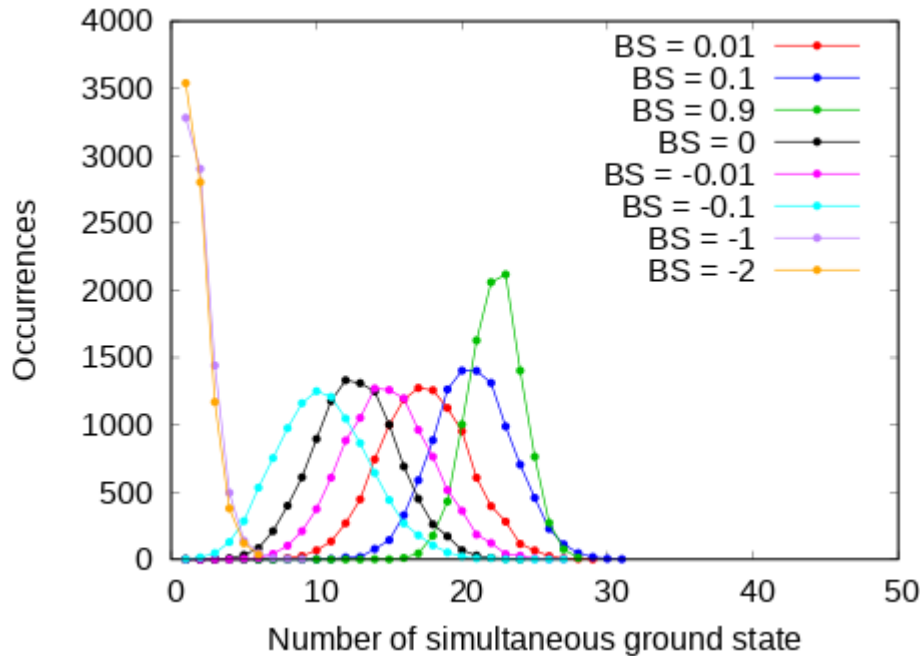
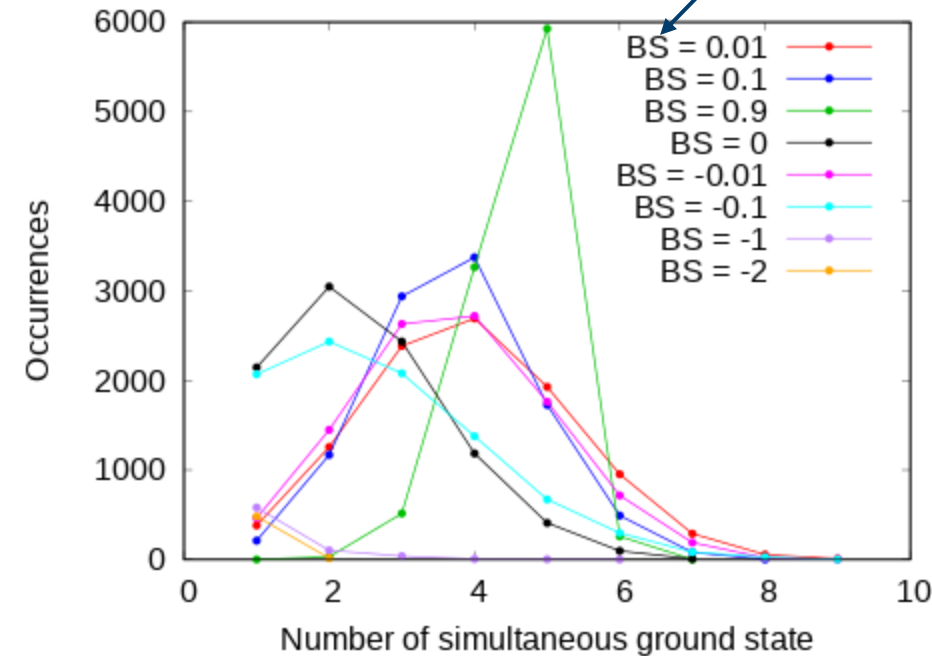
2-SATISFIABILITY: RESULTS

Multiple-copy problem (connected)

10 copies

BS = Bond strength

50 copies



180 logical qubits

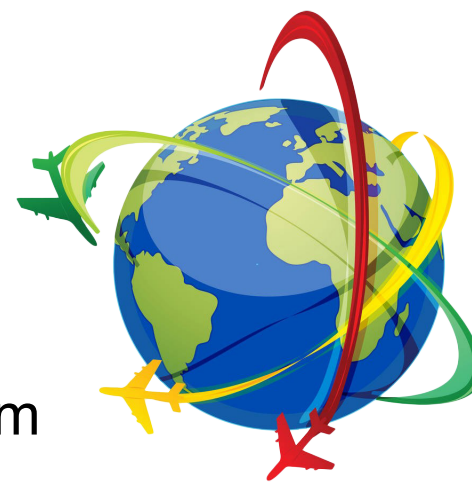
900 logical qubits

* Total number of samples = 10000

- Ground state for the new problems could not be found
- However, Hybrid solver v2 could find the ground state

TAIL ASSIGNMENT

Overview



- Application: Airline planning
- Mathematical formulation: Linear assignment problem

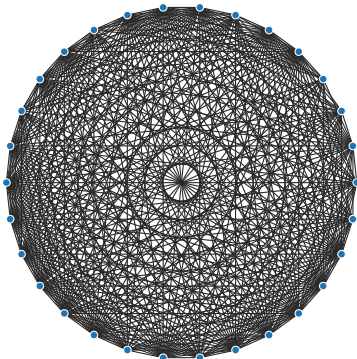
$$\begin{array}{ll} \text{minimize} & \vec{f}^T \vec{x} \\ \text{subject to} & A\vec{x} = \vec{b} \end{array}$$

← encodes cost of assigning aircrafts to routes
← encodes constraints: each aircraft $\leftrightarrow$ one route

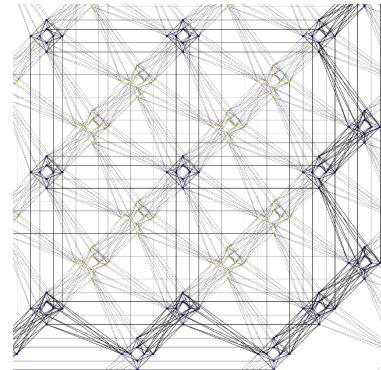
- Reformulation as Ising problem

$$\min_{x_i=0,1} \left(\left(A\vec{x} - \vec{b} \right)^2 + \lambda \vec{f}^T \vec{x} \right) = \min_{s_i=\pm 1} \left(\sum_i s_i + \sum_{i<j} J_{ij} s_i s_j + \text{const} \right)$$

- 25-40 qubits almost fully-connected & 50-120 sparse Ising problems



embedding needed

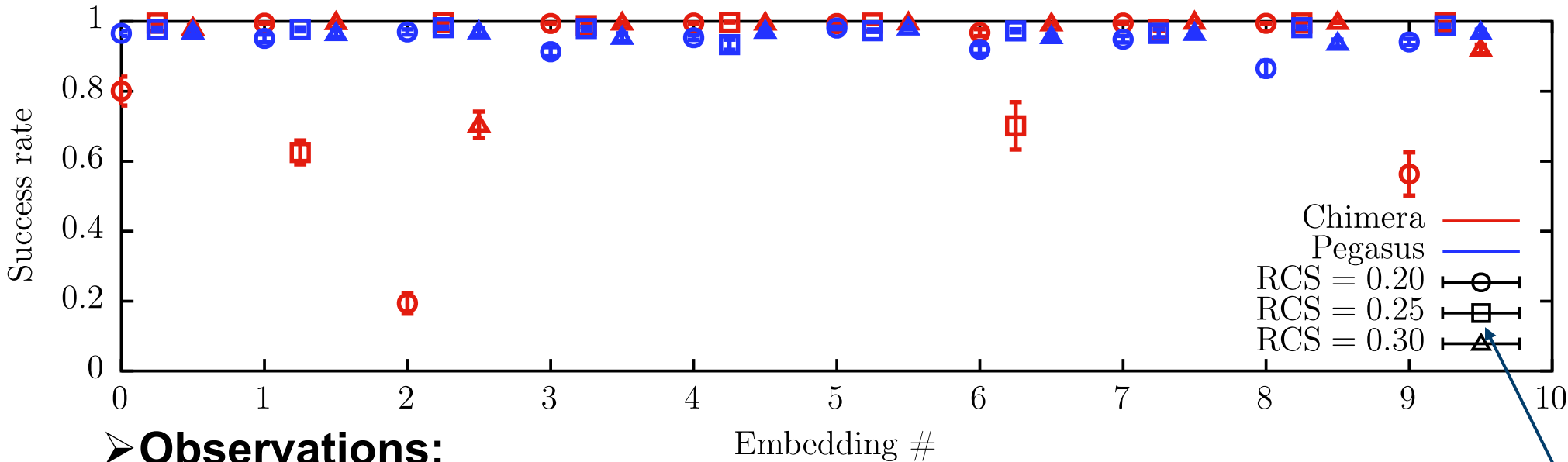


In collaboration with:
Marika Svensson
Jeppesen and
Chalmers University of Technology



TAIL ASSIGNMENT: RESULTS

25 qubits: “simple” problems



RCS = Relative chain strength

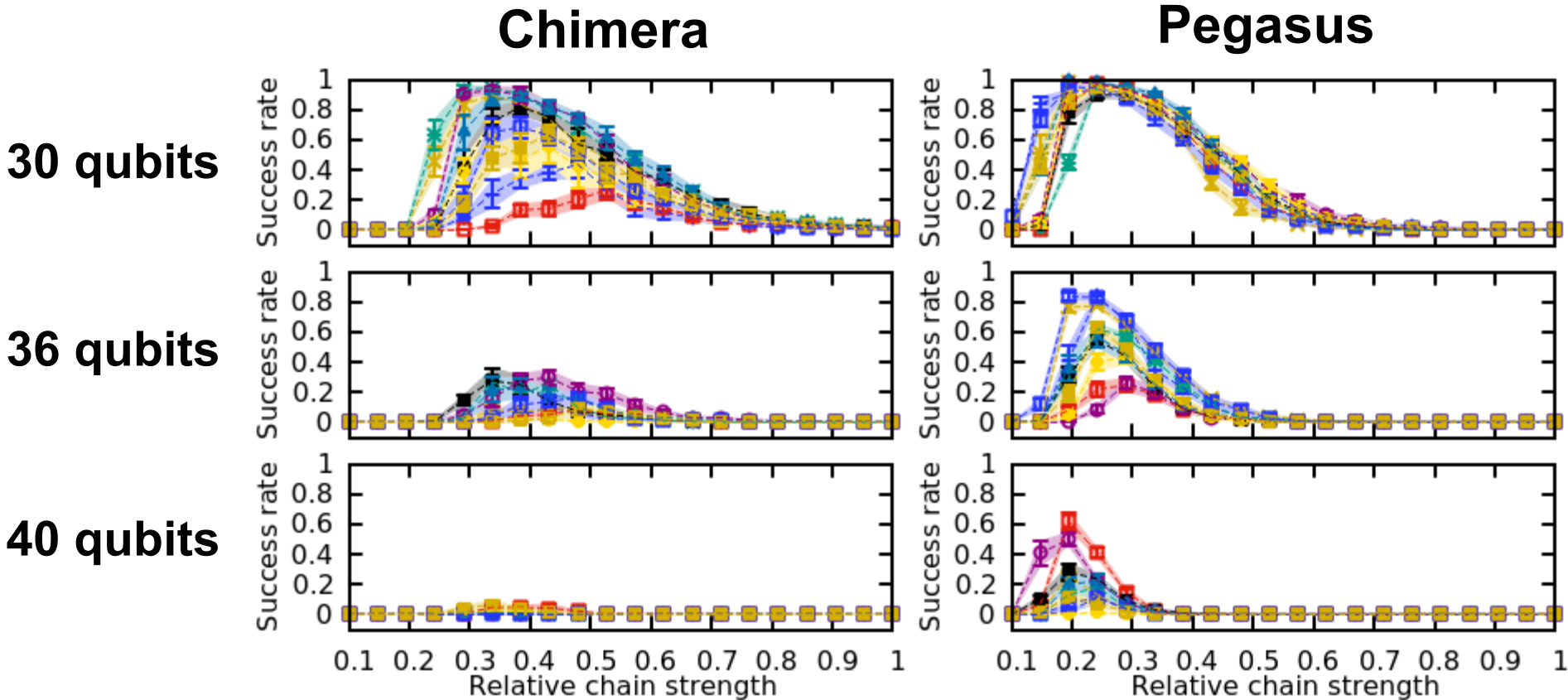
➤ Observations:

- Embeddings on Chimera sometimes give a low success rate
 - Embeddings on Pegasus have on average a high success rate
 - **However:** if the embedding for Chimera is good, Chimera outperforms Pegasus
- **Conjecture:** additional unused couplers on Pegasus disturb the annealing process

TAIL ASSIGNMENT: RESULTS

30-40 qubits: “harder” problems (90% nonzero couplers)

Scan of 10 different embeddings and 20 relative chain strengths:



Observations:

- In general, Pegasus performs better than Chimera
- Peak shifted to the left: Optimal chain strength is smaller

TAIL ASSIGNMENT: RESULTS

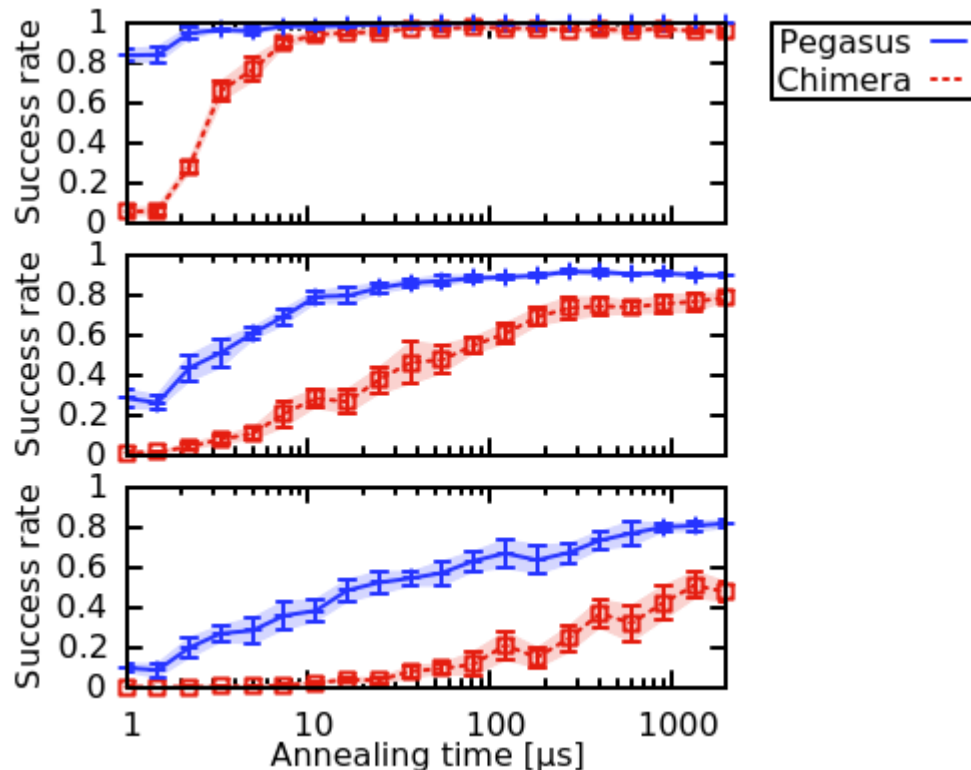
30-40 qubits: “harder” problems (90% nonzero couplers)

For the best embedding and chain strength
found: Scan of the annealing time

30 qubits

36 qubits

40 qubits



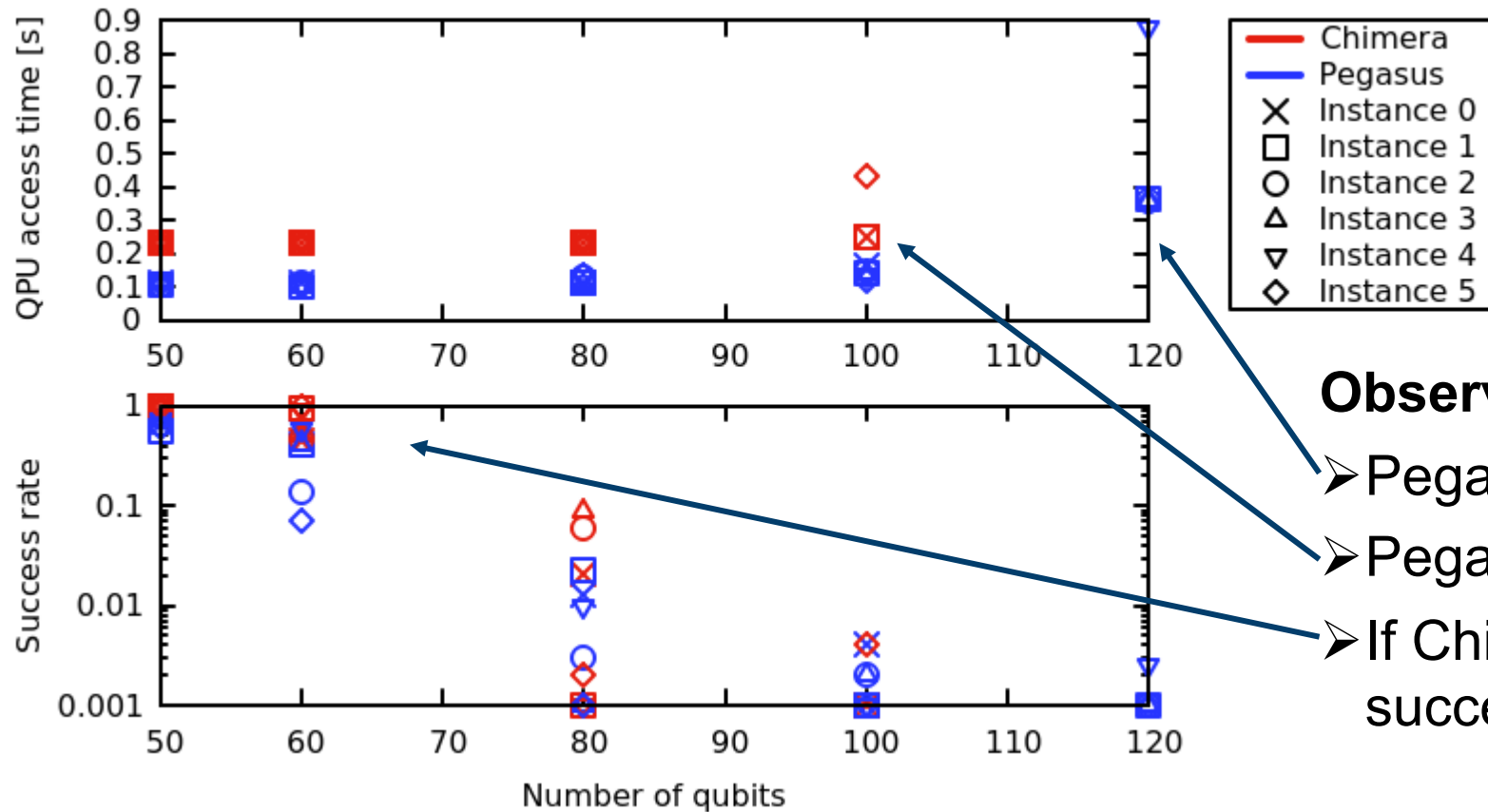
Observations:

- Success rate increases as annealing time is increased
- Larger 30-40 qubits problems have a clear advantage on the D-Wave Advantage

TAIL ASSIGNMENT: RESULTS

50-120 qubits: larger but sparser problems (20% nonzero couplers)

The fastest successful runs that reproducibly gave a solution:



Observations:

- Pegasus solves larger problems
- Pegasus solves problems faster
- If Chimera can solve a problem, the success rate is sometimes higher

CONCLUSIONS

Advantage QPU:

- Problems that are “hard” enough to
 1. make use of the many additional couplers on Pegasus
 2. have embeddings with smaller chain lengths on Pegasus than Chimerashow a clear advantage on the D-Wave Advantage
- Problems that can be solved on both chips are solved faster on the D-Wave Advantage
- **However:** problems that already fit well on Chimera tend to perform worse on Pegasus
 - Conjecture: additional unused couplers on Pegasus disturb the annealing process

Hybrid solver v2:

- Garden problems: Better and ~10x faster than QBSolv (default parameters); better at smaller problems than HSSv1, but worse at bigger ones
- Hard multiple-copy problems, constructed from the 2-SAT problems, cannot be solved by the D-Wave Advantage, but can be solved on Hybrid solver v2