

# Smoothing trajectories of people’s heads

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**Abstract.** This paper compares three methods for smoothing trajectories from controlled experiments to determine the main movement direction: central moving average, spline interpolation of inflection points, and moving convex hull. The often-used method of averaging needs an adaptation due to the velocity but nevertheless is less accurate than the others. The spline method gives best results for high to moderate velocities. The newly introduced method using convex hulls is very robust and offers especially for low velocities good results. Smoothing has a large influence on the calculation of pedestrians’ velocity and thus on the course of the fundamental diagram.

**Keywords:** pedestrian dynamics, experiment, trajectory, smoothing

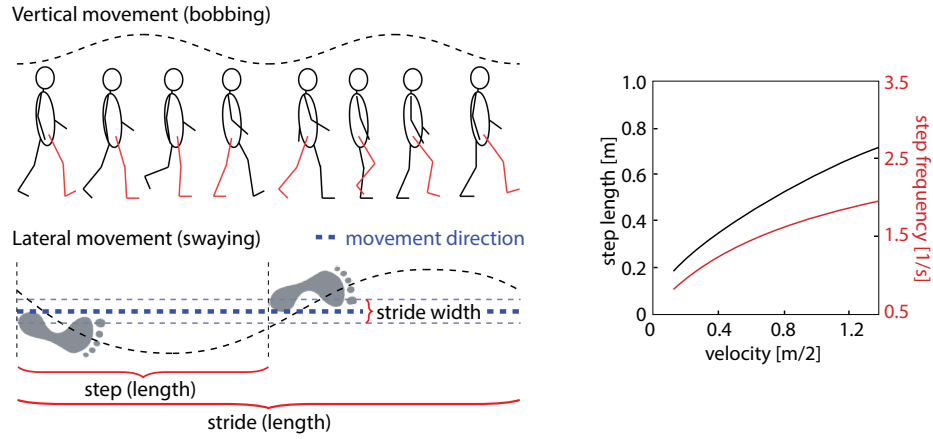
## 1 Introduction

Controlled experiments give the possibility to extract trajectories of the head of each individual person with high accuracy also in dense crowds as long as the head is visible to overhead cameras. These trajectories allow a detailed analysis of the movement, provide a basis for quantifications in legal regulations, guidelines and manuals for the construction of pedestrian facilities and enables the design, calibration and verification of microscopic models.

Caused by swaying and up and down bobbing of human walking these trajectories do not directly show the main movement direction (MMD) also called principal movement. The path of the head is influenced not only by the MMD but also by its bipedal gait and the associated change of the centre of mass (CM) [1]. Physiologically walking is characterized by an inverted pendulum movement in which the CM vaults over a stiff leg with each step [2] so that the path of the CM does not describe the MMD.

The top left of Figure 1 sketches the vertical movement (bobbing) resulting in a height variation for one stride consisting of two steps. In normal walking, the amplitude of vertical CM displacement increases linearly with walking speed from about 0.015 m at low (0.3 m/s) to 0.034 m at high speed (1.6 m/s) [3].

The bipedal gait from one foot to the other with a stride width of 0.05 m to 0.13 m [4] leads also to a lateral movement (swaying) of the head (dashed curved line at the bottom left of Figure 1). The peak-to-peak amplitude (PPA) of the



**Fig. 1.** Sketches of: top left: the vertical movement (bobbing; dashed black line) for one stride consisting of two steps; bottom left: bipedal gait from one foot to the other leads to a lateral movement (swaying; dashed black line) of the head; the thick blue dashed line shows the MMD; right: the dependency between step length and frequency of the gait cycle and velocity

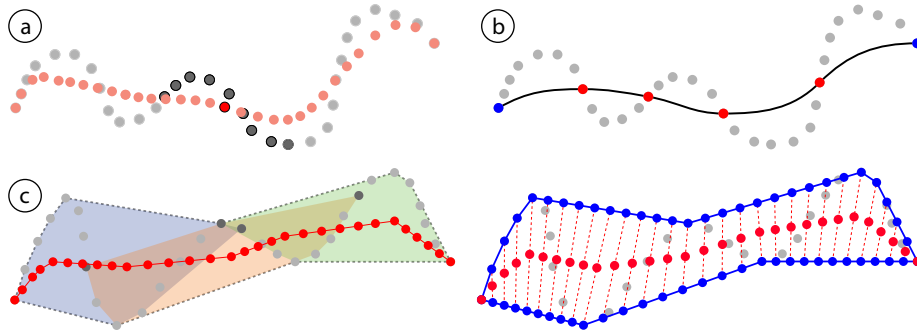
swaying of the head is around 0.05 m for normal walking in free flow condition [5]. For low velocity we see in our data a PPA of up to 0.3 m.

The right of Figure 1 sketches the dependency between step length or frequency of the gait cycle and velocity [6, 7]. Due to the bipedal gait the velocity of body parts is not homogeneous. But for normal walking the head only has a low speed variation of 0.02 m/s [8].

The knowledge of the full body motion enables the understanding of the physical part of pedestrian dynamics as a whole, but for some measures and analysis only derived values lead to a correct understanding. For instance using directly the positions along the trajectory of the head for calculating the velocity of a person leads to an overestimated velocity [9]. Also, lane formation is more difficult to identify, if directly the location of the peoples' head is chosen than the path of the MMD. Microscopic models describing individual movement of pedestrians often represent a person only by a trajectory along their MMD. Thus, to compare simulation results with empirical data on a microscopic level the empirical data has to be smoothed.

This paper describes methods smoothing trajectories of the head to determine the MMD and does not deal with outliers, sudden jerks or a jitter along the trajectory for example coming from an imprecise detection method during field studies without the possibility to use markers [10]. Also colored caps used for detection without structured markers lead to jittering [11]. For these type of errors we refer to methods described in [12], which have to be applied beforehand if necessary.

In the following sections three methods are presented and compared, especially their influence on quantities used for describing pedestrian dynamics.



**Fig. 2.** Sketches of the smoothing methods: a) CMA method forms arithmetic averages (red points) over partial intervals (dark grey points) of the original data (grey points); b) SIP method interpolates the inflection points (red points) by a cubic spline (black curve); c) MCH method uses the envelope of the union of moving convex hulls (colored areas) over a shifting interval along the trajectory; the centre (red points) of the dashed red lines between points on the left and right part of the envelope (blue points) belonging to the same time form the smoothed trajectory

## 2 Methods

**a) Central moving average (CMA)** An often-used method for smoothing data series is the central moving average (see Figure 2 a) also called central rolling average. This method forms arithmetic averages (red points) over partial intervals (dark grey points) of the original data (grey points) equally spaced on either side of the point in the trajectory where the mean is calculated. At the beginning and end of the trajectory the equally spaced interval has to be reduced symmetrically so that the start and end point is part of the resulting smoothed trajectory.

**b) Spline interpolation of inflection points (SIP)** The second examined method chooses the inflection points along the trajectory as knots for a spline interpolation (see Figure 2 b) [13]. As long as there is substantial forward motion the mode of movement is the swinging of the legs which leads to a regular sequence of points of maximum, zero (inflection point, red points), and minimum curvature, which correspond to the times of setting down the right foot, having one foot on the ground while the other just passes the standing leg, and setting down the left foot. Interpolating the inflection points by a cubic spline (black curve) gives the smoothed trajectory. To get a smoothed trajectory for every time of the original trajectory the start and end point of the interpolating spline are the start and end point of the original trajectory (blue points). Furthermore the spline curve is evenly separated between the inflection points corresponding to the number of points in the original trajectory. Because the SIP method is highly susceptible to jittering, a filter usually must be applied beforehand to eliminate these jitters.

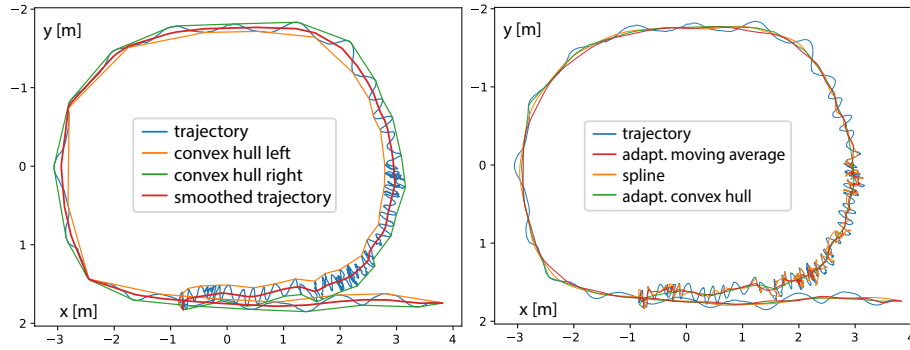
**c) Moving convex hull (MCH)** This new approach for smoothing a trajectory is based on a union of moving convex hulls (see Figure 2 c) over a shifting interval of a defined size along the trajectory. The resulting smoothed trajectory is located between the partial envelopes connected to a single tube around the trajectory. Figure 2 c shows on the left in blue, red, and green three overlapping convex hulls of three time steps with an interval of 16 successive points. The method starts and ends with full sized intervals which automatically includes the beginning and end of the original trajectory (blue and red area). On the right of Figure 2 c the blue line forms the overall envelope of the union of all convex hulls. The left and right part of the envelope is evenly segmented (blue points) between points on the envelope belonging to the original trajectory. The number of additional points corresponds to the number of intermediate points along the original trajectory. The centre of the line between points on the left and right part of the envelope belonging to the same time (red points on dashed red lines) form the resulting smoothed trajectory.

To determine positions to every time step along the spline curve (SIP method) or envelope of the convex hull (MCH method) one could use a projection of the original points to the curve or envelope as well. This would more accurately reflect the velocity of the head within one step in MMD, but other body parts have different progresses of the velocity so that no universal progress can be given and the SIP and MCH method use an even distribution of smoothed points.

All methods can be improved by a preceding step detection to adapt the parameters of the methods. For the CMA method the size of the interval could be set to the length of a stride so that only the period of one stride is continuously contracted. The SIP method would be more reliable, because one could limit the number of inflection points to one per stride located between two steps. The idea behind the MCH method is the construction of a tube just touching the original trajectory at the maximum swaying amplitude. For this the number of successive points of the original trajectory forming a single convex hull has to be the number of points belonging to one stride.

The step detection in video sequences is error prone especially for low velocities. IMU, stereo, or motion capturing systems would give a much more robust detection of steps. Thus, for trajectory extraction from overhead video recordings the approximated velocity can be used to estimate the step length and frequency. The relationship between these variables is sketched on the right of Figure 1. The calculation of this velocity with the CMA method with a fixed large interval covering also low velocities or directly with difference quotients over about one second is reasonable.

Trajectories used for this study have been collected during single file experiments with varying velocity regimes performed with pupils at schools in Germany [14]. The extraction of the data was done automatically from overhead video recordings [15]. For one trajectory (blue) of this data set with varying velocity from stand still to free flow the smoothed trajectory (red) resulting from the MCH method and the appropriate enclosing envelope (green and orange)



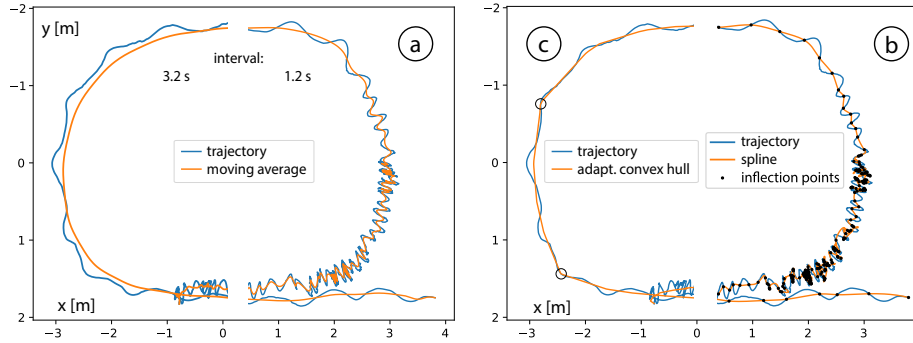
**Fig. 3.** Left: smoothed trajectory (red) from the MCH method and the envelope of the united convex hulls (orange and green); right: smoothing results of the three methods CMA (red), SIP (orange), and MCH (green)

can be seen on the left of Figure 3. The result of all three smoothing methods for this trajectory is shown on the right of Figure 3.

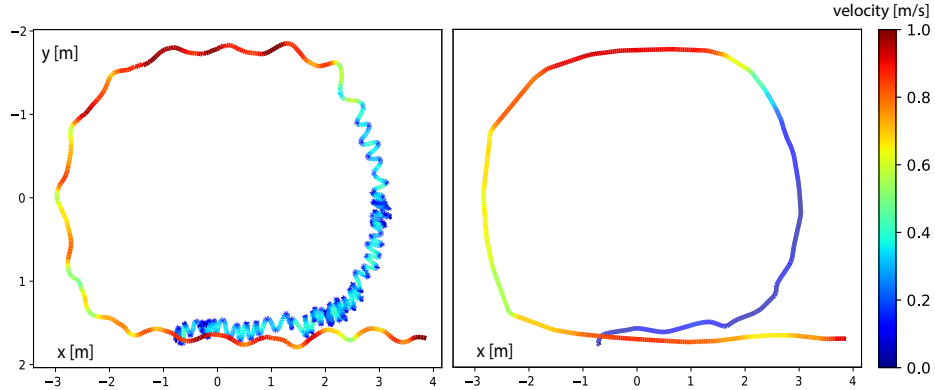
### 3 Results

All three introduced methods have drawbacks for varying conditions. Figure 4 shows some of these constraints.

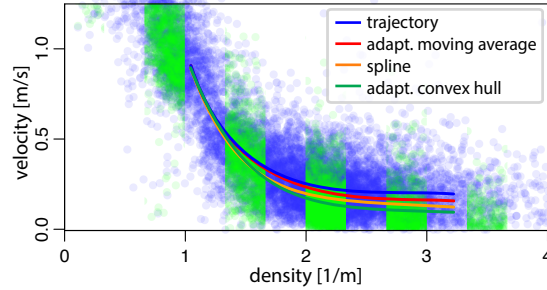
For the CMA method the length of the interval is crucial. A long interval contracts the whole trajectory to their centre. A short interval cannot smooth out the swaying of slow walking. With an adaptation due to the velocity the error is less but still present (see Figure 3).



**Fig. 4.** Constraints of the three smoothing methods a) CMA, b) SIP, and c) MCH; a: depending on the length of the interval (here 3.2s and 1.2s) the smoothed trajectory contracts or is unable to smooth out swaying; b: for low velocities inflection points appear beside the one located between two steps; c: angularity coming from piecewise straight parts of the envelope



**Fig. 5.** Microscopic progression of the velocity along the original and smoothed trajectory (MCH method)



**Fig. 6.** Fundamental diagram as relation between velocity  $v$  and density  $\phi$ : the curves are approximations by a cubic function for  $\phi \in [1, 3.2]$ ; for the original (blue) and the trajectories coming from the MCH method (green) individual values are visualized as underlying stripes for better comparison of the distributions

The SIP method needs a stable walking behaviour with one inflection point while the person is stepping from one onto the other foot. This is not the case for low velocities so that the SIP method cannot smooth out the swaying of slow walking. The limit of the handling velocity is given in the original work with a minimum of 0.3 m/s.

The largest drawback for the MCH method is the angularity of the smoothed trajectory because of a composition of piecewise straight parts of the envelope so that also the smoothed trajectory is a polygonal chain, a connected series of line segments. The length of the interval has to be at least as long as one stride, but too long intervals can mask real changes of the MMD. The choice of the length is not as crucial as for the CMA method and should rather be chosen slightly larger than one stride.

For an quantitative evaluation of the methods following factors are examined on the full data of [14] with varying velocity regimes:

**Distance** between a smoothed and original trajectory should be limited and equally distributed on both sides of the trajectory. The maximum distance is most scattered for the SIP method and has an upper quartile of 0.23 m and a 98.5th percentile of 0.3 m. The upper quartile for the CMA and MCH method is 0.15 m and 0.13 m and the 98.5th percentile 0.17 m and 0.14 m respectively and fits to the observed swaying amplitude.

**Length** difference between smoothed trajectory and the direct path on a straight way should be near zero for single file movement and purposeful walking. The original trajectory shows a median deviation of 0.08 m and all smoothed results approximately 0.02 m, whereby the deviation of the SIP method is a bit more scattered, but the median is the lowest.

**Velocity** along the smoothed trajectory is an important variable, but as mentioned on page 4 there is no universal truth. For measuring the progress in MMD, for example utilized in the fundamental diagram, it is important to use smoothed trajectories as seen in Figure 5 and 6. Otherwise the velocity is overestimated and the swaying has large influence. Despite a typical calculation of the velocity by using a time interval of one second the velocity along the original trajectory is much higher and more scattered for every density value than for all smoothing methods. For example, the average velocity at a density of 2/m is 46% higher than the velocity in MMD using the smoothed trajectories of the MCH method. This relative error increases strongly for higher densities. The CMA and MCH method have the lowest average velocity.

## 4 Conclusion and Outlook

In summary, all smoothing methods are useful for determining the velocity in MMD. The smoothed trajectories should be the basis for the determination of mesoscopic data: average velocities, densities, and fluxes. The derivative of the smoothed trajectory allows a high resolution in time and is less affected by the swaying of heads.

For getting the most reliable results for the path of the MMD, the SIP method gives best results for velocities larger than 0.3 m/s. Below this value the newly introduced MCH method gives a stable path and is more robust than the CMA method. As a recommendation, the best overall result is given by a combination of the SIP and MCH method interpolated when passing the velocity of 0.3 m/s.

For the future one could think of adapted or new methods like using the touch points of the MCH method for determining points of spline interpolation or using frequency analysis.

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