# Tailored time grids for nonlinear scheduling subject to time-variable electricity prices by wavelet-based analysis

Pascal Schäfer,[a] Alexander Mitsos[b,a,c,*]

[a]*Process Systems Engineering (AVT.SVT), RWTH Aachen University, 52074 Aachen, Germany*
[b]*JARA-CSD, 52056 Aachen, Germany*
[c]*Forschungszentrum Jülich, Energy Systems Engineering (IEK-10), 52425 Jülich, Germany*
*amitsos@alum.mit.edu*

## Abstract

Typically, the consideration of nonlinear process models in discrete-time scheduling is limited to short planning horizons and/or coarse discretizations due to a linear scaling of the problem size with the number of considered scheduling intervals. To overcome this limitation, we recently proposed a wavelet-based algorithm focusing on scheduling problems with time-variable electricity prices, which iteratively adapts the time grid (Schäfer et int., Mitsos, doi:10.1016/j.compchemeng.2019.106598). In this work, we extend our approach by presenting a systematic method for the identification of promising initial aggregated time grids based on the analysis of the wavelet representation of the time series of electricity prices. We apply the procedure to a literature example addressing the scheduling of a seawater reverse osmosis (Ghobeity and Mitsos, doi: 10.1016/j.desal.2010.06.041). We demonstrate that substantial reductions in the number of optimization variables in a reduced-space formulation are possible, while furnishing feasible schedules that lead to insignificant deviations below 0.05 % in the objective value compared to the global optimum using the full time grid.

**Keywords**: Demand side management, Discrete-time scheduling, Reduced-space, Global optimization, Adaptive refinement

## 1. Introduction

The adjustment of the electricity consumption to time-variable electricity prices is an important measure to increase the competitiveness of industrial consumers (Mitsos et al., 2018). Consequently, sophisticated methodologies for discrete-time scheduling with time-variables prices have been proposed, mostly aiming at formulating mixed-integer linear programs (MILPs) that can be handled efficiently by state-of-the-art solvers (e.g., Ierapetritou et al., 2002; Mitra et al., 2012; Zhang et al., 2015). In contrast, although many processes are governed by strongly nonlinear characteristics, only few authors tried to consider nonlinear models in discrete-time scheduling (e.g., Ghobeity and Mitsos, 2010), as this leads to nonlinear programs (NLPs) with potentially multiple local solutions. Consequently, solving these problems requires global solution approaches that currently prohibit long planning horizons and/or fine discretizations. To overcome this limitation and allow for nonlinear scheduling with relevant horizons and sufficiently fine discretizations, we recently proposed an iterative algorithm combining three key ideas (Schäfer et al., 2019): a reduced-space scheduling formulation, a time series aggregation, and a wavelet-based grid adaptation procedure.

In this work, we extend our approach by a systematic method to identify a promising initial aggregated time grid. In particular, we perform an analysis of the wavelet representation of the time series of electricity prices to derive the initial grid. The proposed procedure is examined and benchmarked against state-of-the-art solution approaches for a case study. Therein, we consider the scheduling of a seawater reverse osmosis (SWRO) formulated as a mixed-integer nonlinear program (MINLP).

## 2. Case study and solution approaches

### *2.1. Process model and problem description*

We focus on the same case study as Ghobeity and Mitsos (2010), cf. Figure 1. All modeling equations, parameters and operating bounds can be found in their work. The SWRO model comprises eleven variables and ten nonlinear equations. The operation of the SWRO is hence fully determined by specifying one degree of freedom, e.g., the recovery ratio. As in the original reference, we further introduce a disjunction represented by an additional binary variable that allows for shutting down the plant. Discrete-time scheduling of the SWRO consequently corresponds to solving an MINLP with potentially multiple local minima, thus global solution approaches are preferred. We further assume an hourly discretization considering historic time-variable German Day-Ahead spot electricity prices retrieved from EPEX SPOT SE (https://www.epexspot.com/en/). The objective is to achieve lowest electricity costs for fulfilling a given production target, i.e., a fixed cumulated permeate production. Furthermore, the SWRO's operation is constrained by bounds on the key variables: transmembrane pressure, high-pressure pump shaft frequency, recovery ratio and salt concentration in concentrate.

### *2.2. Solution approaches using the full time grid*

When considering the full time grid, i.e., one grid point per hour of the horizon, we apply two different solution approaches for the MINLP. In the first one – referred to as full-space (FS) – all model variables and equations of each scheduling interval are exposed to the optimizer, as it is common practice in the formulation of discrete-time scheduling problems. In this case, model equations simply correspond to equality constraints and operating bounds to box-constraints on selected variables.

In the second approach – referred to as reduced-space (RS) – only a truncated set of

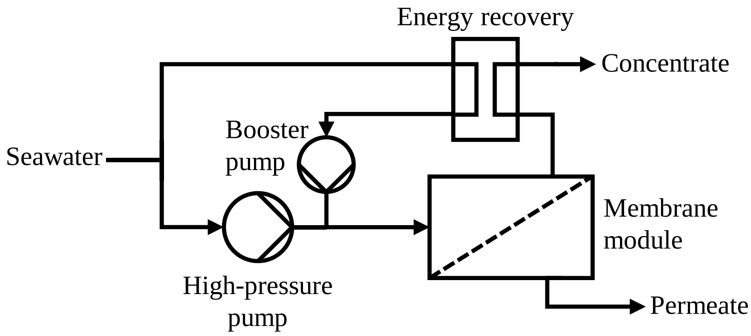


Figure 1: Schematic flowsheet of the considered process configuration for seawater reverse osmosis from Ghobeity and Mitsos (2010).

model variables is exposed to the optimizer; objective and constraints are expressed as functions thereof. This concept has been shown promising for global optimization of process flowsheets (Bongartz and Mitsos, 2017). In the considered case study, we expose two model variables per scheduling interval to the optimizer, although the SWRO's operation would be fully determined by specifying only one, as the model equations cannot be solved analytically. We herein select the high-pressure pump shaft frequencies and the recovery ratios as optimization variables. All other model variables are expressed as explicit functions thereof. One equality constraint per scheduling interval ensures that the selected values of the optimization variables comply with the process model. We remark that, like in the FS formulation, the disjunction introduces an additional binary optimization variable per scheduling interval.

### *2.3. Solution approaches using an aggregated time grid*

Using an RS scheduling formulation allows for the application of our recently proposed time series aggregation scheme (Schäfer et al., 2019), which aims at tailored time grids and thus avoids a global optimization considering the full grid with individual optimization variables in each interval. This is achieved by mapping one optimization variable to multiple intervals with similar electricity prices Thereby, the number of optimization variables in RS is reduced and thus decoupled from the number of considered scheduling intervals, enabling substantial savings in computational time. However, due to the mapping, all scheduling intervals and consequently all constraints are considered further on, ensuring that feasible schedules are furnished. In the computational study below, we make furthermore use of the proposed iterative grid adaptation. Therein, a wavelet transform of the solution from the previous iteration using a coarser grid is conducted and the obtained coefficients are analyzed as proposed by Schlegel et al. (2005), allowing for a systematic adjustment of the mapping procedure by inserting promising new and deleting insignificant grid points.

In this work, we extend our approach by a systematic method to identify promising initial grids for the adaptation algorithm. More precisely, we first perform a wavelet transform of the input time series of electricity prices. Those wavelet coefficients with absolute values above a defined threshold are identified. Then, we use only the set of significant coefficients for the construction of the initial aggregated time grid following the procedure described in our previous work. Note that starting from this initialized grid, the same iterative adaptation algorithm as described above could be applied. However, for illustration purposes, we herein omit this possibility, so that we confine to one single optimization using the initial aggregated grid.

## 3. Computational results

### *3.1. Implementation and solver settings*

The FS formulation (full time grid) is implemented in GAMS version 26.1.0 (GAMS Development Corp.) and corresponding optimization problems are solved globally using BARON version 18.11.12 (Tawarmalani and Sahinidis, 2005) with standard settings. For RS formulations (full time grid, grid adaptation, and grid initialization), the model is implemented as sequential C++ code and global optimizations are conducted using our in-house open-source software MAiNGO (Bongartz et al., 2018). Inside MAiNGO, CPLEX (IBM Corp.) is used for the lower bounding procedure and KNITRO (Exler and Schittkowski, 2007) for the upper bounding. Apart from that, we apply standard settings. For all optimizations, we set the relative optimality tolerance to

0.005. Furthermore, we apply a time limit of 100,000 s of CPU time. For the grid adaptation algorithm, threshold values of 0.0001 for deletion and 0.7 for insertion are used. Concerning the construction of the initial aggregated grid, we apply a threshold value of 0.03. All threshold values are relative to the Euclidean norm of the considered vector of wavelet coefficients. For the grid adaptation approach, three iterations of the algorithm starting from an equally distributed initial grid (6 grid points) are conducted.

### *3.2. Day-ahead scheduling*

First, we consider a day-ahead scheduling, i.e., 24 intervals of one hour, targeting the exploitation of price spreads between day and night. Table 1 summarizes the results for all solution approaches described in Section 2. Note that in all cases, the best feasible schedule is obtained within negligible time. Reported CPU times thus primarily stem from the lower bounding procedures. Due to the good performance of local solvers in the upper bounding even when considering the full time grid (in both FS and RS), the reported solution in this case is considered as the global optimum and thus used as the benchmark for all solutions with aggregated grids. We emphasize that in contrast to the approaches considering the full grid, the approaches using aggregated grids lead to converged solutions within the defined time limit. In particular, savings in computational time when using the aggregated grids are more than two orders of magnitude. Moreover, we find that substantial reductions in the number of considered grid points are possible, while causing only minor deviations in the objective value. For instance, when applying the initial aggregated grid using only eight grid points, a feasible schedule is furnished with a difference in the objective value of ~0.025 % compared to the global optimum. Likewise, the final schedule after the third iteration of the adaptation algorithm starting from an equally distributed grid leads to only ~0.01 % deviations compared to the global optimum by using ten grid points.

Consequently, the corresponding final schedules obtained when using the aggregated grids look highly similar to the globally optimal schedule, as can be seen in Figure 2 (left). In contrast, the intermediate results of the grid adaptation using 6 and 8 grid points respectively lead to inferior schedules that do not make use of the possibility for a temporary shutdown during peak hours. The reason for this finding lies in a distinct price peak at 21 h, which can only be exploited by assigning individual optimization variables to that hour, which is not possible in the first two iterations of the adaptation, as they are limited to aggregating at least four (first) or two (second) intervals.

Table 1: Summary for solution approaches addressing a day-ahead scheduling (24 intervals). Asterisks indicate converged solutions.

| **Solution approach** | **Solver** | **#Grid points** [-] | **CPU time** [s] | **Optimality gap** [%] | **Objective value** [%] |
|---|---|---|---|---|---|
| RS-grid initialization | MAiNGO | 8 | 273 | 0.005* | 100.025 |
| RS-grid adaptation iteration 1 | MAiNGO | 6 | 47 | 0.005* | 100.66 |
| RS-grid adaptation iteration 2 | MAiNGO | 8 | 1,027 | 0.005* | 100.65 |
| RS-grid adaptation iteration 3 | MAiNGO | 10 | 3,055 | 0.005* | 100.01 |
| FS-full time grid | BARON | 24 | 100,000 | 0.006 | 100 |
| RS-full time grid | MAiNGO | 24 | 100,000 | 0.021 | 100 |

Table 2: Summary for solution approaches addressing a week-ahead scheduling (168 intervals). Asterisks indicate converged solutions.

| **Solution approach** | **Solver** | **#Grid points** [-] | **CPU time** [s] | **Optimality gap** [%] | **Objective value** [%] |
|---|---|---|---|---|---|
| RS-grid initialization | MAiNGO | 10 | 22,334 | 0.005* | 100.041 |
| FS-full time grid | BARON | 168 | 100,000 | 0.077 | 100 |

We highlight that we successfully resolve this issue by following the proposed grid initialization procedure. In particular, we thereby a priori identify the most significant parts of the horizon requiring fine discretizations, while relying on coarser discretizations in insignificant parts.

3.3. *Week-ahead scheduling*

We also consider an hourly planning for one week, which allows for further exploiting weekly price patterns, such as lower prices on weekends. For the sake of brevity, we confine ourselves to comparing the proposed procedure for identifying an initial aggregated grid to the FS approach considering the full time grid. A solution summary is given in Table 2. Again, local searches perform exceptionally well, so that the best feasible solution is found in the upper bounding within short time. Thus, the reported solution using the full time grid is again assumed to be the globally optimal schedule.

As in case of day-ahead scheduling, applying the initial aggregated grid from wavelet analysis of the price time series leads to a feasible schedule, while limiting losses in the objective value to <0.05 % compared to the global optimum and schedules look highly similar, cf. Figure 2 (right). Most impressively, this is achieved by using only ten grid points for the scheduling problem, corresponding to a reduction of the temporal dimensionality by 94 %, illustrating the efficacy of the approach for a priori identifying promising tailored aggregated time grids. Moreover, whereas the RS formulation using the aggregated grid results in a converged solution within the time limit, the approach considering the full grid leaves a substantial remaining optimality gap after exceeding the time limit. Comparing Tables 1 and 2 finally illustrates the superior scaling behavior

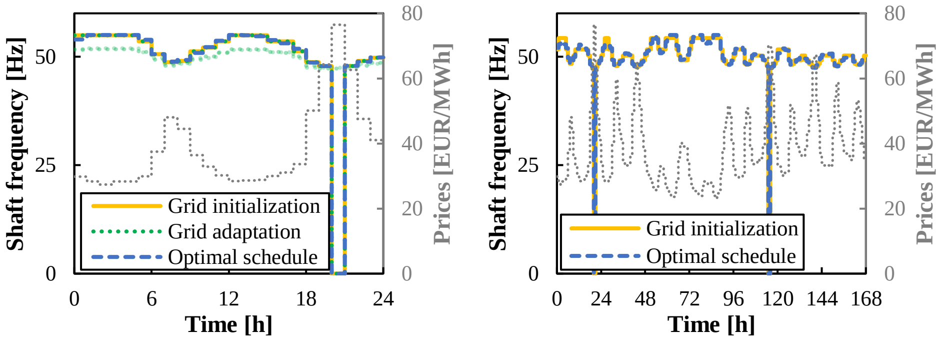


Figure 2: Final production schedules from the different solution approaches for day-ahead (left) and week-ahead (right) scheduling. Orange solid lines: result of a single optimization using an initial aggregated grid following the proposed procedure. Green dotted line: outcome after three iterations of the grid adaptation algorithm when using an equally distributed initial grid (transparent lines correspond to intermediate results). Blue dashed lines: globally optimal production schedule considering the full time grid. Light dotted grey lines: electricity prices.

of solution approaches using aggregated time grids. More precisely, decoupling the number of optimization variables from the number of scheduling intervals avoids the typically exponential scaling with the horizon length when using full time grids.

## 4. Conclusions

We extend our previously proposed algorithm for adaptive grid refinements in scheduling problems with time-variable electricity prices by a systematic method for the identification of promising initial aggregated time grids. The presented case study is suitable for assessing the efficacy of the approach due to a good performance of local solvers on this problem even for long horizons. Our results show that substantial reductions in the number of grid points and hence in the dimensionality of the scheduling problem are possible, enabling promising speed-ups in the optimization, while leading to only insignificant deviations in the objective value. Future work should focus on the application of the procedure to more challenging problems, where generating favourable feasible points is already difficult if considering the full time grid.

## 5. Acknowledgment

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## References

D. Bongartz, A. Mitsos, 2017. Deterministic global optimization of process flowsheets in a reduced space using McCormick relaxations, Journal of Global Optimization 69 (4), 761-796.

D. Bongartz, J. Najman, S. Sass, A. Mitsos, 2018. MAiNGO – McCormick-based Algorithm for mixed-integer Nonlinear Global Optimization. http://permalink.avt.rwth-aachen.de/?id=729717.

O. Exler and K. Schittkowski, 2007. A trust region SQP algorithm for mixed-integer nonlinear programming. Optimization Letters 1 (3), 269-280.

A. Ghobeity, A. Mitsos, 2010. Optimal time-dependent operation of seawater reverse osmosis. Desalination 263 (1), 76-88.

M.G. Ierapetritou, D. Wu, J. Vin, P. Sweeney, M. Chigirinsky, 2002. Cost minimization in an energy-intensive plant using mathematical programming approaches. Industrial & Engineering Chemistry Research. 41 (21), 5262-5277.

S. Mitra, I.E. Grossmann, J.M. Pinto, N. Arora, 2012. Optimal production planning under time-sensitive electricity prices for continuous power-intensive processes. Computers & Chemical Engineering 38, 171-184.

A. Mitsos, N. Asprion, C. A. Floudas, M. Bortz, M. Baldea, D. Bonvin, A. Caspari, P. Schäfer, 2018. Challenges in process optimization for new feedstocks and energy sources. Computers & Chemical Engineering 113, 209-221.

P. Schäfer, A.M. Schweidtmann, P.H.A. Lenz, H.M.C. Markgraf, A. Mitsos. Wavelet-based grid-adaptation for nonlinear scheduling subject to time-variable electricity prices. Computers & Chemical Engineering 132, 106598.

M. Schlegel, K. Stockmann, T. Binder, W. Marquardt, 2005. Dynamic optimization using adaptive control vector parameterization. Computers & Chemical Engineering 29 (8), 1731-1751.

M. Tawarmalani, N.V. Sahinidis, 2005. A polyhedral branch-and-cut approach to global optimization, Mathematical Programming 103 (2), 225-249.

Q. Zhang, I.E. Grossmann, C.F. Heuberger, A. Sundaramoorthy, J.M. Pinto, 2015. Air separation with cryogenic energy storage: Optimal scheduling considering electric energy and reserve markets. AIChE Journal 61 (5), 1547-1558.