

Chapter 1

Orbitronics with uniform and non-uniform magnetic structures

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Abstract

Due to its interdisciplinary attributes, *orbitronics* has spread its branches over different fields in modern condensed matter physics. Its most successful application can be found in manipulating magnetic order originating from both spin or orbital degrees of freedom which has been exploited to fabricate modern fast and low power consuming memory devices. In this chapter we focus on some of the most promising aspects of orbitronics which are driving the recent pioneering researches. We give a brief overview of both theoretical and experimental developments to provide a overview of the evolution of the field. We also analyze different aspects which requires a better understanding and point toward potential future directions which can be helpful in realizing next generation computational devices.

Keywords: Orbitronics, spin-orbitronics, spin-orbit torque, Berry phase, orbital magnetism, skyrmions, Hopfions.

1.1 Introduction

Orbitronics is one of the most diverse field in modern condensed matter physics with an equal impact on both theoretical and experimental community. Broadly speaking, orbitronics deals with the study of electronic orbitals and their interaction with other degrees of freedom such as spin. Due to its widespread nature, it is quite difficult to define the exact domain of this field. In this chapter we are going to provide a brief description of some of the major aspects of orbitronics which are not only at the epicenter of the present studies but also have the potential to revolutionize the future technologies. Based on the nature of magnetism involved, the present researches can be roughly classified into two branches - (i) those using uniform magnetic structures and (ii) those utilising non-uniform magnetic structures. We present these in two sections. The first section focuses on development related to uniform magnetization, namely ferromagnet and antiferromagnets which construct the core of this field and still under immense development. The second section deals with non-collinear magnetic structure which is the heart of the chiral and topological orbitronics and recently have drawn huge attention in the context of next generation computation devices. In each sections we provide a brief overview of present state of art and point toward the possible directions for future studies.

1.2 Orbitronics with uniform magnetization: Ferromagnet and Antiferromagnet

Orbitronics [1] flourished in conjunction with the study of spin-orbit coupling to investigate the possibility of electrical manipulation of magnetic properties or more specifically to manipulate the spin of electrons [2]. Con-

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sequently the term orbitronics is often used as a synonym for *spin-orbitronics*. Initial study of spin-orbitronics was mostly focused on the impact of spin orbit coupling [3] on transport properties such as anomalous Hall effect [4, 5], spin Hall effect [6, 7] to find a mean to generate spin current [8]. The term (*spin*)*orbitronics* therefore is often related the study of anomalous transport properties or Hall effects due to an emergent spin-orbit field which led to new effects such as crystal Hall effect [9, 10, 11] as well as several other kinds of Hall effects like orbital Hall effect [12, 13, 14], skyrmion Hall effect [15, 16] etc. which do not directly rely on spin-orbit coupling. Soon it was realised that the spin current or more precisely the spin accumulation can be utilised to manipulate magnetic moment which gradually found successful application in low power consuming magnetic memory devices [17, 18] known as a spin-orbit torque (SOT) based MRAM or a SOT-MRAM. In this section we are going to focus on aspects of spin-orbitronics regarding uniform magnetization only.

In a SOT-MRAM, the spin current is generated by passing the charge current through a material with strong spin-orbit coupling [19, 20]. The switching torque is generated by non-equilibrium spin accumulation [21, 22] with polarity depending on the direction of the charge current which is an advantage regarding manipulating the switching direction. Using the same principle one can also generate microwave oscillations [23, 24] or the reverse mechanism can be utilised for charge pumping [25, 26, 27]. Research in this field is focused on mainly two directions - (i) understanding the underlying mechanism for efficient spin charge conversion and switching and (ii) looking for materials which possess the relevant interactions. This requires a proper understanding of interfacial physics [28, 29] and also to distinguish the interfacial contribution from the bulk contribution [30] as they follow different physical laws. The recent introduction of topologically non trivial materials have given a new boost in this quest due to their topological connection between bulk and interfacial states. Now a large class of materials like transition metal [31], oxides [32], perovskites [33, 34], topological insulators [35, 36], antiferromagnets [37, 38], Weyl semimetals [39, 40], two dimensional materials [41, 42] and so on are subjected to active study for their spin-orbitronic properties.

1.2.1 Theoretical studies

Theoretical studies in this field are mainly focused on the understanding the interplay between spin and orbital degrees of freedom and resulting non-equilibrium properties as well as new emerging interactions such as Dzyaloshinskii-Moriya interaction (DMI) [43, 44]. The theoretical studies employs a vast spectrum of techniques starting from simple continuum analytic model [45], more involved tight-binding model [46, 47, 48] to rigorous density functional theory (DFT) in both momentum space [49, 50, 51, 52] and real space [53, 54, 55, 56]. Contrary to continuum models, both tight-binding and DFT can deal with extended system which allow them to distinguish between contribution coming from Fermi surface or Fermi sea and thus successfully identify the connection to Berry curvature [57, 58]. The momentum space DFT calculations are mostly done by using maximally localized Wannier states. One of the significant achievement in this regard, is to extend this method over a higher dimensional parameter space [59] which allows to find a deeper connection to mixed Berry curvature (Fig.1.1) [60, 61]. Although being quite close to experimental observation, due to huge computational requirement DFT is not quite successful in dealing with the extrinsic effect like defects and impurities where tight binding methods are still thriving [62, 63].

1.2.2 Heavy metal

Ever since the first observation of room temperature electrical switching of magnetic moment in Pt [64], heavy metals have been the workhorse of SOT devices. The key focus in research with this materials is to improve the efficiency of the charge-spin conversion and the switching mechanism [65, 66, 67, 68]. However, both experimental [69, 70] and theoretical [49] studies show that the driving mechanism behind the generation of the switching field in these systems remains ambiguous. While the Rashba-Edelstein effect [71, 72] is most prominent at the interfaces and causes a field like torque, the spin Hall effect [73] originates from the bulk of the material and plays the key role behind the generation of damping like torque [74, 75]. These two mechanisms not only gives rise to different torques but also respond differently against impurity scattering. Apart from that there can be additional mechanisms such as spin swapping [76, 77] and magnetoelectric effect [57, 58] which may play a crucial role in certain system. A proper analysis of nature and symmetry of the resulting torques [78, 79] in these systems are one of the most crucial aspect of such studies. Active research is now focused on improving the efficiency of these heavy metal based configurations which has been achieved for example, by improved fabrication [80], using interlayer exchange [81], controlled oxidation [82, 83], using ferroelectric material [84], ferrimagnet [85], inserting light metal [86] etc. and shows promising results not only at sub Kelvin temperature but also at room temperature.

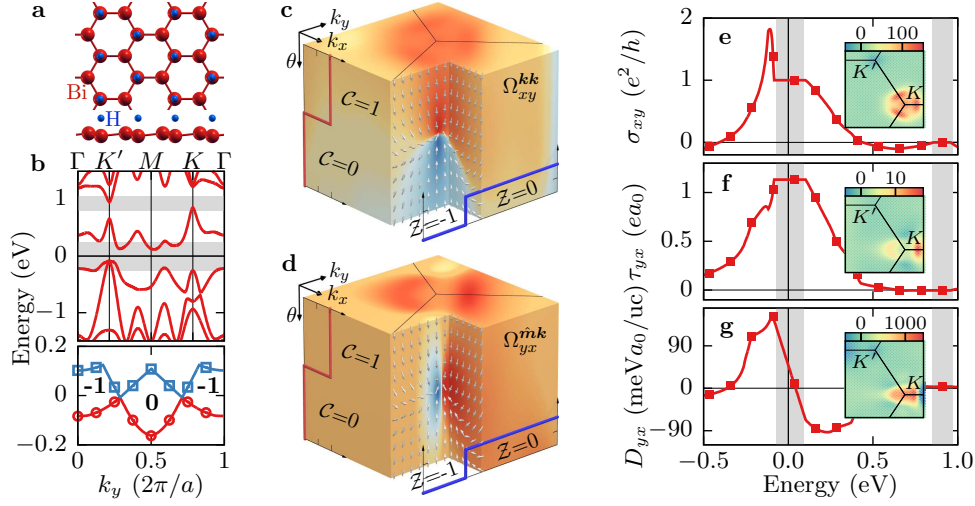


Figure 1.1: Magnetoelectric properties of a mixed Weyl semimetal from DFT calculation [60]. (a) Crystal structure and (b) band structure and Chern number (bold integer) of hydrogenated Bi layer. (c,d) Mixed Berry curvature field (Ω) and mixed Chern number (Z). Energy dependence of (e) anomalous Hall coefficient, (f) anomalous Hall angle and (g) spin Hall angle. [Reprint from [60]]

1.2.3 Topological Insulator

The advent of topological insulator marks a new milestone in spin-orbitronics. Their strong interfacial spin-orbit coupling makes them a very promising candidate for charge-spin conversion and magnetization switching. First observation of efficient magnetization switching with topological insulator Bi_2Se_3 was made by Mellnik et.al. in 2014 [87] which was within less than five years of their experimental [88] discovery. Successive studies [89, 90] showed that not only they possess superior efficiency [91, 92, 93], but it also can be manipulated with an applied electric field [94]. While a standard heavy metal based device requires 10^7 A/cm^2 current density to operate [95, 73], a topological insulator based device can operate at 10^5 A/cm^2 at room temperature [96, 97, 98, 99]. However the source of their superior efficiency is not quite well understood. DFT based studies show that their surface spin-momentum locking is completely destroyed in presence of a magnetic elements [100, 101], yet they produce efficient switching of magnetic moment even in sputtered form [98, 102]. The answer to some of these puzzles are hidden in the connection between their bulk and surface due to their non-trivial topology [46]. As a result the transport properties can be deeply influenced by quantum states which are far below Fermi level [47] (also known as Fermi sea contribution) as well as by the states close to Fermi level (also known as Fermi surface contribution) (Fig.1.2(c)). A proper understanding of these feature requires a thorough study of their electronic properties as well as their connection to their topological properties. A significant amount of study is still under progress not only to understand the underlying mechanism and improve the device, but also in characterizing the materials [103, 104, 105] theoretically.

	$\sigma (\Omega^{-1} \text{m}^{-1})$	$\sigma_S (\frac{\hbar}{2e} \Omega^{-1} \text{m}^{-1})$	θ_S	$J_{SW} (\text{A cm}^{-2})$
$\text{Bi}_x\text{Se}_{1-x}$ [98]	0.78×10^4	1.5×10^5	18.62	4.3×10^5
Bi_2Se_3 [96]	9.43×10^4	0.15×10^5	0.16	3×10^6
Bi_2Se_3 [97]	2.43×10^4	0.43×10^5	1.75	6×10^5
$\beta\text{-Ta}$ [73]	5.3×10^5	-0.8×10^5	-0.15	5.5×10^6
Pt [95]	4.2×10^6	3.4×10^5	0.08	$2.85 \times 10^{7-8}$

Table 1.1: Conductivity(σ), spin Hall conductivity(σ_S), spin-Hall angle (θ_S) and switching current (J_{SW}) for different systems [98].

1.2.4 Antiferromagnet

Study of spin-orbitronics witnessed a new horizon with the introduction of *antiferromagnetic spintronics* [37, 106]. Due to their insensitivity to an applied magnetic field antiferromagnets have not attracted any attention regarding an application in spintronics devices. To manipulate magnetic order parameter using electrical current one needs to produce an alternating non-equilibrium spin component (Fig.1.2(b), [47, 107]) which can move the order parameter rather than the individual magnetic moment. This has been a major obstacle from orbitronics point of view as no anti-ferromagnetic material was known to produce such alternative spin

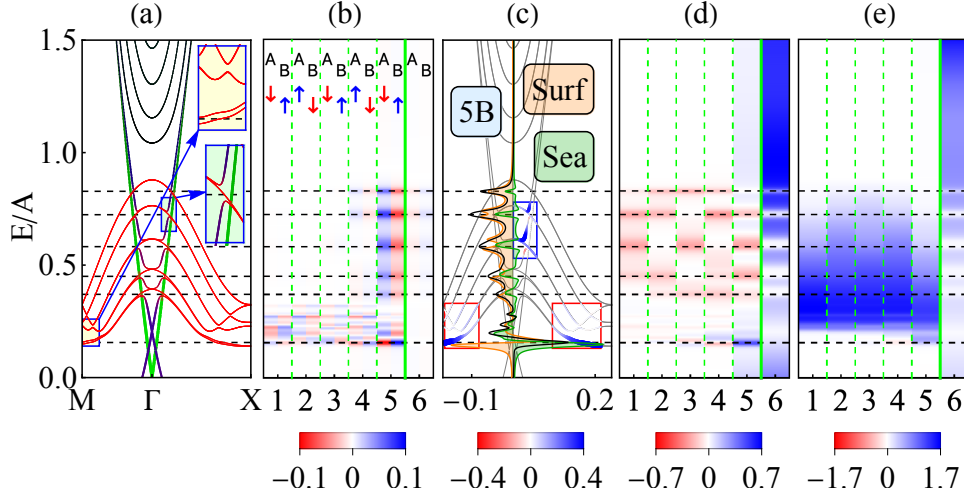


Figure 1.2: (a) Band structure and non-equilibrium (b) S_x , (d) S_y and (e) longitudinal conductance (σ_{xx}) in a 3D topological insulator - antiferromagnet heterostructure. (c) shows the Fermi surface and Fermi sea contribution to S_x coming from B sublattice of first antiferromagnet layer. The blue color denotes the equilibrium S_z component. Adapted from [47]

density. In 2014 Železný et. al. [108] first proposed utilizing spin-orbit torque to manipulate the order parameter of a noncentrosymmetric antiferromagnet which was experimentally demonstrated in 2015 by Wadley et. al. [109] in CuMnAs. In 2016 ultrafast switching is reported in Mn₂Au [110]. In 2017 Olejnik [111] demonstrated the first functional room temperature antiferromagnetic memory. Apart from being immune to an external magnetic field, antiferromagnets also show faster switching speed [112, 113, 114, 111, 115]. These studies gradually extended to a wide range of systems such as Mn₂Au [116, 117], Pt/NiO [118], synthetic antiferromagnet CoGd [119], epitaxially grown collinear antiferromagnet L1₀ – IrMn [120] etc.. The rising interest naturally takes the study to the next stage, namely *Topological antiferromagnetic spintronics* [39] which is closely associated with Dirac and Weyl Fermions [40]. Antiferromagnets naturally captures significant interest in the context of topological insulators because of their ability to preserve the surface Dirac cone. Theoretically such possibility was first described by Mong [121] which was later found to be existed in GdPtBi [122, 123]. Theoretical models [124, 107, 47] shows interesting features regarding electrical switching and robustness against impurity. Recent experiments [125, 126] also shows impressing results from spin-orbitronics in topological insulator antiferromagnet hetero structures. Contrary to topological insulators, antiferromagnets from Weyl family have shown much better performance [127, 128] specially non-collinear antiferromagnets [129, 130]. Electrical switching of antiferromagnetic Weyl semimetal Mn₃Sn has been achieved recently [131] at room temperature with a switching current $\sim 10^6\text{--}10^7\text{ A/cm}^2$. Such progress clearly shows that antiferromagnetic spintronics is one of the most promising and active field in present spintronics community.

1.2.5 Spin-orbitronics in two dimensions

The study of two dimensional materials like graphene, silicene, germanene and the transition metal dichalcogenides (TMDC) like MoS₂, WSe₂ or WS₂ has developed independently due to their potential application in electronics [136]. Compared to theoretical community, initially graphene failed to draw significant attention regarding spintronics application since its inherent spin orbit coupling is quite small [137]. Soon it was found that the spin orbit coupling can be enhanced with proximity effect [138] which can be utilized to improve the spin transport [139, 140, 141]. These new findings gradually led towards *graphene spintronics* [142, 143]. One of the main advantage of graphene over other solid state device is their potential application in flexible devices [144]. In fact curvature [145], strain [146] and twist [147] themselves can give rise to unique properties to their spin-orbit coupling. Besides their honeycomb lattice structure further can couple valleys with their spin degrees of freedom [148] which allow them to show different spin-orbitronic properties in different direction. When a little buckling is added to the planer structure, for example as in silicene or germanene [149, 150], their spin-orbit coupling becomes even more versatile giving rise to several new topological states. Soon, owing to this additional feature, transition metal dichalcogenide (TMDC) dominates over their flat counter parts [151]. Recent studies shows a significant improvement of SOT due to inclusion of TMDC in layered structure [152, 153, 154] which can be further controlled by crystal symmetry [155] or a gate voltage [156]. Another class of materials which presently has captured enormous attention is the two dimensional van-der Waals magnets [157, 135]. Several candidates like MnSe₂, Fe₃GeTe₂, Cr₂Ge₂Te₆, CrI₃ have already been experimentally verified. Their magnetic properties

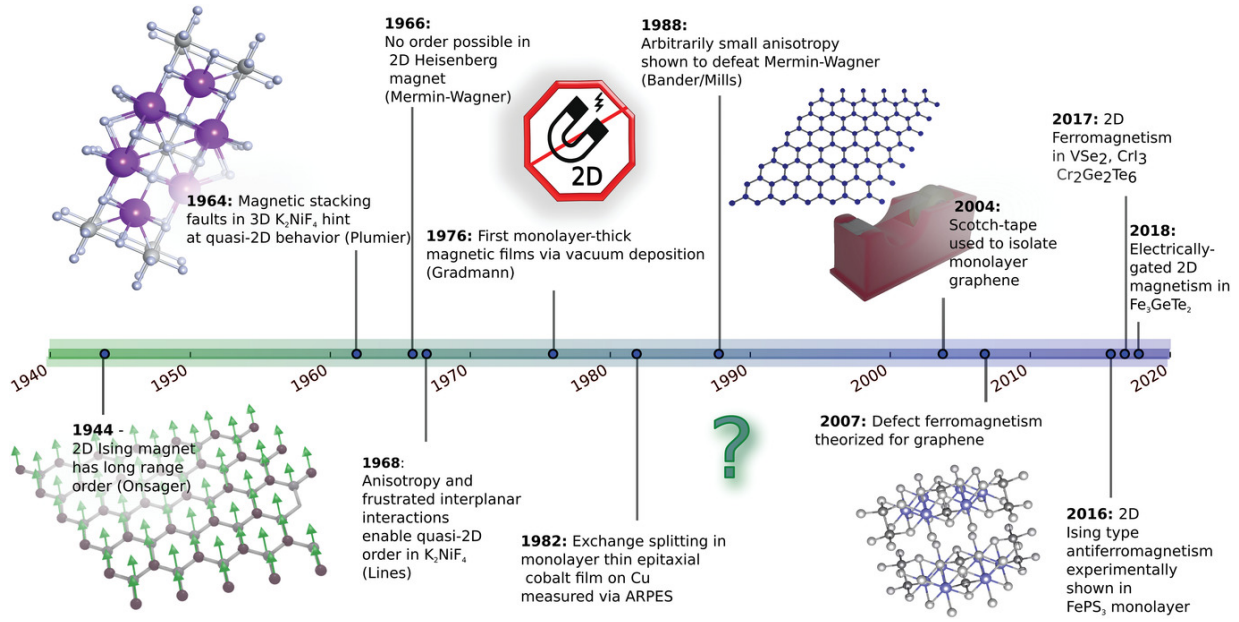


Figure 1.3: Timeline of selected important theoretical and experimental results in the field of 2D magnetism beginning with Onsager's solution in 1944, and culminating in the discovery of intrinsic 2D vdW materials in 2016. (Reprint from [135])

can further be manipulated by using strain [158, 159] or doping [160]. Theoretical [161] and experimental studies [162, 163] have shown the possibility of electrical manipulation of their magnetic properties.

1.3 Future Perspective

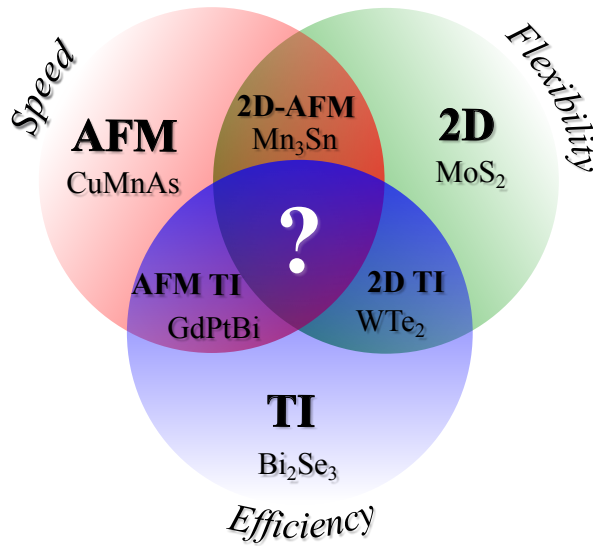


Figure 1.4: Venn diagram for different classes of spin-orbitronics materials.

Beside immense theoretical interest, study in spin-orbitronics is also strongly driven by the requirement of next generation memory device which not only be useful for the present architecture but also establish the cornerstone of the upcoming neuromorphic computing [164, 134]. SOT based devices already have shown great success in realizing artificial neuron [165, 166] and other neural network based computation [167]. Two major objectives in this regards are to find more energy efficient transport and faster switching. These mechanisms are already spotted in different materials even in room temperature. For example, topological insulators show higher charge-spin conversion ratio and already crossed the long anticipated minimum switching current $2MA/cm^2$ by one order of magnitude. Antiferromagnets are quite successful in THz regime. Two dimensional materials on the other hand are favored for their high tunability and ease of integration. That naturally brings us to an

obvious question - can these properties coexist in a single material. For example GdPtBi (three dimensional) and Mn₃Sn are found to retain both non-trivial topological properties and antiferromagnetism. Whether their topological properties can help to attain a magnetic switching at smaller current is yet to be discovered. Besides due to their multilevel response [133] antiferromagnetic memories can be a successful candidate for neuromorphic computing [168]. Search for more antiferromagnetic materials can be quite beneficial for further development in this direction (Fig.1.4). Another very important aspect is to understand how these properties get modified in presence of other materials or impurities. For example, although spin-Hall conductivity is an intrinsic property, it can be manipulated to a great extent by tuning the doping concentration [169, 170]. Realizing these possibilities requires a thorough theoretical study as well. One major challenge here is to improve the present database [171] to identify suitable candidates with a desired nontrivial topology. From an architecture point of view, the only drawback of SOT device is its three terminal configuration which limits its storage density. Recently demonstration of two terminal configuration [172] has taken away this hurdle and open new possibilities for high density SOT memory devices. Switching with sputtered topological insulator and Weyl semimetal [173] is quite promising regarding fabrication since the switching mechanism is robust against structural imperfection. Experimentally surface roughness also has been found to enhance anomalous Hall resistance in platinum [174] which indicates that a detailed study of extrinsic mechanism and its role in enhancing SOT is strongly required from both theoretical and experimental point of view. 2D materials also provide a versatile playground for spin-orbitronics as their properties can be manipulated to a great extent either by tuning substrate, using selective doping or by mechanical deformation. Recent discoveries of two dimensional van-der Waals ferromagnets have opened new possibilities. Search for new materials as well as fabricating new heterostructures has stimulated a large number of ongoing research. Recent studies [175, 176] clearly indicate that the exact nature of the spin-orbit coupling and spin orbit torque in these materials are still under the mist. A more detailed study in this field can open new horizons for next generation spintronics device (Fig.1.5).

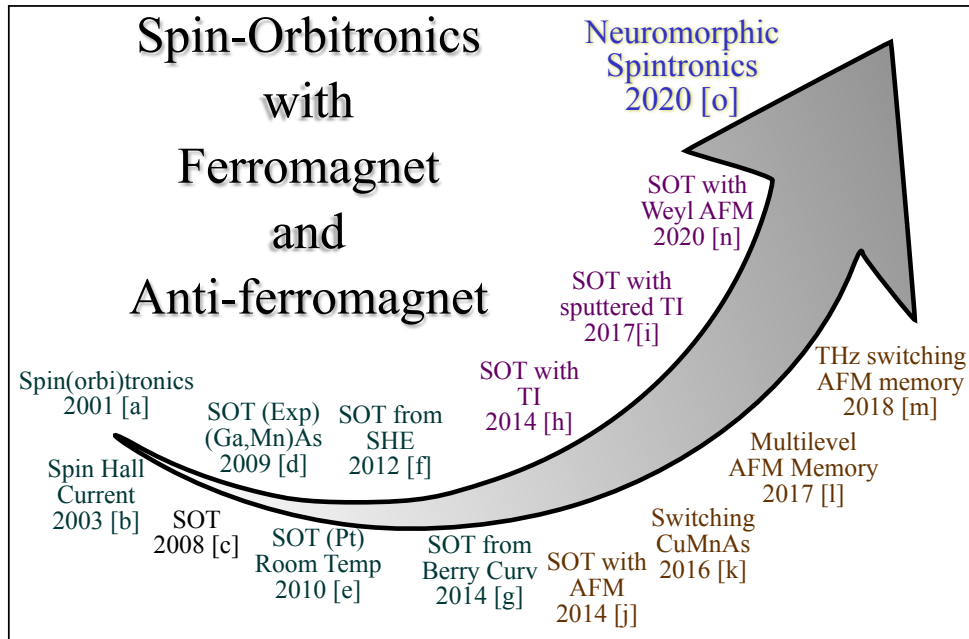


Figure 1.5: Evolution of spin-orbitronics with ferromagnets and antiferromagnets in 21st century. [a][2], [b][8], [c][21], [d][132], [e][64], [f][73], [g][57], [h][87], [i][98], [j][108], [k][109], [l][133], [m][111], [n][131], [o][134].

1.4 Orbitronics with non-uniform magnetization: The emergence of chiral and topological orbital magnetism

Last several years have witnessed significant amount of scientific excitement in the field of spin-orbitronics. Many of the breakthroughs stem from the realisation that the topological phases in real and reciprocal space provide an outstanding effect on spin-dependent transport, such as the topological and chiral Hall effects [177, 178], the anomalous Hall effect [179, 180, 181], and Orbital Hall effect [182]. In addition they greatly influence material properties such as orbital magnetism (OM) [183, 184, 185, 186, 187, 188, 189, 190, 191, 192], which is one of the central concepts in this field. In particular, the advent of Berry phase concepts in condensed matter, which is directly connected to the momentum-space [193, 194, 195] and real-space [193, 194, 195] topology, provides fundamentally new technique to tune OM over orders of magnitude [188] that is crucial to the design of low-

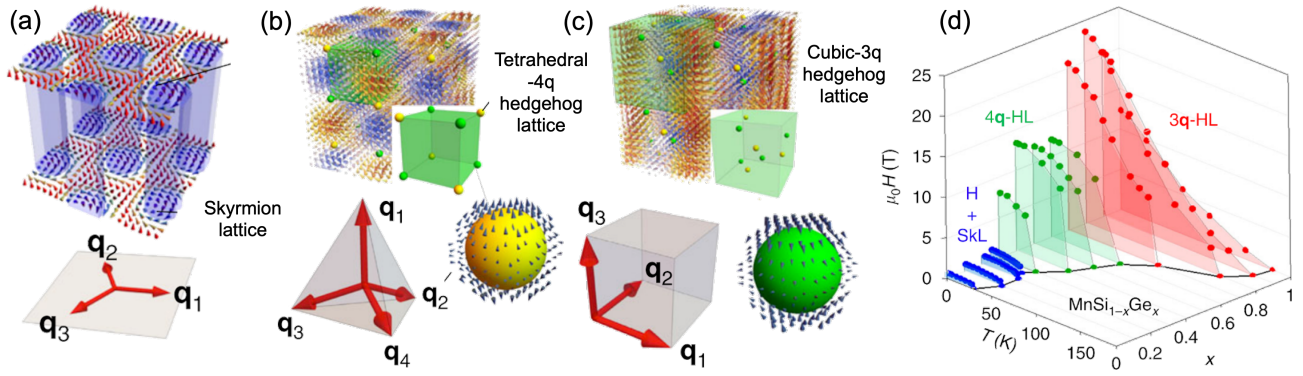


Figure 1.6: Multiple- q spin textures observed in B20 chiral magnets $\text{MnSi}_{1-x}\text{Ge}_x$: (a) 2D skyrmion lattice, (b) 3D tetrahedral-4 q hedgehog lattice, and (c) 3D cubic-3 q hedgehog lattice. (d) Variation of phase boundary between ferromagnetic and helimagnetically ordered states as a function of magnetic field H , temperature T , and x in $\text{MnSi}_{1-x}\text{Ge}_x$. Adopted from [208].

energy dissipation, faster magnetization switching, and to tolerance of disorder [196, 197], as well as to generate efficient spin-orbit torque (SOT) [57, 58, 60]. Similar effect can be produced by the non-collinear magnetism as well which can be used as alternative for SOC to generate spin or orbital current [198, 182].

At the core of these effects lies Berry phase which can be viewed as an emergent electromagnetic fields, that affect motion of the electrons in solids and thus give rise to large variety of the topology-related phenomena. Such phases are distinguished by Berry curvature Ω that arises in momentum-space due to specific band crossings acting as the magnetic monopoles (i.e. the sources or sinks of the gauge field) or in real-space due to the gauge field produced by the non-coplanarity of the underlying magnetic texture [199]. Furthermore, Berry curvatures in real-space Ω^{RR} and momentum-space Ω^{kk} interact with each other via spin orbit interaction (SOI) [200, 195]. Such interaction gives rise to mixed Berry curvature Ω^{Rk} which, as it was shown earlier, can modify the density of states in phase-space and become a primary origin of DMI [201], Chiral orbital moment [188] and Chiral Hall effect [178]. In this section, we discuss the current status of theoretical and experimental works on realizing a novel phenomena that emerge from the topological phases in solids. In particular, our focus is the orbital magnetism, which due its direct connection to the topology in momentum-space [193, 194, 195] and in real-space [202, 203], provides new tools to control magnetism in solids [188, 182] and leads to novel magnetic interactions favoring 3D magnetic textures in the ground state without any assistance of an external magnetic field [190]. Finally, we discuss several open problems and future directions with regard to the role of topology in the presence of nonlinearity.

1.4.1 Real space topology.

The role of real-space topology has attracted considerable attention in the search for complex states in condensed matter. The magnetic domain walls, Vortices, Merons, Skyrmions and recently growing into popularity Hopfions, are the main objects of this field. Magnetic skyrmions, two-dimensional (2D) localized solitons, are prominent and the most studied example, because their specific magnetic texture stabilized by an asymmetric exchange interaction, the so-called Dzyaloshinskii–Moriya interaction (DMI), makes them stable against transformation into trivial states, e.g., the ferromagnetic ground state. Most investigations in the field were focused on materials for which a strong DMI can be expected, fulfilling the requirements of nano-meter size, room temperature stability at zero magnetic field, controllable dynamics and manipulation of skyrmions. Since the first discovery of the skyrmions ten years ago [204, 205, 206, 207], significant progress to fulfil these requirements was made, however, skyrmion-based logic and memory devices are still not in the marker. In this context, novel materials holding mixed non-trivial typology that can be manipulated by electric or magnetic fields, and, as was shown recently, give rise to novel phenomena [190, 188, 182, 178], might be a great alternative to the DMI-based materials.

To substantiate the importance of the non-trivial real-space typology we begin with an example of intensively scrutinized B20 magnet MnGe for which puzzling three-dimensional (3D) skyrmion lattices with size of a few nanometers were observed experimentally [209, 210], but could not be reproduced by any theoretical calculations so far [211, 212]. Such short-range magnetic modulations in MnGe also gives rise to large topological Hall effect and the topological Nernst effect [213, 214]. In addition, as it is illustrated in Fig. 1.6, $\text{MnSi}_{1-x}\text{Ge}_x$ alloy reveals a nontrivial phase transitions between skyrmion, 3 q and 4 q -lattices as function of the chemical composition x [208]. In contrast to the 2D skyrmions of a hundred nm size observed in most other B20 compounds like FeGe , the model based on the exchange interaction and DMI fails to explain 3D textures of such short-period in MnGe and $\text{MnSi}_{1-x}\text{Ge}_x$ alloy [215, 211, 212]. Therefore, understanding of the nature of the magnetic interactions that

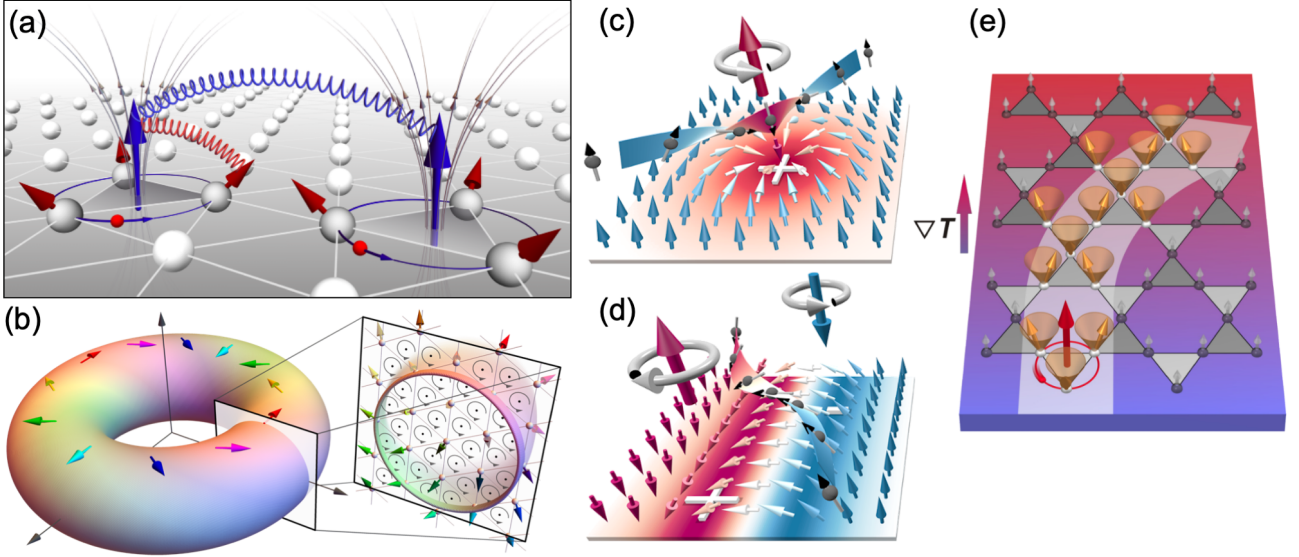


Figure 1.7: (a) The illustration shows the formation of topological orbital moments (large blue arrows) as a result of circular currents (thin blue arrows) flowing through groups of three atoms in a crystal lattice with a specific spin structure (red arrows). The magnetic fields (black arrows) induced by such orbital currents, mediate interactions of the TOMs with each other (blue strings) and with the magnetic moments of neighbouring atoms (red strings). Adopted from [190]. (b) Representation of the magnetic structures of a doughnut-shaped hopfion, together with a hopfion cross section. Coloured arrows represent the in-plane orientations of the magnetic moments. The cross-section shows the circular currents (grey arrows) formed by the spin arrangement within each triplet. Schematic depiction of (c) TOM and (d) COM (circular arrows) due to the emergent field as electrons (gray spheres) are adiabatically traversing over skyrmion and spin spiral, respectively. Adopted from [188]. (e) Generation and drag of orbital angular momentum (red arrow) by magnons (orange arrow) in a temperature gradient ∇T . Adopted from [216].

can stabilise such magnetic textures without using an external magnetic field might provide a powerful tool to explore plethora of novel materials with the nontrivial topology, beyond magnetic skyrmions.

On the other hand, a small sized magnetic texture, such as in B20 MnGe, give rise to a pronounced Berry curvature and thus to the associated with it topological Hall effect and topological orbital magnetism. The topological orbital moment (TOM) is proportional to the scalar spin chirality, $\mathbf{L}_{ijk}^{\text{TO}} \sim \chi_{ijk} \boldsymbol{\tau}_{ijk} = \mathbf{S}_i \cdot \mathbf{S}_j \times \mathbf{S}_k \boldsymbol{\tau}_{ijk}$, where $\boldsymbol{\tau}_{ijk}$ is the surface normal of the triangle spanned by the neighboring magnetic moments $\mathbf{S}$ (classical spin moments) located at sites i , j , and k , see Fig. 1.7 (a). This microscopic mechanism occurs even without SOC and as it was shown in case of MnGe the TOM might be much larger compared to the orbital moment induced due to SOC [190]. The emergence of topological orbital magnetization has been shown in recent years also in various other systems [183, 184, 185, 186, 187, 188, 189]. In addition, TOM might be induced even in collinear FM or AFM structures utilizing magnonic excitations, which in combination with a temperature gradient provides also an efficient transport of electronic orbital angular momentum by magnons [216], see Fig. 1.7(e). This finding might be interesting for the combined functionalities of magnonics and orbitronics in the realm of spintronics applications. In this context, novel mechanisms allowing to generate and manipulate TOM via electromagnetic excitations, e.g., magnetic or electric field, are of great interest for the orbitronics. As an example, coupling between an external magnetic field and TOM, known as ring exchange, may induce chirality density even in collinear FM or collinear AFM structures [217, 218, 219].

The emergence of the TOM due to the non-collinearity has also a great influence on the magnetic structure itself even without external magnetic field, as it was recently demonstrated by discovering topological-chiral magnetic interactions mediated by TOM [190]. These interactions come in two flavours. The first type is spin-chiral interaction (SCI), which arises as a result of a direct coupling between the TOM and local spins, mediated by the SOC, $\sim \xi \mathbf{S}_i \cdot \mathbf{L}_i^{\text{TO}}$. The second type is chiral-chiral interaction (CCI), which is the interaction between TOM's, $\mathbf{L}_{ijk}^{\text{TO}} \cdot \mathbf{L}_{ijk}^{\text{TO}} \sim \chi_{ijk}^2$ (where dominant contribution to the total energy originates from TOM's between the same three sites i , j , and k) and it does not depend on SOC, see Figs. 1.7(a). Since both interactions are proportional to the scalar spin chirality χ_{ijk} they favour the emergence of non-coplanar magnetic structures with scalar spin chirality of specific sign even without an external magnetic field. As it was shown recently [190] both contributions to the total energy in MnGe are not small but can dominate over the celebrated DMI in selecting the chiral ground state. Thus these interactions might provide possibility for solving the open question of the recently observed complex 3D magnetic structures in B20-type chiral magnet MnGe.

On the other hand, because the spin-chirality relates to the curvature of the magnetization field in the continuum limit, one flavour of the topological–chiral interaction reverts to the Faddeev model with solutions for the magnetic hopfions [190], see Fig. 1.7(b). Such 3D magnetic solitons [220, 221, 222, 223] have been recently growing in popularity due to a great potential for innovative spintronic applications [224, 225, 226, 227] and brain-inspired computing. This is in particular because hopfion exhibits rich dynamics and, in contrast to 2D skyrmions, it has longitudinal motion along the current direction [226]. However, only little is known about solids where such 3D magnetic solitons may exist and a complete theoretical model for the underlying magnetic interactions is remarkably elusive until now. From the future perspective, topological–chiral interactions as well as other higher order magnetic interactions that have been recently discovered [228, 229], might motivate to explore their role not only for the magnetic hopfions, but also for other magnetic object to which the scalar spin chirality is inherent, e.g., skyrmions, magnons, as well as their material design and relevance for field- and current-driven dynamics.

In addition to the topological orbital moment, as was recently demonstrated in terms of Rashba model, a combination of the inversion symmetry breaking with the SOI and magnetic texture, gives rise to the Chiral orbital moment (COM) [188], see Fig. 1.7(d). This type of orbital magnetization is proportional to the emergent Chiral field $\mathbf{L}^{\text{CO}} \sim \mathbf{B}_R^{\text{eff}}$ (Rashba field), which is $\mathbf{B}_R^{\text{eff}} \sim \alpha_R \nabla \cdot \mathbf{n}$ directly proportional to the strength of the spin–orbit interaction (as given by α_R) and chirality of the underlying texture $\nabla \cdot \mathbf{n}$. Such contribution to the OM, in contrast to TOM, is non-vanishing even for one-dimensional spin textures and it appears to be very promising at surfaces and interfaces with respect to achieving a large magnitude of the chiral field in chiral spin textures. Remarkably, the spin–orbit interaction in various magnetic textures can be used to tune the orbital magnetism over orders of magnitude by merging the real-space topology with the topology in reciprocal space. And finally, systems with a strong coupling between momentum space topology and mixed space topology give rise to chiral Hall effect [178], which requires symmetry breaking either by SOC or due to the emergent field in non-coplanar magnetic textures. This novel Hall effect, similarly to the chiral orbital moment, not only emerges for one-dimensional spin textures such as spin spirals and domain walls, where THE is zero, but, also it is prominent in canted ferromagnets and antiferromagnets [230]. Furthermore, it was recently realised that chiral Hall effect together with crystal Hall effect [11, 231, 10] are two main contributions to the AHE of canted spin systems with net vector spin chirality [230].

1.4.2 Momentum-space topology.

Berry phase effects stemming from the momentum-space topology are currently under intensive investigations as they hold great opportunities for the efficient magnetization control [60, 196, 197], topological transport phenomena [179, 180, 181] and technological applications in magneto-electric [233], magneto-optic [234], and topological spintronics. In particular, momentum-space topology has profound effect on OM and it can be manipulated using electromagnetic excitations. According to modern theory of the OM, the contribution of the momentum-space topology to the OM can be viewed also as a dynamical effect, which in particular has pronounce non-local character in the vicinity of the band crossing in the electronic structure. To give an example, Fig. 1.8(a) and (b) illustrate a contrasting features of the OM between two cases: 1) when the non-local contribution to the OM is included via modern theory of OM, and 2) when it is ignored and only local part is computed using so called atom-centered approximation (ACA). In particular, such non-local contribution to the OM due to momentum-space topology is crucial in different heterogeneous systems, such as a Mn monolayer deposited on W(001) [185]. Here we briefly discuss some recent studies of materials allowing to manipulate the OM by assessing its non-local (topological) contribution via magnetic or electric field.

One example of the materials revealing non-trivial topology in momentum space are intensively studied electronic systems with flat bands, Dirac and Weyl semimetals, twisted bilayers, and topological insulators [39, 196, 197, 189]. In particular, recent studies indicate that breaking the time-reversal or inversion symmetry in such systems give rise to a pronounced Berry curvature and associated orbital polarization that can be manipulated via the magnetic or electric field. For instance, the magnetic field applied to kagome magnet $\text{Co}_3\text{Sn}_2\text{S}_2$ generates an orbital moment that is opposite to the field direction [235]. A valley-contrasting Berry curvature in 2D Dirac materials couples to an external electromagnetic excitations, i.e., circularly polarized light, magnetic field, and electric current [236, 237] and thus enabling to control the valley degree of freedom that carries an orbital magnetic moment [187]. By breaking the three-fold rotational symmetry via a uniaxial stress in MoS_2 gives rise to a pure electrical generation of valley magnetization as a response to an in-plane electric field due to the Berry curvature dipole [237, 238]. Such scenario has been recently proposed for the magnetization switching in the twisted bilayer graphene [196, 197]. Also, symmetry breaking in twisted bilayer graphene induces large Berry curvatures of the flat bands so that a tiny electric current can generate a large OM [197]. The experimental signatures of such momentum-space topological magnetic moments, with theoretically predicted anomalously large Valley splitting [187] in bilayer graphene were recently observed experimentally [239, 240, 189]. Orbital textures in momentum-space were experimentally detected also in 2D topological insulators using angle-resolved photoelectron spectroscopy [241]. In addition, orbital magnetization might result from

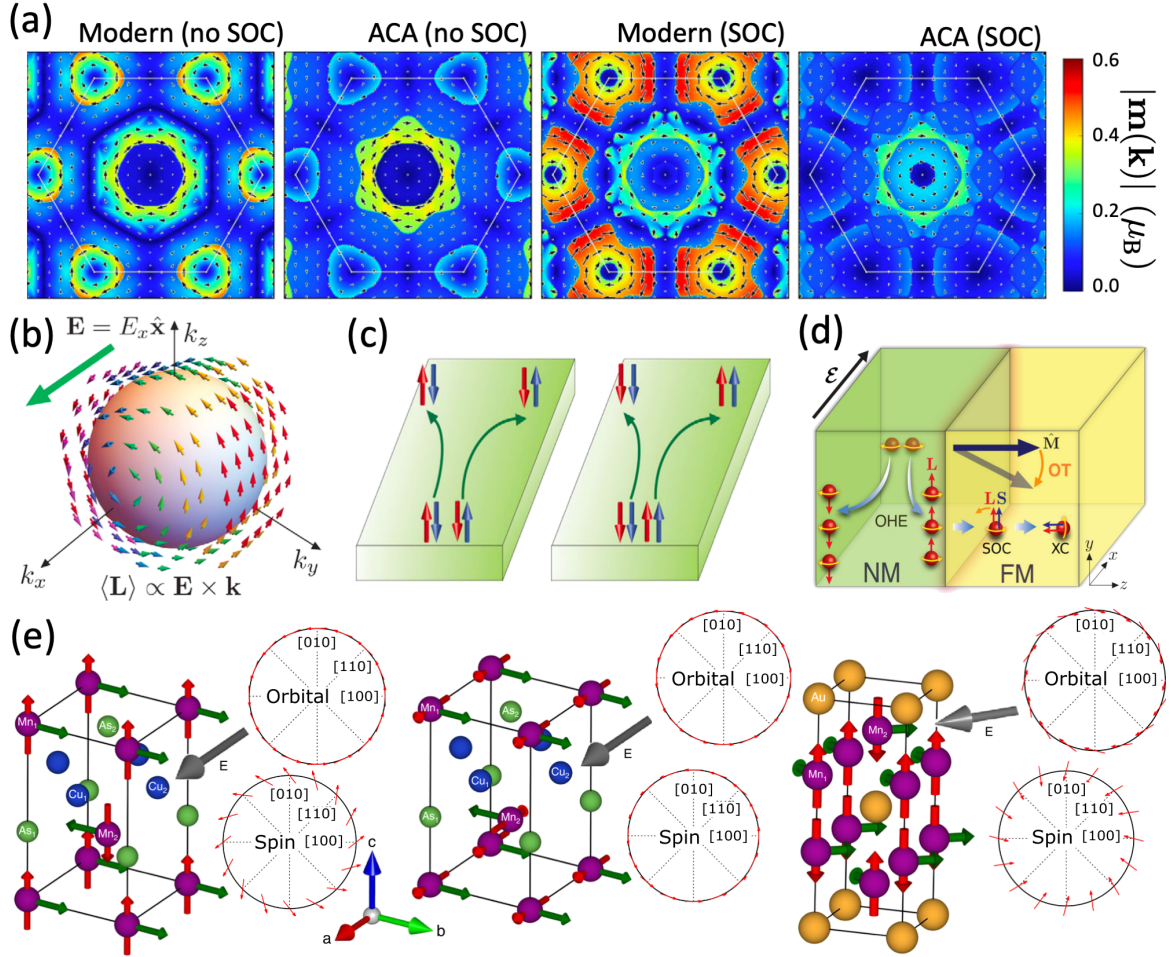


Figure 1.8: (a) Distribution of the in-plane OM $\mathbf{m}(\mathbf{k})$ in k -space of BiAg₂ monolayer obtained from first principles calculations using Berry phase theory (modern) or atom-centered approximation (ACA) without and with taking into account spin-orbit interaction. From Ref. [13]. (b) Orbital magnetization in three-dimensional $\mathbf{k}$ -space (small arrows), which in combination with applied electric field $\mathbf{E}$ gives rise to the orbital moment $\langle \mathbf{L} \rangle$ normal to $\mathbf{E} \times \mathbf{k}$. This leads to finite OHE. (c) SOC give rise to the SHE, which occurs in the same (left panel) or opposite direction of OHE (right panel). Figs. (b) and (c) are taking from Ref. [182]. (d) Orbital torque (OT) emerging in NM/FM bilayer: an external electric field $\mathbf{E}$ gives rise to OHE in NM, which injects $\mathbf{L}$ to the FM, where it couples to the spin $\mathbf{S}$, and thus exerts torque on the magnetization $\mathbf{M}$. From Ref. [232]. (e) Rashba-Edelstein effect in antiferromagnetic CuMnAs and Mn₂Au. Applied an electric field $\mathbf{E}$ (gray arrow) acts on the initial magnetization of Mn atoms (red arrows) inducing a nonequilibrium magnetization (green arrows). Inserts show symmetry of the induced spin and orbital magnetizations (small red arrows) as a function of the static electric-field direction. From Ref. [192]

spontaneous spin-polarization combined with SOC in magnetically-doped topological insulator, so called spin Chern insulators or from spontaneous orbital currents in moiré superlattices of bilayer graphene, known as orbital Chern insulators. This effects have the same origin as quantized anomalous Hall effect [196, 242], and, as it was recently proposed the ferromagnetism in such systems resulting from the spontaneous orbital moments could enable magnetic state reversal in the presence of a magnetic field to be achieved purely electrically [243].

Another prominent realization of the momentum-space topology in controlling the OM is the orbital Rashba effect [13]. This effect arises in the centrosymmetric systems through momentum-space orbital texture even without spin-orbit coupling, see Fig. 1.8(a). The crucial role for the magnitude and variation of the orbital textures is played by the Berry phase theory, proclaiming the orbital Rashba effect as a promising platform for surface orbitronics. Furthermore, it is remarkable that momentum-space orbital texture give rise to an orbital Hall effect (OHE), see Fig. 1.8(c), which occurs even without spin-orbit coupling (SOC) and is converted into the spin Hall effect (SHE) [244] through SOI [182, 245]. Therefore, both OM and SHE might be significantly enhanced due to momentum-topology in some materials even if SOC is small. On the other hand, similar to the SHE that by inducing spin current in non magnet (NM) imposes the spin-orbit torque on local magnetic moments of the on-top FM, the OHE allows for electrical generation of a transverse orbital current in NM generating an orbital torque on local magnetic moments of the FM [232], Fig. 1.8(d). Thus the orbital torque

also might provide a venue to achieving high torque efficiency in spintronic devices.

Recently an orbital analogue of the Edelstein effect was proposed for the chiral crystals [191] and in the noncentrosymmetric antiferromagnets CuMnAs and Mn₂Au [192], Fig. 1.8(e). Such effect induces an OM due to charge current and it can be drastically enhanced, compared to spin Rashba-Edelstein effect, in materials such as Weyl semimetals with the Weyl nodes close to the Fermi level [61, 191]. Therefore, such effect is proposed for the current induced magnetization switching, as alternative to Spin-Hall effect and the Edelstein effect requiring for that materials with strong SOC. Also, it might be used to manipulate the domain wall motion and domain flips. Also, interestingly the magnetization control can be achieved without SOC using spin polarization in collinear antiferromagnet, such as MnF₂, because flowing electrons along certain crystallographic directions contributes to spin splitting similarly as SOC does [246].

1.4.3 Conclusion

The topological phases emerging in real-space and momentum-space have demonstrated a plethora of novel phenomena allowing to control and manipulate orbital magnetism, spin and orbital dependent transport. In particular, strong effect of Berry phase on orbital magnetism and its close relationship to the Hall effect suggests that that non-collinear spin systems, such as magnetic skyrmions, hedgehog lattices, and hopfions, may exhibit a rich landscape of orbital magnetism emerging from scalar spin chirality rather than from spin-orbit interaction (SOI). In addition the emergence of TOM due to the non-coplanarity of the magnetic structure, give rise to the topological-chiral magnetic interactions [190] that has great influence on the magnetic structure itself favouring formation of 3D magnetic structures even without magnetic field. This finding raise a number of important fundamental questions, such as the relevance of these interactions for the ground state of existing materials that exhibit diverse magnetic orders as well as this also concerns the systems in which emerging homochiral magnetic structures were previously thought to be the result of the Dzyaloshinskii-Moriya interaction (DMI). Furthermore, while these interactions mediated by the emergent orbital magnetism were found to be absolutely vital in bulk of B20 MnGe, their role for the stabilization of 2D and 3D textures in other materials of bulk and surface form, as well as their influence on the dynamical properties of ferromagnetic, chiral, and antiferromagnetic systems is not explored yet. In addition, one flavour of the topological-chiral interactions in combination with exchange interaction was found to stabilise magnetic hopfions. Such 3D magnetic solitons have great potential for innovative spintronic applications, however materials of their existence have not been found yet.

On the other hand, the emergence of TOM requires no SOC, therefore finding mechanisms allowing to generate it in FM or AFM using electromagnetic excitations, such as electric current or electric field, as well as manipulation and transfer of TOM through solid is of great interest for the spin-orbitronics. As an example, recently it was shown that magnons in combination with temperature gradient can be used to induce and drag TOM in FM and AFM [216]. This effect might pave a way towards the combined functionalities of magnonics and orbitronics applications. In addition, presence of SOI in materials with broken inversion symmetry and with non-collinear magnetic structure give rise to chiral orbital moment COM [188] which appears to be very promising with respect to achieving a large magnitude in chiral spin textures emerging at surfaces and interfaces. In analogy to the topological-chiral interactions mediated by TOM that were found in bulk of B20 MnGe, similar interactions mediated by COM might also emerge at the surfaces and interfaces. Taking into account that COM is proportional to strength of Rashba SOI α_R , which can be tuned by electric field, such hypothetical interactions are very promising for the efficient magnetization switching as well for their contribution towards formation and control of the chiral magnetism using the electric field.

Also, recent studies indicate that breaking the time-reversal or inversion symmetry in electronic systems with flat bands, Dirac and Weyl semimetals, bilayer graphene, and topological insulators gives rise to a pronounced Berry curvature and associated orbital polarization that can be manipulated via the magnetic or electric field. For example, it was shown that orbital textures in momentum space might appear naturally in systems with a broken inversion symmetry even without SOC. Such textures contributes to the OM, give rise towards Orbital Rashba and Orbital Hall effects. Orbital textures can be generated also using electric current in chiral FM or non-centrosymmetric AFM due to the orbital Edelstein effect.

These findings indicate that the topological effects originated from the momentum-space curvature can be easily modified by electromagnetic excitations. On the other side, real-space curvature is a characteristic of the magnetic particles and textures, and it might contribute to their stability via topological-chiral interactions. Therefore, considering these two aspects, another interesting direction in realm of spin-orbitronics is the interplay between topological effects emerging from real-space and momentum space topologies. As an example, the interplay between momentum space topology Ω^{kk} and mixed space topology Ω^{Rk} is an origin of DMI [201], contributes to the antidamping torque [57, 58, 60], give rise to Chiral orbital moment (COM) [188] and Chiral Hall effect [178, 230]. Also, coupling between both topologies might provide an efficient tool to excess and manipulate non-trivial topological phases, thus finding magnetic materials possessing Ω^{kk} and Ω^{RR} are of great interest. One prominent example of such materials are Heusler compounds, allowing to realize a wide variety of topological phases through Berry curvature design [247]. In particular, the topological properties of Heusler

compounds can be manipulated by various electromagnetic excitations, giving rise, for instance, to anomalous, spin, and topological Hall effects. Besides this momentum-space Berry curvature, Heusler compounds possess also a real-space topological states such as magnetic skyrmions and antiskyrmions. Also recently it was shown that large contribution to the THE in Heusler compounds exhibiting antiskyrmion-type spin structures stems from the Berry curvature not only in the real-space Ω^{RR} but also from the Berry curvature in the momentum-space Ω^{kk} [248].

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