Precision calculations in non-perturbative QCD (I):
Effective Field Theories (EFT), analyticity and dispersion relations

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Motivation

Strategies to treat strongly interacting systems:

→ Phenomenological models
degrees of freedom: quarks or hadrons;
Advantage: Typically guided by clear physical picture
Disadvantage: Uncontrolled unertainty

→ Effective Field Theory
degrees of freedom: quarks or hadrons;
Perturbative or Non-Perturbative
Advantage: Controlled uncertainty (expansion in Q/Λ)
Disadvantage: Limited range of applicability

→ Dispersion theory
degrees of freedom: hadrons
Advantage: model independent
Disadvantage: Needs proper inputs

→ Lattice QCD

The Concept of EFTs

S. Weinberg, PhysicaA96(1979)327

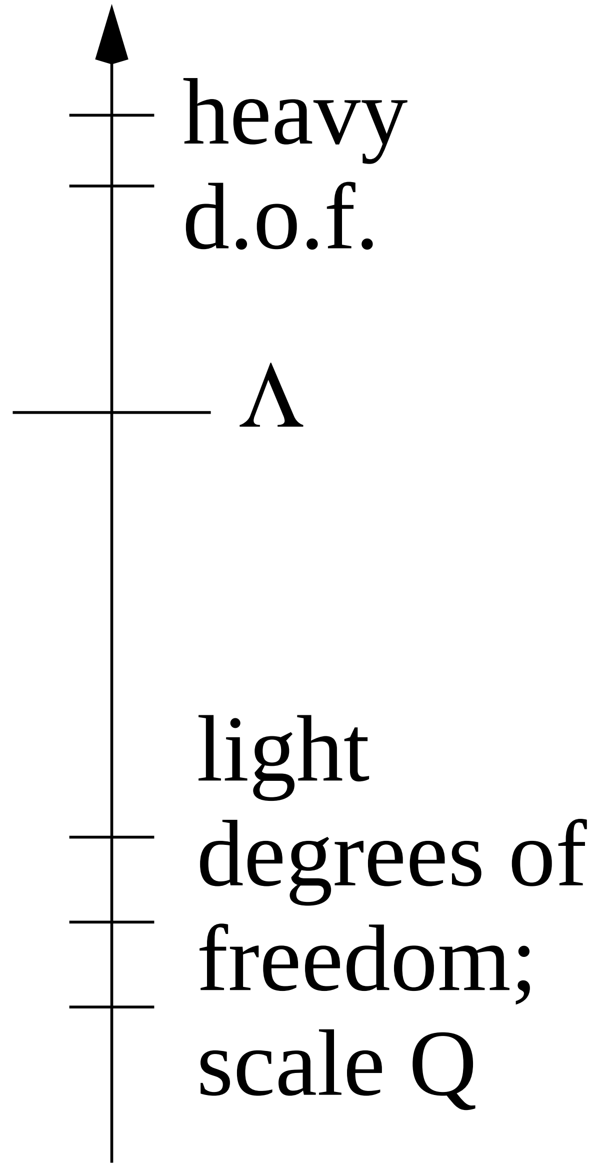
Applicable if there is separation of scales

→ Treat light degrees of freedom completely (at scale Q)

→ Collect heavy degrees of freedom in local counter terms

→ Expansion parameter: Q/Λ

$\Longrightarrow$ Allows for uncertainty estimate



Tailormade for particular system

Limited range of applicability

Non-pert. systems: Expand potential & resum

S. Weinberg, PLB251(1990)288

Dispersion Theory (hadrons as d.o.f)

see also talks by J.R. Pelaez and B. Mousallam in this session

Starting point: Im-part of form factor F_i

$$\mathrm{Im}(F_i) = \sum_k t^*_{ik}\sigma_k F_k \quad \rightarrow \text{Dispersion Integral(s)}$$

$$F_i(s) = \frac{1}{\pi}\int dz \frac{\mathrm{Im}(F_i(z))}{z - s - i\epsilon}$$

for single channel $\rightarrow$ Watson theorem and Omnès function

e.g. for two channels:

$$t = \begin{pmatrix} \frac{\eta e^{2i\delta}-1}{2i\sigma_\pi} & g e^{i\psi} \\ g e^{i\psi} & \frac{\eta e^{2i(\psi-\delta)}}{2i\sigma_K} \end{pmatrix} \text{ and } \Omega_{ij}(s) = \frac{1}{\pi}\int_{s_{\mathrm{th}}}^{\infty} \mathrm{d}z \, \frac{(t)^*_{ik}(z)\sigma_k(z)\Omega_{kj}(z)}{z-s-i\epsilon}$$

and

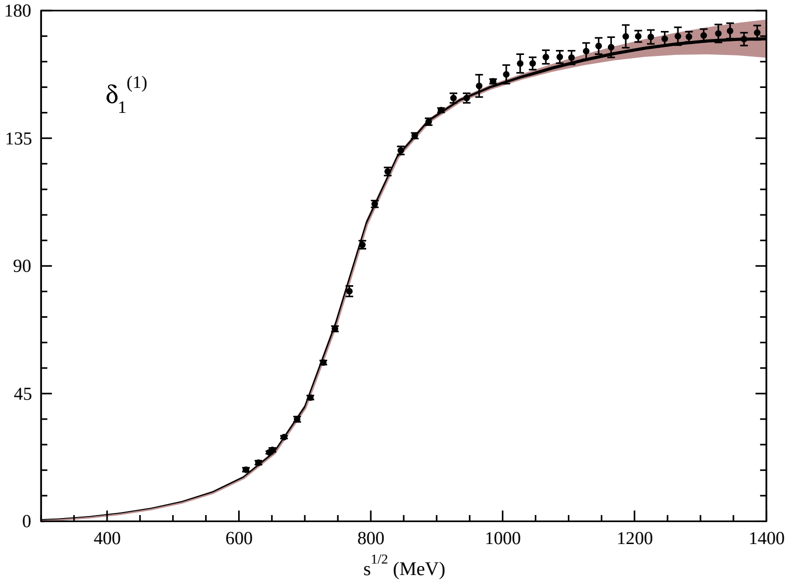
$$\boxed{F_k(s) = \Omega_{ki}(s) M_i(s)}$$

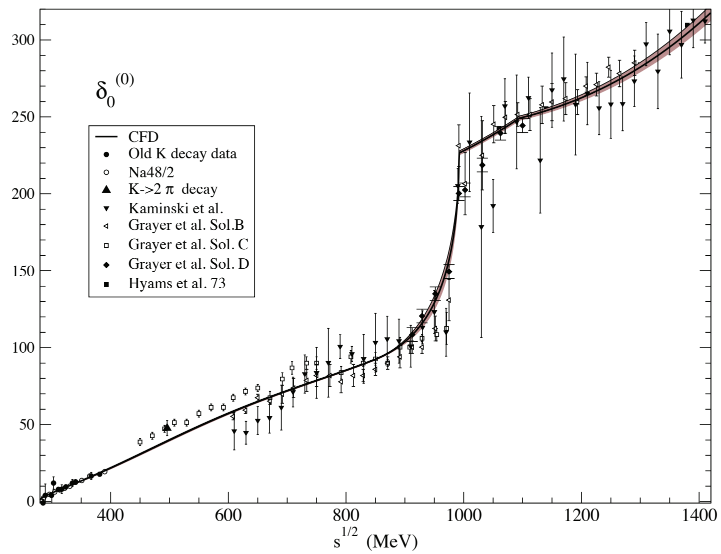
where the $\Omega_{ij}(s)$ are universal and

the $M_i(s)$ are reaction dependent functions

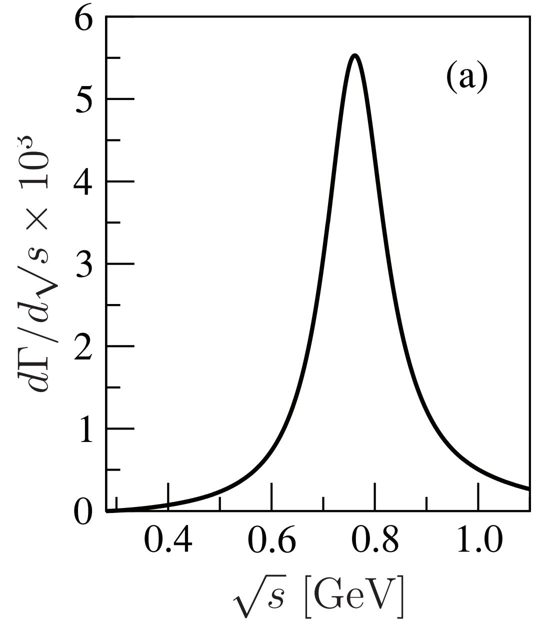
$\Longrightarrow$ Can be extended to higher energies by adding resonances

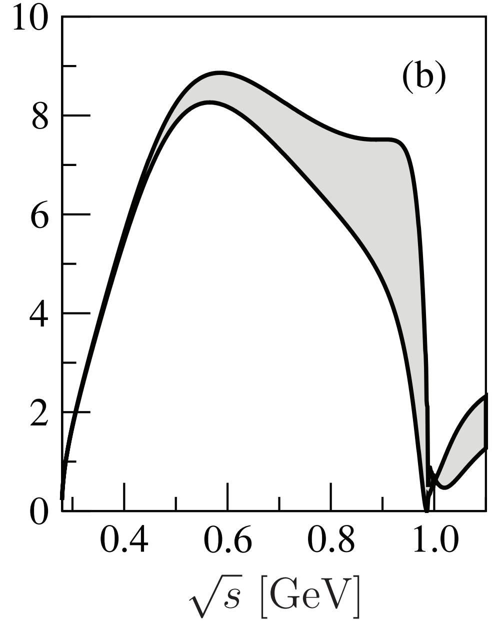
Example $\Omega(s)$: $\pi\pi$ S- and P-waves

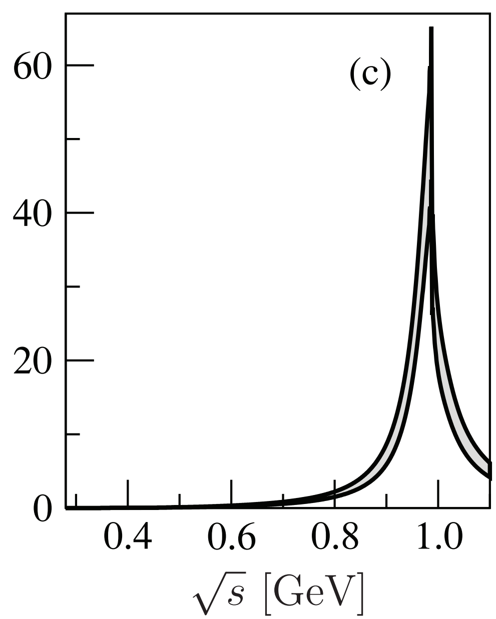




give for $\Omega_1^1(s)(p+p')^\mu = \langle\pi\pi|\bar{q}\gamma^\mu q|0\rangle$ $\Gamma_\pi^n(s) = \langle\pi\pi|(\bar{u}u+\bar{d}d)/2|0\rangle$ $\Gamma_\pi^s(s) = \langle\pi\pi|\bar{s}s|0\rangle$







δ: Garcia-Martin et al., PRD83(2011)074004; FF's: Daub et al., JHEP01(2013)179

Examples from T1.1 (leaks into T2.1)

→ EFTs with Quarks and Gluons as d.o.f.s

- ▷ Precision calc. with NREFTs (N. Brambilla)
- ▷ Heavy Quarkonium Production in pNRQCD (H.S. Chung)
- ▷ Effective field theory for double heavy baryons (J. Soto)

→ EFTs for (exotic) hadrons

- ▷ Isospin violation in $\psi \to \Lambda\bar{\Sigma}^0 + \mathrm{c.c.}$ (A. Mangoni)
- ▷ The molecular nature of some exotic hadrons
- ▷ talks by M.-L. Du, E. Oset, U.-G. Meißner, V. Baru
- ▷ Interplay of Quark- and two hadron states
- ▷ Charm mesons in a hot pion bath
- ▷ Triangle singularities in heavy ion collisions

→ Dispersion Theory (+EFT)

- ▷ Dispersive study of πK and $\pi\pi \to \bar{K}K$ (J.R. Pelaez)
- ▷ High energy extension of πK amplitudes
- ▷ Revisiting the a_0 in $\gamma\gamma$ scattering (B. Mousallam)

High energy extension of dispersive approach

C.H., PLB715 (2012) 170; Ropertz et al., EPJC78 (2018) 1000, L. von Detten et al., in preparation

We need unitary formalism that

→ Matches smoothly onto dispersive representation;

→ Allows for the inclusion of additional resonances;

→ Allows for the inclusion of additional channels.

Assumption: Additional channels couple via resonances only

$$T = T_0 + \tilde{\Omega}\,[1 - V_R\,\Sigma]^{-1}\,V_R\,\tilde{\Omega}^t \quad \& \quad F = \tilde{\Omega}\,[1 - V_R\,\Sigma]^{-1}\,M$$

with t & Ω from above, embedded into enlarged channel space:

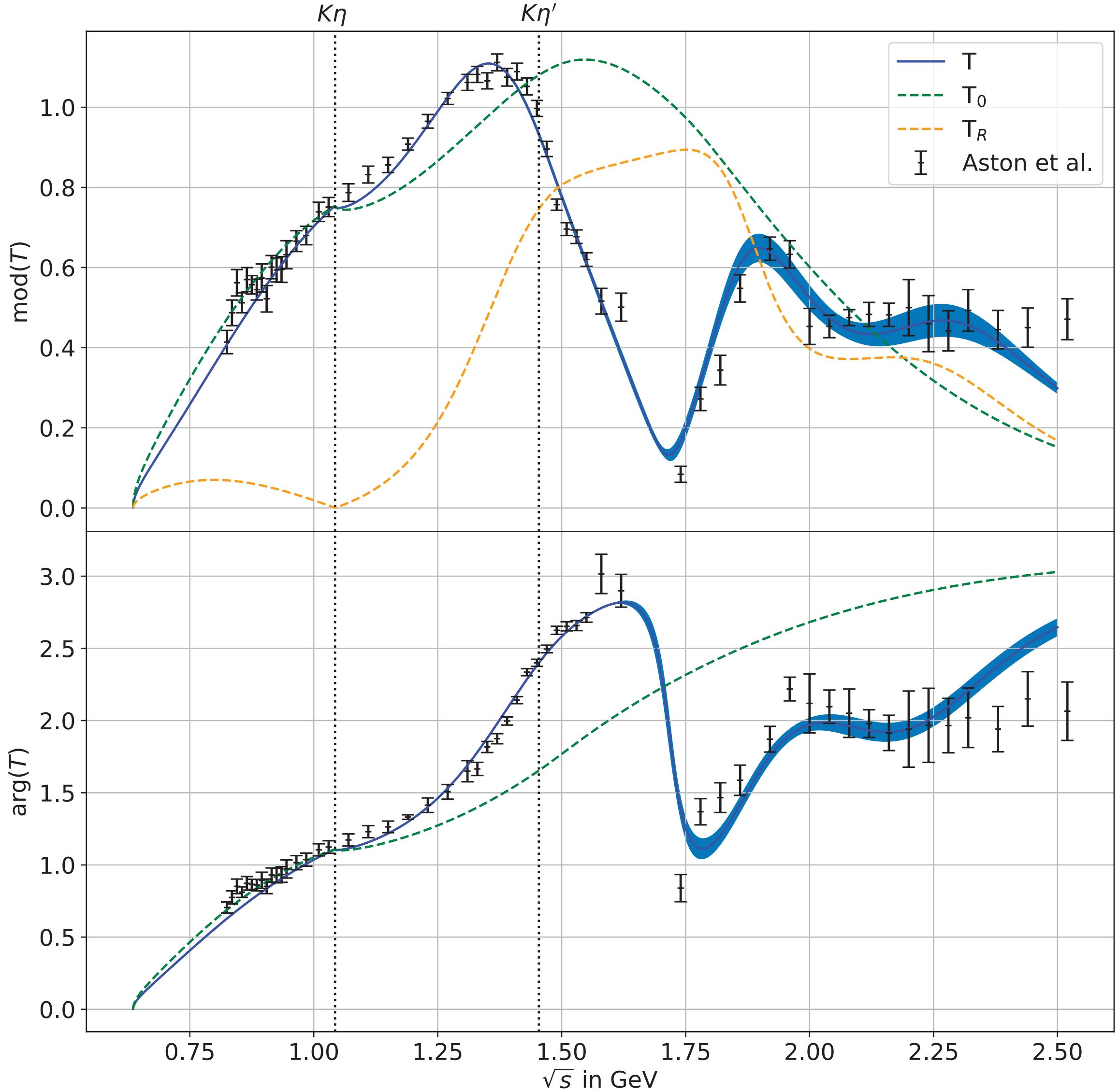
$(T_0)_{ik} = t_{ik}$ ($\tilde{\Omega}_{ik} = \Omega_{ik}$) for $i, k \leqslant 2$ and 0 (1) otherwise

$$\Sigma_{ij}(s) = \frac{s}{\pi}\int_{s_{\rm th}}^{\infty}\frac{{\rm d}z}{z}\,\frac{\tilde{\Omega}^*_{ki}(z)\sigma_k(z)\tilde{\Omega}_{kj}(z)}{z-s-i\epsilon}$$

V_R = resonance potential. Previous form recovered for $V_R \equiv 0$

L. von Detten et al., in preparation

2 resonance poles; Input phase: J.R. Pelaez and A. Rodas, PRD93(2016)074025



Pole extraction (Padé) for the higher pole:

Re($\sqrt{s_p} = (1878 \pm 10)$ MeV

Im($\sqrt{s_p} = (222 \pm 6)/2$ MeV

using $s_0 = 1.877$ GeV

PDG lists for (M, Γ):

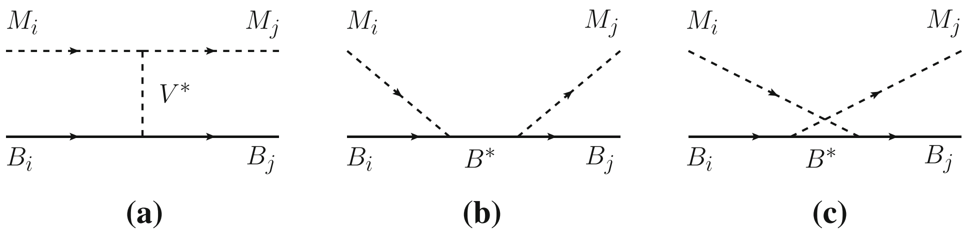
$(1945 \pm 22, 201 \pm 90)$ MeV

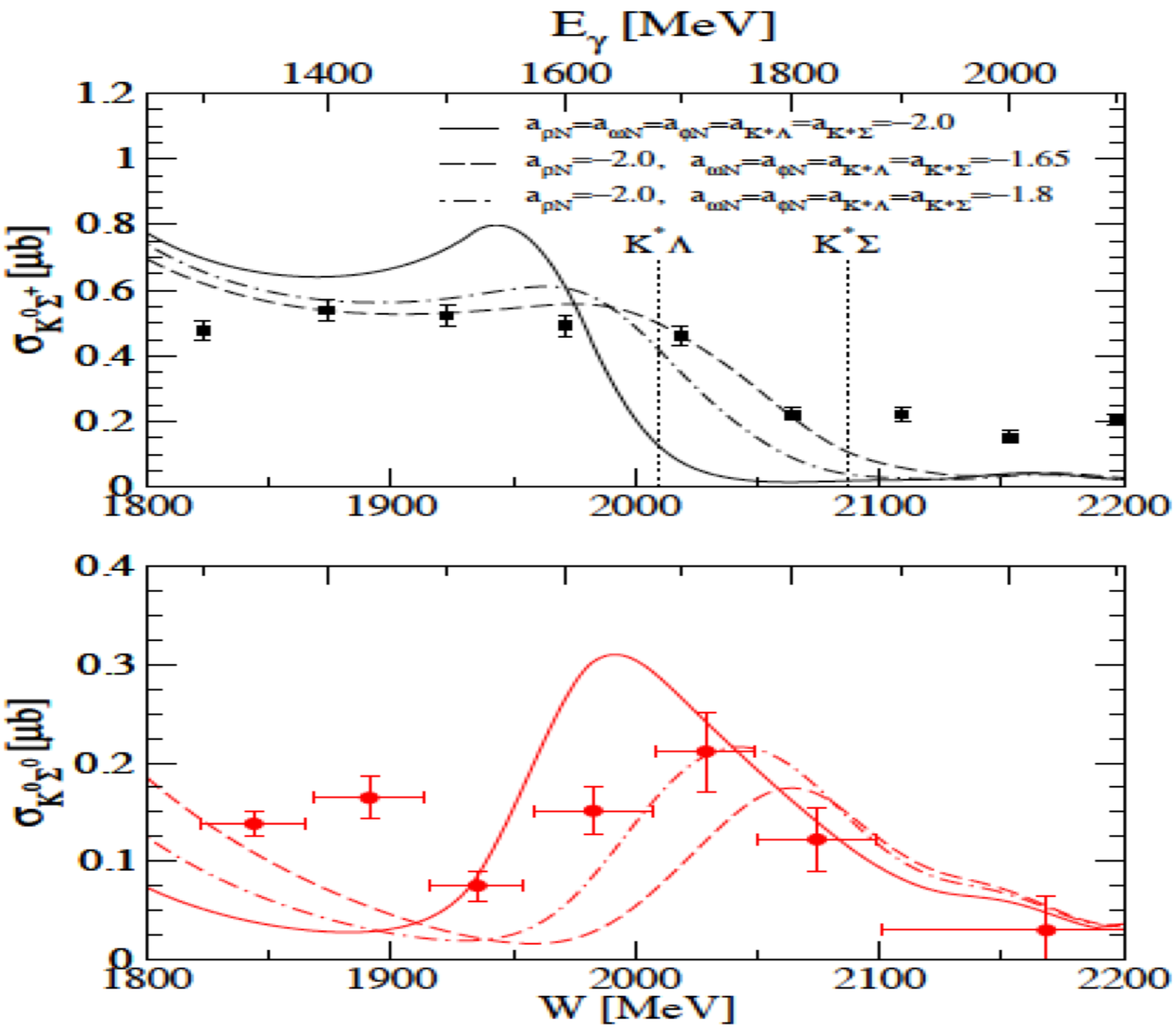
We currently work on extraction for $K_0^*(1430)$.

The molecular nature of some exotic hadrons
(Meson-baryon composite states from unitarized effective meson-baryon interactions)

A. Ramos, A. Feijoo, Q. Llorens,
G. Montaña, Few Body Syst. 61 (2020) 4, 34

Employing conveniently unitarized effective meson-baryon interactions, we **reveal the existence of a N* resonance around 2 GeV**, having a YK* composite nature



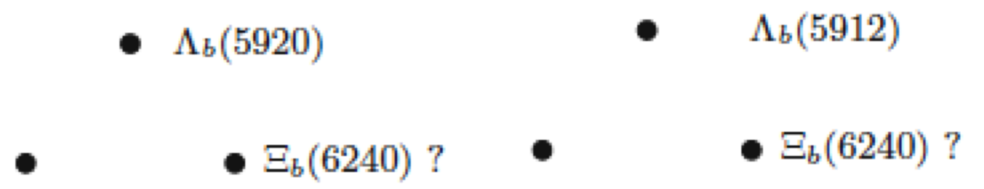


- very sensitive to coupled-channel interference effects
- LO calculation still shows sizable regulator dependence

UNIVERSITAT DE BARCELONA

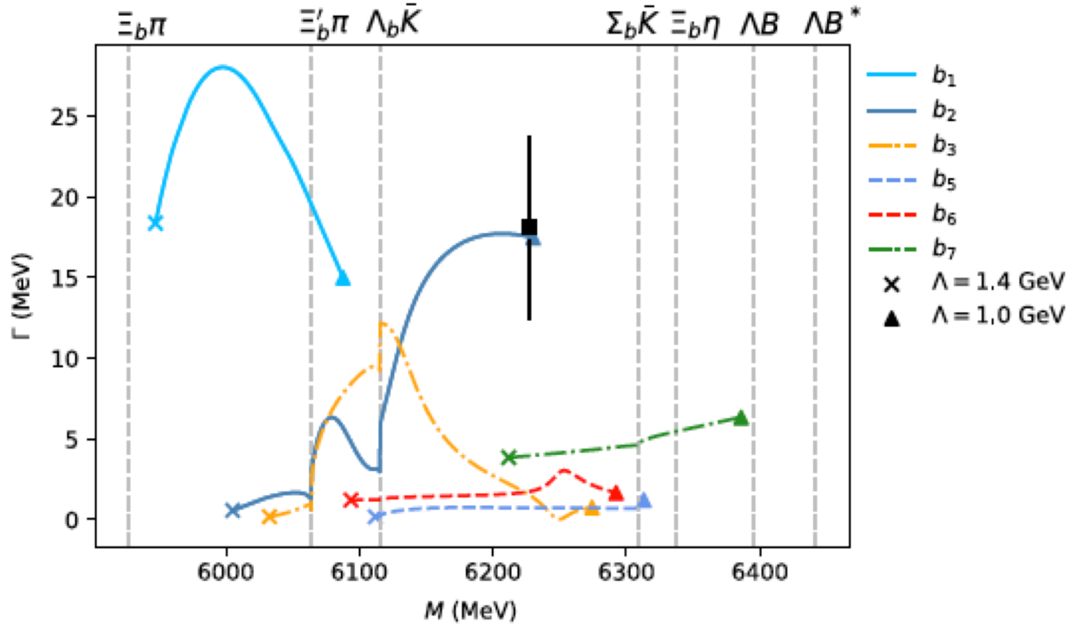
J.Nieves, R.Pavao and L.Tolos,
Eur. Phys. J. C 80 (2020) no.1, 22

Detailed analysis of the **robustness of the molecular interpretation of several experimental excited Ξ_c and Ξ_b states**, using a coupled-channel unitarized model that is based on heavy-quark spin symmetry.

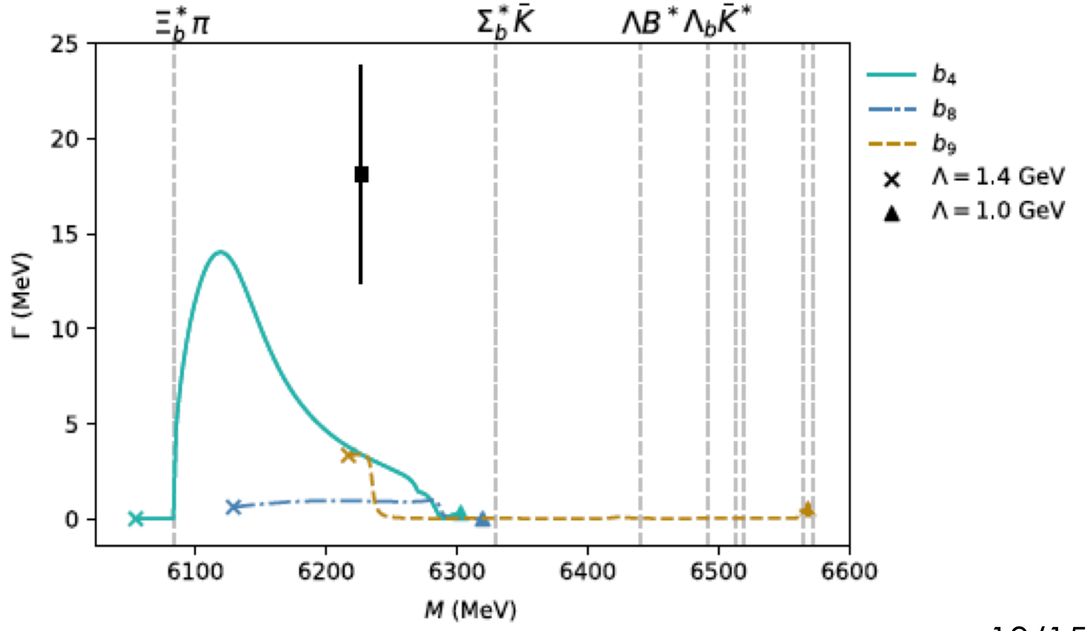


Bottom baryon states with J=1/2 and J=3/2 within SU(3) **3*** irreps. The question mark indicates our predictions.

Evolution of the masses and widths of the dynamically-generated Ξ_c and Ξ_b states, as we vary the cutoff. The squares and their associated errorbars show the masses and widths of the experimental observed states.



(a) $J = 1/2$



(b) $J = 3/2$

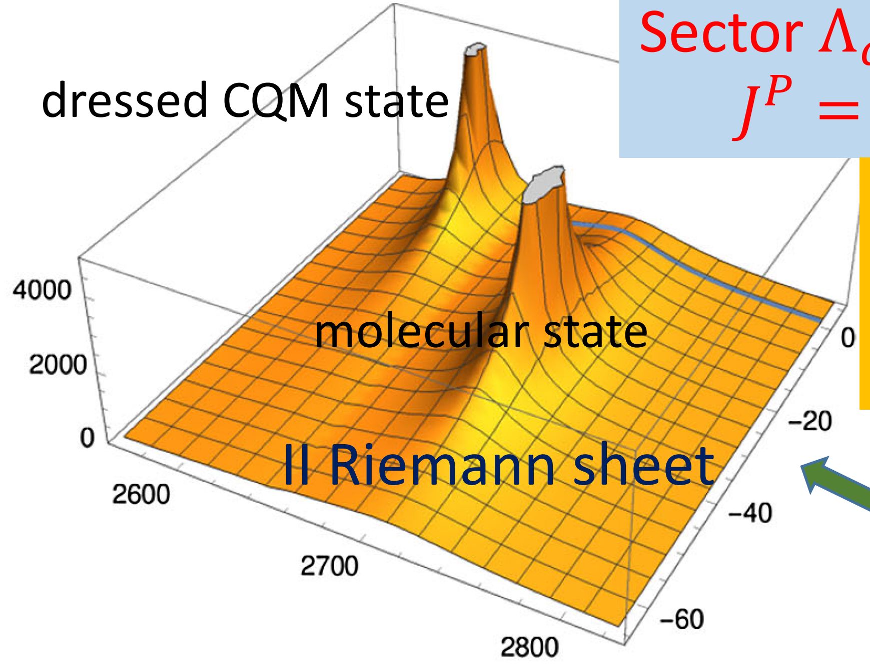
Sector $\Lambda_c(2625)$ $J^P = 3/2^-$

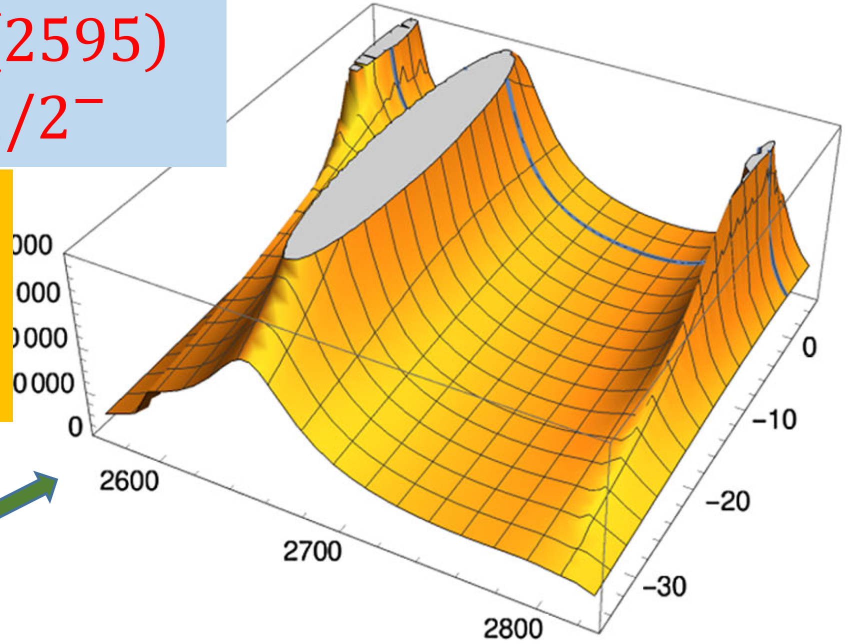
Sector $\Lambda_c(2595)$ $J^P = 1/2^-$

Nature of $\Lambda_c(2625)$ and $\Lambda_c(2595)$

Nieves+Pavao:PRD 101 (2020) 014018

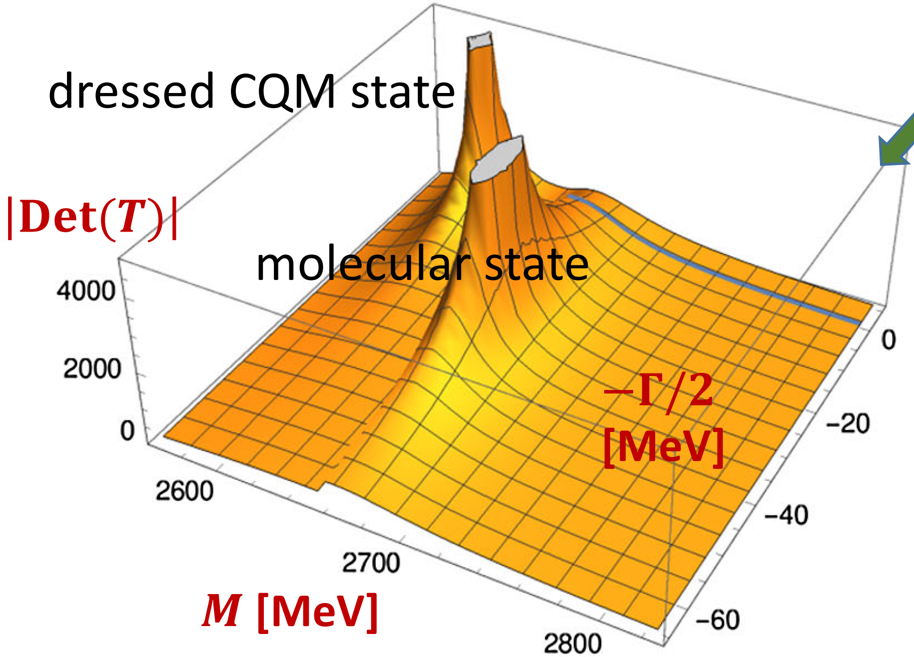
- ✓ $\Sigma_c\pi, \Sigma_c^*\pi, DN, D^*N$ **(HQSS)**
- ✓ **Constituent Quark Model (CQM)**



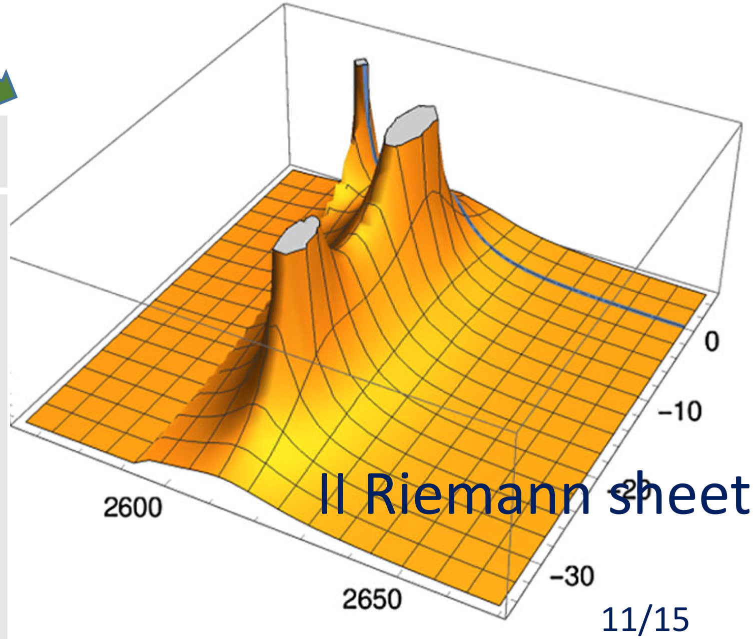


RS: subtraction at a common scale

RS: common UV cutoff 650 MeV



$J^P = 3/2^-$	$J^P = 1/2^-$
In both Renormalization Schemes (RS) , the **dressed CQM state describes fairly well the $\Lambda_c(2625)$ resonance**	**Three poles**, which positions and interpretations depend on the RS and the **interplay between CQM and meson-baryon degrees of freedom**



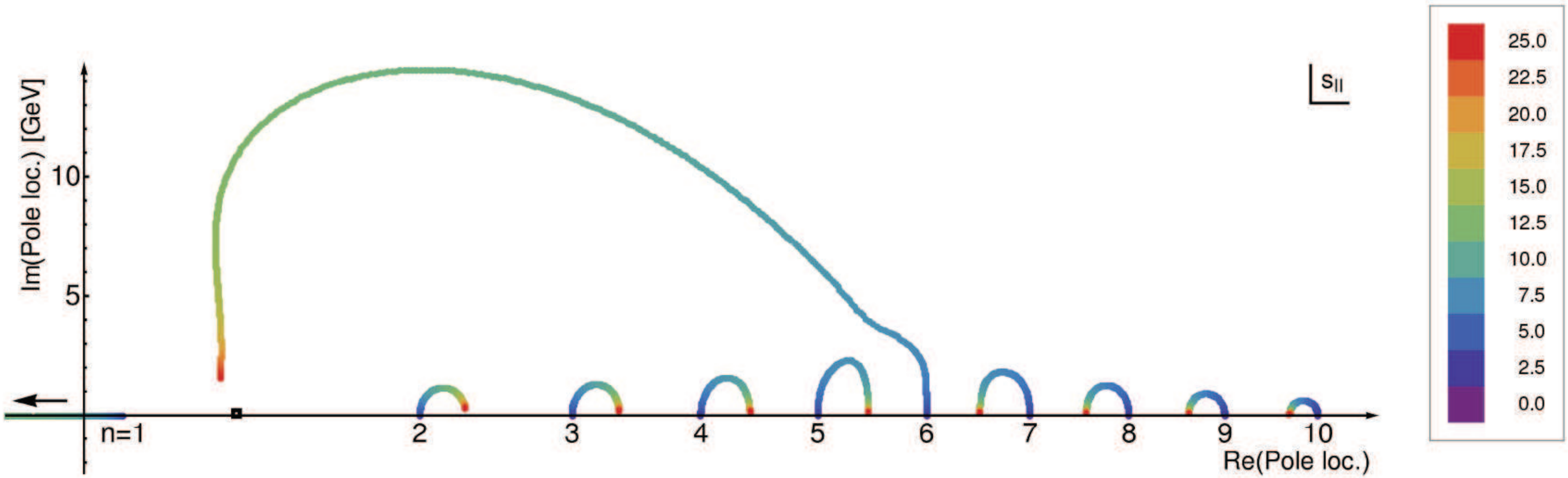
... or do the quark model states decouple?

B. O. Kerbikov et al. ITEP-61-1978; E. van Beveren et al., PLB641(2006)265
I. K. Hammer, CH and A. V. Nefediev, EPJA52(2016)330

→ Start from a large number of compact states

→ Couple to continuum channel

→ Study pole trajectories for increasing coupling

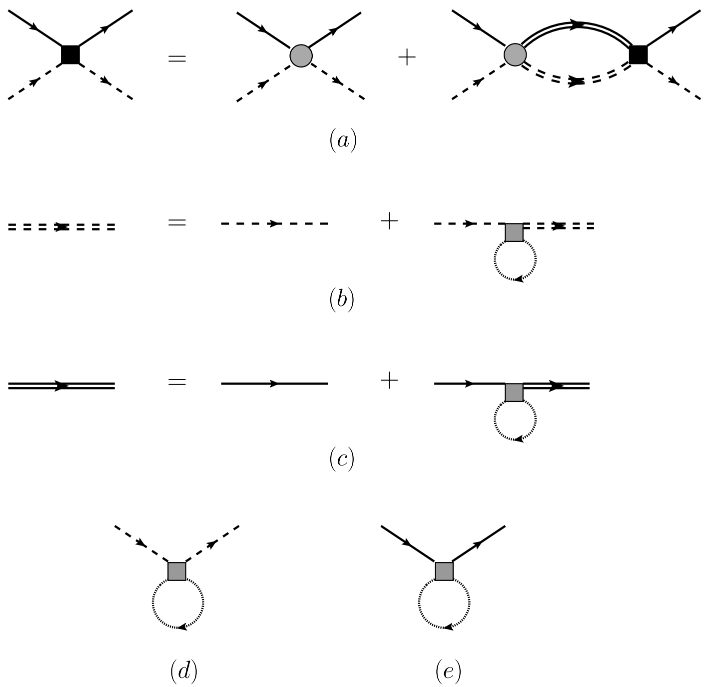


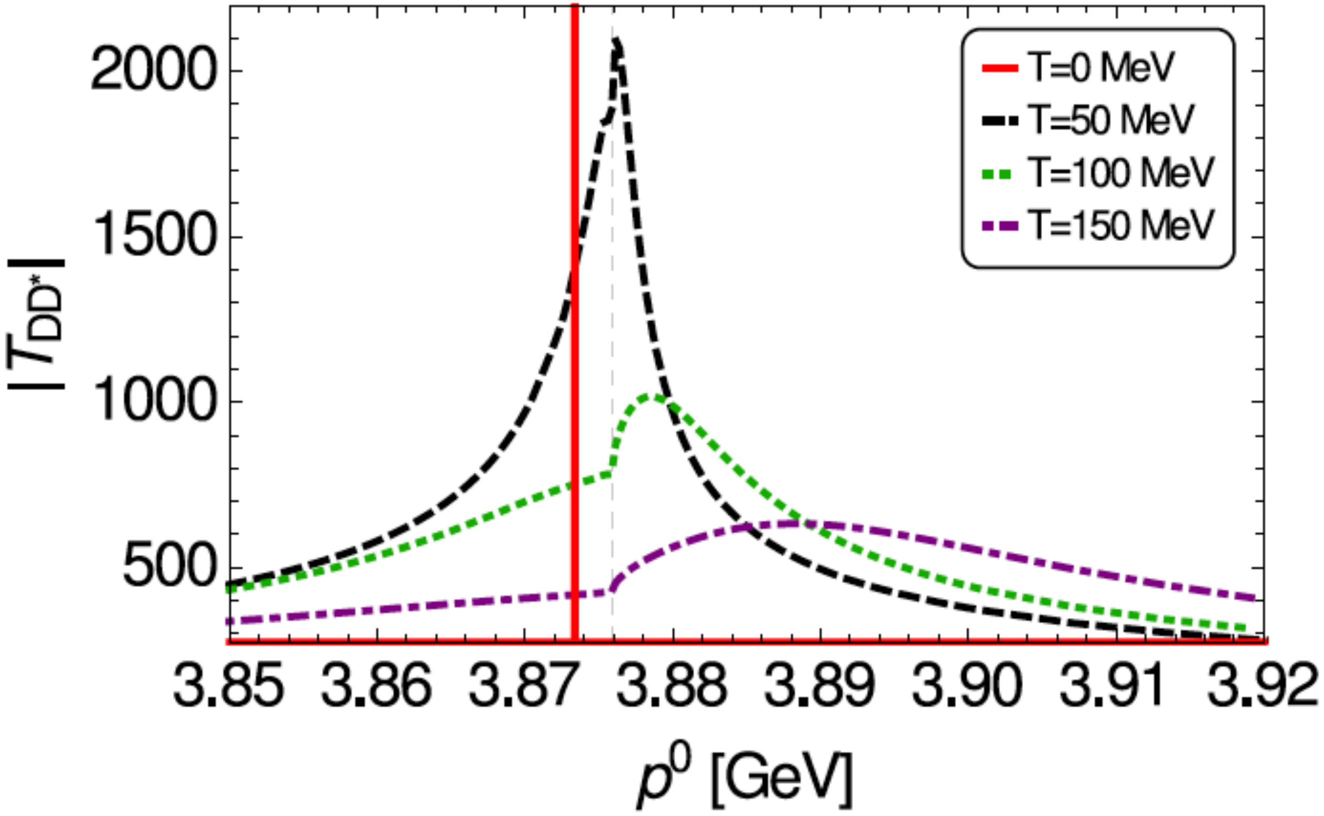
⟹ For large couplings: • A few collective states,

• decoupled compact states

Charm mesons in a hot pion bath

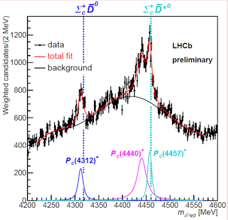
M. Cleven, V.K. Magas, A. Ramos
Phys.Lett. B799 (2019) 135050





We study the properties of the **X(3872) in a pionic medium** and find that, if it was a **DDbar* molecule**, it would develop a **subtantial width, of the order of a few tens of MeV** in hot pionic environments **at temperatures 100-150 MeV**.

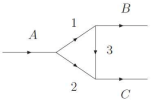
Heavy ion collisions blur triangle singularities (@UCMadrid)

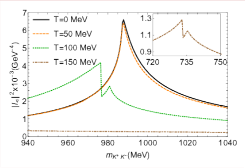


Q: given such an experimental peak,

- is there an underlying particle?
- or is it rather a scattering effect such as a triangle singularity?

- Contribution: we computed (in the Matsubara formalism) several hadron triangle diagrams at finite T





- And showed that the triangle integral is severely affected and blurred dropping out of the apparent spectrum in RHIC/ALICE at T=150 MeV (except perhaps for pions in the loop)

Luciano Abreu and Felipe J. Llanes-Estrada, 2008.12031

Summary

→ The hadron spectrum is still not known completely

→ There is a lot of evidence for states beyond the most simple realisation of the quark model

→ EFTs are the systematic approach to make progress

- ▷ On the quark level (see talk later in this meeting)
- ▷ On the hadron level (this talk and later in this meeting)

→ To access the hadron spectrum 'resummation' necessary

- ▷ by solving differential equation
- ▷ by solving LS equation
- ▷ by employing dispersion theory

→ Further information from lattice QCD

- ▷ to fix low energy constants
- ▷ provides information on quark mass dependence

$\Longrightarrow$ important synergies!