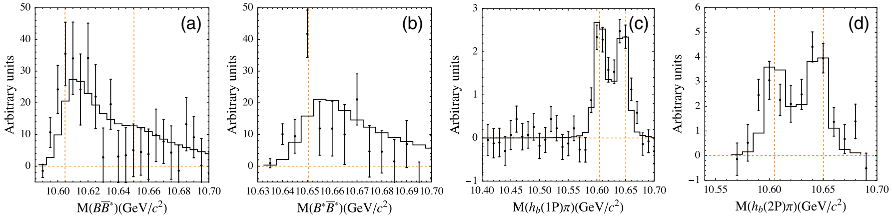
DIPION TRANSITIONS AND BOTTOMONIUMLIKE STATES

| C. Hanhart with V. Baru, E. Epelbaum, A.A. Filin, R.V. Mizuk, A.V. Nefediev and S. Ropertz |

REMINDER



Data: Belle, PRL108(2012)122001 & PRL116(2016)212001

Curves: model A (point interactions only), Q. Wang et al., PRD98(2018)074023

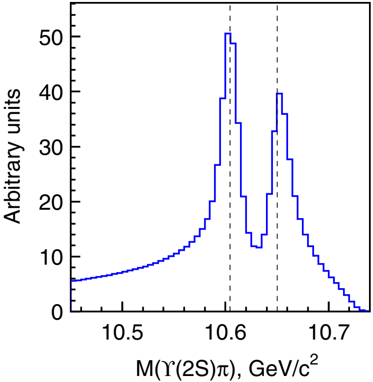
- Charged states $Z_b(10610)$ and $Z_b(10650)$ seen in
  - $\Upsilon(10860) \to \pi[\pi\Upsilon(nS)]$ (n=1,2,3) $[b\bar{b}]_{\mathrm{Spin1}} \to [b\bar{b}]_{\mathrm{Spin1}}$
  - $\Upsilon(10860) \to \pi[\pi h_b(mS)]$ (m=1,2) $[b\bar{b}]_{\mathrm{Spin1}} \to [b\bar{b}]_{\mathrm{Spin0}}$
  - $\Upsilon(10860) \to \pi[B^{(*)}\bar{B}^{(*)}]$
- masses very close to the $B^*\bar{B}$ and $B^*\bar{B}^*$ thresholds, respectively
- decay almost exclusively to open bottom channels

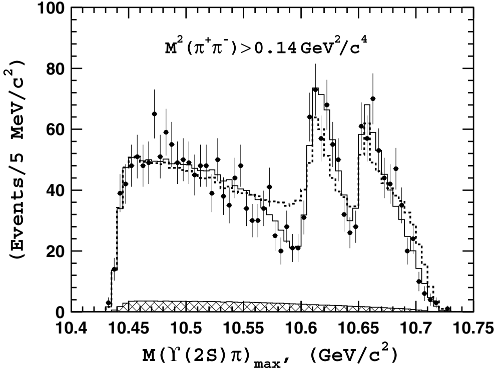
Excellent candidates for hadronic molecules

A. E. Bondar et al. PRD84(2011)054010

THE $\pi\pi\Upsilon$ FINAL STATES

Both $B^{(*)}\bar{B}^{(*)}$ and $\pi\pi$ interaction matter simultaneously.





Data: Belle, PRD91(2015)072003

Goal: Proper inclusion of both interactions and their interplay

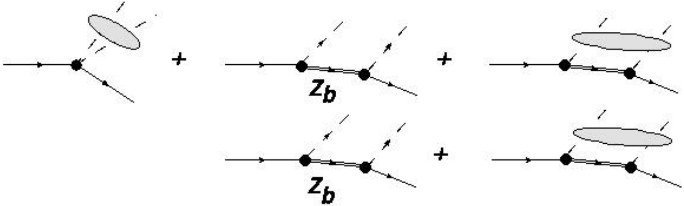
Tools: Dispersion theory + chiral perturbation theory

DEFINITION OF THE PROBLEM

The difference between the transitions to $h_b(mP)$ and $\Upsilon(nS)$:

$\Upsilon(5S) \to \Upsilon(nS)\pi\pi$

$\Upsilon(5S) \to h_b(mP)\pi\pi$



To reach the $S = 0$ h_b states: Z_b states needed as doorway

$\Longrightarrow$ Signal only in the Z_b mass range

The $S = 1$ $\Upsilon(nS)$ states can be reached directly:

$\Longrightarrow$ Direct transitions feed amplitude outside Z_b peaks

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DISCLAIMER

We here focus on the proper inclusion of the $\pi\pi$ interaction

V. Baru et al., PRD103(2021)034016

with special emphasis on how to quantify imaginary parts

in contrast to D.A.S. Molnar et al. PLB 797(2019)13485 (for Z_c states)

We therefore:

- use a simplified model for the Z_b-states

$\Longrightarrow$ Contact interactions only; no one-pion exchange

- use Z_b-parameters from an earlier fit to $B^{(*)}\bar{B}^{(*)}$ and $h_b(mP)\pi$

Fit A of Q. Wang et al., PRD98(2018)074023

$\Longrightarrow$ No combined fit of all channels (yet)

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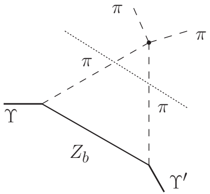
THE KHURI-TREIMAN FORMALISM

N. N. Khuri and S. B. Treiman, PR119(1960)1115; revised since F. Niecknig, B. Kubis and S. P. Schneider, EPJC72(2012)2014



- Amplitude $\hat{M} = \hat{M}^R + \hat{M}^L$, where $\hat{M}^R$ ($\hat{M}^L$) has only a right (left) hand cut
- $m_{\pi\pi}^{\text{max.}} > 1$ GeV $\Longrightarrow \pi\pi - K\bar{K}$ coupled system needed

$\hat{M}$ can be reconstructed dispersively — for $\pi\pi$ S-wave

$$\hat{M}(s) = \hat{M}^L(s) + \frac{\hat{\Omega}_0(s)}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{\Omega}_0^{-1}(s')\hat{T}_0(s')\hat{\sigma}(s')\hat{M}_0^L(s')}{s' - s - i0}.$$

with $\sigma_{ij}(s) = \delta_{ij}(1 - s_i^{\text{th}}/s)^{1/2}$ and the Omnès matrix

$$\hat{\Omega}_0(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{T}_0^{\,*}(s')\hat{\sigma}(s)\hat{\Omega}_0(s')}{s' - s - i0}$$

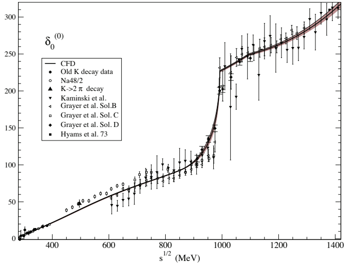
Input needed: $\hat{T}_0(s)$ and $\hat{M}^L(s)$ $\quad \left(M_0^L(s) = \frac{1}{2}\int_{-1}^{+1} dz\, M^L(s,t,u) \right)$

$\pi\pi - K\bar{K}$ SCATTERING

$$\hat{T}_0 = \begin{pmatrix} T_{\pi\pi\to\pi\pi} & T_{\pi\pi\to K\bar{K}} \\ T_{K\bar{K}\to\pi\pi} & T_{K\bar{K}\to K\bar{K}} \end{pmatrix} = \begin{pmatrix} (\eta e^{2i\delta}-1)/2i\sigma_\pi & g e^{i\psi} \\ g e^{i\psi} & (\eta e^{2i(\psi-\delta)}-1)/2i\sigma_K \end{pmatrix}$$

where $\eta = \sqrt{1 - 4g^2\,\sigma_\pi\,\sigma_K\,\theta(s-4m_K^2)}$.

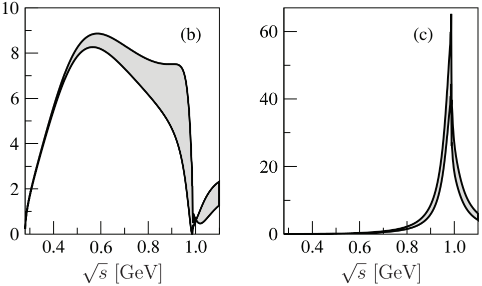
L. Y. Dai and M. R. Pennington, PRD90(2014)036004



R. Garcia-Martin et al., PRD83(2011)074004

$\Longrightarrow$

$\Gamma_\pi^n(s) = \langle\pi\pi|(\bar{u}u+\bar{d}d)/2|0\rangle$ $\Gamma_\pi^s(s) = \langle\pi\pi|\bar{s}s|0\rangle$



Tested in $B_d \to J/\psi\pi\pi/K\bar{K}$ & $B_s \to J/\psi\pi\pi/K\bar{K}$

J. T. Daub et al., JHEP02(2016)009

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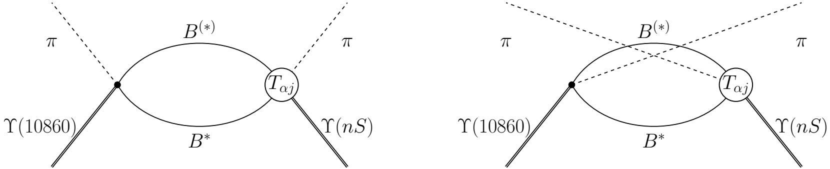
LEFT-HAND CUT CONTRIBUTION



$$M^L = U(t) + U(u) = \int_{\mu^2_{\rm min}}^{\mu^2_{\rm max}} d\mu^2 \rho(\mu^2) M^L_{\rm stable}(t, u; \mu) \; .$$

The information on the structure/nature of the Z_b states is in

$$\rho(\mu^2) = -\frac{1}{\pi} \mathrm{Im}\, U(\mu^2)$$

and we get for Khuri-Treiman integral (anomalous contrib. not shown)

$$\hat{I}_0(s) = \int_{\mu^2_{\rm min}}^{\mu^2_{\rm max}} d\mu^2 \rho(\mu^2) \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{\Omega}_0^{-1}(s') \hat{T}(s') \hat{\sigma}(s') \hat{M}^L_{\rm stable\, 0}(s'; \mu)}{s' - s - i0}$$

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REMARKS ON THE IM.-PARTS

To ensure convergence and to suppress high energies

$\hat{I}_0$ needs subtractions:

$$\hat{I}_0^{(n)}(s) = \hat{\mathcal{P}}_{n-1}(s) + \frac{s^n}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^n} \frac{\hat{\Omega}_0^{-1}(s')\hat{T}(s')\hat{\sigma}(s')\hat{M}_0^L(s')}{s' - s - i0}$$

where the coefficients of $\hat{\mathcal{P}}_{n-1}(s)$ are in general complex,

since $\hat{M}_0^L(s')$ has imaginary part D.A.S. Molnar et al. PLB 797(2019)13485 (for Z_c states)

However, the integral over $\mathrm{Im}(\hat{M}_0^L(s'))$ is finite, s.t.:

$$\mathrm{Im}\hat{\mathcal{P}}_{n-1}(s) = \sum_{k=0}^{n-1} \frac{s^k}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^{(k+1)}} \hat{\Omega}_0^{-1}(s')\hat{T}(s')\hat{\sigma}(s')\,\mathrm{Im}\hat{M}_0^L(s')$$

and we only need to fit $\hat{R}_{n-1}(s) = \mathrm{Re}\hat{\mathcal{P}}_{n-1}(s)$ (we chose $n = 2$)

Its structure is fixed by matching to LO ChPT Y.H. Chen et al., PRD95(2017)034022

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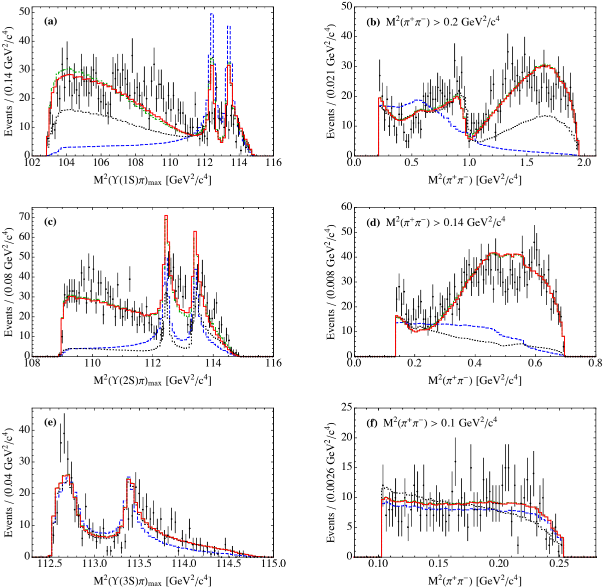
RESULTS



We fit 3 parameters (c_1, c_2 (ChPT), N) to the 2D Dalitzplot

- Z_b*s* only
- + KT integral
- + polynomial
- + *D*-wave

Good description

small # of para.

syst. deviations

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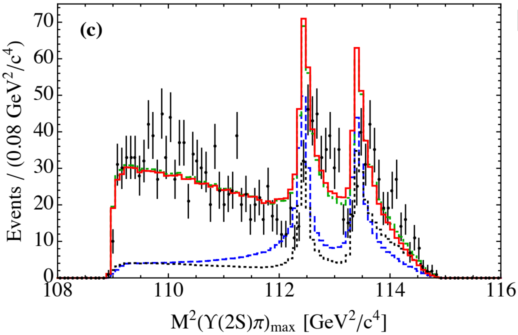
SUMMARY/OUTLOOK

A consistent inclusion of crossed channel effects is possible!

We use input from $\pi\pi - K\bar{K}$ scattering

$\Longrightarrow$ The total number of parameters is small



Next steps:

- Use improved Z_b description (with OPE)
- Perform combined fits of all channels
- Study also $\Upsilon(4S)$ and $\Upsilon(3S)$ decays

This will allow for a high accuracy extraction of the Z_b pole parameters and reliable prediction for the spin partner states