Simultaneously Optimizing Bidding Strategy in Pay-as-Bid-Markets and Production Scheduling

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Abstract

The volatility of renewable energy sources leads to the development of electricity markets with different time horizons, where flexible consumers can monetize their flexibility to stabilize the electric grid. From an electricity-user perspective, the optimal monetization strategy and choice of markets is difficult to identify. We consider simultaneous participation in an ancillary service market with pay-as-bid mechanism and a day-ahead market. We develop a formulation based on Benders decomposition that decouples the participation in both markets. This allows to optimally distribute flexibility to bids in an ancillary service market and participation in day-ahead market via scheduling optimizations. In particular, our formulation allows the optimization problem to be solved with general-purpose nonlinear programming solvers. We demonstrate that using the proposed decomposition is computationally more efficient than our previously published enumeration-based approach.

keywords: Demand-Side Management, Ancillary Service Markets, Benders Decomposition, Model-Based Bidding Strategies

1 Introduction

Renewable energy sources have the potential to solve the climate crisis (Houghton, 2009), but are fluctuating in time. An established way to help synchronize supply and demand in electricity grids powered by fluctuating renewable energy suppliers is demand-side management (Mitsos et al., 2018; Strbac, 2008; Gellings, 1985), which refers to all efforts to utilize consumer flexibility for grid stability.

Flexible electricity users can monetize their flexibility in a variety of markets: For example, Schäfer et al. consider combined participation in a day-ahead market and primary balancing reserve market (Schäfer et al., 2019b), and extend their approach to also participating in the secondary balancing reserve market (Schäfer et al., 2019a). Otashu and Baldea (2018) aim for participation of a chlor-alkali process in a 15-minute market and Dowling et al. (2017) consider both a hierarchy of ancillary service markets addressing different time scales and a hierarchy of spot markets from day-ahead market to real-time trading. Finally, Zhang et al. (2016) also consider specially negotiated contracts such as discount and penalty contracts. As done in (Dowling et al., 2017), the markets can be categorized into markets for ancillary services and spot markets. Ancillary services, i.e., providing balancing reserve capacity, are often sold in auction-based markets. If a bid is accepted, the electricity user permanently has to reserve sufficient flexibility to serve potential balancing requests within a tight time frame. Alternatively, flexible electricity users can exploit price fluctuations on a spot market to schedule overproduction during times of cheap electricity and reduce consumption when electricity becomes expensive (Ramin et al., 2018; Zhang and Grossmann, 2016; Castro et al., 2011, 2009). The question of how much flexibility to monetize in which market is a complex optimization problem (Bohlayer et al., 2018; Klæboe and Fosso, 2013), as answering the question requires simultaneous consideration of an optimal bidding strategy and the influence of that bidding strategy on the ability of the process to reduce its production cost on a spot market.

The markets around the globe, both for ancillary services and spot markets, are evolving (Eid et al., 2016; Hu et al., 2018). The considered time horizons for ancillary service bids, payment mechanisms for accepted bids, qualifications of participation in ancillary service markets and spot markets and several other details are adapted, in part based on research with the goal to harmonize the markets. We follow the German market structure presented in our previous work (Schäfer et al., 2019b): a pay-as-bid balancing reserve market and the day-ahead market. Therein, we developed a method to optimally distribute flexibility between these two markets. Our solution approach in (Schäfer et al., 2019b) relies on explicit enumeration of scenarios of acceptance and rejection of bids on the balancing market. Because of combinatorial explosion, such an algorithm rapidly becomes intractable, even on modern computers. To facilitate computationally more efficient solution algorithms for the simultaneous optimization of bidding strategy and production scheduling, we improve on both the formulation and solution approach presented in our previous work (Schäfer et al., 2019b).

Our novel formulation is based on the idea of Benders decomposition (BD) (Benders, 1962) to separate the production scheduling problem from the optimal bidding problem. BD relies on strong duality of the subproblems of the decomposition. In our case, the subproblem is the scheduling problem, which is often formulated as Mixed-Integer Linear Problem (MILP) (Zhang and Grossmann, 2016) because such models have the capability to model a broad variety of situations and powerful solution algorithms exist for this class of problems. For example, Zhang et al. (2016) model continuous process networks, Kelley et al. (2018)

formulate MILP-models to integrate scheduling and control, and Raman and Grossmann (1991) formulate logic constraints as mixed-integer linear constraints. The scheduling problem in (Schäfer et al., 2019b) is also formulated as MILP. Despite all the advantages that come with this problem class, strong duality rarely holds for MILPs. Therefore, it is not enough to reformulate the interaction between scheduling problem and bidding problem. We also reformulate the scheduling problem itself to a continuous linear program (LP), so that strong duality holds.

The key contributions of this work are:

- We reformulate the scheduling model in a way that strong duality holds.
- We exploit the property of strong duality to embed optimal scheduling information in a bidding strategy model. As a consequence, explicit enumeration is not required to find an optimal bidding strategy anymore.
- We reconstruct results from (Schäfer et al., 2019b) and show that our new formulation enables solution algorithms that can be more efficient than explicit enumeration.

2 Background

Production processes are subject to a variety of constraints to satisfy quality standards, safety requirements, timing of demand and supply, and storage space. Other constraints arise from physics and technical limits. An optimal schedule satisfies all these constraints and chooses the degrees of freedom of the process in a way that optimizes the process aim. Typically, many degrees of freedom of energy-intensive processes can be found in the production profile, i.e. the distribution of utilized production capacity over time can be chosen by process operators. Historically, such profiles have often been a flat nominal production rate to keep the process control simple. However, increasingly fluctuating electricity prices give rise to demand for optimal production scheduling, in particular for energy-intensive processes. Research in this field is done for a broad variety of applications: seawater desalination (Jabari et al., 2019; Ghobeity and Mitsos, 2014), air separation (Basán et al., 2020;

Caspari et al., 2020; Tsay et al., 2019), chlor-alkali processes (Otashu and Baldea, 2019; Brée et al., 2019), and cooling of buildings (Sadat-Mohammadi et al., 2020) are some recent examples. Often, the uncertainty of electricity prices is accounted for with stochastic or robust programming in the references mentioned above. Fluctuating electricity prices also affect the revenue of power generators, so optimal scheduling for electricity generation and distribution is also studied (Alirezazadeh et al., 2020; Bostan et al., 2020; Thaeer Hammid et al., 2020). Generic production processes are considered in (Schäfer et al., 2020) and (Castro et al., 2011). A common ground of optimal scheduling research findings is developed in (Maravelias, 2012), while (Harjunkoski et al., 2014) focuses more on future challenges in the field.

We will integrate the scheduling problem into a bidding strategy problem with a BD. This approach is well suited for computationally decoupling components of an optimization problem without compromising solution quality and has had tremendous success in optimization problems in virtually all application areas, including electricity grids (Mansouri et al., 2020; Saberi et al., 2020), logistics (Alkaabneh et al., 2020; Fischetti et al., 2017) and production scheduling (Fang et al., 2021; Michels et al., 2019). While BD is still advancing – see (Rahmaniani et al., 2017) for a recent review – we will only need the basic concepts which connect subproblems that are pure LPs (Benders, 1962) or convex nonlinear programs (NLP) with constraint qualifications (Geoffrion, 1972) to a master problem.

Our master problem will be the problem of finding the optimal amount of balancing capacity to offer and the optimal ask price for each offer as presented in (Schäfer et al., 2019b). Therein, an aluminum electrolysis process participating in the German primary balancing market is considered. The following tradeoffs need to be optimized: a larger amount of offered balancing capacity – when accepted – generates more revenue from the balancing market, but reduces the flexibility available for the process to exploit price fluctuations on a spot market through an optimal production schedule. The ask price also directly influences the revenue generated on the balancing market. However, since the grid operators are more likely to accept lower prices, a large ask price comes with the risk of being rejected, thus jeopardizing all potential balancing market revenue.

In the literature, optimal bidding strategies are studied in different fields of application. Power production from conventional thermal power plants (Plazas et al., 2005; Conejo et al., 2002) and from hydropower (Faria and Fleten, 2011; Fleten and Kristoffersen, 2007) were among the first considered applications. Through aggregation of several entities which alone would not be allowed to enter restricted markets, consumer pools were enabled to participate in those markets, e.g. residential pools (Nizami et al., 2020) or industrial aggregators (Ottesen et al., 2018). On an even larger scale, (Khajeh et al., 2019) aggregated several electrical microgrids to a price-making entity of the market.

3 An Improved Formulation to Integrate Optimization of Bidding Strategy and Flexible Scheduling

In (Schäfer et al., 2019b), we developed a model of an aluminum electrolysis participating in a pay-as-bid balancing market and a day-ahead spot market. We reported that a monolithic formulation, including both the bidding and the scheduling problem, does not allow to compute a global solution with state-of-the-art solvers. Instead, we exploited that the variables which couple bidding and scheduling are the amounts of reserved balancing capacity. Because of market regulations, the offered capacities have to have integer values. Therefore, there is only a finite number of feasible values that are coupling bidding and scheduling, which facilitates an enumeration. We pre-computed the solution of all possible scheduling problems before using these solutions in an explicit enumeration of the acceptance/rejection scenarios of the bids. Then, the remaining NLP to find the optimal ask prices is solved to global optimality for each scenario. Using the pre-computed scheduling results this way requires that only integer-valued points are considered also in the bidding problem. Thus, the explicit enumeration is necessary, general-purpose MINLP solution algorithms can not be applied. The latter would require the relaxation of the integrality requirements, which is not possible since optimal schedules can only be pre-computed for a finite number of scenarios. Our novel formulation introduces bounds for the scheduling problem through BD. These bounds are valid for fractional values of offered balancing capacities. Thus, our formulation enables the use of general-purpose MINLP solution algorithms.

BD is a decomposition of a problem into a master problem (here the bidding problem) and a subproblem (here the scheduling problem), where only few terms are coupling the master and the subproblem (here the reserved balancing capacity). The subproblem can be formulated parametrically in the coupling terms. For an effective decomposition, the subproblem has to be easy to solve once the coupling terms are fixed. From such a subproblem solution, a cut can be generated. These Benders-cuts (BCs) are inequalities that are added to the master problem. They describe a lower bound on the objective value of the subproblem as a function of the coupling terms, which are decision variables in the master problem. Such a bound is sharp for at least the value the coupling terms were fixed to in the run that generated the cut. Moreover, the lower bound is valid for all values of coupling terms, including fractional values after relaxing integrality constraints.

As we will show, by representing the scheduling problem with BCs in the bidding problem, the latter can be solved as one Mixed-Integer Nonlinear Problem (MINLP). This was not possible in (Schäfer et al., 2019b) because the tabulated solutions for the scheduling problem were only available for integral values of the coupling terms. As we demonstrate in our case study, we can compute the solution for the optimal bidding problem with integrated scheduling computationally more efficiently.

3.1 The Process Model of Aluminum Production

We already presented a decomposition of the simultaneous optimization of bidding strategy and production scheduling to a bidding problem and a scheduling problem in (Schäfer et al., 2019b). Although it was not used for a BD, the scheduling problem was already formulated parametrically in the amount of balancing capacity that had to remain reserved.

We review the most important aspects of the aluminum production process and its model. The process in general is very simple: a bauxite-based liquid electrolyte feeds an electrolysis cell, where electrical power is used to form aluminum from aluminum ions. The aluminum is heavier than the electrolyte and sediments to the bottom of the electrolysis cell, where it can be collected. The plant that runs this process is located in Essen, Germany and operated by TRIMET Aluminium SE (Schäfer et al., 2019b). In our model, the power consumption of the process can vary between 75% and 125% of the nominal power consumption. As the process was optimally designed for nominal operation, production at rates above or below 100% of nominal power consumption is less efficient (the specific electricity requirement increases). This effect is modeled with a cubic relation between power consumption and resulting aluminum production. Due to efficiency losses for both over- and underproduction, the aluminum production curve as a function of power consumption is concave. Also, if the production rate exceeds or subceeds 100%, the accumulated thermal energy in the process increases or decreases. To prevent damage in the electrolysis cells, the accumulated thermal energy must not exceed two days of maximum power uptake worth of accumulated energy. Similarly, to prevent freezing of the electrolyte in the cells, the accumulated thermal energy must not fall below two days of minimum power uptake worth of thermal energy reduction.

Within the given bounds for the power consumption, the aluminum electrolysis can switch from one power consumption level to another within seconds, as already pointed out by Todd et al. (2008). Since primary balancing requests require a response within seconds, the ability to change power consumption fast is a prerequisite for the process to provide primary balancing capacity. The agility of the process also allows us to describe the process as a quasi-stationary model with noncontinuous production changes. The time is discretized in hourly intervals because we consider participation in a day-ahead market providing hourly prices. The scheduling problem minimizes cost for electricity and opportunity cost for lost aluminum production due to efficiency losses.

Other processes with dynamic operational limits could also be modeled: if linear ramping constraints are used, the resulting description would still lead to an LP scheduling problem. However, such processes will probably have to participate in other markets, that pose less tight response requirement, for example, the secondary balancing market that is also considered in (Schäfer et al., 2019a).

3.2 Scheduling Problem Formulation with Strong Duality

A prerequisite for computing valid BCs is that strong duality holds for the scheduling problem. While this property typically does not hold in MILP formulations, it always holds for continuous LPs and convex NLPs with constraint qualifications. LPs as subproblems are used in the original BD (Benders, 1962), convex NLPs are used in generalized BD (Geoffrion, 1972). We formulated the scheduling problem as MILP in (Schäfer et al., 2019b); now we reformulate it as LP to prepare the decomposition. In particular, we deconstruct the usage of binary variables in the scheduling model used in (Schäfer et al., 2019b) and then develop a reformulation that provides a continuous LP.

3.2.1 Origin of Binary Variables

In (Schäfer et al., 2019b), we linearized the cubic relation between power consumption and aluminum production to avoid a nonlinear optimization problem with a piecewise linearization on disjunctive intervals, which requires the introduction of binary variables to keep track of the linearization intervals. This results in the following description of the aluminum production rate:

$$\dot{m}_t = \sum_{i \in \mathcal{I}} y_{i,t} \cdot b_i + \sum_{i \in \mathcal{I}} P_{i,t}^{(lin)} \cdot s_i, \tag{1}$$

$$y_{i,t} \in \{0, 1\} \qquad \forall i \in \mathcal{I}; \quad \forall\, t \in \{1, \cdots, T\}, \tag{2}$$

$$\sum_{i \in \mathcal{I}} y_{i,t} = 1 \qquad \forall\, t \in \{1, \cdots, T\}, \tag{3}$$

where $\dot{m}_t$ denotes aluminum production rate in time slot t, electrical power input is denoted with $P_{i,t}^{(lin)}$, and the linearization parameters slope and intercept are denoted with s_i and b_i. Finally, $y_{i,t}$ are indicators for the relevant linearization interval i from the set of linearization intervals $\mathcal{I}$ in time slot t. A scheduling horizon of one week with hourly resolution leads to $T = 168$. In combination, (2) and (3) ensure that in each time slot t exactly one $y_{i,t}$ is equal to one. Thus, the first sum in (1) has exactly one nonzero addend. The entries of the power input $P^{(lin)}$ obeys the following bounds:

$$P_{i,t}^{(lin)} \geq y_{i,t} \cdot P_{MIN,i}^{(lin)} \qquad \forall\ i \in \mathcal{I}; \quad \forall\ t \in \{1, \cdots, T\}, \tag{4}$$

$$P_{i,t}^{(lin)} \leq y_{i,t} \cdot P_{MAX,i}^{(lin)} \qquad \forall\ i \in \mathcal{I}; \quad \forall\ t \in \{1, \cdots, T\}, \tag{5}$$

with Big-M parameters $P_{MIN,i}^{(lin)}$ and $P_{MAX,i}^{(lin)}$, leading to exactly one nonzero addend of the second sum in (1).

3.2.2 Reformulation Without Binary Variables

For a formulation without binary variables, we do not keep track of linearization intervals. Rather, we can interpret the linear functions defined by slope s_i and intercept b_i as globally valid inequalities, below which the aluminum production rate has to stay. Every inequality is the tightest one on the interval it was designed for, because the original nonlinear function has a concave curvature. Conversely, an inequality is redundant on every interval it was not designed for. The linear inequalities for the production rates $\dot{m}_t$ are:

$$\dot{m}_t \leq b_i + P_t \cdot s_i \qquad \forall\ i \in \mathcal{I}; \quad \forall t \in \{1, \cdots, T\}. \tag{6}$$

In (6), the parameters s_i and b_i are the same as in the corresponding (1) from the mixed-integer scheme. (2) and (3) do not have counterparts, since this linearization approach does not require binary variables. Also, upper and lower bounds for the power input P_t do not depend on the linearization intervals. Simple box constraints are derived from technical considerations:

$$P_t \geq P_{MIN} \quad \forall\ t \in \{1, \cdots, T\}, \tag{7}$$

$$P_t \leq P_{MAX} \quad \forall\ t \in \{1, \cdots, T\}. \tag{8}$$

How is it possible to replace equality constraints with inequality constraints, as we propose? Admittedly, with our reformulation, we changed the feasible set of the scheduling problem. Now that we are using inequalities to model the relation between electrical power input and aluminum output, a nonnegative, but nonphysically low aluminum production is feasible for any electrical power input that obeys the bounds for power input. However, the objective of the scheduling problem is designed to minimize production cost including opportunity cost for not produced products. To minimize this opportunity cost, any optimal point is on the upper bound of feasible aluminum production – as long as the aluminum price is positive, which is a reasonable assumption for any industrially produced good. The upper bound of feasible aluminum production is exactly the feasible set of the MILP formulation that used equalities. Mathematically, this is because the projection of the objective function to the $\dot{m}_t$-P_t-plane always has a positive component along the $\dot{m}_t$-axis, so it always points towards maximum production for products with a positive price. Figure 1 sketches the new feasible set with globally valid inequalities and visualizes the projection of the objective function to illustrate that every possible optimal point is also a feasible point of the MILP-model from (Schäfer et al., 2019b).

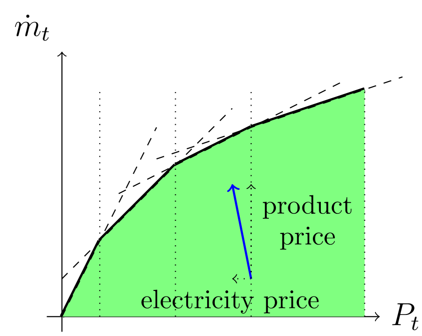

Figure 1: The lines defined by the s_i and b_i are globally valid inequalities. Each line is sketched solid on the interval i it was designed for, and dashed on neighboring intervals. In the P_t-$\dot{m}_t$-plane, the piecewise linear relation composed of the solid line segments would be the feasible region of the integer-based formulation; the green area would be the feasible region of the integer-free formulation. All possible optimal points remain on the solid line segments since the objective function (projected with blue arrow) favors maximum allowed production, as it always has a component in positive $\dot{m}_t$-direction for positive product prices.

Our formulation of the scheduling problem now has exactly the same optimal points as the MILP-formulation presented in (Schäfer et al., 2019b), but additionally has the property that strong duality holds. As a consequence, we can now formulate BCs that underestimate the optimal cost of production, even when integrality requirements are relaxed. A possible alternative to the linearization we propose is to stick to the nonlinear formulation of aluminum production. Thereby, a convex NLP would arise, which could be used to generate the required cuts as well by using generalized BD. However, as in (Schäfer et al., 2019b), we decided to use a linearization because it reduces the complexity of the problems to solve.

The real-world prerequisite for such a modeling approach without integers was the curvature of the relation between production and electrical power input. This is a pattern that can be found in many real-world flexible processes, because plants are still designed and optimized for one specific operation point of nominal operation. Then, any deviation from the nominal operation comes with efficiency losses. When a stronger deviation from the design point leads to more efficiency losses, a concave relation between production and electrical power input emerges. In turn, the problem of optimally scheduling a flexible process might be nonlinear, but it stays a convex problem. Further, the absence of logical or other non-continuous phenomena allows to formulate the scheduling problem without using binary or integer variables.

3.3 Embedding Solutions of Scheduling Problems in the Bidding Problem

When strong duality holds, optimal objectives of primal and dual problem are equal. BD is based on the idea that the dual problem corresponding to the primal subproblem has a feasible set that is independent from the values of the coupling terms. Thus, the dual problem can be described by its extreme points and extreme rays, and the coupling term values only determine the direction of the objective function. In LPs, an optimal solution point of a feasible and bounded problem can always be found on an extreme point of the feasible set. So, to embed the subproblem in the master problem, it suffices to describe all the extreme points and extreme rays of the dual feasible set in the master problem. The objective function of the dual problem on all those points is a lower bound for the optimal value of the primal problem – and at least one point's dual objective is equal to the optimal value of the primal subproblem. However, this is only theoretically interesting because the number of extreme points and extreme rays of a problem grows exponentially with the problem size. Usually, when BD is successful, the reason is that a provably optimal point is found by only adding a small fraction of extreme points and extreme rays of the dual feasible set to the master problem rather than performing an exhaustive enumeration.

The term that couples the bidding problem and the scheduling problem is the amount of balancing capacity that has to remain reserved. Due to market restrictions, it is a bounded integer quantity. In the primal scheduling model, the reserved capacity directly affects the box constraints of electrical power inputs P_t and the production loss due to efficiency losses in off-design production when the balancing capacity is called. Because the coupling term is a bounded integer, it can only take finitely many feasible values.

In the perspective of the dual scheduling problem, the integrality requirements of the coupling term imply that only a finite number of objective functions is possible in the dual scheduling problem. Thus, rather than enumerating all extreme rays and extreme points of the dual feasible set, we enumerate all possible objective functions of the dual scheduling problem. This yields all possible optimal schedules which can be represented in the bidding problem through BCs. In fact, one could iterate between master and subproblem like Benders originally suggested (Benders, 1962). Such an algorithm would add representations of optimal schedules iteratively and hope to find a provably optimal point before computing all possible optimal schedules. Whether that is efficient depends on the computational effort to solve the master problem relative to the computational effort to solve the subproblem.

A BC can be generated from the optimal solution of a scheduling problem by fixing the amount of balancing capacity that has to remain reserved. This term is just a parameter of the scheduling problem, but it does not explicitly exist in the bidding problem. There, it is uncertain which offers will be accepted and therefore what balancing capacity has to remain reserved. In the following paragraphs, we describe how we solve this issue by defining scenarios for which bids are accepted.

3.3.1 Scenarios for Accepted Bids

In the bidding problem, the degrees of freedom are – for each bid b allowed to be placed – one amount of balancing capacity P_b^{PRL} in MW and a corresponding ask price A_b in €/MW. To break symmetry, we sort the bids in ascending order of prices, i.e.

$$A_1 \leq A_2 \leq \cdots \leq A_B, \tag{9}$$

where B is the number of bids allowed to be placed in the model. An offer will be accepted when the ask price is below the maximum price the grid operators will accept, $A^{acc,max}$. In the bidding problem, $A^{acc,max}$ is a continuous random variable. With a suitable stochastic model for the maximum accepted ask price $A^{acc,max}$, such as the one presented in (Schäfer et al., 2019b), the probability that a bid with ask price A_b will be accepted can be computed with

$$\mathbb{P}(\text{bid } b \text{ accepted}) = \mathbb{P}(A^{acc,max} \geq A_b) = 1 - \mathbb{P}(A^{acc,max} \leq A_b). \tag{10}$$

After ordering the bids according to (9), we can define discrete scenarios by aggregating all values for $A^{acc,max}$ between two ask prices plus two more scenarios to describe the case where the maximum accepted ask price is below respectively above all ask prices A_b. We will refer to the scenario in which all bids from the first to the b^{th} bid are accepted and all other bids are rejected as scenario $b^{(acc)}$. From (10), we derive the probability for any of these scenarios as:

$$\mathbb{P}_{B^{(acc)}} = 1 - \mathbb{P}(A^{acc,max} \leq A_B) \tag{11}$$

$$\begin{aligned} \mathbb{P}_{b^{(acc)}} &= \mathbb{P}(A_b \leq A^{acc,max} \leq A_{b+1}) \\ &= \mathbb{P}(A^{acc,max} \leq A_{b+1}) - \mathbb{P}(A^{acc,max} \leq A_b) \quad \forall\, b = \{1, \cdots, B-1\}, \end{aligned} \tag{12}$$

$$\mathbb{P}_{0^{(acc)}} = 1 - \sum_{\tilde{b}=1}^{B} \mathbb{P}_{\tilde{b}^{(acc)}} \tag{13}$$

where $\mathbb{P}_{0^{(acc)}}$ denotes the probability of scenario $0^{(acc)}$ – in which all bids are rejected–, $\mathbb{P}_{b^{(acc)}}$ denotes the probability of scenario $b^{(acc)}$, and $\mathbb{P}_{B^{(acc)}}$ denotes the probability that all bids are accepted, i.e. the probability of scenario $B^{(acc)}$. Now, the bidding problem is able to optimize the expected value of scheduling cost and revenue from balancing markets by weighting the optimally scheduled production cost and the PRL revenue from each scenario with the probability of that scenario.

3.3.2 Representing Costs of Optimal Schedules in the Bidding Problem with BC

The amount of balancing capacity that has to remain reserved is the sum of the offered capacities P_b^{PRL} of all accepted bids. This sum is the term that couples bidding problem and scheduling problem. Because the coupling terms are scenario-dependent, the BC that relate coupling terms and the representation of optimal production cost are also formulated for each scenario $b^{(acc)}$, except for the scenario $0^{(acc)}$ in which all bids are rejected:

$$C^{(sched)}_{b^{(acc)}} \geq \rho_c - \mu_c \cdot \sum_{b=1}^{b^{(acc)}} P_b^{PRL} \quad \forall\, c \in \{0, \cdots, 10\}, \quad \forall\, b^{(acc)} \in \{1^{(acc)}, \cdots, B^{(acc)}\} \tag{14}$$

where $C^{(sched)}_{b^{(acc)}}$ represents the optimal production cost in scenario $b^{(acc)}$ of the bidding problem. In case that all offers are rejected, we do not need to specify any BC; the optimally scheduled production cost for the full flexibility available on the day-ahead market is computed before the solution of the bidding problem and specified as parameter $C_{0^{(acc)}}^{(sched)}$. The coefficients ρ_c and μ_c can be computed according to (Benders, 1962) from the dual solution of the scheduling problem in which c MW had to remain reserved for balancing requests. Note that the scheduling problem only depends on this reserved balancing capacity. In particular, the optimal schedule and the corresponding dual solution are independent of whether the reserved balancing capacity was just due to one single bid or the sum of several bids. As a consequence, the coefficients ρ_c and μ_c are independent of the scenario $b^{(acc)}$.

By adding BCs to the bidding problem, we have represented all possible optimal schedules in the bidding problem with a few linear inequalities rather than the fully blown scheduling problem. The remaining bidding problem has to optimize the offered capacities P_b^{PRL} and corresponding ask prices A_b. General-purpose (MI)NLP solvers can construct and use valid continuous relaxations of the bidding problem because it contains BCs that describe valid lower bounds for the optimally scheduled production cost for all fractional values of offered capacity. Therefore, this formulation can be solved by implicit enumeration algorithms. Like explicit enumeration, such algorithms have an exponential runtime in the worst case, but in practice they are often more efficient than explicit enumeration.

4 Case Study

We repeat the case study of an aluminum production process presented in (Schäfer et al., 2019b), which we briefly reviewed in Subsection 3.1, and simultaneously optimize bidding strategy and production scheduling for this setup. We compare solving the original formulation by an explicit enumeration scheme to solving our reformulation with a general-purpose MINLP solver. Coming from the bidding model and scheduling model presented in (Schäfer et al., 2019b), we replace the parts of the scheduling model described in Section 3.2.1 with our reformulation presented in Section 3.2.2. The amount of reserved balancing capacity determines both the flexible range of electrical power input and the production losses due to efficiency losses when a balancing request is issued by the grid.

We compute all possible cuts prior to solving the bidding-MINLP. We solve the scheduling

LPs for all feasible balancing capacities and compute all corresponding ρ_c and μ_c to build all required cuts (14). This guarantees that we only have to solve one MINLP. Also, we can derive tight bounds for the objective and the variables $C^{(sched)}_{b^{(acc)}}$ which represent the optimally scheduled production cost in the bidding MINLP. The effort for solving the scheduling LPs is negligible compared to solving the MINLP. In other setups (less complex bidding and more complex scheduling problem), an algorithm with few iterations between master and subproblems might be suitable, as discussed in Section 3.3.

Following (Schäfer et al., 2019b), we take market data from the first four weeks of 2018 in Germany to describe primary balancing market and the day-ahead market. We run the case study for the number of allowed bids B ranging from 1 to 3, again adopting form (Schäfer et al., 2019b). Note that in practice, one bid on the primary balancing market suffices to find a good solution (Schäfer et al., 2019b), but we want to examine the benefit of our reformulation of the problem also for the computationally more complex case of $B = 3$.

For (mixed-integer) nonlinear optimization problems, such as our bidding problem, it is desirable to solve for a global optimum. For this purpose, we choose the global deterministic optimization solver BARON (Tawarmalani and Sahinidis, 2005; Zhou et al., 2018). We also tried to solve the bidding problem with our solver MAiNGO (Bongartz et al., 2018), which is based on McCormick-relaxations (McCormick, 1976; Mitsos et al., 2009; Tsoukalas and Mitsos, 2014) rather than the auxiliary variable method. However, MAiNGO performed inferior to BARON. We do not know the reason for the performance differences, but remark that BARON and MAiNGO differ substantially in many aspects, for example, MAiNGO uses McCormick-relaxations (McCormick, 1976; Mitsos et al., 2009; Tsoukalas and Mitsos, 2014), while BARON uses the auxiliary variable method. Further differences are the methods to treat bilinear products, and whether the code is open-source or commercially available, to name some more examples. All calculations were carried out on a Server equipped with an Intel(R) Xeon(R) Silver 4112 CPU @2.60 GHz and 256GB RAM. We used a single thread and called BARON through the GAMS 28.2-interface (GAMS Development Corporation, 2019).

Our code, including a MATLAB script to compute the BC coefficients from the scheduling problems, is publicly available on `http://permalink.avt.rwth-aachen.de/?id=963420`.

5 Results and Discussion

We solved the problem of simultaneous optimization of bidding strategy in pay-as-bid markets and production scheduling with the explicit enumeration strategy from (Schäfer et al., 2019b) and as monolithic MINLP enabled by our reformulation. As convergence criterion, we define a relative optimality gap of 0.1%. Such a tight relative tolerance is desired because the absolute production costs are several hundred thousand euros per week. Thus, even relatively small savings correspond to a significant absolute cost reduction. Tighter tolerances would not be justified, given the approximate model.

The two algorithms find comparable solutions in all studied setups. As shown in Table 1, the bidding strategies sometimes differ slightly, but the relative differences in the optimal objectives are significantly smaller than the optimality tolerance and as such, both solutions are optimal. Figure 2 shows the optimal schedules for two scenarios from week 4: If no bid is accepted, the full process flexibility is available and will be exploited by the process. If some flexibility has to remain reserved as balancing capacity, the feasible domain for P_t becomes tighter. The schedule shown in the bottom of Figure 2 is the optimal schedule for both explicit enumeration and MINLP formulation in week 4 with $B = 2$, if both bids are accepted. In this case, the MINLP solution is to bid 6 MW in the first bid and 1 MW in the second bid, whereas the explicit enumeration approach would be to split the bids into 5 MW and 2 MW. However, if both bids are accepted, the total reserved balancing capacity –which determines the optimal schedule– is 7 MW either way. Since the LP formulation developed in Section 3 has the same optimal points as the MILP formulation in (Schäfer et al., 2019b), the optimal schedules are independent of the chosen formulation.

In Table 2, we present the CPU time required to execute the solve statements for both approaches (provided by the GAMS (GAMS Development Corporation, 2019) parameter *etSolve*). The scheduling step that takes place before the explicit enumeration, respectively

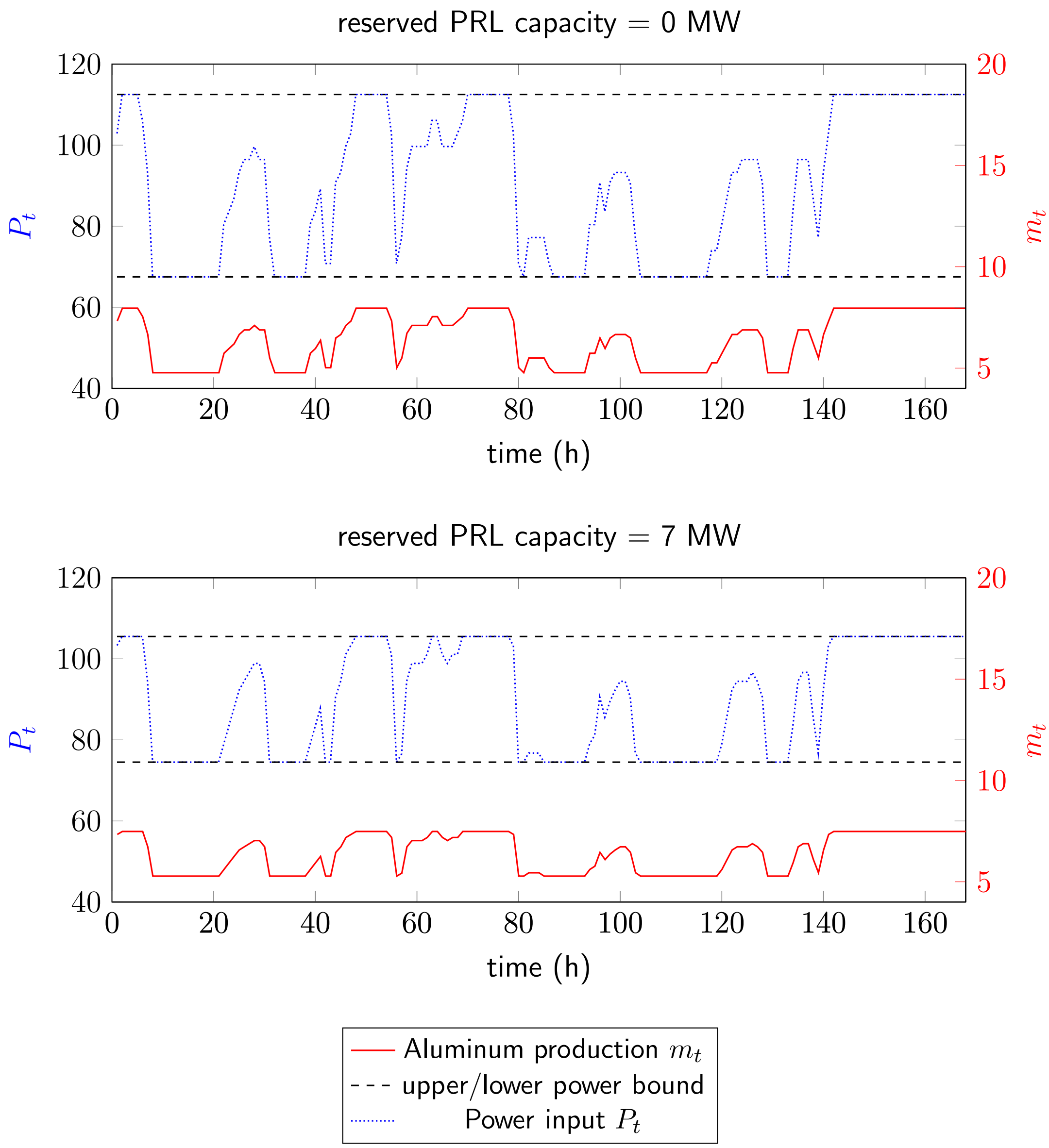

Figure 2: Optimal schedules in week 4; in the top for the scenario where no bid is accepted, and in the bottom where the total reserved balancing capacity is 7 MW.

the MINLP-solution process, takes around 60 seconds for all 4 weeks with the MILP formulation from (Schäfer et al., 2019b) and 13 seconds for all 4 weeks with our LP-reformulation from Section 3.2. Table 2 shows that our reformulation decreases the time required for global optimization of a bidding strategy with simultaneous consideration of production scheduling with access to a day-ahead market. Our MINLP approach often reduces the time explicit enumeration requires by more than 70% and up to 87%. Only in setups with $B = 3$, the performance of our MINLP approach is mixed, for which we do not have a clear explanation. With increasing B, the MINLP problem size, i.e, the number of variables and constraints increases. While the increase is mostly linear in B, as each additional bid gives one additional acceptance scenario to consider, the computational effort for solving a linearly growing problem grows exponentially. In contrast, in the explicit enumeration scheme, the number of NLPs that have to be solved grows combinatorially, and the NLP size grows linearly. It is hard to judge if these effects contribute to the mixed performance of our reformulation relative to the explicit enumeration scheme. Another potentially relevant aspect might be the tolerances: as we argued before, 10^{-3} is quite a tight tolerance for global optimization solvers, but it is justified in our context. In our preliminary studies, we have seen that with a tolerance of 10^{-2}, for both the NLPs in explicit enumeration, and the MINLP resulting from our reformulation, our MINLP is still 50%-90% faster than the explicit enumeration for all the scenarios with $B = 3$. This is an indication that the tight tolerances have a larger impact on the one large MINLP formulation, than on the many smaller NLPs that are solved during explicit enumeration.

The runtime until the optimal solution is found when BARON solves our MINLP formulation is given in the last column of Table 2. Often, the optimal solution is already found in BARON's preprocessing step. This means that BARON quickly finds the optimal solution and spends the vast majority of the runtime proving its optimality by refining the lower bound. We did not try to solve the problem with local solvers because the globally optimal solution can be considered a local solution as soon as it is found, i.e. within less than one second. Such an algorithmic behavior is common in deterministic global optimization. The existence of a global lower bound is the second advantage of general-purpose MINLP solvers over explicit enumeration in which no lower bound is available. With the lower bound, a guaranteed optimality gap for the solution is available as soon as an incumbent solution is found. Another advantage of the MINLP formulation over the explicit enumeration approach is that just one call to BARON is required to solve one MINLP, which might reduce solver-calling-related computational overhead.

In explicit enumeration, the time to find the optimal solution depends on the enumeration order, which is arbitrary. Therefore, we only show the time to complete the enumeration in Table 2, not the time until the optimal solution is found. In this case study, the differences between the optimal objectives of different scenarios can be up to 2.5% of the optimal expected cost. This seemingly small difference is in fact very relevant because the majority of cost savings is achieved by optimally scheduling the production to exploit price differences in the day-ahead market. When we compare optimal objectives of different scenarios, the schedule is already optimized in both cases and the difference is only affected by the bidding strategy. However, note that the absolute numbers in this case study are several hundred thousand € production cost per week. Therefore, the bidding strategy has a significant effect on the absolute cost and it is highly attractive to solve for the optimal strategy. Thus, a third advantage of our MINLP-formulation over explicit enumeration is its ability to reliably identify the optimal solution early in the algorithm. Particularly for the setups with $B = 3$, where the MINLP-formulation can take more time to terminate with proven optimality than explicit enumeration, it is important to note that the MINLP-formulation finds the optimal solution reliably in less than a second.

As the results presented in (Schäfer et al., 2019b) used a very tight relative optimality gap of 10^{-5}, we compared the performance of our approach to the explicit enumeration for this tight tolerance as well. Except once, we found exactly the same solutions with the two approaches, with our approach being more efficient for all but some setups with $B = 3$. In one of these setups, the MINLP approach did not converge to a relative optimality gap of 10^{-5} within 4 CPU-hours. As discussed before, problem size and a potentially larger impact of tight tolerances on the runtime to solve one large MINLP than on the runtime to solve many smaller NLPs are possible explanations for the degradation of MINLP performance compared to explicit enumeration.

6 Conclusion

Based on BD, we developed a reformulation of the problem of simultaneous optimization of bidding strategy in pay-as-bid markets and production scheduling that enables the solution with general-purpose MINLP solvers. A case study on the German primary balancing market and the German day-ahead-market of 2018 showed that the reformulation enables reductions of the computational runtimes to find and proof a globally optimal solution, often to a third or less of the time an explicit enumeration scheme takes. Further algorithmic advantages enabled by our reformulation are that optimal solutions are identified quickly and inherently come with an optimality gap as soon as they are identified with branch-and-bound-based algorithms.

The reformulation we presented is applicable in all pay-as-bid markets. Examples for such markets from 2021 are the secondary and tertiary balancing market in Germany, also known as automatic Frequency Restoration Reserve and manual Frequency Restoration Reserve. In pay-as-cleared market the optimal bidding strategy is simpler to identify without solving optimization problems of the kind we discussed: Setting an ask price equal to the marginal cost of providing balancing capacity for the process always generates the optimal revenue from a pay-as-cleared market for a price-taking market participant.

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Table 1: Optimal Results for 1 to 3 maximum allowed bids B

B	week		explicit Enumeration	general-purpose MINLP
1	1	optimal expected cost	237 k€	237 k€
		P_1^{PRL} and A_1	7 MW, 3623 €/MW	6 MW, 3597 €/MW
	2	optimal expected cost	516 k€	516 k€
		P_1^{PRL} and A_1	10 MW, 3005 €/MW	10 MW, 3005 €/MW
	3	optimal expected cost	454 k€	454 k€
		P_1^{PRL} and A_1	10 MW, 2665 €/MW	10 MW, 2665 €/MW
	4	optimal expected cost	427 k€	427 k€
		P_1^{PRL} and A_1	6 MW, 2420 €/MW	6 MW, 2420 €/MW
2	1	optimal expected cost	237 k€	237 k€
		P_1^{PRL} and A_1	5 MW, 3574 €/MW	5 MW, 3574 €/MW
		P_2^{PRL} and A_2	3 MW, 3891 €/MW	4 MW, 3965 €/MW
	2	optimal expected cost	516 k€	516 k€
		P_1^{PRL} and A_1	6 MW, 2968 €/MW	6 MW, 2968 €/MW
		P_2^{PRL} and A_2	4 MW, 3094 €/MW	4 MW, 3094 €/MW
	3	optimal expected cost	454 k€	454 k€
		P_1^{PRL} and A_1	7 MW, 2631 €/MW	8 MW, 2641 €/MW
		P_2^{PRL} and A_2	3 MW, 2824 €/MW	2 MW, 2866 €/MW
	4	optimal expected cost	427 k€	427 k€
		P_1^{PRL} and A_1	5 MW, 2403 €/MW	6 MW, 2420 €/MW
		P_2^{PRL} and A_2	2 MW, 2627 €/MW	1 MW, 2692 €/MW
3	1	optimal expected cost	236 k€	237 k€
		P_1^{PRL} and A_1	4 MW, 3554 €/MW	4 MW, 3554 €/MW
		P_2^{PRL} and A_2	3 MW, 3773 €/MW	3 MW, 3773 €/MW
		P_3^{PRL} and A_3	2 MW, 4140 €/MW	3 MW, 4240 €/MW
	2	optimal expected cost	516 k€	516 k€
		P_1^{PRL} and A_1	5 MW, 2960 €/MW	0 MW, 2976 €/MW
		P_2^{PRL} and A_2	3 MW, 3042 €/MW	7 MW, 2976 €/MW
		P_3^{PRL} and A_3	2 MW, 3142 €/MW	3 MW, 3116 €/MW
	3	optimal expected cost	454 k€	454 k€
		P_1^{PRL} and A_1	5 MW, 2613 €/MW	7 MW, 2631 €/MW
		P_2^{PRL} and A_2	3 MW, 2713 €/MW	0 MW, 2631 €/MW
		P_3^{PRL} and A_3	2 MW, 2866 €/MW	3 MW, 2824 €/MW
	4	optimal expected cost	427 k€	427 k€
		P_1^{PRL} and A_1	4 MW, 2388 €/MW	6 MW, 2420 €/MW
		P_2^{PRL} and A_2	2 MW, 2529 €/MW	0 MW, 2420 €/MW
		P_3^{PRL} and A_3	2 MW, 2770 €/MW	1 MW, 2692 €/MW

Table 2: Runtimes of Explicit Enumeration and Benders Decomposition

B	week	solution time expl. Enum. [in s]	solution time MINLP [in s]	time until BARON identifies optimal MINLP-solution [in s]
1	1	11	2.8	0.19
	2	10	1.3	0.14*
	3	10	2.1	0.25
	4	10	1.9	0.14
2	1	94	25	0.19
	2	86	19	0.20*
	3	79	13	0.20*
	4	80	15	0.22*
3	1	898	3076	0.34
	2	719	206	0.31*
	3	673	701	0.28*
	4	503	794	0.42*

The star ($*$) marks cases in which BARON already identifies the optimal solution during preprocessing.