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$N\bar{N}$ interaction from chiral effective field theory and its application to neutron-antineutron oscillations

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Abstract. An $\bar{N}N$ potential is introduced which is derived within chiral effective field theory and fitted to up-to-date $\bar{N}N$ phase shifts and inelasticities, provided by a proper phase-shift analysis of available $\bar{p}p$ scattering data. As an application of this interaction neutron-antineutron oscillations in the deuteron are considered. In particular, results for the deuteron lifetime are presented, evaluated in terms of the free-space $n-\bar{n}$ oscillation time, utilizing that $\bar{N}N$ potential together with an NN interaction likewise derived within chiral effective field theory.

1 Introduction

Recently the Jülich-Bonn group has established a high-precision $\bar{N}N$ potential within chiral effective field theory (EFT) [1]. Starting point for that work was Ref. [2] where a new generation of NN potentials derived in the framework of chiral EFT was presented. In that publication a novel local regularization scheme was introduced and applied to the pion-exchange contributions of the NN force. Furthermore, an alternative scheme for estimating the theoretical uncertainty was proposed that no longer depends on a variation of the cutoffs [3, 4]. Those concepts were adopted and implemented in the study of the $\bar{N}N$ interaction [1]. Specifically, an $\bar{N}N$ potential was derived up to next-to-next-to-next-to-leading order ($\mathrm{N^3LO}$) in the perturbative expansion, thereby extending a previous work by the Jülich-Bonn group that had considered the $\bar{N}N$ force up to $\mathrm{N^2LO}$ [5]. In both cases the strength parameters of the contact terms that arise in the EFT framework [4, 6] (i.e. the low-energy constants or simply LECs) have been fixed by a fit to the phase shifts and inelasticities provided by a proper phase-shift analysis of $\bar{p}p$ scattering data [7].

As an application of this interaction neutron-antineutron oscillations in the deuteron have been considered [8]. Neutron-antineutron ($n-\bar{n}$) oscillations involve a change of the baryon number (B) by two units ($|\Delta B| = 2$). An experimental observation would allow a glimpse on physics beyond the standard model, see e.g. [9–11]. Since in such oscillations B is violated the process satisfies one of the Sakharov conditions [12] that have been formulated in order to explain the observation that there is more matter than anti-matter in the universe [13].

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The key quanitity in this subject is the free $n-\bar{n}$ oscillation time, $\tau_{n-\bar{n}}$. The presently best experimental limit on it is $\tau_{n-\bar{n}} > 0.86 \times 10^8$ s ≈ 2.7 yr (with 90 % C.L.) [14]. Additional information can be deduced from studies of $n-\bar{n}$ oscillations in a nuclear environment. Corresponding experiments have been performed, e.g., for ^{56}Fe [15], ^{16}O [16], and for the deuteron (^{2}H) [17], while others are planned [18]. In such a case the oscillation process is suppressed as compared to the free situation. The pertinent lifetime $\tau_{\rm nuc}$ is commonly expressed in terms of the one in free space as [10]

$$\tau_{\rm nuc} = R\,\tau_{n-\bar{n}}^2\,, \tag{1}$$

where R is an intranuclear suppression factor, also called reduced lifetime, that depends on the specific nucleus. It can be calculated from nuclear theory and then can be used to relate the measured lifetimes of those nuclei with the free $n-\bar{n}$ oscillation time [10], see, e.g., Refs. [19–23].

For a long time the suppression factors published in 1983 [20] have been used as standard by experimentalists in the interpretation of their measurements [15, 17]. For example, in case of the deuteron the corresponding value is $R \sim (2.40-2.56)\times 10^{22}$ s^{-1}, a prediction based on the phenomenological antinucleon-nucleon ($\bar{N}N$) potentials by Dover and Richard [24, 25]. Recently, however, those values have been challenged in a work by Oosterhof et al. [26]. In that study an effective field theory for the $|\Delta B| = 2$ interaction is constructed and the quantity R is evaluated within the power counting scheme proposed by Kaplan, Savage, and Wise (KSW) [27, 28] for the nucleon-nucleon (NN) and $\bar{N}N$ interactions. The value of R for the deuteron obtained in that approach is $(1.1 \pm 0.3)\times 10^{22}$ s^{-1}, about a factor 2 smaller than the one by Dover et al. [20].

In the light of this controversal situation a new calculation of the suppression factor for the deuteron has been performed by us [8]. It was prompted by the aim to utilize the discussed modern chiral interactions for the involved NN [2] and $\bar{N}N$ [1] systems. Specifically, in case of the $\bar{N}N$ interaction most of the available precise $\bar{p}p$ scattering data (mostly from the LEAR facility at CERN [29]) have appeared only after the publication of the potentials used in Ref. [20]. Therefore, an update is long overdue. Of course, the main motivation was the aforementiond discrepancy reported in Ref. [26] and the prospect to find a plausible explanation for that difference.

The paper is structured in the following way: In Sect. 2 the employed $\bar{N}N$ potential is introduced and some results for $\bar{n}p$, relevant for the calculation of $n-\bar{n}$ oscillations in the deuteron, are presented. In Sect. 3 a basic description of the formalism for evaluating the $n-\bar{n}$ oscillations in the deuteron is provided. Results for the oscillations, specifically for the suppression factor R, are presented in Sect. 4. The paper closes with a brief summary.

2 The $\bar{N}N$ interaction in chiral EFT

A detailed description of the derivation of the chiral $\bar{N}N$ potential using the Weinberg power counting can be found in Ref. [1]. For information on the NN potential employed in the present calculation see Ref. [2]. As already indicated above, the chiral potential contains pion exchanges and a series of contact interactions with an increasing number of derivatives. Up to N^{3}LO there are contributions from one-, two- and three-pion exchanges. Those are identical to the ones that appear in the NN potential [2]. However, there is a sign change in case of an odd number of exchanged pions due to its negative G parity, i.e. $V^{\bar{N}N} = -V_\pi + V_{2\pi} - V_{3\pi}$, with V_π etc. being the corresponding contributions to the NN force. On the other hand, the contact interaction, $V_{cont}^{\bar{N}N}$, cannot be taken over simply from the NN case. Those contact terms represent effectively the short-range part of the interaction and, therefore, the G parity

of the indivudal contributions remains unresolved. Thus, the strength parameters associated with the arising contact terms, the LECs, need to be determined in a fit to $\bar{N}N$ data. We fix them by fitting to the phase shifts and inelasticity parameters of the PWA of Zhou and Timmermans [7]. How this is done is described in detail in Ref. [1]. Note that there are more independent LECs in the $\bar{N}N$ case than in NN because in the former system there is no restriction from the Pauli principle. In addition, in our approach $\bar{N}N$ annihilation (into multi-meson channels) is likewise parameterized by contact terms, see Refs. [1, 5] for explicit expressions.

Once the potential is established, the reaction amplitude is obtained from the solution of a relativistic Lippmann-Schwinger (LS) equation. It reads in partial-wave projected form

$$\begin{aligned} T_{L''L'}(p'', p'; E_k) &= V_{L''L'}(p'', p') \\ &+ \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'', p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p, p'; E_k) \,. \quad (2) \end{aligned}$$

Here p'' and p' are the (moduli of the) center-of-mass $\bar{N}N$ momenta in the final and initial states, respectively, and $E_k = \sqrt{M_p^2 + k^2}$, where k is the on-shell momentum. We adopt a relativistic scattering equation so that our amplitudes fulfill the relativistic unitarity condition at any order, as done also in the NN sector [3, 6]. On the other hand, relativistic corrections to the potential are calculated order by order. They appear first at N^{3}LO in the Weinberg scheme, see Appendix A in Ref. [1]. L, L', etc., specifies the orbital angular momentum, considering that the $\bar{N}N$ system can be in uncoupled (spin-singlet or triplet) states where $L'' = L' = L = J$ or in coupled partial waves with $L'', L', L = J - 1, J + 1$.

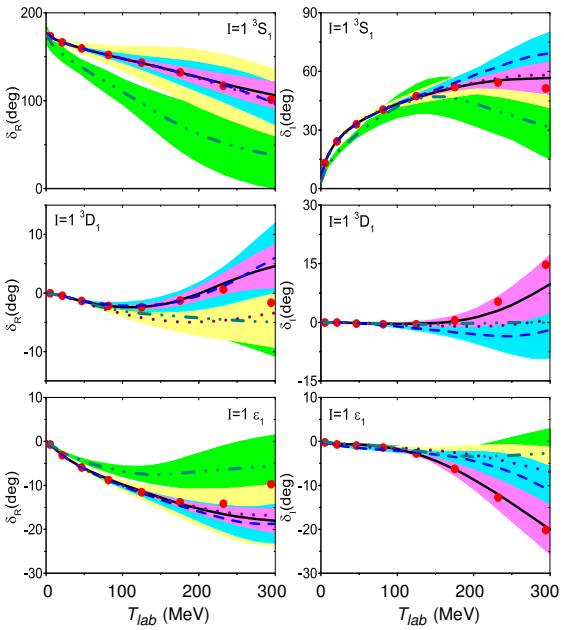

Figure 1. Real and imaginary parts of the phase shifts in the $^3S_1$-$^3D_1$ partial wave with isospin $I = 1$. Results at N^{3}LO (black solid line), N^{2}LO (blue dashed line), and NLO (red dotted line) are shown. The estimated uncertainty is indicated by bands: N^{3}LO (magenta), N^{2}LO (cyan), and NLO (yellow). The filled circles represent the solution of the $\bar{p}p$ PWA [7].

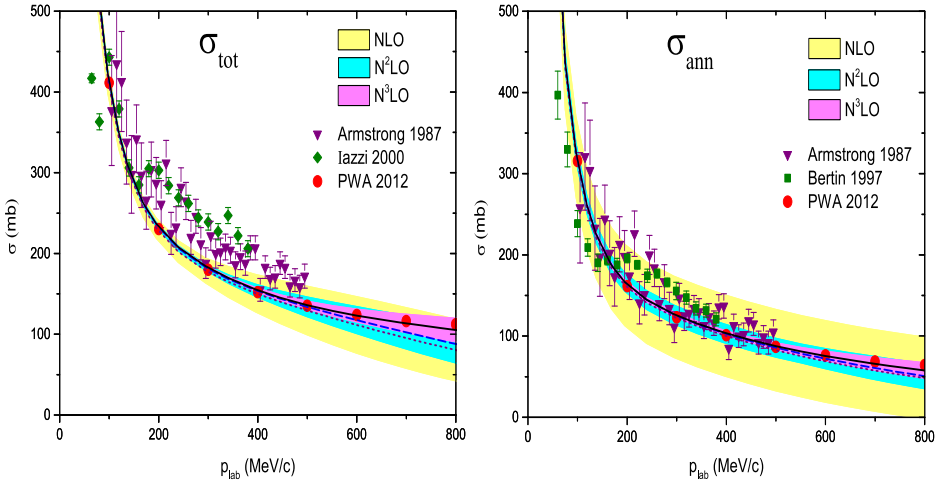

Figure 2. Total (σ_{tot}) and integrated annihilation (σ_{ann}) cross sections for $\bar{n}p$ scattering. Same description of curves and bands as in Fig. 1.

Since the integral in the LS equation (2) is divergent for the chiral potentials [3, 6] a regularization needs to be introduced. For that the regularization scheme of Ref. [2] is utilized, where a local regulator is used for the pion-exchange contributions and a nonlocal regulator for the contact terms:

$$V_{n\pi}(q) \to V_{n\pi}(r) \times f_R(r) \to V_{n\pi}^{reg}(q); \qquad (\vec{q} = \vec{p}\,' - \vec{p})$$
$$V_{cont} = V(p', p) \to V(p', p) \times f_\Lambda(p', p) = V_{cont}^{reg} \,. \tag{3}$$

The explict form of the regulation functions is $f_R(r) = \left[1 - exp(-r^2/R^2)\right]^6$ and $f_\Lambda(p, p') = exp(-(p'^2 + p^2)/\Lambda^2)$. The cutoff radius R is varied in the range R = 0.7-1.2 fm [2] where $\Lambda = 2/R$ is used for relating the momentum-space cutoff parameter with the cutoff radius.

A complete overview of our results for $\bar{N}N$ scattering up to N^{3}LO in chiral EFT can be found in Ref. [1] while possible $\bar{N}N$ bound states are discussed in Ref. [30]. Here we focus on quantities that are relevant for the study of $n - \bar{n}$ oscillations in the deuteron. Specifically, we look at the $^3S_1$-$^3D_1$ phase shifts in the $\bar{n}p$ (isospin $I = 1$) channel (Fig. 1) and the $\bar{n}p$ cross section, see Fig. 2. Note that data for the latter reaction were not included in the PWA [7] because they are less precise than the ones for $\bar{p}p$. The bands in Figs. 1 and 2 represent the estimated uncertainty of our results. Here we follow the suggestion of Ref. [2] and use as measure the expected size of the higher-order corrections together with the actual size of the higher-order corrections, see Ref. [1] for details.

In the study of $n - \bar{n}$ oscillations below we employ also an earlier (N^{2}LO) $\bar{N}N$ potential published by our group. A description of that interaction can be found in Ref. [5]. We consider these two $\bar{N}N$ interactions because they are based on rather different regularization schemes. In the earlier potential [5] a non-local exponential exponential regulator was employed for the whole potential while, as outlined above, in the N^{3}LO interaction [1] a local regulator was adopted for the evaluation of the one- and two-pion contributions. Comparing the pertinent results allows us to shed light on the question in how far the choice of the regulator influences the predictions. For exploring the sensitivity of the results to the deuteron wave function we employ also those from two meson-exchange potentials [31, 32], besides the ones calculated consistently within chiral EFT.

3 Formalism for $n-\bar{n}$ oscillations

The evaluation of $n-\bar{n}$ oscillations in the deuteron is done along the formalism presented in Refs. [19, 20]. However, since our study is performed in momentum space we describe the main steps below. The starting point is the eigenvalue (Schrödinger) equation [19]

$$\begin{pmatrix} H_0+V_{np} & V_{n\bar{n}} \\ V_{n\bar{n}} & H_0+V_{\bar{n}p} \end{pmatrix} \begin{pmatrix} |\psi_{np}\rangle \\ |\psi_{\bar{n}p}\rangle \end{pmatrix} = (E-i\Gamma/2) \begin{pmatrix} |\psi_{np}\rangle \\ |\psi_{\bar{n}p}\rangle \end{pmatrix} . \tag{4}$$

Here, V_{np} and $V_{\bar{n}p}$ are the potentials in the np and $\bar{n}p$ systems and $|\psi_{np}\rangle$ and $|\psi_{\bar{n}p}\rangle$ are the corresponding wave functions. The systems are coupled via $V_{n\bar{n}}$ which is given by the $n-\bar{n}$ transition matrix element $\delta m_{n\bar{n}}$ where the latter is proportional to the inverse of the $n-\bar{n}$ oscillation time, i.e. $V_{n\bar{n}} = \delta m_{n\bar{n}} = \hbar/\tau_{n\bar{n}}$ [10].

To leading order the $\bar{n}p$ component $|\psi_{\bar{n}p}\rangle$ obeys the equation

$$(H_0 + V_{\bar{n}p} - E_d)|\psi_{\bar{n}p}\rangle = -V_{n\bar{n}}|\psi_d\rangle \,, \tag{5}$$

where E_d is the unperturbed energy of the deuteron and $|\psi_d\rangle$ is the corresponding deuteron wave function. The decay width of the deuteron, Γ_d, is then [19]

$$\Gamma_d = -2\, V_{n\bar{n}}\, \mathrm{Im}\langle\psi_d|\psi_{\bar{n}p}\rangle \,. \tag{6}$$

We solve Eq. (5) in momentum space. Performing a partial wave decomposition and taking into account the coupling of the ${}^3S_1$ and ${}^3D_1$ channels, the above integral equation reads

$$(2E_p - E_d)\,\psi^L_{\bar{n}p}(p) + \sum_{L'} \int \frac{dq\, q^2}{(2\pi)^3} V^{L,L'}_{\bar{n}p}(p,q)\,\psi^{L'}_{\bar{n}p}(q) = -V_{n\bar{n}}\,\psi^L_d(p) \,, \tag{7}$$

with $L, L' = 0, 2$. Note that E_d is the total energy corresponding to the deuteron, i.e. $E_d - 2m_N = 2\sqrt{m_N^2-\kappa^2} - 2m_N = -B_d$ where B_d is the standard binding energy of 2.225 MeV and $\kappa = \sqrt{m_N B_d} \simeq 45.7$ MeV is the binding momentum. The deuteron wave function is normalized by

$$\int dp p^2 \left[(\psi^0_d(p))^2 + (\psi^2_d(p))^2\right] = 1 \,, \tag{8}$$

and the width is provided by

$$\Gamma_d = -2\, V_{n\bar{n}}\, \mathrm{Im} \sum_L \int dp p^2\, \psi^L_d(p)\, \psi^L_{\bar{n}p}(p) \,. \tag{9}$$

The deuteron lifetime τ_d is given by $\tau_d = \hbar/\Gamma_d$. The interesting quantity is the so-called reduced lifetime R [19, 20, 23] which relates the free $n-\bar{n}$ oscillation lifetime with that of the deuteron,

$$\tau_d = R\,\tau^2_{n\bar{n}} \quad \text{i.e.} \quad R = \frac{\hbar}{\Gamma_d \tau^2_{n\bar{n}}} \,. \tag{10}$$

4 Results and discussion

Our results for the reduced lifetime R for the deuteron are summarized in Table 1. They are based on our N^{3}LO interaction with cutoff $R_0 = 0.9$ fm [1] and the N^{2}LO interaction with cutoff $\{\Lambda, \tilde{\Lambda}\} = \{450, 500\}$ MeV [5]. For technical details see the corresponding publications. Besides the predictions for R based on the chiral $\bar{N}N$ interactions we list also the values given

in Ref. [20] where the $\bar{N}N$ potentials DR_1 and DR_2 by Dover-Richard [24, 25] have been utilized. Furthermore we include results from the calculation of Oosterhof et al. performed directly within EFT on the basis of the KSW approach. In this case R can be represented in a compact analytical form which reads up to NLO [26]

$$R = -\frac{\kappa}{m_N}\,\frac{1}{\mathrm{Im}\,a_{\bar{n}p}}\,\frac{1}{1+0.4+2\kappa\,\mathrm{Re}\,a_{\bar{n}p}-0.13\pm 0.4}\,. \tag{11}$$

Obviously, the only parameter here is the $\bar{n}p\ ^3S_1$ scattering length. All other quantities that enter are well established NN observables, cf. Ref. [26] for details. Note that in that paper, the scattering length $\mathrm{Re}\,a_{\bar{n}p}$ was taken from Ref. [1].

Table 1. Reduced lifetime R calculated for the χEFT $\bar{N}N$ potentials from Refs. [1, 5], together with information on the pertinent $\bar{n}p\ ^3S_1$ scattering length. Results for the Dover-Richard potentials DR_1 and DR_2 are taken from Ref. [20]. The corresponding scattering lengths are from Ref. [33]. Predictions based on Eq. (11), i.e. on the KSW approach applied in Ref. [26], are indicated too.

	χEFT N^{2}LO [5]	χEFT N^{3}LO [1]	DR_1 [20]	DR_2 [20]
R [s^{-1}]	2.49×10^{22}	2.56×10^{22}	2.56×10^{22}	2.40×10^{22}
(Eq. (11))	$(1.1\pm0.3)\times10^{22}$	$(1.2\pm0.3)\times10^{22}$	$(1.4\pm0.4)\times10^{22}$	$(1.3\pm0.3)\times10^{22}$
$a_{^3S_1}$ [fm]	$0.44-\mathrm{i}\,0.91$	$0.44-\mathrm{i}\,0.96$	$0.87-\mathrm{i}\,0.66$	$0.89-\mathrm{i}\,0.71$

Table 1 reveals that the values for R predicted by the chiral $\bar{N}N$ interactions are fairly similar to those obtained for the DR potentials in the past. By contrast, the results based on the framework employed by Oosterhof et al. [26] are rather different. Since the scattering length from the N^{3}LO chiral $\bar{N}N$ interaction [1] is utilized in that work, the large discrepancy observed in Ref. [26] is certainly not due to differences in Im $a_{\bar{n}p}$ but must be primarily a consequence of the different approaches.

For investigating the sensitivity of our results to the used ingredients we performed various exploratory calculations. Specifically, we employed the NLO and N^{2}LO variants of the considered $\bar{N}N$ (and NN) interactions. The corresponding predictions for R were found to lie within a range of $(2.48-2.65)\times10^{22}$ s^{-1}. If one takes this variation as measure for the uncertainty due to the nuclear structure, i.e. the NN and $\bar{N}N$ interactions (wave functions), a value of roughly $R=(2.6\pm0.1)\times10^{22}$ s^{-1} can be deduced. Application of the method proposed in Ref. [2] for estimating the uncertainty to the calculation based on the $\bar{N}N$ interaction from 2017 [1], say, leads to a slightly smaller uncertainty. We have also varied the deuteron wave functions alone. As an extreme case we even took wave functions from phenomenological NN potentials derived in an entirely different framework, namely in the meson-exchange picture [31, 32]. Also here the obtained values for R remained within the range given above. Finally, omitting the D-wave component of the deuteron wave function in our calculation causes a 5 % variation. It leads to an increase of R and, thus, does not bring the result closer to the values presented by Oosterhof et al. Overall, we confirm the observation by Dover et al. that the predictions for R are fairly insensitive to the details of the employed $\bar{N}N$ potentials [20], provided that these potentials describe the $\bar{p}p$ data at low energies.

Admittedly, we do not have a ready explanation for the difference of our results (and those of Ref. [20]) to the ones of Oosterhof et al. [26]. However, we believe that it is due to the fact that in the latter work the width Γ_d is evaluated following the perturbative scheme

developed by Kaplan, Savage, and Wise [28]. In that scheme there is no proper deuteron wave function. Rather one works with an effectively constructed wave function that is represented in terms of an irreducible two-point function [26, 28]. This seems to work well for some electromagnetic form factors of the deuteron, at least at low momentum transfer [28, 34]. On the other hand, the quadrupole moment of the deuteron is overestimated by 40 % [28], which hints that the properties of the wave function at large distances (small momenta) are not that well represented in this scheme. Clearly, this should have an impact on the quantity studied in the present work as well. Note that a comparable agreement (mismatch) with regard to the KSW scheme has been also observed in studies of the electric dipole moment (magnetic quadrupole moment) of the deuteron [35–37]. In any case, one should not forget that there is convergence problem of the KSW approach for NN partial waves where the tensor force from pion exchange is present [38]. It affects specifically the $^3S_1$-$^3D_1$ channel where difficulties appear already for momenta around 100 MeV/c, see [38] and also the discussions in Refs. [4, 39].

5 Summary

In this presentation we have introduced an $\bar{N}N$ potential [1], established within chiral effective field theory, which has been fitted to up-to-date $\bar{N}N$ phase shifts and inelasticities provided by a phase-shift analysis of available $\bar{p}p$ scattering data [7].

As an application of this interaction neutron-antineutron oscillations in the deuteron have been considered. In particular, results for the deuteron lifetime have been presented, evaluated in terms of the free-space $n - \bar{n}$ oscillation time, utilizing that $\bar{N}N$ potential in combination with an NN interaction likewise derived within chiral effective field theory. The value obtained for the so-called reduced lifetime R which relates the free-space $n - \bar{n}$ oscillation time $\tau_{n-\bar{n}}$ with the deuteron lifetime is found to be $R = (2.6 \pm 0.1) \times 10^{22}$ s^{-1}, where the quoted uncertainty is due to the NN and $\bar{N}N$ interactions (wave functions).

Our prediction for R agrees with the value obtained by Dover and collaborators almost four decades ago [20] but deviates from recent EFT calculations, based on the perturbative scheme proposed by Kaplan, Savage, and Wise [26], by about a factor of 2. A possible explanation for the difference could be that the KSW scheme does not involve a proper deuteron wave function. Rather this ingredient is represented effectively in terms of an irreducible two-point function. It is known from past studies that the KSW approach fails to describe quantities that depend more sensitively on the wave function like, for example, the quadrupole moment of the deuteron [28].

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