The effect of shear-induced migration on crossflow filtration of colloids

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References:
1. Park and Nägele, *JCP*, 2020
2. Park and Nägele, *Membranes*, 2021
3. Park, Dhont, and Nägele (manuscript in preparation)

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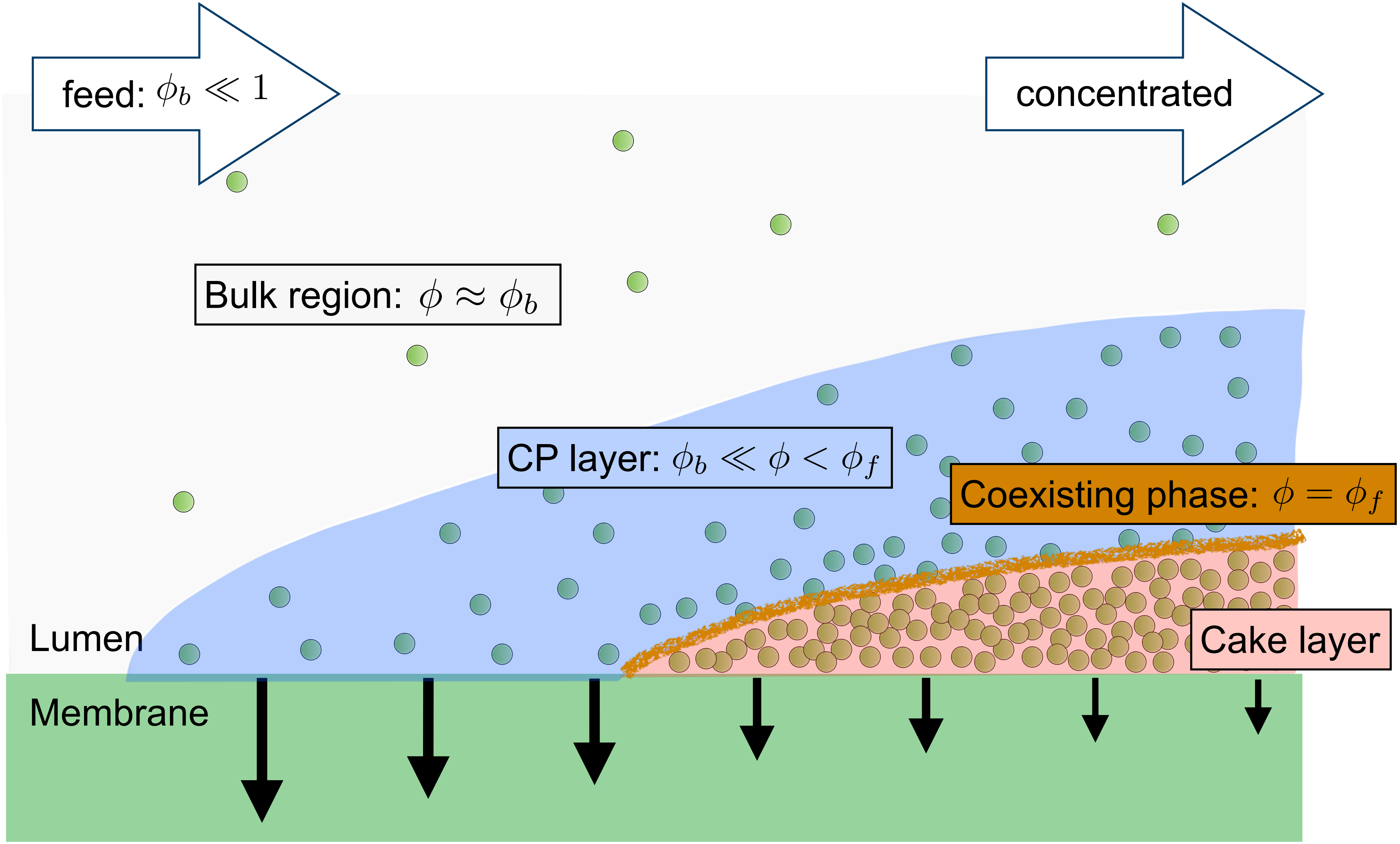
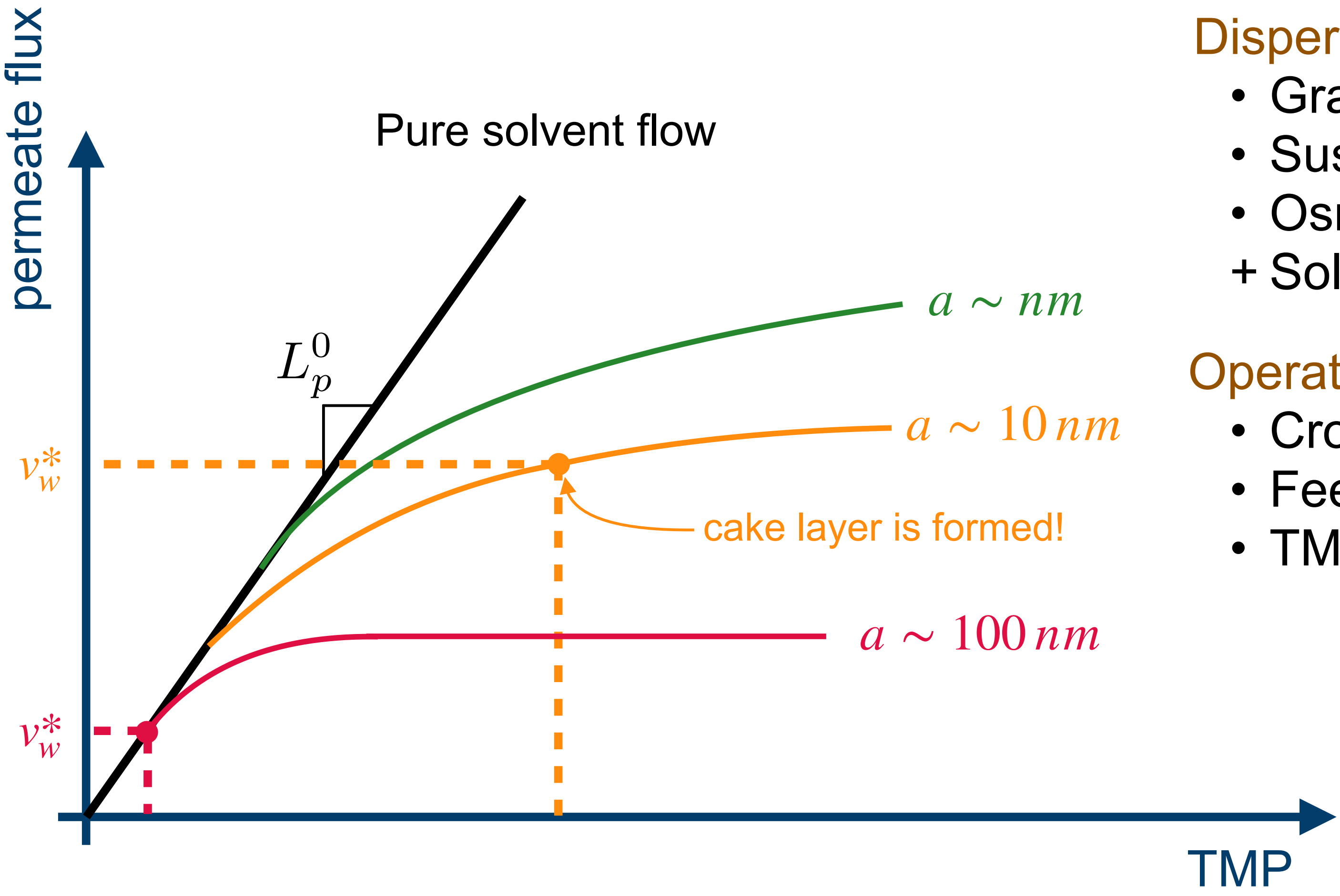
Crossflow filtration



Conditions for cake layer formation



Membrane properties:
- Hydraulic permeability
- Membrane dimensions

Dispersion properties:
- Gradient diffusion coefficient
- Suspension viscosity
- Osmotic pressure

+ Solidification (cake layer)

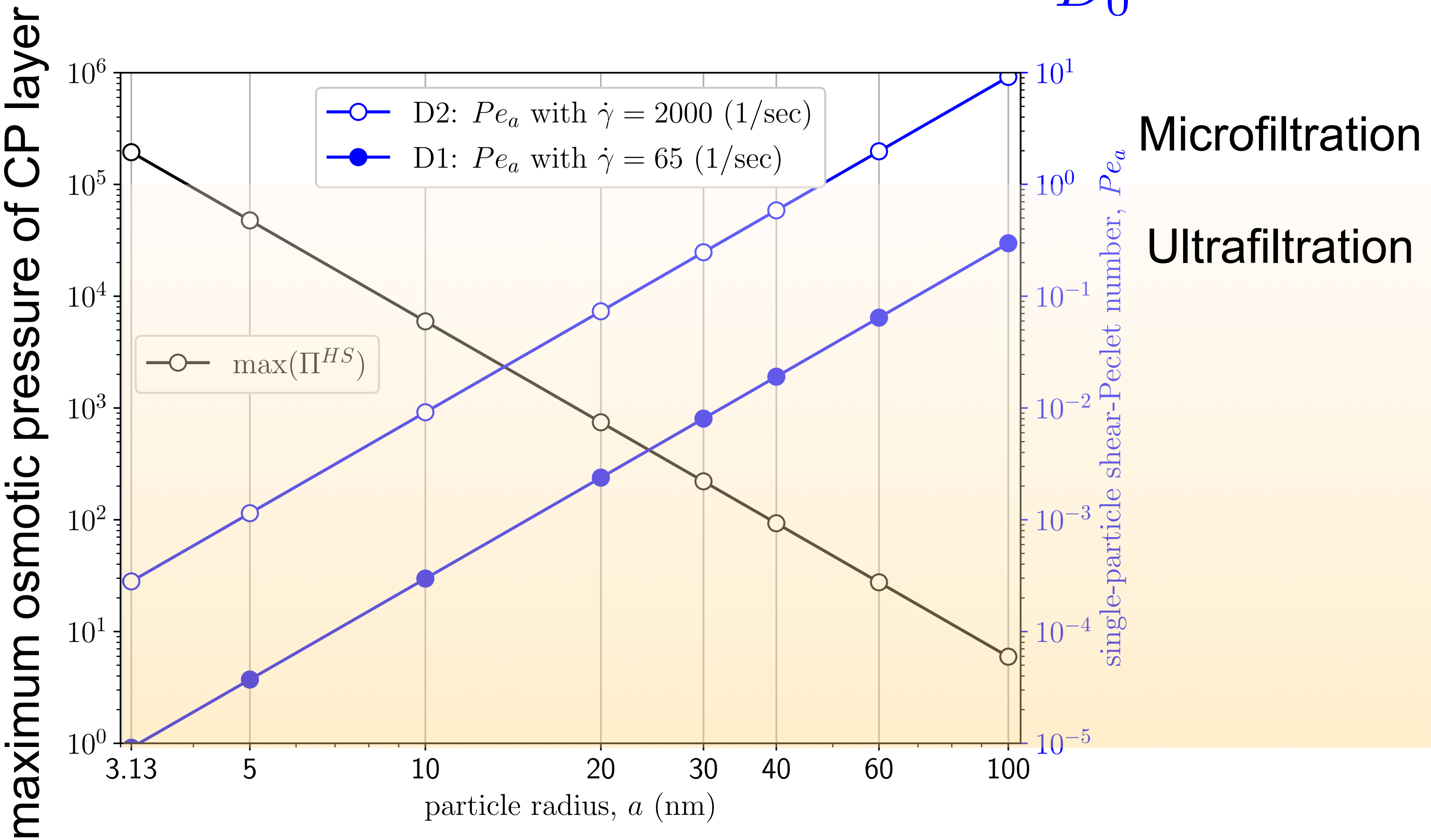
Operating conditions:
- Crossflow velocity
- Feed concentration
- TMP

Effect of particle size

Ultrafiltration and microfiltration

$$\Pi(\phi_f; a) = \frac{k_B T}{(4/3)\pi a^3}\phi_f Z(\phi_f) \propto \frac{1}{a^3}$$

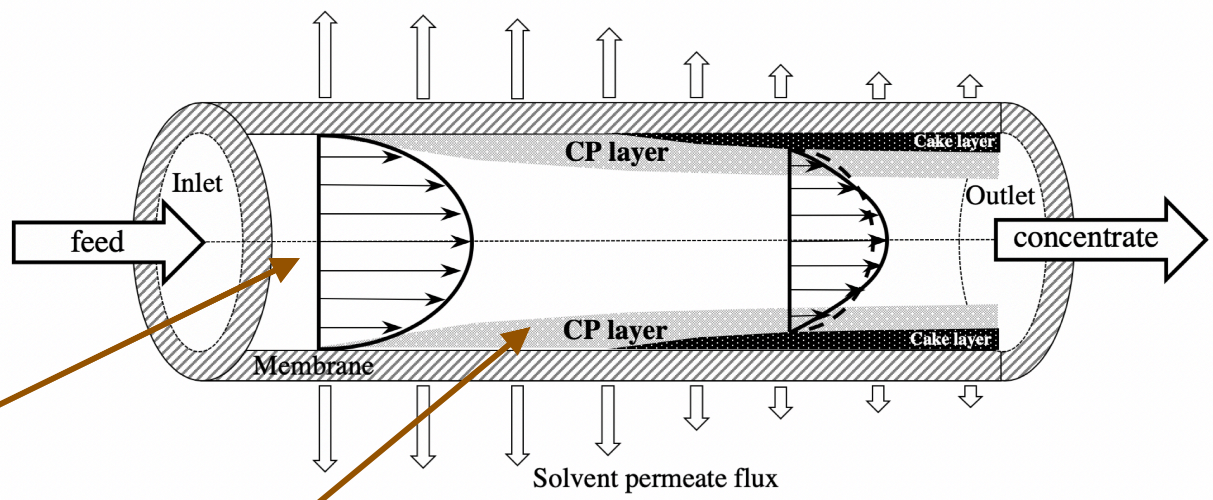
$$Pe_a = \frac{\dot{\gamma} a^2}{D_0} \propto a^3$$



mBLA method

Park and Nägele, *JCP*, 2020
Park and Nägele, *Membranes*, 2021

CF model



Advection-diffusion equation (ϕ)

Effective Stokes equation ($\mathbf{V}, P$)

Darcy-Starling equation (v_w)

Bulk (outer) solution

- $\phi^{out}(y,z) \approx \phi_b$ (about $10^{-3} \sim 10^{-4}$ in this study)
- Similar to pure solvent flow profile (except v_w)

$$k^2 = 16\left(\frac{L}{R}\right)^2 \frac{\eta_s L_P^0}{R}$$

Boundary layer (inner) solution

- Wall concentration ϕ_w can be up to $\phi_f \approx 0.494$.
- Strong influence of $(\phi, \dot{\gamma})$ on dispersion properties

$$Pe_R = \frac{v_w R}{D_0} \gg 1$$

+ Global particle flux conservation law ⇒ $\phi_w(z)$ and $L_p(z)$

Note: mBLA method provides

- semi-analytic expression for $(\mathbf{V}, P, \phi)$ inside lumen
- In good agreement with FEM result (and fast!)

Simplified relation: flux and concentration

without non-linear axial variation of flow profile, osmotic pressure, and cake at outlet

$k^2 \ll 1$ $\Pi/\mathrm{TMP} \approx 0$ $\phi_w(z=L) = \phi_f$

$$\text{Eq. } (*) \cdots \quad \frac{Pe_R^*}{\Psi^{1/3}(Pe_R^*)} \approx \left[2\, Pe_L \frac{\phi_f}{\phi_b}\right]^{1/3}$$

$$Pe_R^* = \frac{R^2/D_0}{R/v_w^*}$$

$$Pe_L = \frac{R^2/D_0}{L/u^0}$$

Effect of different dispersion properties:

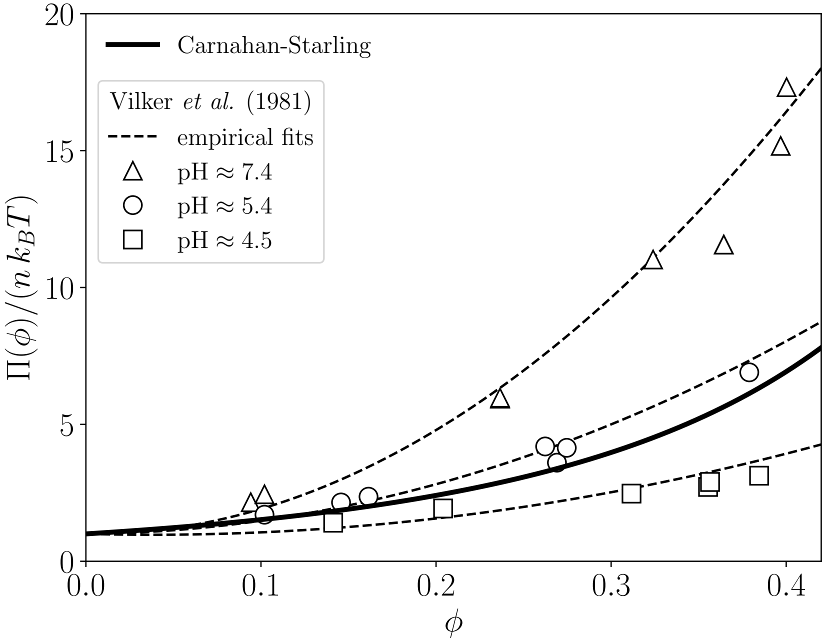
$$\Psi(Pe_R^*) \approx \frac{(Pe_R^*)^2}{2} \int_0^1 d\tilde{y}\,(1-\tilde{y})(2-\tilde{y}) \left(\frac{1}{\eta^*}\int_0^{\tilde{y}} \frac{\eta_s}{\eta(\phi)}\, d\tilde{y}'\right) \exp\left(-Pe_R^* \int_0^{\tilde{y}} \frac{D_0}{D(\phi)}\, d\tilde{y}'\right)$$

viscosity effect
(from axial velocity)

diffusivity effect
(from concentration)

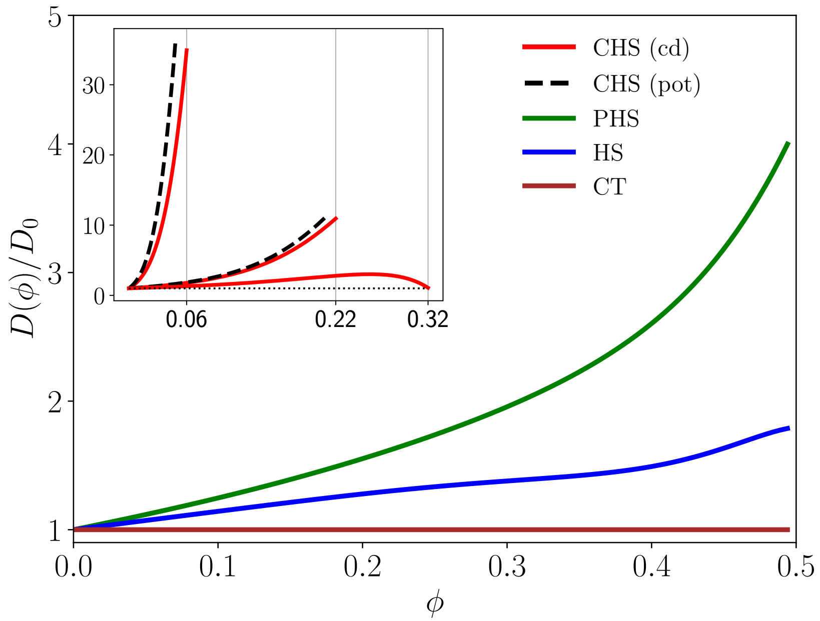
- For constant transport properties (CT): $\Psi^{CT}(Pe_R^*) \approx 1$
- Similar to standard earlier description of CT (film theory + mass transfer coefficient)

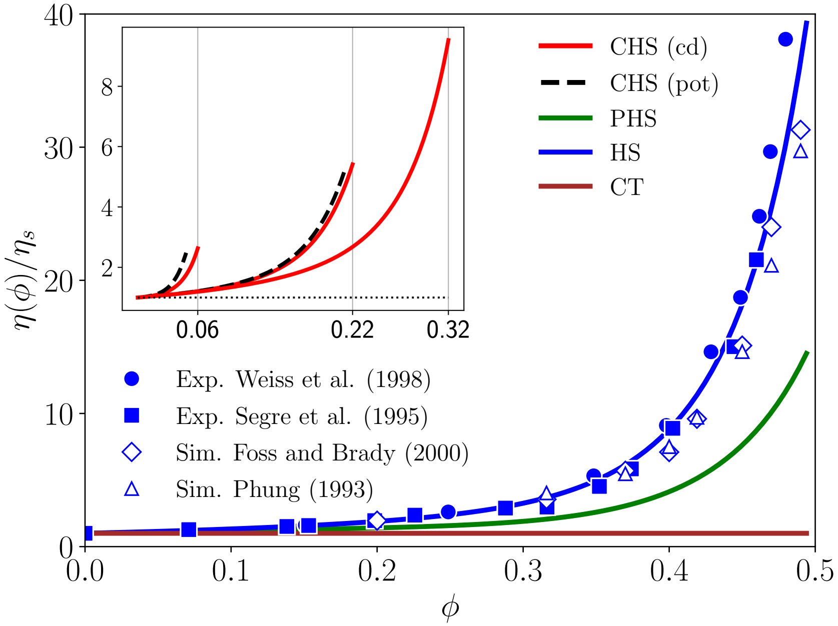
Equilibrium dispersion properties



We stay focused on the neutral dispersions:

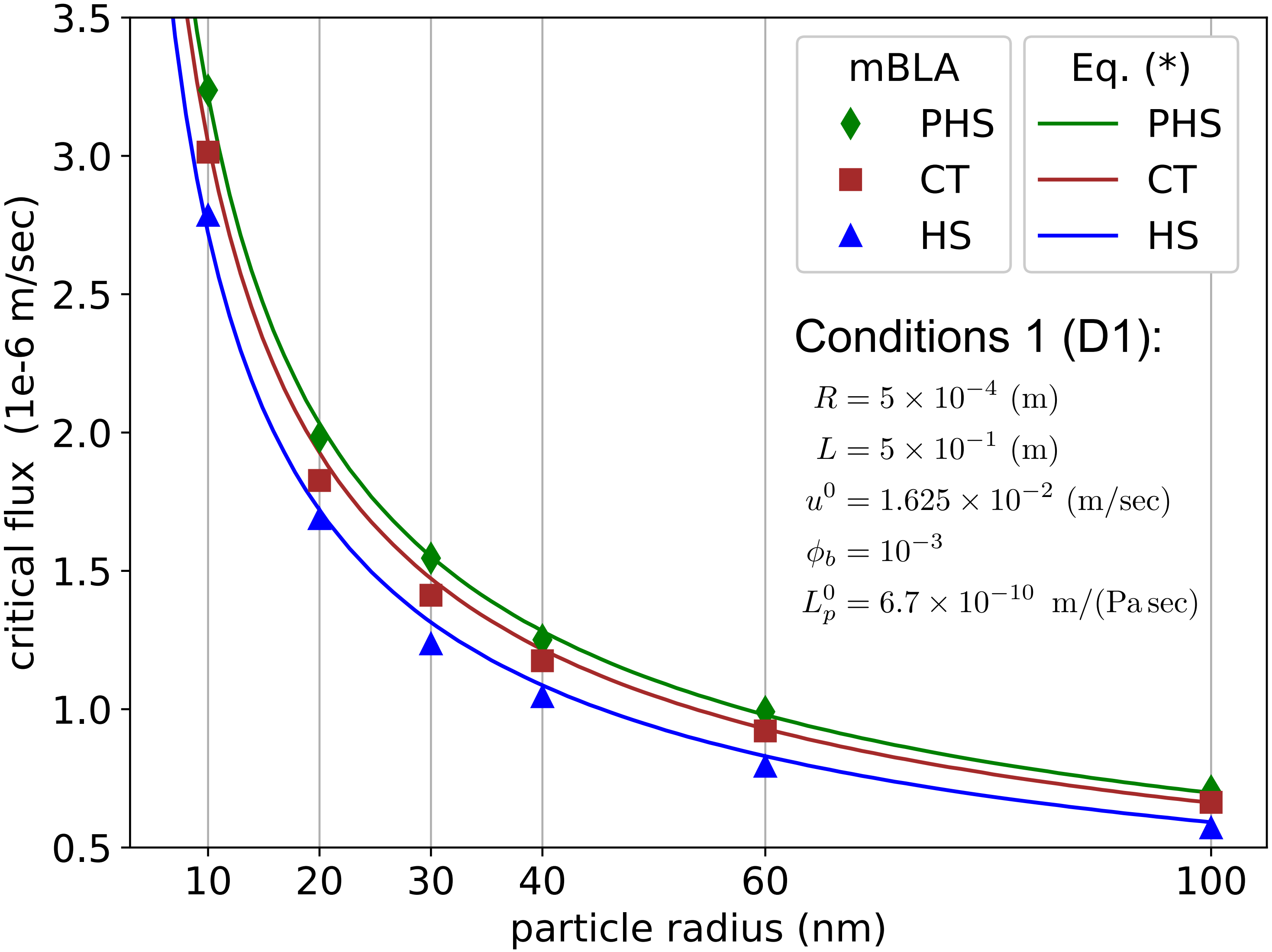
- HS = hard spheres
- PHS = solvent-permeable hard spheres
- CT = constant transport properties





Critical flux vs. particle size

PHS/CT/HS under conditions 1 (D1)



Shear-induced diffusivity $(Pe_a \lesssim 10)$

- At equilibrium freezing concentration: $\phi_f = \phi_f^{eq}$
- No osmotic pressure contribution: $\frac{\Pi(\phi_f, Pe_a)}{\text{TMP}} \approx 0$

$$40 \text{ nm} \leq a \leq 200 \text{ nm}$$

- Without shear-thinning effect: $\eta = \eta^{KD}(\phi)$
- Equilibrium sedimentation coefficient: $K_{sed} = K_{sed}^{eq}(\phi)$
- Shear-curvature effect is negligible:

$$-j_y^{diff} = D_{yy}(\phi, \dot{\gamma})\frac{\partial \phi}{\partial y} + \xi(\phi, \dot{\gamma})\frac{\partial \dot{\gamma}}{\partial y}$$

- yy-component collective diffusion tensor:

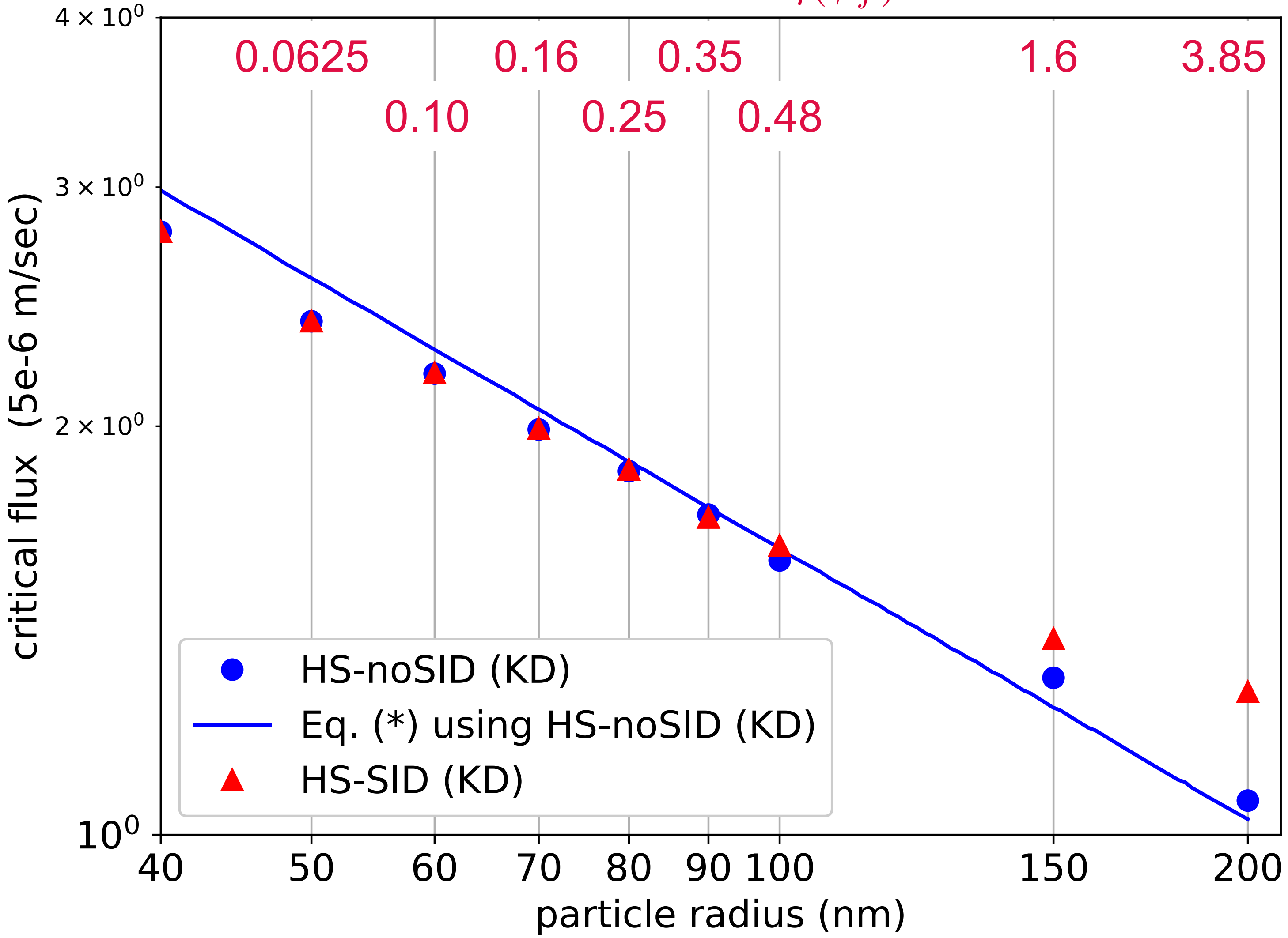
$$D_{yy}(\phi, Pe_a) = D_0 K_{sed}^{eq}(\phi)\left[1 + 4\frac{\partial}{\partial \phi}\left(\phi^2 g_{iso}^c(\phi, Pe_a)\right)\right]$$

Jin ... Dhont, *JCP* (2014)

Effect of shear-induced diffusivity

HS in equilibrium, HS with SID

$$Pe_a(y=0, z=L) \approx \frac{2u^0}{R}\frac{1}{\eta(\phi_f)}$$



Condition 2 (D2):

$$R = 1 \times 10^{-4} \text{ (m)}$$

$$L = 5 \times 10^{-2} \text{ (m)}$$

$$u^0 = 0.1 \text{ (m/sec)}$$

$$\phi_b = 10^{-4}$$

Concluding remarks

- Concentration-polarization and cake layers are important filtration subjects
- Predicting the critical flux is a long-standing key problem not fully solved to date
- mBLA method provides accurate semi-analytic flow and concentration profiles
- Simplified critical flux result prediction in Eq. (*) is accurate and related the new mBLA method to earlier less accurate methods
- Explore effect of shear-induced diffusivity on the critical flux
- Work in progress: Shear-thinning and osmotic pressure effects

References:

1. Park and Nägele, *JCP*, 2020
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Thank you

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SFB985: Functional microgel and microgel system

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