Chiral theory of ρ-meson gravitational form factors

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(Received 27 September 2021; accepted 24 December 2021; published 20 January 2022)

The low-energy chiral effective field theory of vector mesons and Goldstone bosons in an external gravitational field is considered. The energy-momentum tensor is obtained, and the gravitational form factors of the ρ-meson are calculated up to next-to-leading order in the chiral expansion. This amounts to considering tree-level and one-loop order diagrams. The chiral expansion of the form factors at zero momentum transfer, as well as of the slope parameters, is also performed. Also, the long-range behavior of the energy and internal force distributions is obtained and analyzed.

DOI: 10.1103/PhysRevD.105.016018

I. INTRODUCTION

The linear response of a hadron to a change of the background space-time metric is described by the gravitational form factors (GFFs). For the first time, the GFFs for spin-0 and spin-1/2 particles were introduced and discussed in detail in Refs. [1,2], for spin-1 particles in Ref. [3], and for hadrons with arbitrary spin in the recent work of Ref. [4]. The GFFs contain rich information about the internal structure of hadrons, such as the distribution of the spin [5], the energy distribution [6], and the elastic pressure and shear force distributions [7]. For recent reviews, see Refs. [8,9].

Our aim here is to study the GFFs of the spin-1ρ-meson in chiral effective field theory (EFT). Hadrons with spin $S > 1/2$ are not spherically symmetric. The spin, energy, and force distributions acquire higher multipole components (quadrupole, etc.) [4,10–13]. The higher multipole energy and force distributions carry valuable information about the mechanisms of the hadron's binding. For example, the large-N_c picture of baryons as chiral solitons implies certain relations between the quadrupole energy and the force distributions [11,14]. Experimental checks of these relations would allow one to reveal the nature of higher spin baryons.

The GFFs of the ρ-meson were computed in Refs. [15–17] using various approaches. More recently, the gluon part of the GFFs was obtained in lattice QCD calculations [18]. Here, we investigate the dependence of the ρ-meson GFFs on the soft scales (pion mass, small momentum transfer) using chiral EFT. To this end, following the logic of Ref. [19], we first write down the chiral effective action for the ρ- and ω-mesons and pions in an external gravitational field. Next, we obtain the corresponding energy-momentum tensor (EMT) and compute the chiral corrections to the GFFs of the ρ-meson. The corresponding calculation, in particular, allows us to obtain the large distance behavior of the energy and force distributions. The results of our study can also be used in chiral extrapolations of the lattice-QCD simulations down to the physical values of the pion masses.

Chiral EFTs with heavy degrees of freedom encounter a nontrivial power-counting problem [20]. In the one-nucleon sector of baryon chiral perturbation theory, this problem can be solved by applying the heavy-baryon approach [21,22] or a suitably chosen renormalization condition [23–26]. Because of the small nucleon-delta mass difference, the Δ resonance can also be consistently included in the framework of EFT [27–30].

The treatment of the ρ-meson in chiral EFT is complicated as it decays into two pions with masses that vanish in the chiral limit. Because of this, for energies of the order of the ρ-meson mass, loop diagrams develop large imaginary parts. In distinction to the baryonic sector, these large power-counting-violating contributions cannot be absorbed in the redefinition of the parameters of the Lagrangian as long as the usual renormalization procedure is used. Still, the problem can be handled [31] by using the complex-mass renormalization scheme [32,33], which is an extension of the on-mass-shell renormalization scheme to unstable particles. For more details on and different approaches to these problems, see e.g., Refs. [34–42].

Our work is organized as follows. In Sec. II we write down the action corresponding to the effective Lagrangian up to next-to-leading order and obtain the pertinent EMT. In Sec. III we briefly discuss the renormalization and the power counting. The definition and the calculations of the GFFs of the ρ-meson are presented in Sec. IV, which also contains various chiral expansions of the obtained results. Section V is devoted to the discussion of the energy and the force distributions, and we summarize the obtained results in Sec. VI.

II. EFFECTIVE LAGRANGIAN AND THE ENERGY-MOMENTUM TENSOR

Using the results of Refs. [31,43], we consider the following action of ρ- and ω-mesons and pions using the parametrization of model III of Ref. [34] (where the ρ-meson vector fields transform inhomogeneously under chiral transformations), interacting with an external gravitational field $g^{\mu\nu}$:

$$\begin{aligned}S = \int d^4x\sqrt{-g}\bigg\{&\frac{F^2}{4}g^{\mu\nu}\mathrm{Tr}(D_\mu U(D_\nu U)^\dagger) + \frac{F^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\\ &-\frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\mathrm{Tr}(\rho_{\mu\alpha}\rho_{\nu\beta}) + g^{\mu\nu}\left[M_R^2 + \frac{c_x\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)}{4}\right]\mathrm{Tr}\left[\left(\rho_\mu - \frac{i\Gamma_\mu}{g}\right)\left(\rho_\nu - \frac{i\Gamma_\nu}{g}\right)\right]\\ &-\frac{1}{4}g^{\mu\nu}g^{\alpha\beta}(\partial_\mu\omega_\alpha - \partial_\alpha\omega_\mu)(\partial_\nu\omega_\beta - \partial_\beta\omega_\nu) + g^{\mu\nu}\frac{M_\omega^2\omega_\mu\omega_\nu}{2} + \frac{g_{\omega\rho\pi}}{2\sqrt{-g}}\epsilon^{\mu\nu\alpha\beta}\omega_\nu\mathrm{Tr}(\rho_{\alpha\beta}u_\mu)\\ &+[v_1 + v_2\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)]R\,\mathrm{Tr}(\rho_\mu\rho^\mu) + v_3R^{\mu\nu}\mathrm{Tr}(\rho_\mu\rho_\nu)\\ &+v_4R\,\mathrm{Tr}(\rho_{\alpha\beta}\rho^{\alpha\beta}) + v_5R^{\mu\nu}g^{\alpha\beta}\mathrm{Tr}(\rho_{\mu\alpha}\rho_{\nu\beta}) + v_6R^{\mu\nu\alpha\beta}\mathrm{Tr}(\rho_{\mu\nu}\rho_{\alpha\beta})\bigg\},\end{aligned} \tag{1}$$

where

$$\begin{aligned}U &= u^2 = \exp\left(\frac{i\vec{\tau}\cdot\vec{\pi}}{F}\right),\\ \rho^\mu &= \frac{\vec{\tau}\cdot\vec{\rho}^\mu}{2},\\ \rho^{\mu\nu} &= \partial^\mu\rho^\nu - \partial^\nu\rho^\mu - ig[\rho^\mu,\rho^\nu],\\ \chi &= 2B_0(s+ip),\\ D_\mu U &= \partial_\mu U - ir_\mu U + iUl_\mu,\\ \Gamma_\mu &= \frac{1}{2}[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger - i(u^\dagger r_\mu u + ul_\mu u^\dagger)],\\ u_\mu &= i[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger r_\mu u - ul_\mu u^\dagger)].\end{aligned} \tag{2}$$

Terms involving the Riemann tensor $R^{\mu\nu\alpha\beta}$, the Ricci tensor $R^{\mu\nu}$, and the Ricci scalar R (for definitions of these quantities, see e.g., Ref. [44]) are those with nonminimal coupling of the ρ-meson fields to gravity, which are relevant for the considered order of accuracy. The parameter B_0 is proportional to the scalar vacuum condensate, and s, p, $l_\mu = v_\mu - a_\mu$, and $r_\mu = v_\mu + a_\mu$ are external sources, while F denotes the pion-decay constant in the chiral limit. Further, M_R^2 and M_ω^2 are the (complex) pole positions of ρ and ω propagators in the chiral limit, and the $v_i (i = 1,\ldots,6)$, g, c_x, and $g_{\omega\rho\pi}$ are coupling constants. For the $\rho\pi\pi$ coupling we use [45]

$$M_R^2 = ag^2F^2, \tag{3}$$

which, in the case of $a = 2$, amounts to the KSFR relation [46–48]. Although phenomenologically $a \simeq 2$, in what follows we will explicitly keep this parameter. All parameters of the effective Lagrangian are to be interpreted as renormalized ones. We apply the complex-mass scheme and do not show counterterms explicitly; however, their contributions are taken into account in calculations of the quantum corrections to the physical quantities. Notice that all couplings (including the v_i, the parameters of nonminimal couplings to gravity, resulting mainly from the compositeness of the ρ-meson) are assumed to be dominated by the scale of the strong interaction $\sim$1 GeV.

Applying the standard formula for the EMT of matter fields interacting with the metric fields [44],

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}, \tag{4}$$

to the action of Eq. (1), we obtain, in flat space-time,

$$
\begin{aligned}
T_{\mu\nu} = {} & \frac{F^2}{4}\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger + D_\nu U (D_\mu U)^\dagger) \\
& - \eta_{\mu\nu}\left\{\frac{F^2}{4}\mathrm{Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\} \\
& - 2\eta^{\alpha\beta}\mathrm{Tr}(\rho_{\mu\alpha}\rho_{\nu\beta}) + 2\left[M_\rho^2 + \frac{c_x \mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)}{4}\right]\mathrm{Tr}\left[\left(\rho_\mu - \frac{i\Gamma_\mu}{g}\right)\left(\rho_\nu - \frac{i\Gamma_\nu}{g}\right)\right] \\
& - \eta_{\mu\nu}\left\{-\frac{1}{2}\mathrm{Tr}(\rho_{\alpha\beta}\rho^{\alpha\beta}) + \left[M_\rho^2 + \frac{c_x \mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)}{4}\right]\mathrm{Tr}\left[\left(\rho_\alpha - \frac{i\Gamma_\alpha}{g}\right)\left(\rho^\alpha - \frac{i\Gamma^\alpha}{g}\right)\right]\right\} \\
& - \eta^{\alpha\beta}(\partial_\mu\omega_\alpha - \partial_\alpha\omega_\mu)(\partial_\nu\omega_\beta - \partial_\beta\omega_\nu) + M_\omega^2\omega_\mu\omega_\nu \\
& - \eta_{\mu\nu}\left\{-\frac{1}{4}(\partial_\alpha\omega_\beta - \partial_\beta\omega_\alpha)(\partial^\alpha\omega^\beta - \partial^\beta\omega\alpha) + \frac{M_\omega^2\omega_\alpha\omega^\alpha}{2}\right\} \\
& + 2(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\{[v_1 + v_2\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)]\mathrm{Tr}(\rho_\alpha\rho^\alpha) + v_4\mathrm{Tr}(\rho_{\alpha\beta}\rho^{\alpha\beta})\} \\
& + (\eta_{\mu\alpha}\eta_{\nu\beta}\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta - \eta_{\mu\alpha}\partial_\nu\partial_\beta - \eta_{\nu\alpha}\partial_\mu\partial_\beta)[v_3\mathrm{Tr}(\rho^\alpha\rho^\beta) + v_5\eta_{\lambda\sigma}\mathrm{Tr}(\rho^{\alpha\lambda}\rho^{\beta\sigma})] \\
& + 4v_6\eta^{\alpha\lambda}\eta^{\beta\sigma}\partial_\lambda\partial_\sigma\mathrm{Tr}(\rho_{\mu\beta}\rho_{\nu\beta}), \qquad (5)
\end{aligned}
$$

where $\eta_{\mu\nu}$ is the metric tensor in Minkowski space.

III. RENORMALIZATION AND POWER COUNTING

To perform the renormalization we express the bare quantities in terms of renormalized ones and counterterms and apply the complex-mass renormalization scheme [32,33]. We parametrize the pole of the ρ-meson dressed propagator in the chiral limit as $M_R^2 = (M_\chi - i\Gamma_\chi/2)^2$, where M_χ and Γ_χ are the pole mass and width of the ρ-meson in the chiral limit, respectively. Both are input parameters within our formalism.

Following Ref. [31], we fix the mass counterterm and the wave function renormalization constant by requiring that, in the chiral limit, M_R^2 coincides with the pole position of the dressed propagator and the residue is equal to unity. The renormalized complex mass M_R appears in the propagator, and the counterterms are included perturbatively. Notice that in the complex-mass renormalization scheme, the counterterms are also complex quantities. However, this does not lead to a violation of unitarity as one might naively expect [49,50]. Let us demonstrate this using the example of the renormalization of the ρ-meson mass. The Lagrangian is given in terms of bare parameters, and physical quantities can also be calculated in terms of these parameters within some ultraviolet regularization scheme. The physical mass of a stable particle, as well as the mass and width of an unstable particle, can be obtained from the corresponding two-point function by finding its pole position. Defining the self-energy of the ρ-meson as the sum of all one-particle irreducible diagrams contributing to the two-point function of the ρ-meson field operators, we parametrize this quantity as

$$i\Pi^{\mu\nu}(p) = i(g^{\mu\nu}\Pi_1(p^2) + p^\mu p^\nu \Pi_2(p^2)). \qquad (6)$$

The equation determining the pole position z of the two-point function written in terms of the bare parameters has the form

$$z - M_0^2 - \Pi_1(z, M_0, M_\pi, \cdots) = 0, \qquad (7)$$

where M_0 is the bare mass of the ρ-meson, M_π is the pion mass, and the ellipses denote other parameters of the Lagrangian and also the ultraviolet regulator. The solution to Eq. (7) has the form

$$z = f(M_0, M_\pi, \cdots) \equiv M_0^2 + \text{corrections}. \qquad (8)$$

We denote the quantity z in the chiral limit by M_R^2 and invert Eq. (8) for $M_\pi = 0$ to obtain

$$M_0^2 = f^{-1}(M_R^2, 0, \ldots) \equiv M_R^2 + \hbar\delta M^2(\hbar, M_R^2, \cdots), \qquad (9)$$

where M_R^2 and δM^2 are both complex, while M_0^2 is real. We have indicated the explicit factor of $\hbar$ to emphasize that within the formalism employed in the current work, after substituting Eq. (9) in the Lagrangian, we treat the second term on the right-hand side (after further expanding it in powers of $\hbar$) together with the loop diagrams, i.e., perturbatively.

For the effective Lagrangian, we apply the standard rules counting the pion mass and the derivatives acting on pion fields as small quantities, while the derivatives acting on the heavy vector mesons count as large quantities of order $\mathcal{O}(1)$. The large mass of the ρ-meson that does not vanish in the chiral limit violates the simple correspondence between the power counting for the Lagrangian and the power counting for the loop diagrams, thus leading to a considerable complication. One needs to investigate all possible flows of the external momenta through the internal lines of the considered loop diagram. Next, assigning powers to propagators and vertices, one needs to determine the chiral order for a given flow of external momenta. Finally, the smallest order resulting from all possible assignments should be defined as the chiral order of the given diagram [31]. To assign the corresponding chiral order to a diagram for a given flow of external momenta, we apply the following rules: Taking q as a small quantity like the pion mass or small external momenta, pion propagators count as $\mathcal{O}(q^{-2})$ if not carrying large external momenta while $\mathcal{O}(q^0)$ otherwise. A vector-meson propagator counts as $\mathcal{O}(q^0)$ if it does not carry large external momenta and as $\mathcal{O}(q^{-1})$ if it does. The vector-meson mass counts as $\mathcal{O}(q^0)$, the width of the vector mesons as well as the pion mass count as $\mathcal{O}(q^1)$. Interaction vertices generated by the effective Lagrangian of the order n do not automatically count as $\mathcal{O}(q^n)$ but rather need to be assigned orders according to a given flow of large and small external momenta. As the contributions of loops involving only vector meson propagators can be absorbed systematically in the redefinition of the parameters of the effective Lagrangian, such loop diagrams need not be included at low energies.

IV. GRAVITATIONAL FORM FACTORS OF THE ρ-MESON: DEFINITIONS AND CALCULATION

The GFFs of a spin-1 particle were defined for the first time in Ref. [3]. Here, we follow the conventions and notations of Ref. [12], in which the GFFs of a spin-1 particle were defined as

$$\begin{aligned}\langle p',\sigma'|T_{\mu\nu}|p,\sigma\rangle = {}&\epsilon^{*\alpha'}(p',\sigma')\epsilon^{\alpha}(p,\sigma)\left[2P_\mu P_\nu\left(-\eta_{\alpha\alpha'}A_0(t)+\frac{P_\alpha P_{\alpha'}}{m^2}A_1(t)\right)+2[P_\mu(\eta_{\nu\alpha'}P_\alpha+\eta_{\nu\alpha}P_{\alpha'})+P_\nu(\eta_{\mu\alpha'}P_\alpha+\eta_{\mu\alpha}P_{\alpha'})]J(t)\right.\\ &+\frac{1}{2}(\Delta_\mu\Delta_\nu-\eta_{\mu\nu}\Delta^2)\left(\eta_{\alpha\alpha'}D_0(t)+\frac{P_\alpha P_{\alpha'}}{m^2}D_1(t)\right)\\ &\left.+\left[\frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\alpha'}+\eta_{\mu\alpha'}\eta_{\nu\alpha})\Delta^2-(\eta_{\nu\alpha'}\Delta_\mu+\eta_{\mu\alpha'}\Delta_\nu)P_\alpha+(\eta_{\nu\alpha}\Delta_\mu+\eta_{\mu\alpha}\Delta_\nu)P_{\alpha'}-4\eta_{\mu\nu}P_\alpha P_{\alpha'}\right]E(t)\right],\end{aligned}\tag{10}$$

where $\Delta = p_f - p_i$, $P = (p_f + p_i)/2$, m is the mass (note that we reserve the symbol M for the ρ and ω masses) of the spin-1 particle, and the polarization vector $\epsilon_\alpha(p,\sigma)$ satisfies the condition

$$\sum_\sigma \epsilon_\alpha(p,\sigma)\epsilon_\beta(p,\sigma) = -\eta_{\alpha\beta} + \frac{p^\alpha p^\beta}{m^2}. \tag{11}$$

For the reader's convenience we collect in Table I other notations for the GFFs of spin-1 particles used in the literature.

As the ρ-meson is an unstable particle, we extract its gravitational form factors from the residue at the complex double pole of the three-point correlation function of the EMT and the vector-meson fields [53]. In this case, m^2 in the above formulas is the complex pole position of the corresponding dressed propagator.

In the current work we consider contributions of tree-level and one-loop diagrams to the gravitational form factors of the ρ-meson (see Fig. 1). At tree level, there are contributions of higher-order terms in the effective

TABLE I. Notations for GFFs of spin-1 particles used in the literature.

Reference [12] and this work	A_0	A_1	D_0	D_1	J	E
Holstein [3]	F_1	$4F_5$	$-2F_2$	$8F_6$	F_3	$-2F_4$
Abidin *et al.* [15]	A	$-2E$	C	$-8F$	$A+B$	D
Taneja *et al.* [51]	$\mathcal{G}_1$	$-2\mathcal{G}_2$	$-\mathcal{G}_3$	$-2\mathcal{G}_4$	$\frac{1}{2}\mathcal{G}_5$	$-\frac{1}{2}\mathcal{G}_6$
Cosyn *et al.* [13]	$\mathcal{G}_1$	$-2\mathcal{G}_2$	$-\mathcal{G}_3$	$-2\mathcal{G}_4$	$\frac{1}{2}\mathcal{G}_5$	$-\frac{1}{2}\mathcal{G}_6$
Cosyn *et al.* [52] generalized form factors	$A^a_{2,0}$	$-2C^a_{2,0}$	$-4F^a_2$	$-8G^a_2$	$\frac{1}{2}B^a_{2,0}$	$D^a_{2,1}$

TABLE II. Values of form factors at $t = 0$.

	$A_0(0)$	$A_1(0)$	$D_0(0)$	$D_1(0)$	$J(0)$	$E(0)$
This work	1	$\sim 2.56+0.01i$	$\sim -7.77-0.08i$	$\sim 21.4-6.0i$	1	$\sim -0.39-0.03i$
[14]	1	~ 1.2	~ 0	~ 0.8	1	~ 0.15
[16]	1	~ 0.4	1	~ -3.1	1	~ 0.5

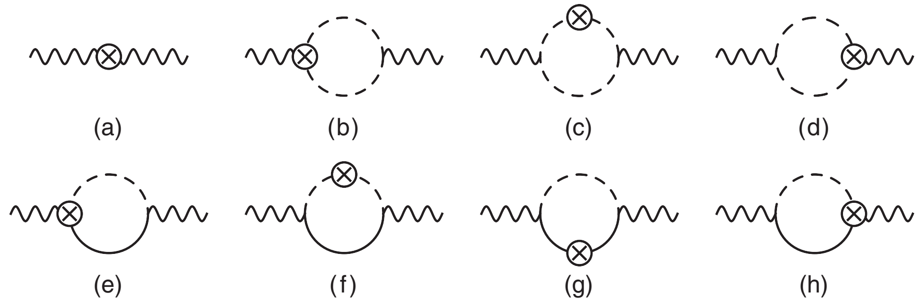

FIG. 1. Tree-level and one-loop diagrams contributing to the ρ-meson gravitational form factors. The dashed, solid, and wiggly lines correspond to the pion, the ω-meson, and the ρ-meson, respectively. The crossed vertex denotes an insertion of the EMT.

Lagrangian. As the higher-order Lagrangian is not available yet, we include these contributions to the form factors parametrized in the general form as polynomials of the pion mass and the momentum transfer squared. To calculate the loop diagrams we apply dimensional regularization and use the program FeynCalc [54,55]. To calculate the various expansions of the loop integrals, we apply the method of dimensional counting of Ref. [56]. To simplify the analytic expressions, we take $M_\omega^2 = M_R^2$, which is a good approximation given the accuracy of this work. Below, we specify the chiral expansions of the form factors at $t = 0$ and of the slope parameters and also provide expressions for the form factors in the small-t region in the chiral limit. Within the accuracy of our calculations, the pion mass at leading order in the chiral expansion can be replaced by its full expression M_π.

The chiral expansion of the form factors at zero momentum transfer has the form

$$
\begin{aligned}
A_0(0) &= 1,\\
A_1(0) &= 8v_6 M_R^2 + X_{A_1} M_\pi^2 + \frac{g_{\omega\rho\pi}^2 M_R}{48\pi F^2} M_\pi - \frac{g_{\omega\rho\pi}^2(1+4v_6 M_R^2)}{8\pi^2 F^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3),\\
J(0) &= 1,\\
D_0(0) &= 1 + 4v_1 + 8v_4 M_R^2 + X_{D_0} M_\pi^2 - \frac{(a + 3g_{\omega\rho\pi}^2(v_1 + 2v_4 M_\rho^2))}{12\pi^2 F^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3),\\
D_1(0) &= -8(4v_4 + v_5 + v_6) M_R^2 + \frac{g_{\omega\rho\pi}^2 M_R^3}{60\pi F^2}\frac{1}{M_\pi} + \frac{M_R^2(5g_{\omega\rho\pi}^2 - 8a)}{60\pi^2 F^2} \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi),\\
E(0) &= 1 - v_3 - v_5 M_R^2 + X_E M_\pi^2 + \frac{g_{\omega\rho\pi}^2 M_R}{96\pi F^2} M_\pi + \frac{((6a_3 + 6v_5 M_R^2 - 5)g_{\omega\rho\pi}^2 - 4a)}{96\pi^2 F^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3).
\end{aligned}
\tag{12}
$$

Here, X_{F_i} (as well as Y_{F_i}, Z_{F_i}, and W_{F_i} below) are some linear combinations of renormalized complex-valued low-energy constants from the higher-order effective Lagrangian.

The above equations provide the dependence of the GFFs at zero momentum transfer on the pion mass. They can be used for extrapolations of the lattice-QCD results for the GFFs to the physical values of the pion masses. In recent lattice calculations of the gluon part of GFFs for the ρ-meson [18], it was found, in particular, that the value of $D_1(0)$ is compatible with zero, albeit with large error bars [$D_1(0) = 0.0 \pm 0.7$]. From our calculations we see that $D_1(0) \sim 1/M_\pi$ for small pion masses. This singular contribution alone leads to a large value of $D_1(0) \approx 4$ for the physical pion mass. This value of the singular part of the GFF, unfortunately, cannot be directly compared to the results of the lattice simulations of Ref. [18] as only the gluon part of the GFF D_1^g was computed in that paper. However, we expect that the singular $\sim 1/M_\pi$ part is also present in $D_1^g(0)$ with a slightly modified coefficient. This suggests that the chiral extrapolation of lattice results of Ref. [18] for the pion mass to its physical point should be studied with great care.

For the numerical estimates we use $g_{\omega\rho\pi} = 1.478$ from Ref. [57] and adopt the physical values for the various meson masses and the pion decay constant instead of the corresponding chiral-limit values (the differences are beyond the accuracy of our calculations); namely, we take $M_R = 0.775 - 0.075i$ GeV, $M \sim M_\pi = 0.1395$ GeV, $F \sim F_\pi = 0.0924$ GeV, and $a = 2$. For the unknown couplings we employ, for illustrative purposes, $v_1 = -v_3 = -1$, $v_4 = -v_2 = -v_5 = -v_6 = -1\ \text{GeV}^{-2}$, and $X_i = Y_i = Z_i = W_i = 0$.

Notice that due to the ρ-meson being unstable, our results for the form factors also take complex values for vanishing and negative t, in contrast to the model calculations of Refs. [14,16]. Our values for the form factors at vanishing t are given in Table II, together with the corresponding results of Refs. [14,16]. Since our results depend on unknown low-energy constants, this comparison does not allow us to draw any conclusions about the reliability of the model calculations of the quoted references. Notice also that the real part of $D_0(0)$ is negative only because we chose to substitute v_1 and v_4 with negative signs.

In Fig. 2, we plot the real and imaginary parts of our calculated form factors for the above specified values of the parameters.

Defining the slopes s_F as the coefficients of linear terms in the Taylor expansion of the form factors, $F(t) = F(0) + s_F t + \cdots$, we obtain, for their chiral expansions, the following results:

$$\begin{aligned}
s_{A_0} &= \frac{v_5}{2} + Z_{A_0} M_\pi^2 - \frac{g_{\omega\rho\pi}^2}{64\pi F^2 M_R} M_\pi - \frac{g_{\omega\rho\pi}^2 (6 v_5 M_R^2 + 7)}{192\pi^2 F^2 M_R^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3),\\
s_{A_1} &= Y_{A_1} - \frac{g_{\omega\rho\pi}^2 M_R}{480\pi F^2} \frac{1}{M_\pi} + \mathcal{O}(M_\pi),\\
s_J &= \frac{1}{2}(v_5 + 2v_6) + Z_J M_\pi^2 - \frac{17 g_{\omega\rho\pi}^2 + a}{1152\pi^2 F^2} - \frac{g_{\omega\rho\pi}^2}{192\pi^2 F^2} \ln\frac{M_\pi}{M_R} + \frac{5 g_{\omega\rho\pi}^2}{768\pi F^2 M_R} M_\pi\\
&\quad - \frac{g_{\omega\rho\pi}^2 (3 v_5 M_R^2 + 6 v_6 M_R^2 - 5) - 2a}{96\pi^2 F^2 M_R^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3),\\
s_{D_0} &= -4v_4 - \frac{v_5}{2} + Z_{D_0} M_\pi^2 + \frac{(35 + 24 i\pi) a - 36 g_{\omega\rho\pi}^2}{1440\pi^2 F^2} + \frac{g_{\omega\rho\pi}^2 M_R}{160\pi F^2} \frac{1}{M_\pi} + \frac{5 g_{\omega\rho\pi}^2 + 4a}{240\pi^2 F^2} \ln\frac{M_\pi}{M_R}\\
&\quad + \frac{43 g_{\omega\rho\pi}^2}{3840\pi F^2 M_R} M_\pi + \frac{5 g_{\omega\rho\pi}^2 (48 v_4 M_R^2 + 6 v_5 M_R^2 + 7) - 8a}{960\pi^2 F^2 M_R^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3),\\
s_{D_1} &= Y_{D_1} + \frac{g_{\omega\rho\pi}^2 M_R^3}{560\pi F^2} \frac{1}{M_\pi^3} - \frac{(11 g_{\omega\rho\pi}^2 - 12a) M_R^2}{840\pi^2 F^2} \frac{1}{M_\pi^2} - \frac{131 g_{\omega\rho\pi}^2 M_R}{13440\pi F^2} \frac{1}{M_\pi} - \frac{35 g_{\omega\rho\pi}^2 + 64a}{840\pi^2 F^2} \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi),\\
s_E &= \frac{1}{2}(v_5 + 2v_6) + Z_E M_\pi^2 + \frac{(35 + 12 i\pi) a - 92 g_{\omega\rho\pi}^2}{5760\pi^2 F^2} - \frac{g_{\omega\rho\pi}^2 M_R}{1920\pi F^2} \frac{1}{M_\pi} + \frac{4a - 5 g_{\omega\rho\pi}^2}{960\pi^2 F^2} \ln\frac{M_\pi}{M_R}\\
&\quad + \frac{19 g_{\omega\rho\pi}^2}{15360\pi F^2 M_R} M_\pi - \frac{5 v_5 g_{\omega\rho\pi}^2 M_R^2 + 10 v_6 g_{\omega\rho\pi}^2 M_R^2 + 2a}{160\pi^2 F^2 M_R^2} M_\pi^2 \ln\frac{M_\pi}{M_R} + \mathcal{O}(M_\pi^3).
\end{aligned} \tag{13}$$

We see from the above expressions that some of the slopes have strong singularities for small pion masses. This again underlines the need for a careful analysis of the chiral extrapolation of lattice data.

It is also instructive to study the t-dependence of GFFs in the chiral limit $M_\pi = 0$. In the next section, the corresponding results will allow us to derive the large distance asymptotics of the energy and force distributions. The expressions of the form factors in the small-t region in the chiral limit have the form

$$\begin{aligned}
A_0(t) &= 1 + \frac{v_5}{2} t + W_{A_0} t^2 + \frac{7 g_{\omega\rho\pi}^2}{4096 F^2 M_R} (-t)^{3/2} + \frac{5 g_{\omega\rho\pi}^2}{768\pi^2 F^2 M_R^2} t^2 \ln\left(-\frac{t}{M_R^2}\right),\\
A_1(t) &= 8 v_6 M_R^2 + Y_{A_1} t + \frac{3 g_{\omega\rho\pi}^2 M_R}{1024 F^2} \sqrt{-t},\\
J(t) &= 1 + \frac{1}{2}(v_5 + 2v_6) t - \frac{9 g_{\omega\rho\pi}^2 + a}{1152\pi^2 F^2} t - \frac{g_{\omega\rho\pi}^2}{384\pi^2 F^2} t \ln\left(-\frac{t}{M_R^2}\right),\\
D_0(t) &= 1 + 4v_1 + 8 v_4 M_R^2 - \left(4v_4 + \frac{v_5}{2}\right) t - \frac{5 g_{\omega\rho\pi}^2 M_R}{1024 F^2} \sqrt{-t} + \frac{5 g_{\omega\rho\pi}^2 + 4a}{480\pi^2 F^2} t \ln\left(-\frac{t}{M_R^2}\right) + \frac{3(7 + 40 i\pi) a - 200 g_{\omega\rho\pi}^2}{7200\pi^2 F^2} t,
\end{aligned}$$

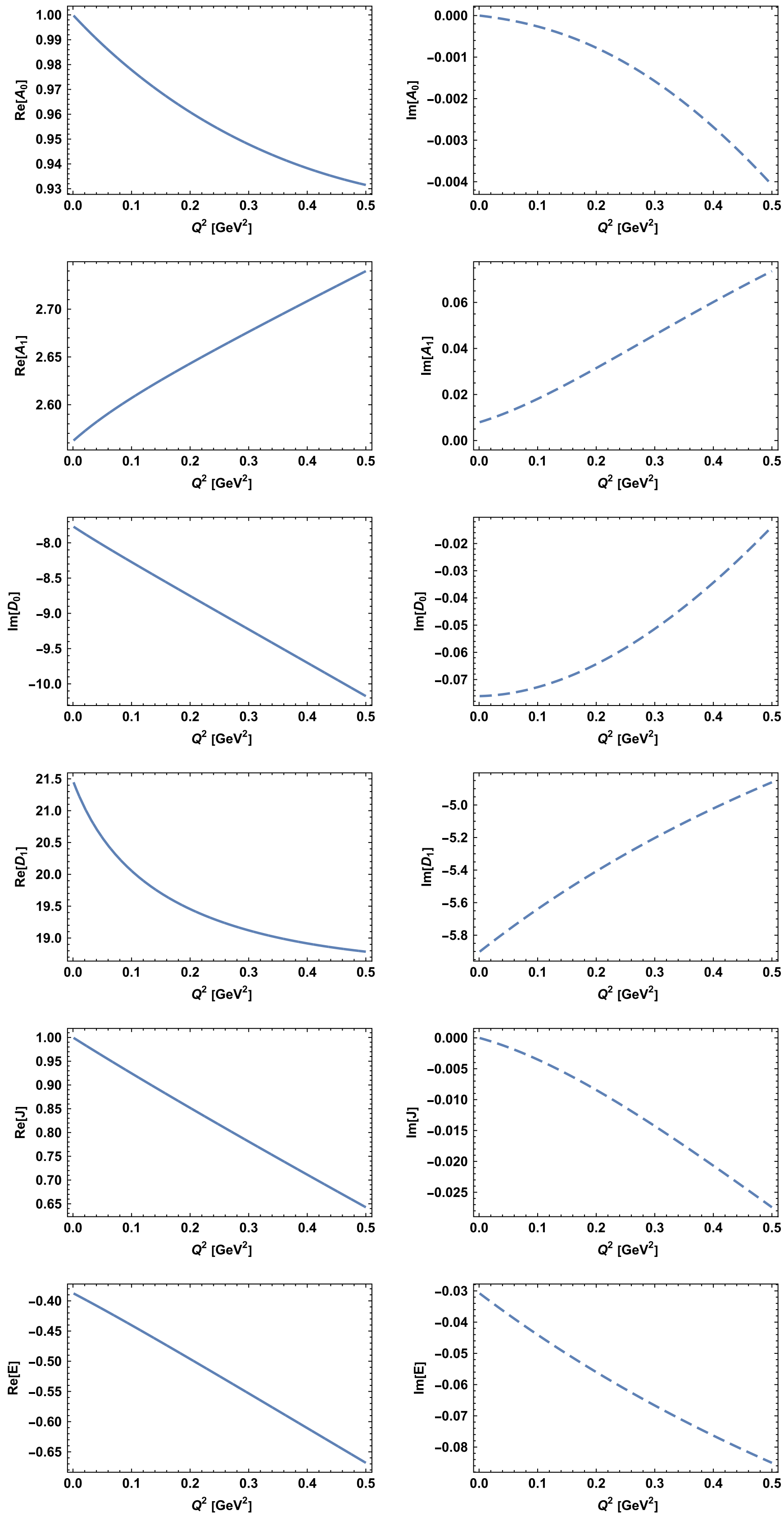

FIG. 2. Real and imaginary parts of the gravitational form factors of the ρ-meson as functions of $Q^2 = -t$.

$$D_1(t) = -8(4v_4 + v_5 + v_6)M_R^2 + \frac{M_R^2(47a - 20g_{\omega\rho\pi}^2)}{450\pi^2 F^2} + \frac{3g_{\omega\rho\pi}^2 M_R^3}{256F^2}\frac{1}{\sqrt{-t}} + \frac{(5g_{\omega\rho\pi}^2 - 8a)M_R^2}{120\pi^2 F^2}\ln\left(-\frac{t}{M_R^2}\right),$$

$$E(t) = 1 - a_3 - v_5 M_R^2 + \frac{1}{2}(v_5 + 2v_6)t + \frac{a(60\pi i - 9) - 245g_{\omega\rho\pi}^2}{28800\pi^2 F^2}t + \frac{g_{\omega\rho\pi}^2 M_R}{1024F^2}\sqrt{-t} + \frac{4a - 5g_{\omega\rho\pi}^2}{1920\pi^2 F^2}t\ln\left(-\frac{t}{M_R^2}\right). \quad (14)$$

Notice that while not all analytic (at $t = 0$) terms can be absorbed into renormalization of the coupling constants, all power-counting-violating pieces are systematically removed.

V. LARGE DISTANCE BEHAVIOR OF ENERGY AND FORCE DISTRIBUTIONS

It is particularly interesting to look at the energy distribution and mechanical properties such as the elastic pressure and shear force distributions inside the ρ-meson. These fundamental distributions are encoded in the static EMT defined in the Breit frame as [7]

$$T^{\mu\nu}(\vec{r}, \sigma', \sigma) = \int \frac{d^3\Delta}{(2\pi)^3 2E} e^{-i\vec{\Delta}\cdot\vec{r}} \langle p', \sigma' | \hat{T}^{\mu\nu}_{\rm QCD}(0) | p, \sigma\rangle. \quad (15)$$

Here, $\hat{T}^{\mu\nu}_{\rm QCD}(0)$ is the QCD EMT operator of the matrix element that is computed between hadron states with spin projections σ, σ' and momenta $p^0 = p^{0\prime} = E = \sqrt{m^2 + \vec{\Delta}^2/4}$, and $p^{i\prime} = -p^i = \Delta^i/2$. The 00 component of the static EMT contains the information about the energy distribution, the $0i$ components encode the spin distribution, while the ik components provide us with the distributions of elastic pressure and shear forces inside the hadron [7].

Various components of the static EMT for hadrons with arbitrary spin can be decomposed into multipoles of the hadron's spin operator. The expansion to the quadrupole order has the following form [10–12,17][1]:

$$T^{00}(\mathbf{r}) = \varepsilon_0(r) + \varepsilon_2(r)\hat{Q}^{pq}Y_2^{pq} + \ldots, \quad (16)$$

$$T^{ik}(\mathbf{r}) = p_0(r)\delta^{ik} + s_0(r)Y_2^{ik} + \left(p_2(r) + \frac{1}{3}p_3(r) - \frac{1}{9}s_3(r)\right)\hat{Q}^{ik}$$
$$+ \left(s_2(r) - \frac{1}{2}p_3(r) + \frac{1}{6}s_3(r)\right)2[\hat{Q}^{ip}Y_2^{pk} + \hat{Q}^{kp}Y_2^{pi} - \delta^{ik}\hat{Q}^{pq}Y_2^{pq}]$$
$$+ \hat{Q}^{pq}Y_2^{pq}\left[\left(\frac{2}{3}p_3(r) + \frac{1}{9}s_3(r)\right)\delta^{ik} + \left(\frac{1}{2}p_3(r) + \frac{5}{6}s_3(r)\right)Y_2^{ik}\right] + \ldots. \quad (17)$$

Here, the ellipses denote the contributions of 2^nth multipoles with $n > 2$. They are absent for a spin-1 particle. The quadrupole operator is a $(2J+1)\times(2J+1)$ matrix:

$$\hat{Q}^{ik} = \frac{1}{2}\left(\hat{J}^i\hat{J}^k + \hat{J}^k\hat{J}^i - \frac{2}{3}J(J+1)\delta^{ik}\right), \quad (18)$$

which is expressed in terms of the spin operator $\hat{J}^i$. The spin operator can be expressed in terms of SU(2) Clebsch-Gordan coefficients (in the spherical basis):

$$\hat{J}^{\mu}_{\sigma'\sigma} = \sqrt{J(J+1)}C^{J\sigma'}_{J\sigma 1\mu}. \quad (19)$$

Furthermore, we introduce the irreducible (symmetric and traceless) tensor of rank n made out of $\mathbf{r}$:

$$Y_n^{i_1 i_2 \ldots i_n} = \frac{(-1)^n}{(2n-1)!!}r^{n+1}\partial^{i_1}\ldots\partial^{i_n}\frac{1}{r}, \quad (20)$$

i.e.,

$$Y_0 = 1, \qquad Y_1^i = \frac{r^i}{r}, \qquad Y_2^{ik} = \frac{r^i r^k}{r^2} - \frac{1}{3}\delta^{ik}, \quad \text{etc.} \quad (21)$$

Note that only the monopole quantities $\varepsilon_0(r)$, $p_0(r)$, and $s_0(r)$ are left after spin averaging. The functions $\varepsilon_0(r)$ and $\varepsilon_2(r)$ correspond to the spin-averaged energy density and to the quadrupole deformation of the energy density inside the hadron, respectively. There is an obvious relation $\int d^3r\varepsilon_0(r) = m$. Also, it is obvious that $\varepsilon_2(r) = 0$ for hadrons with spin 0 and 1/2 (that is why such hadrons can be called spherically symmetric).

From the equilibrium condition for the stress tensor, $\partial_k T^{ik}(\mathbf{r}) = 0$, one can easily obtain the equations for the functions $p_n(r)$ and $s_n(r)$:

[1]In what follows, we shall suppress the hadron's spin indices σ, σ' when their position is obvious. Also, we employ here the parametrization of the static stress tensor that differs from that of Refs. [12,17] by a simple redefinition. The corresponding relations are given in the Appendix of Ref. [11].

$$\frac{d}{dr}\left(p_n(r)+\frac{2}{3}s_n(r)\right)+\frac{2}{r}s_n(r)=0, \quad \text{for } n=0,2,3. \tag{22}$$

To see the physical meaning of the quadrupole force distributions $p_{2,3}(r)$ and $s_{2,3}(r)$, it is instructive to look at the force acting on the infinitesimal radial area element dS_r ($d\vec{S}=dS_r\vec{e}_r+dS_\theta\vec{e}_\theta+dS_\phi\vec{e}_\phi$). With the help of the parametrization of Eq. (17) and the relation of the force to the stress tensor, $dF_i=T_{ik}dS_k$, we obtain

$$\frac{dF_r}{dS_r}=p_0(r)+\frac{2}{3}s_0(r) + \hat{Q}^{rr}\left(p_2(r)+\frac{2}{3}s_2(r)+p_3(r)+\frac{2}{3}s_3(r)\right), \tag{23}$$

$$\frac{dF_\theta}{dS_r}=\hat{Q}^{\theta r}\left(p_2(r)+\frac{2}{3}s_2(r)\right),$$
$$\frac{dF_\phi}{dS_r}=\hat{Q}^{\phi r}\left(p_2(r)+\frac{2}{3}s_2(r)\right). \tag{24}$$

We see that in contrast to spherically symmetric hadrons, the radial area element experiences not only normal forces but also tangential ones. The strengths of the tangential forces are governed by $p_2(r)$ and $s_2(r)$; the quadrupole force distributions $p_3(r)$ and $s_3(r)$ contribute to the spin-dependent part of the radial force.

Using the result for the t-dependence of GFFs in the chiral limit of Eq. (15) obtained in the previous section, we can easily calculate the analytic expressions for the large distance behavior (in the chiral limit) of the energy and force distributions defined in Eqs. (16) and (17):

$$\varepsilon_0(r)=\frac{g^2_{\omega\rho\pi}}{32\pi^2F^2}\frac{1}{r^6}-\frac{3(2a+5g^2_{\omega\rho\pi})}{32\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$\varepsilon_2(r)=\frac{3g^2_{\omega\rho\pi}}{128\pi^2F^2}\frac{1}{r^6}+\frac{21(4a-5g^2_{\omega\rho\pi})}{256\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$p_0(r)=-\frac{g^2_{\omega\rho\pi}}{96\pi^2F^2}\frac{1}{r^6}+\frac{(16a+15g^2_{\omega\rho\pi})}{144\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$s_0(r)=\frac{g^2_{\omega\rho\pi}}{32\pi^2F^2}\frac{1}{r^6}-\frac{7(16a+15g^2_{\omega\rho\pi})}{384\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$p_2(r)=\frac{g^2_{\omega\rho\pi}}{32\pi^2F^2}\frac{1}{r^6}+\frac{5(20a-13g^2_{\omega\rho\pi})}{192\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$s_2(r)=-\frac{3g^2_{\omega\rho\pi}}{32\pi^2F^2}\frac{1}{r^6}-\frac{35(20a-13g^2_{\omega\rho\pi})}{512\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$p_3(r)=-\frac{9g^2_{\omega\rho\pi}}{128\pi^2F^2}\frac{1}{r^6}-\frac{7(8a-5g^2_{\omega\rho\pi})}{48\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),$$
$$s_3(r)=\frac{27g^2_{\omega\rho\pi}}{128\pi^2F^2}\frac{1}{r^6}+\frac{49(8a-5g^2_{\omega\rho\pi})}{128\pi^3F^2M_R}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right). \tag{25}$$

In Refs. [8,9,58] it was conjectured that for the stability of a mechanical system, the spin-averaged pressure and shear forces should satisfy the inequality

$$\frac{2}{3}s_0(r)+p_0(r)\geq 0, \tag{26}$$

which corresponds to positivity of the radial pressure. From the derived large distance behavior of the $p_0(r)$ and $s_0(r)$, we see that the inequality Eq. (26) is indeed satisfied. However, the ρ-meson decays in our theory. The terms in the large distance expansion (25) which "know" about the instability of the particle are proportional to the $\rho\pi\pi$ coupling constant squared $\sim a$. It is interesting to note that the corresponding terms violate the stability condition of Eq. (26); also, the corresponding terms in the spin-averaged energy density $\varepsilon_0(r)$ violate its positivity. A detailed study of the relations between mechanical stability conditions and the decay of unstable particles will be given elsewhere.

VI. SUMMARY

To summarize, we have applied chiral EFT to vector mesons and Goldstone bosons in the presence of an external gravitational field. Using standard definitions, we obtained the expressions of the EMT in the flat background metric. As first noticed in Ref. [43], terms in the effective Lagrangian involving the gravitational curvature, which vanishes in the flat background, give nontrivial contributions to the EMT. This also happens for the case at hand. Therefore, in order to keep track of all relevant contributions to the EMT in flat space-time, it is necessary to consider the effective Lagrangian in curved space-time. In the next step, we calculated the gravitational form factors of the ρ-meson at next-to-leading order. This involved the calculation of tree-level and one-loop diagrams. To get rid of ultraviolet divergences and power-counting-violating pieces, we applied the complex-mass renormalization scheme, which allows one to also subtract the large imaginary parts from loop diagrams. The matrix element of the EMT for this spin-1 hadron is parametrized using six independent structures. We do not give the rather lengthy expressions of the obtained expressions of the gravitational form factors of the ρ-meson in terms of standard loop functions,[2] but focus on the chiral expansion of the form factors at zero momentum transfer and of their slopes. These expressions should be useful for lattice extrapolations of the corresponding results by taking the pion mass to its physical value. We also presented the expansion of the form factors in the small-t region in the chiral limit. Using these expressions, we further calculated the large-distance behavior of the energy distribution and the internal forces. The obtained results are consistent with the stability condition of a mechanical system.

[2]These are available from the authors upon request.

ACKNOWLEDGMENTS

The authors thank Julia Panteleeva for checking some equations. This work was supported in part by BMBF (Grant No. 05P18PCFP1), Georgian Shota Rustaveli National Science Foundation (Grant No. FR17-354), the NSFC and the Deutsche Forschungsgemeinschaft through the funds provided to the Sino-German Collaborative Center TRR110 "Symmetries and the Emergence of Structure in QCD" (NSFC Grant No. 12070131001, DFG Project-ID 196253076-TRR 110), and by the EU Horizon 2020 research and innovation programme, the STRONG-2020 project under Grant Agreement No. 824093. The work of U. G. M. was also supported in part by the Chinese Academy of Sciences (CAS) through a President's International Fellowship Initiative (PIFI) (Grant No. 2018DM0034), by the VolkswagenStiftung (Grant No. 93562).

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