PEDESTRIAN FUNDAMENTAL DIAGRAMS AND CONTINUITY EQUATION

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Juliane Adrian, Ann Katrin Boomers, Sarah Paetzke and Armin Seyfried

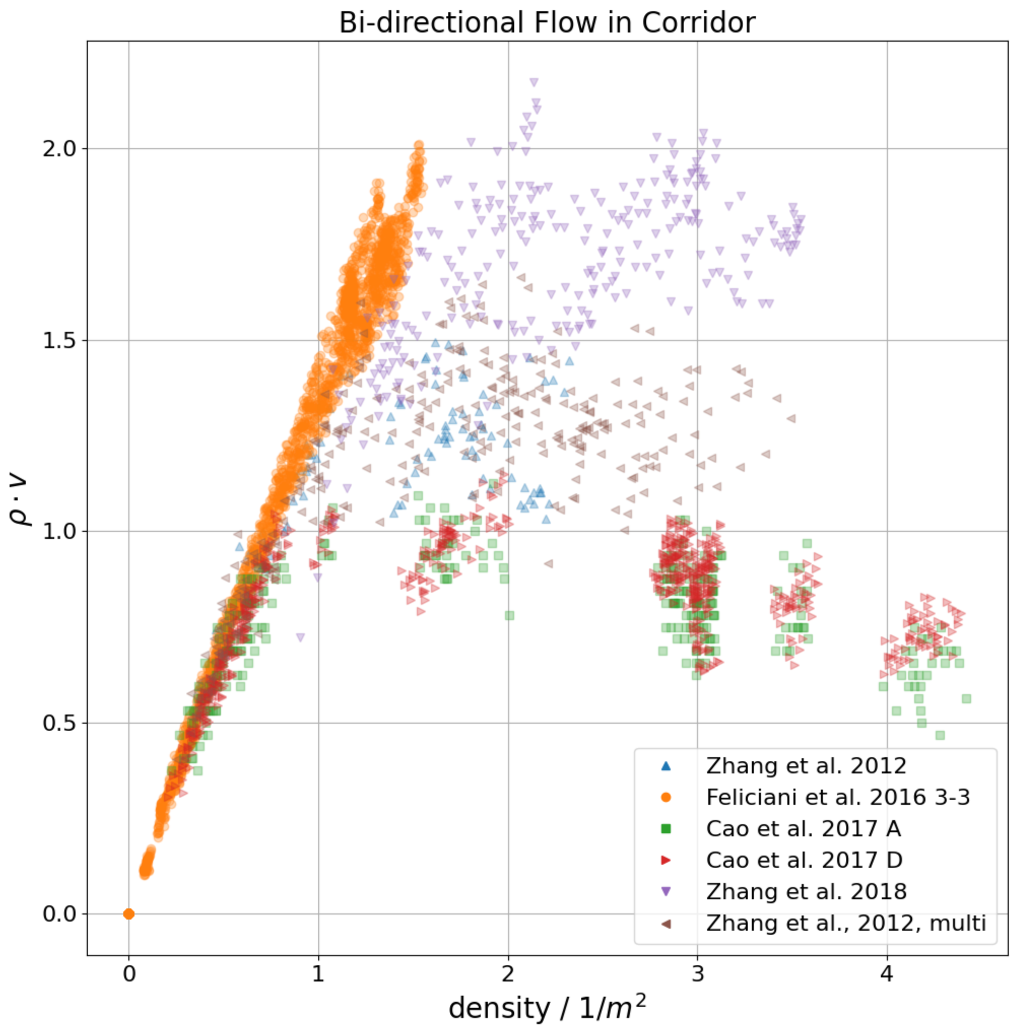
INTRODUCTION

Fundamental diagram

- Relation between speed, flow and density
- Describes the performance of pedestrian facility
- Connects performance (flow, speed) with level of service (density)

Problems

- Large deviations in the literature
- Human factors
- Uni-, bi- and multidirectional flow
- Different methods to measure speed, density and flow are used
- Until now a consensus on a standard measurement method is missing



CONTINUITY EQUATION AND FLOW EQUATION

Continuity equation

Continuity equation is a local form of a conservation law

In a system where pedestrians move, the equation ensures that no pedestrians appear or disappear. Teleportation like in Star Trek is excluded!

Measurements not in consistency with the continuity equation could lead to an over- or underestimation of flow, density or speed.

CONTINUITY EQUATION AND FLOW EQUATION

Continuity equation

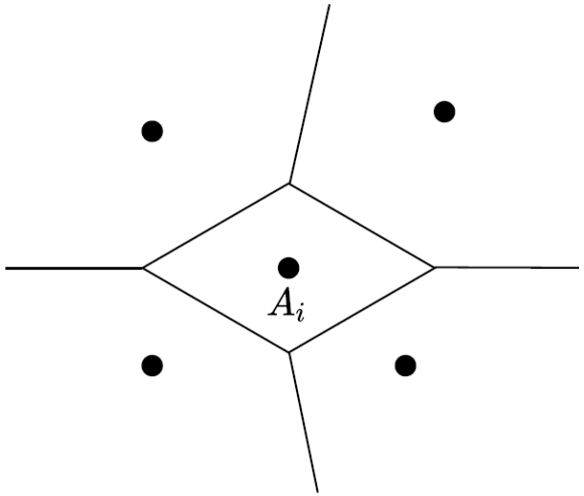
$$\frac{dN}{dt} = \int_{l_0} \vec{\boldsymbol{j}}\,\vec{n}\,dy + \int_{l_1} \vec{\boldsymbol{j}}\,\vec{n}\,dy \text{ with } \vec{\boldsymbol{j}} = \boldsymbol{\rho}\,\vec{\boldsymbol{u}}$$

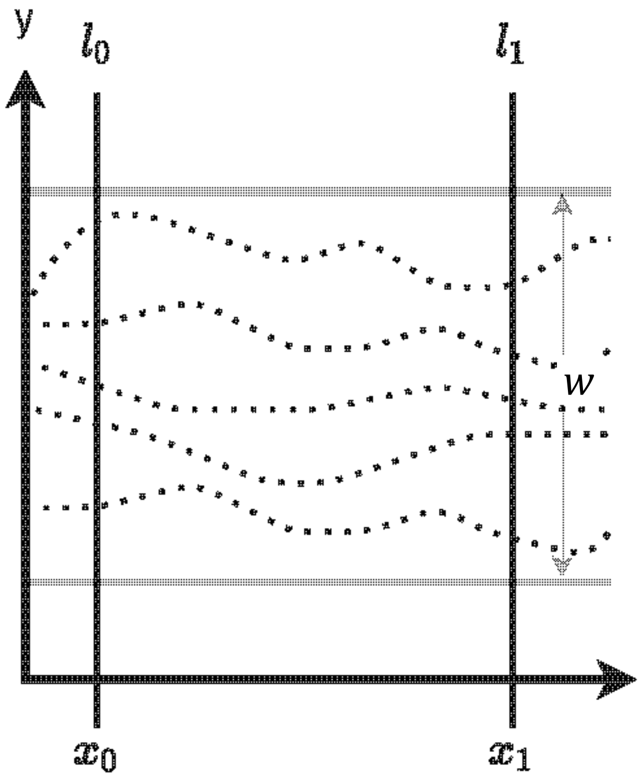
where $\vec{\boldsymbol{j}}, \boldsymbol{\rho}$ and $\vec{\boldsymbol{u}}$ are fields e.g. calculated by Voronoi decomposition

$$\boldsymbol{\rho} = \sum_i \rho_i \text{ with } \rho_i(\vec{x}) = \begin{cases} {}^{1}/_{A_i} : \vec{x} \in A_i \\ 0 : else \end{cases} [1/m^2]$$

$$\vec{\boldsymbol{u}} = \sum_i \vec{\boldsymbol{u}}_i \text{ with } \vec{\boldsymbol{u}}_i(\vec{x}) = \begin{cases} \vec{v}_i : \vec{x} \in A_i \\ 0 : else \end{cases} [m/s]$$

$$\vec{\boldsymbol{j}} = \sum_i \vec{\boldsymbol{j}}_i \text{ with } \vec{\boldsymbol{j}}_i(\vec{x}) = \begin{cases} {}^{\vec{v}_i}/_{A_i} : \vec{x} \in A_i \\ 0 : else \end{cases} [1/ms]$$





CONTINUITY EQUATION AND FLOW EQUATION

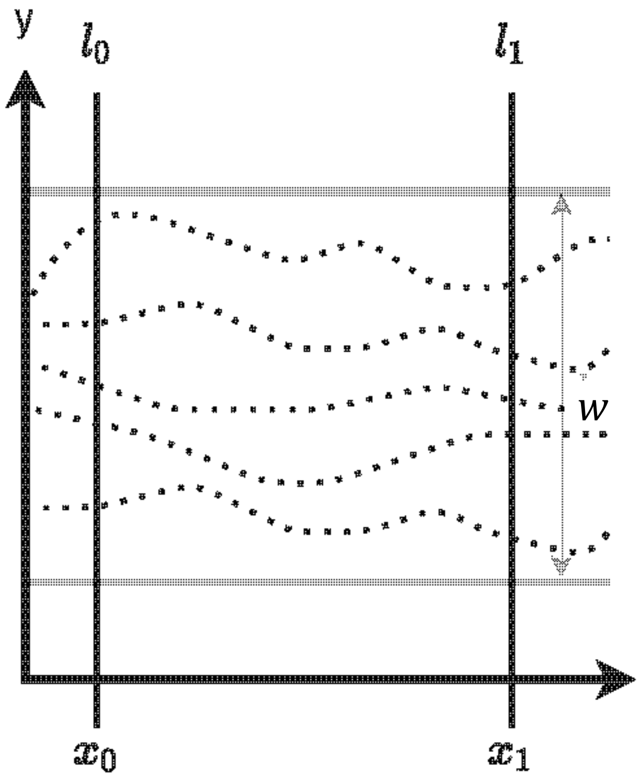
For a time interval $\Delta t = t_N - t_0$ the flow $J_{l_0} = {}^{\Delta N}/_{\Delta t}$ at the line l_0 calculated by $\boldsymbol{\rho}$ and $\vec{\boldsymbol{u}}$ is given by

$$J_{l_0} = \int_{y_1}^{y_2} \vec{\boldsymbol{j}}(x_0, y)\, \vec{n}\, dy = \int_{y_1}^{y_2} \boldsymbol{\rho}(x_0, y) dy \int_{y_1}^{y_2} \vec{\boldsymbol{u}}(x_0, y) \vec{n}\, dy \quad (1)$$

Classical flow equation: $J_{l_0} = \rho_A \cdot s_{A,l_0} \cdot w \quad (2)$

The comparison of (1) and (2) points to the following problems:

- in (1) the integral over $\boldsymbol{\rho}$ is a line density, in (2) ρ_A is an area density
- multiplication with w in (2) implies ρ and s are homogenous along l_0
- in (1) the integral over $\vec{\boldsymbol{u}}$ considers **normal** and **negative** components.
  In (2) classical definitions, like e.g. $s_A = \frac{1}{N_A} \sum_{i=1}^{N_A} |\vec{v_i}|$ negative contributions are neglected leading to an overestimation of the flow



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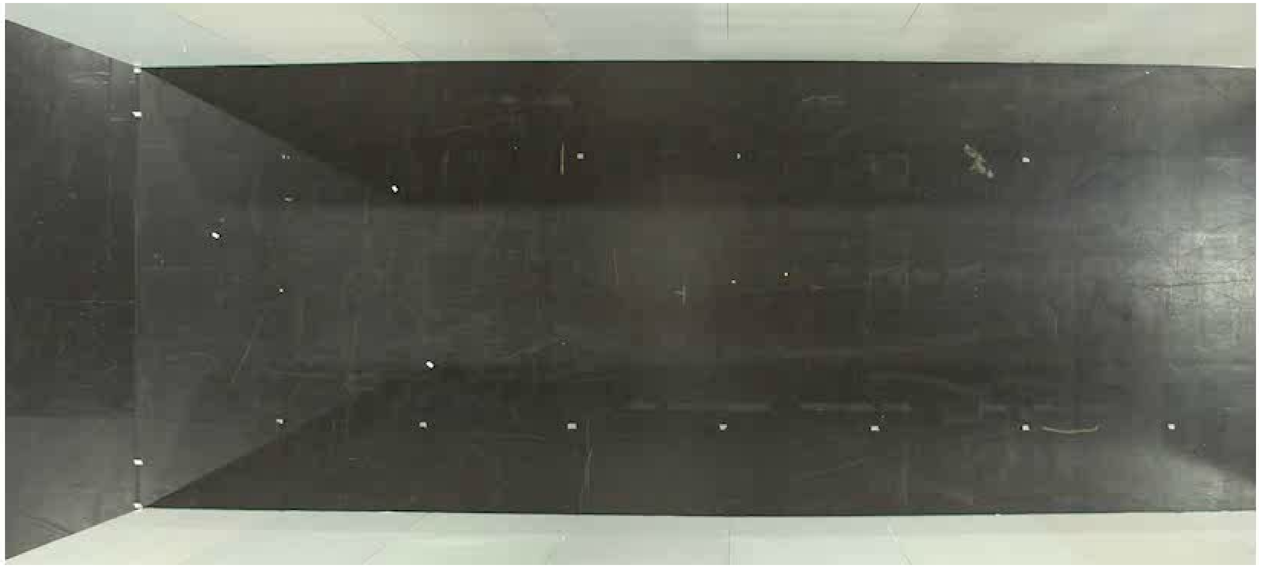
MEASUREMENTS METHODS

How we can improve the consistency of measurements basing on trajectory data with the continuity equation?



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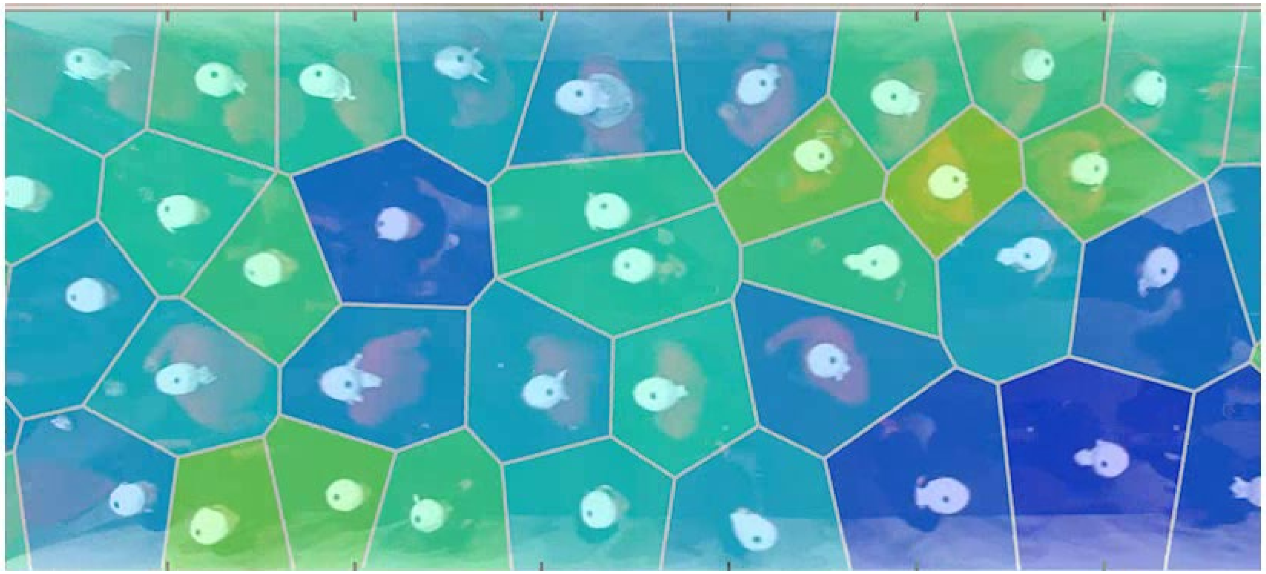
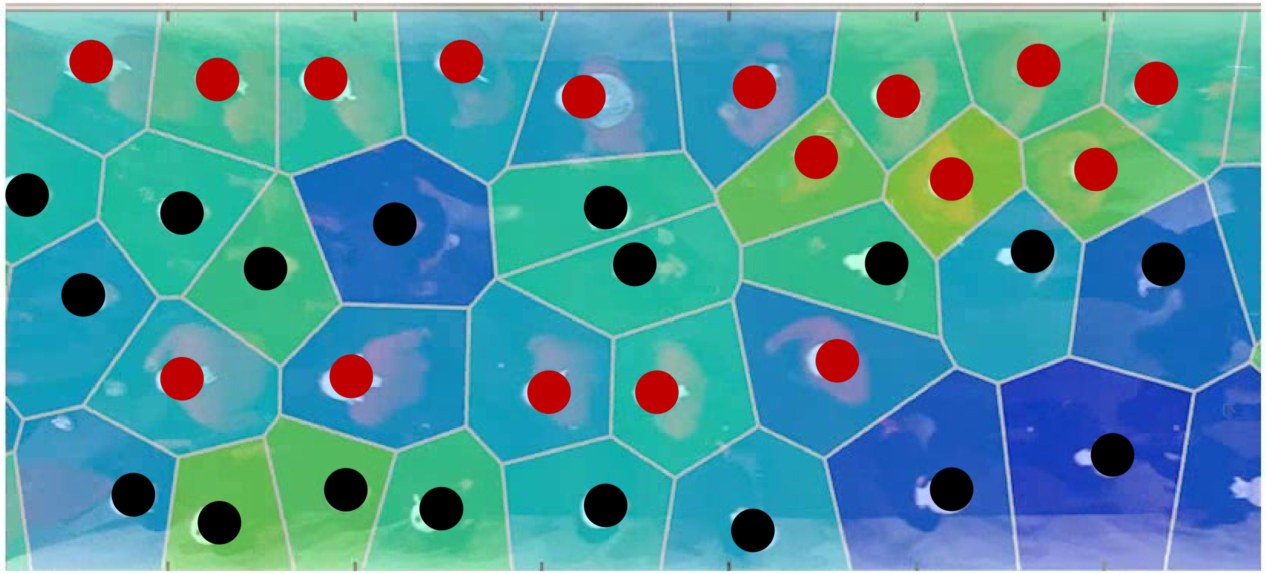


MEASUREMENTS METHODS

To consider negative contributions v and ρ are measured separately for stream I and II

$$\rho = \rho^{I} + \rho^{II}$$

$$v = v^{I} + v^{II}$$



● stream I

● stream II

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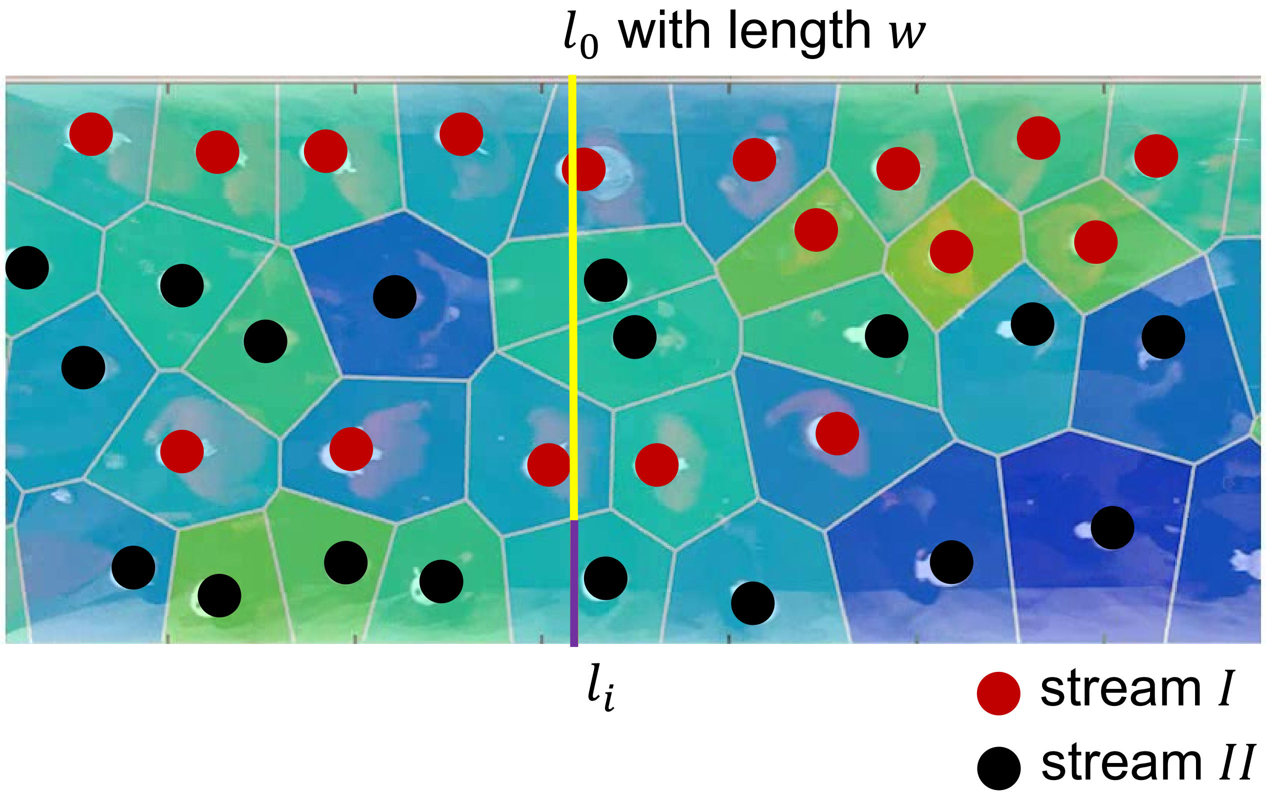
$\frac{l_i}{w}$ take inhomogeneities along l_0 into account.



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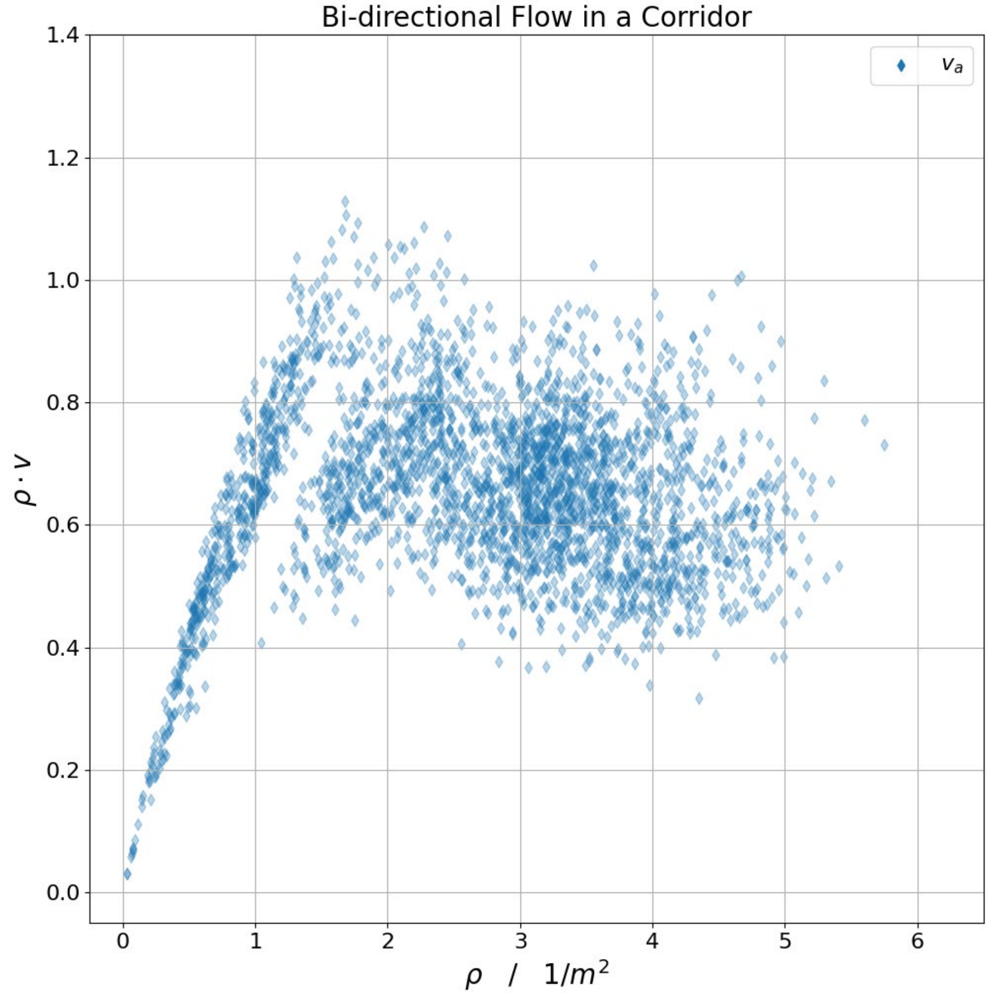

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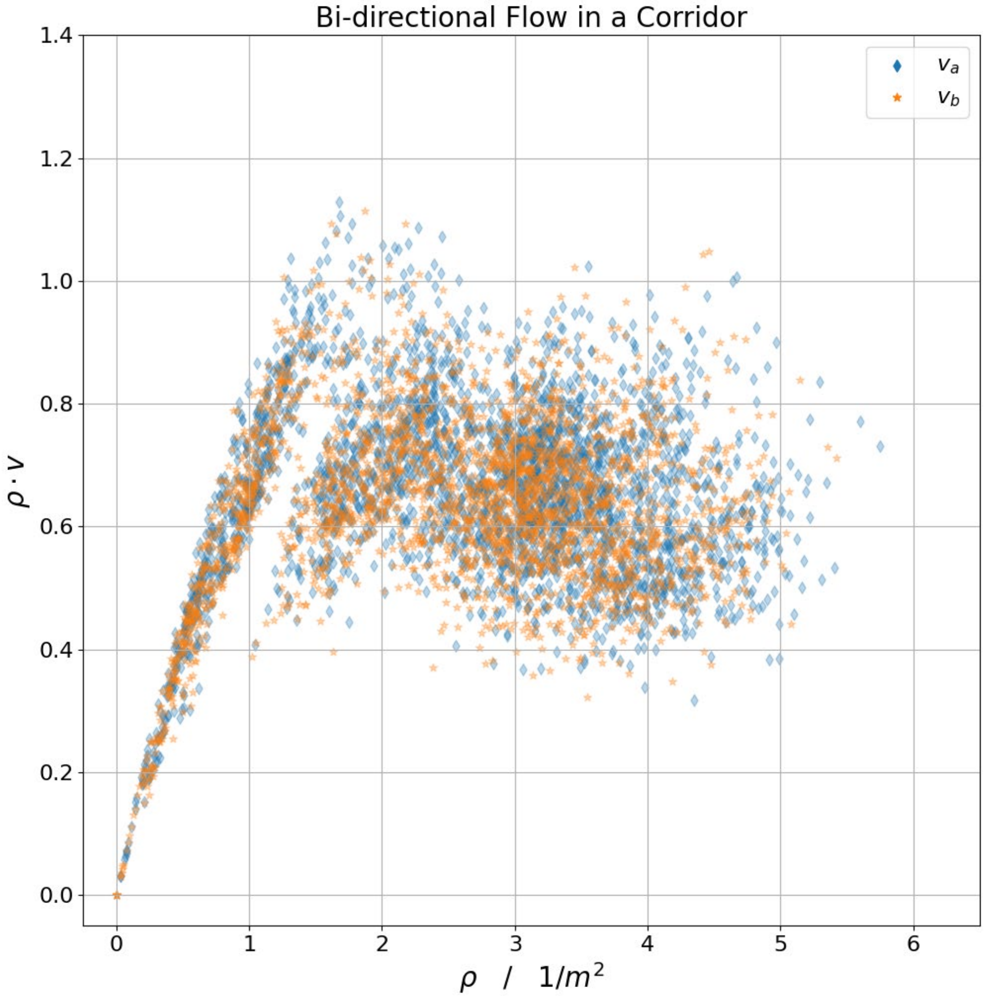

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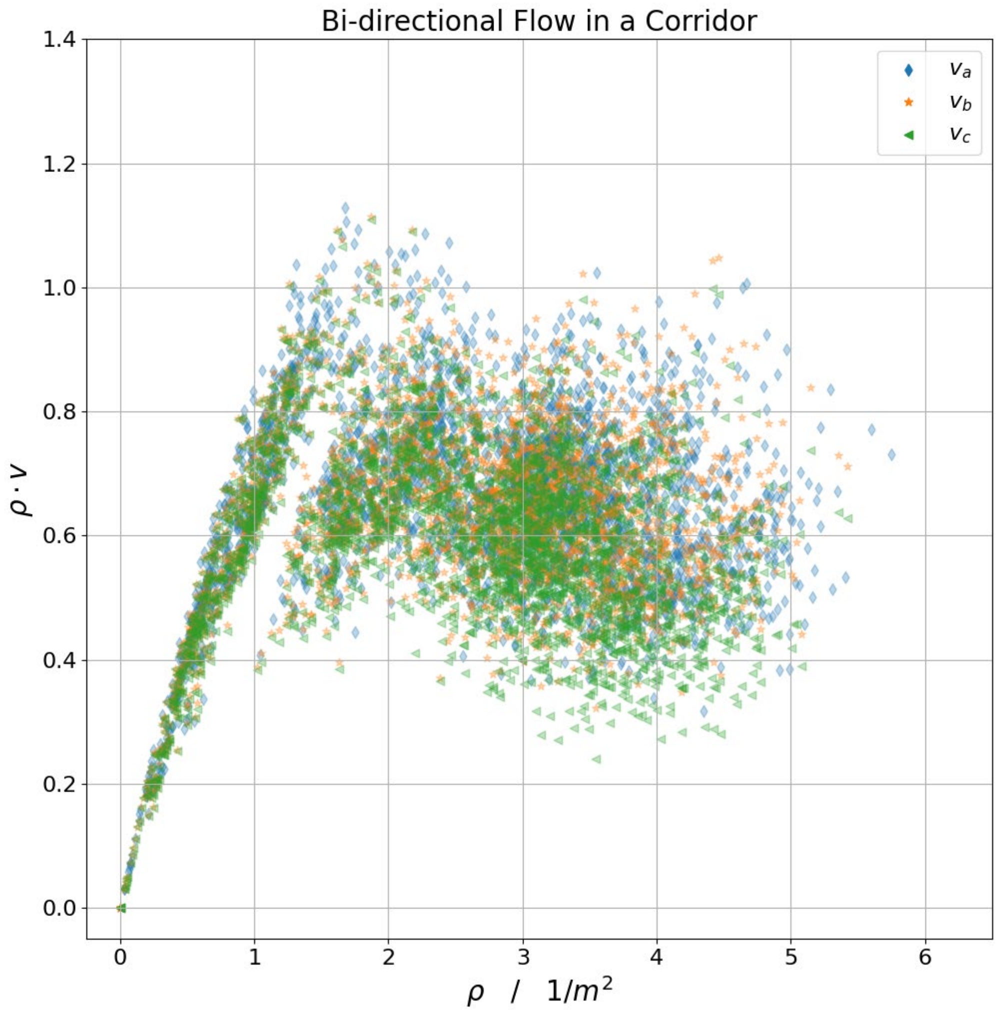

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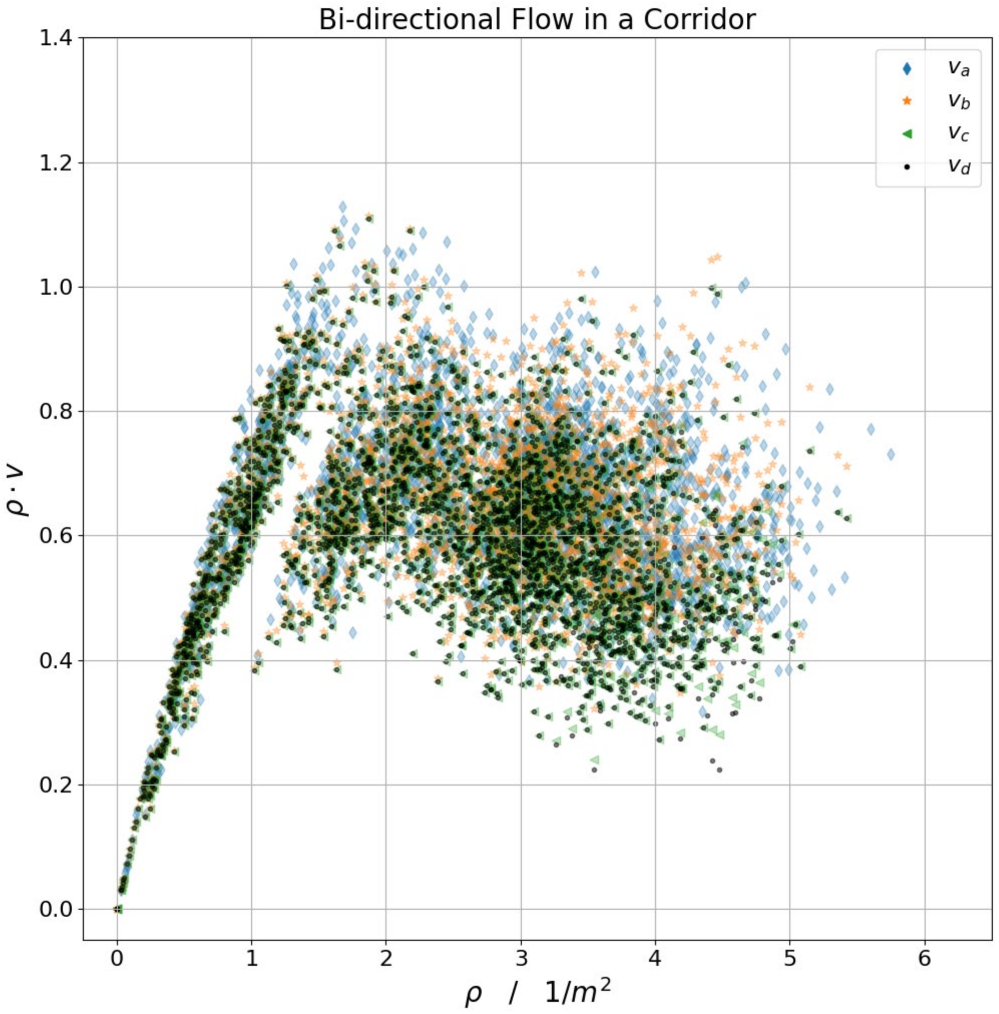

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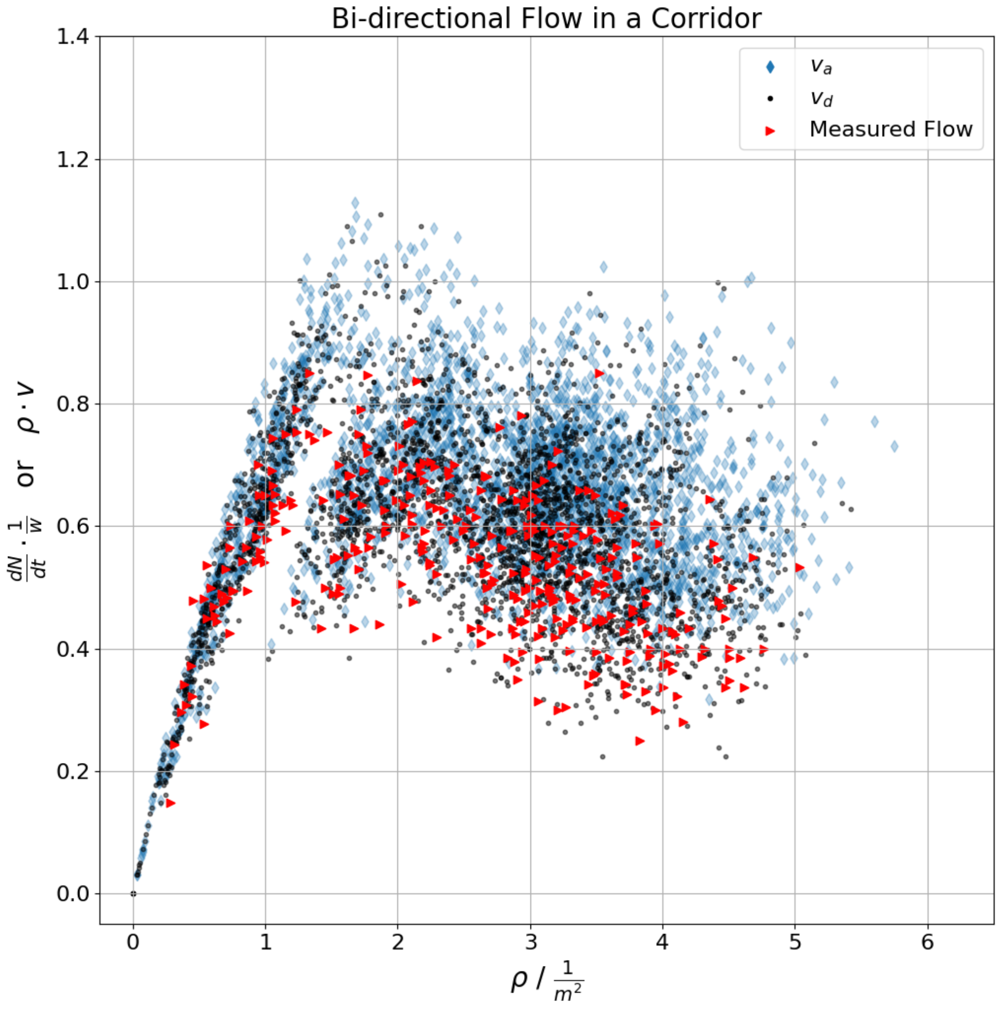
Comparison of v_d^m with $J_{l_0}(\rho) = {}^{\Delta N}/_{\Delta t}$ and $\Delta t = 5\ s$



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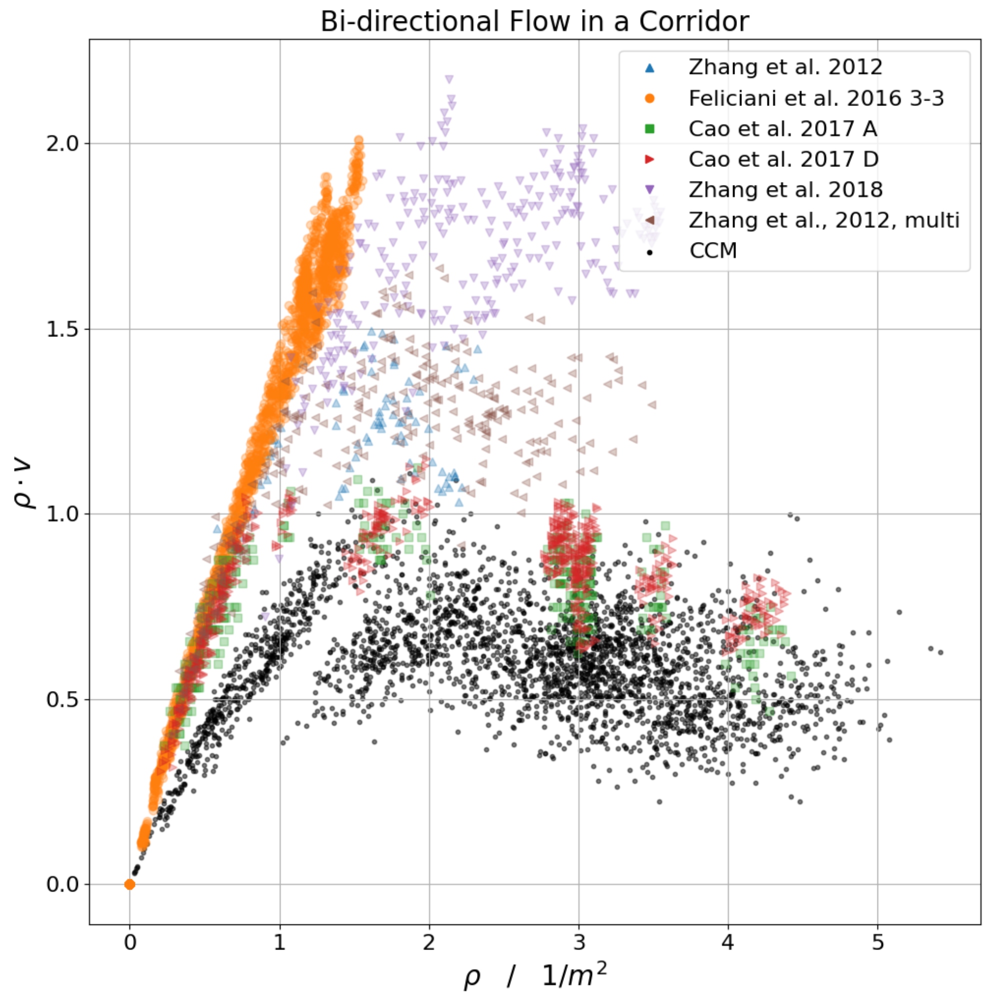
COMPARISON WITH THE LITERATURE

Observations

- Slope of free flow branch is smaller
- Densities measured at a line show higher values
- Values for the maximal flow are lower
- ‘Classical’ methods overestimates the capacity and underestimates the density!



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SUMMARY

Lessons learned

- Conformity to the continuity equation is relevant in measurements of the fundamental diagram
- The flow equation is not well defined. This can lead to the fact that the number of pedestrians is not conserved
- Current standard measurement methods lead to overestimation of capacity and an underestimation of density. Both effects are critical for safety-related assessments

Outlook

- Measurement of multidirectional streams at crossings
- Measurements of systems with stop and go waves

Thanks to the coauthors

- Juliane Adrian, Ann Katrin Boomers and Sarah Paetzke