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## 1. Collective dynamics by active particles

- Self-organized collective phenomena in animal swarms, fish schools, pedestrians, robots, colloids, bacteria, road traffic, ...
- Motility Induced Phase Separation: positive feedback between aggregation and slowing down
- Moving bands in flocking models, Active Ising models, charged colloids
- Succession of de- and acceleration: stop-and-go waves in pedestrian, vehicular and bicycle traffic

## 2. Stop-and-Go: Deterministic Models

- Instability of the homogeneous configuration
- Phase transition & fine tuning of parameters, limit-cycle solution only for non-linear models
- E.g. Optimal-Velocity model  $\ddot{x}_i = \frac{1}{\tau} (V(\Delta x_i) - \dot{x}_i)$  linearly stable for  $\tau > 1/(2V')$

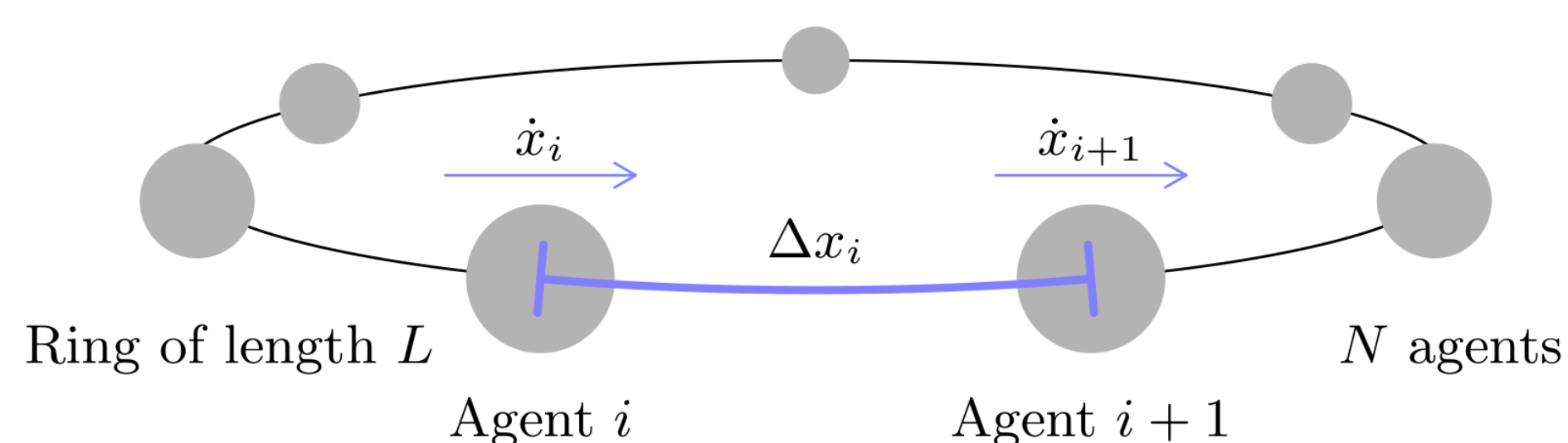


Figure 1: Scheme for the considered system of  $N$  particles on a ring of length  $L$ .

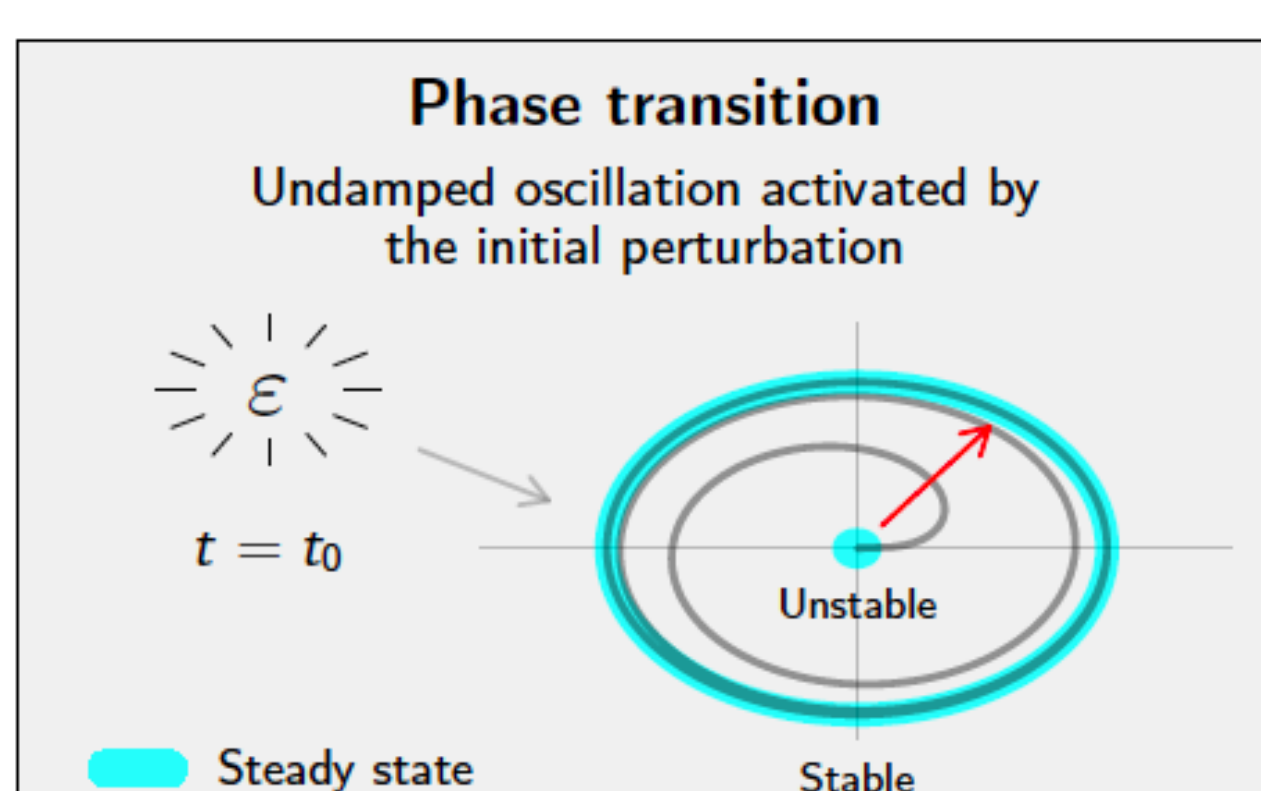
## 3. Stop-and-Go: Stochastic Models

- The stochastic microscopic motion model is

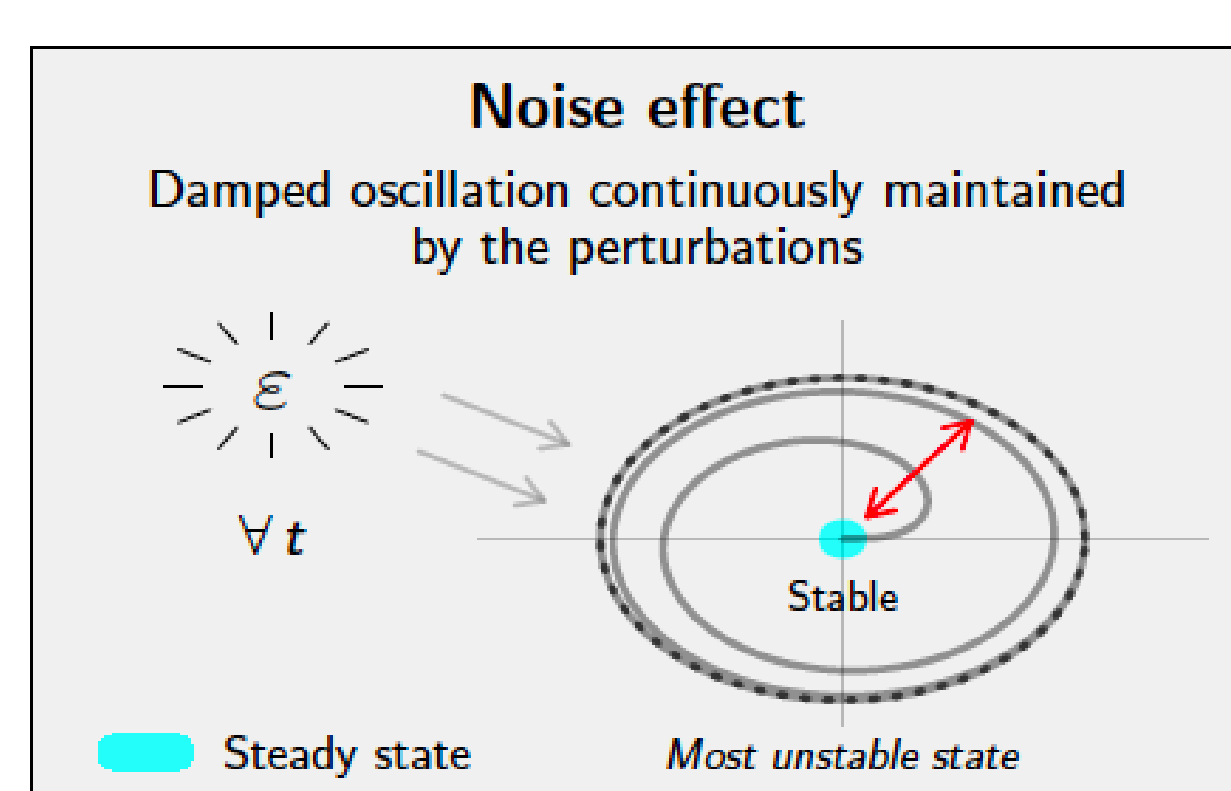
$$\begin{cases} dx_i = [\lambda(\Delta x_i - \ell) + z_i]dt, \\ dz_i = [\gamma(\Delta x_i - \Delta x_{i+1}) - \beta z_i]dt + \sigma dW_i, \end{cases} \quad (1)$$

where the  $W_i(t)$  are independent Wiener processes

- Purely linear, stochastic differential system: Minimal deterministic part and stochastic Gaussian noise
- No instability, phase transition, or non-linearity needed for phase separation  $\rightarrow$  Noise Induced Stop-and-Go dynamics!



(a)



(b)

Figure 2: Illustrative scheme for the modeling of stop-and-go dynamics with phase transition in the periodic solution (a) and the noise-induced oscillating behavior (b).

## 4. Results

- The sufficient stability condition is

$$-\beta\lambda < 2\gamma < \beta\lambda \quad (2)$$

- change dynamics from persisting waves ( $\gamma > 0$ ; reaction rate), to dissipating waves ( $\gamma < 0$ ; anticipation rate)
- Under stability exactly solvable: the model is a Gaussian process with a single stationary distribution
- Expectation tends to space-homogeneous solution  $\mathbb{E}[\mathcal{X}(t)] \rightarrow 0$  as  $t \rightarrow \infty$
- Stop-and-Go waves as non-trivial fluctuations in co-variance matrix  $\Sigma(\infty)$
- Linear stability can not ensure the absence of jams!

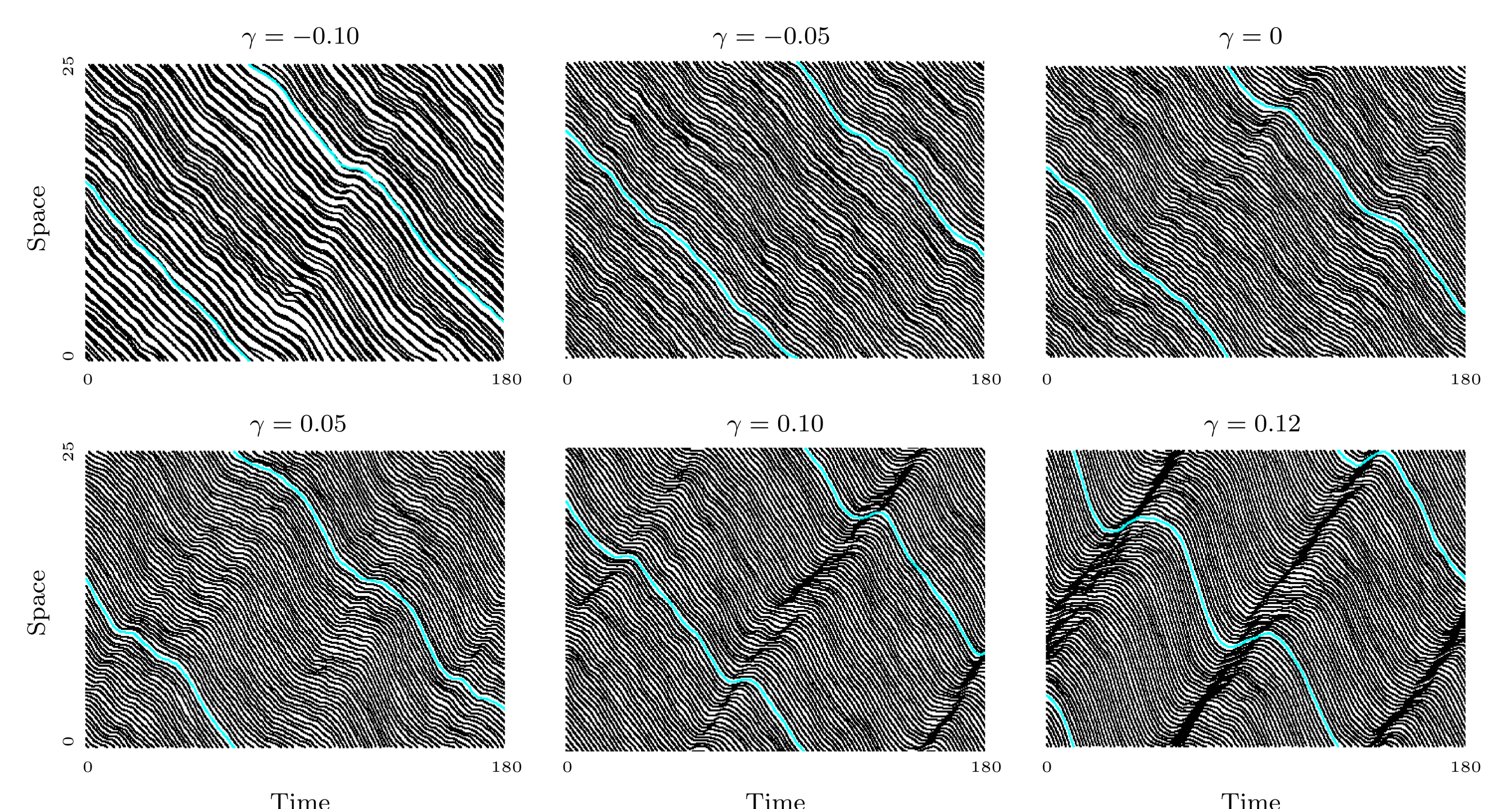


Figure 3: Examples of trajectories of 50 particles in stationary states for  $\lambda = 1$ ,  $\beta = 0.2$ , and different values of  $\gamma$ . The dynamics range from 2 by 2 coupled configurations as  $\gamma$  tends to  $\gamma_c^1$ , homogeneous configurations for moderated negative values for  $\gamma$ , to configurations with a single stop-and-go wave as  $\gamma$  tends to  $\gamma_c^2$ . Note that the trajectories with no noise ( $\sigma = 0$ ) are systematically strictly homogeneous (i.e. parallel and equidistant).

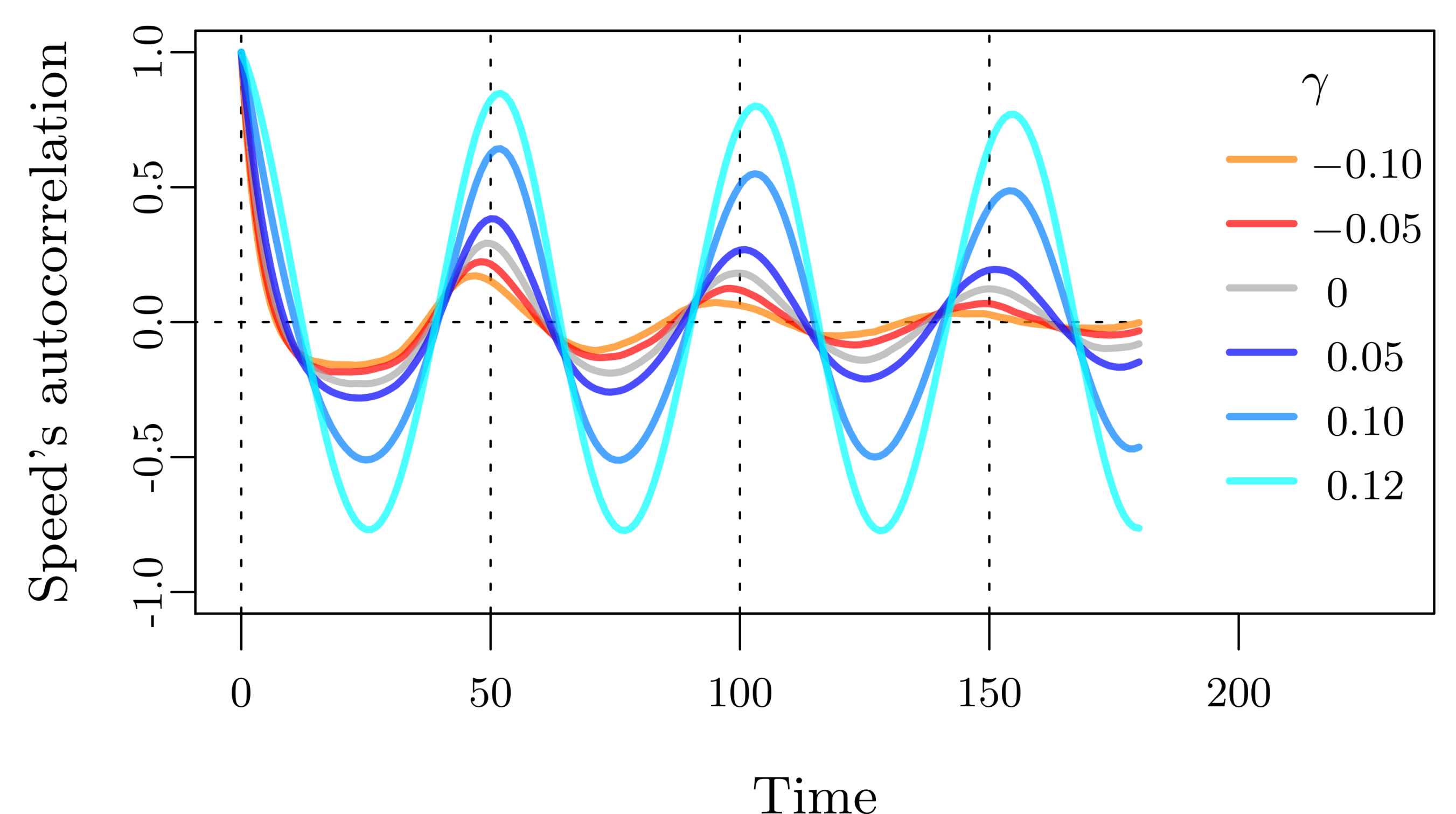


Figure 4: Auto-correlation in time of a single vehicle in stationary state for different stable values for the parameter  $\gamma = -0.1, 0.05, 0, 0.05, 0.1, 0.12$  (the stability condition being  $\gamma_c^1 = -0.1 < \gamma < \gamma_c^2 \approx 0.1285$ ). The autocorrelation functions oscillate, especially as  $\gamma$  tends to  $\gamma_c^2$ . This attests for the presence of periodic stop-and-go dynamics.

## References

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