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Resilience in Spectral Deferred Corrections

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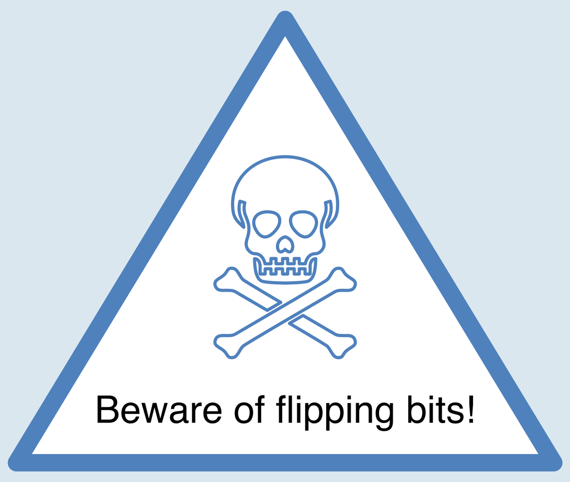
Resilience



Radiation can trigger current in transistor:

Main cause of faults: Bits can be flipped by solar or cosmic radiation!
Many other sources of faults: ageing hardware, voltage flashovers, …

Murphy's Law:
$P_{\text{failiure}} \propto N_{\text{potential failiure}}$
→ More parallelism ⟹ less reliability!

Error correction codes protect main memory, but caches remain exposed
In 2007 on largest machine at the time: **Uncorrectable faults in L1 cache about every 8h** [1]

Conventional generic resilience (checkpointing, replication) will not work on exascale machines!
Solution: Non-generic but more efficient algorithm-based fault-tolerance

Spectral Deferred Corrections (SDC)

Initial value problem in Picard form:
$$u(t) = u(t_0) + \int_{t_0}^{t} f(u(\tau))\, d\tau$$

Discretize with spectral quadrature:
$$\mathbf{u} = \mathbf{u}_0 + \Delta t Q F(\mathbf{u})$$

Preconditioned iteration with simpler (lower triangular) quadrature rule:
$$\left(I - \Delta t Q_\Delta F\right)\left(\mathbf{u}^{k+1}\right) = \mathbf{u}_0 + \Delta t\left(Q - Q_\Delta\right) F\left(\mathbf{u}^k\right)$$

Popular preconditioner LU-decomposition for stiff problems [2]:
$Q_\Delta = U^{\mathrm{T}}$ with $LU = Q^{\mathrm{T}}$

- Order can be equal to iteration count, depending on preconditioner
- Parallel-in-time extensions easy due to iterative nature
- Very malleable by choice of preconditioner(s)
- Iterating until a residual tolerance is reached is already a valid resilience strategy, but we can do even better!

Adaptivity

Estimate local error and adapt time step size to match preset tolerance as closely as possible

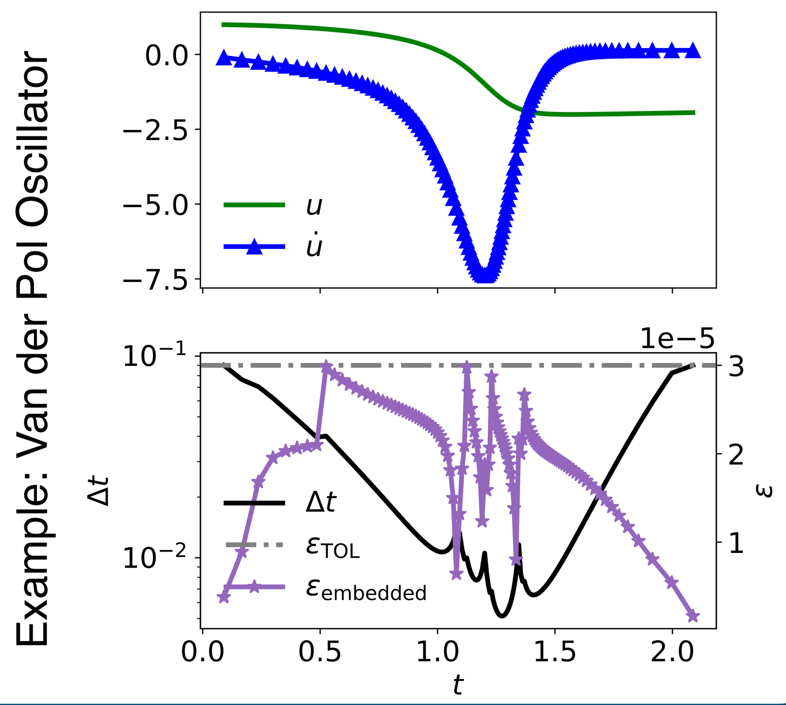
Scheme of order k at step n:
$$\frac{e_n'}{e_n} = \left(\frac{\Delta t_n'}{\Delta t_n}\right)^{k+1}$$

Now (falsely) assume that also
$$\frac{e_{n+1}}{e_n} = \left(\frac{\Delta t_{n+1}}{\Delta t_n}\right)^{k+1}$$

Replace: $e_{n+1} = \epsilon_{\mathrm{TOL}}$ and
estimate e_n with embedded method:
$\epsilon = \|u^{(k-1)} - u^{(k)}\|$
(superscript is iteration number)

Adaptivity step size update:
$$\Delta t_{n+1} = \beta \Delta t_n \left(\frac{\epsilon_{\mathrm{TOL}}}{\epsilon}\right)^{1/k}, \beta = 0.9$$

Fix faulty assumptions by only moving on to next step if $\epsilon \leq \epsilon_{\mathrm{TOL}}$.
These restarts give increased resilience!

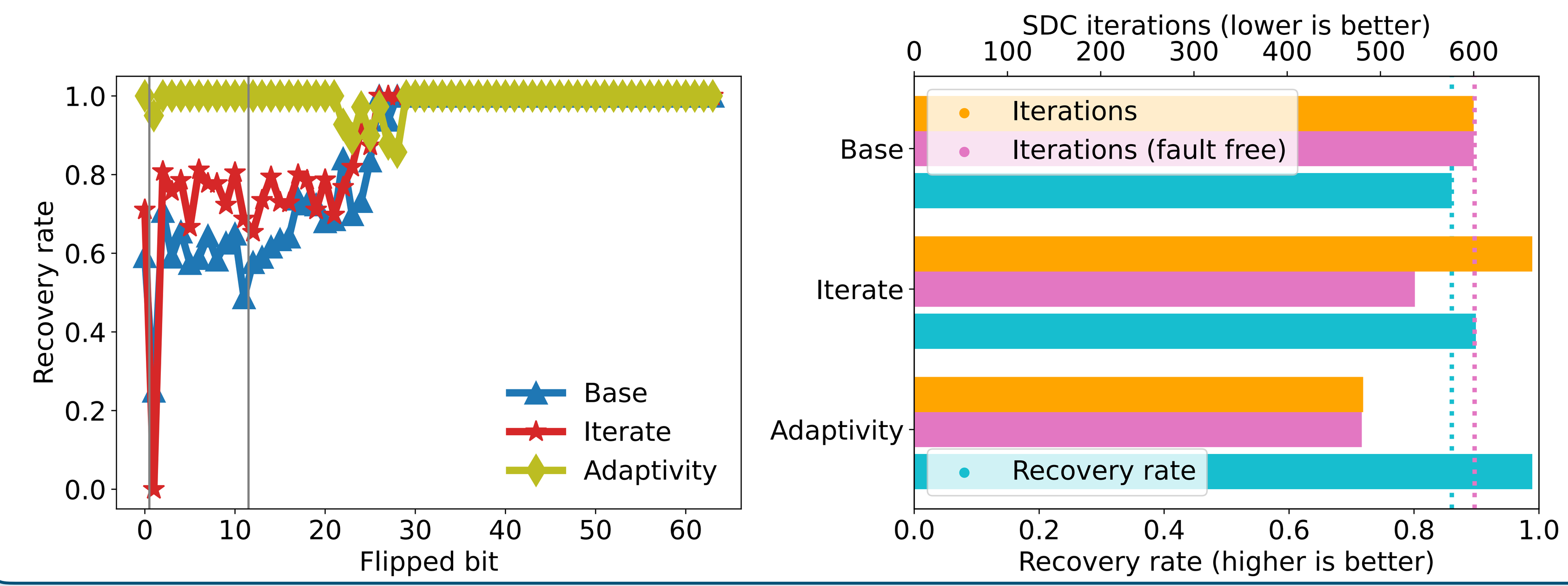


Resilience Statistics

Experiment:
- Flip random bit in the solution while solving Van der Pol problem
- Accept as recovered when final error is close to that of a fault free run

Schemes:
- Base: Fixed iteration count, fixed step size
- Iterate: Iterate until reaching residual tolerance, fixed step size
- Adaptivity: Fixed iteration count, continually adjust step size

Winner: Adaptivity: provides high resilience, while also boosting efficiency!

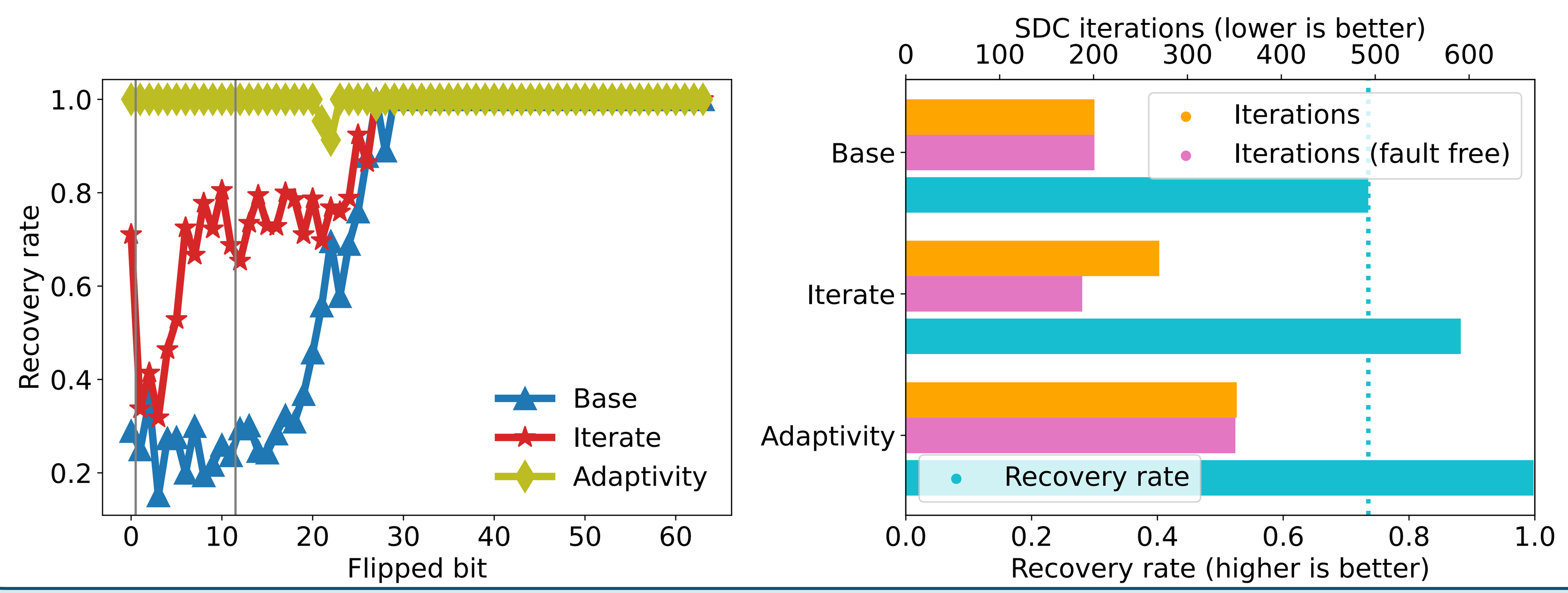


Diagonal Preconditioners

Diagonal preconditioners → Update quadrature nodes simultaneously in SDC iteration
⟹ **simple-to-implement small-scale time-parallelism**
Beware: Convergence properties with different preconditioners highly problem dependent! [3]

Experiment: van der Pol with diagonal preconditioner "MIN" from [3]:
Fewer iterations needed for convergence to same accuracy and parallelizable!

Same conclusions on resilience as with LU preconditioner:



Summary

Please keep faults in mind when developing code!
Do not despair: Iterative nature of SDC lends itself well to do unobtrusive resilience.
Non-generic and non-bulky resilience strategies can be implemented in your code as well!

Adaptivity is very promising!
+ Can boost resilience and efficiency at the same time, while being simple to implement
- Unclear how to extend this to parallel-in-time multi-step SDC, but various versions possible

Diagonal preconditioners are worth a try if you do SDC!
+ SDC iteration becomes embarrassingly parallel across the collocation nodes
- Convergence is highly problem dependent and not always properly mathematically founded

References

[1]: J. N. Glosli, D. F. Richards, K. J. Caspersen, R. E. Rudd, J. A. Gunnels and F. H. Streitz, "Extending stability beyond CPU millennium: a micron-scale atomistic simulation of Kelvin-Helmholtz instability," SC '07: Proceedings of the 2007 ACM/IEEE Conference on Supercomputing, 2007, pp. 1-11, doi: 10.1145/1362622.1362700.

[2]: Weiser, M. Faster SDC convergence on non-equidistant grids by DIRK sweeps. Bit Numer Math 55, 1219–1241 (2015). https://doi.org/10.1007/s10543-014-0540-y

[3]: Speck, R. Parallelizing spectral deferred corrections across the method. Comput. Visual Sci. 19, 75–83 (2018). https://doi.org/10.1007/s00791-018-0298-x

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