Investigating quantum annealing strategies to solve the Fermi-Hubbard model

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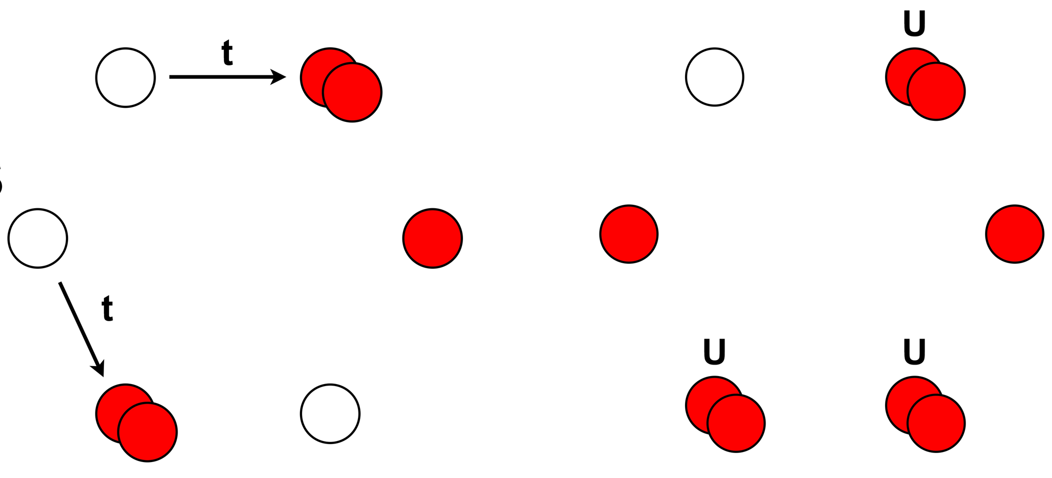
Goal

To find the ground-state energy of the Hubbard Hamiltonian for various fillings and total Spin-Z

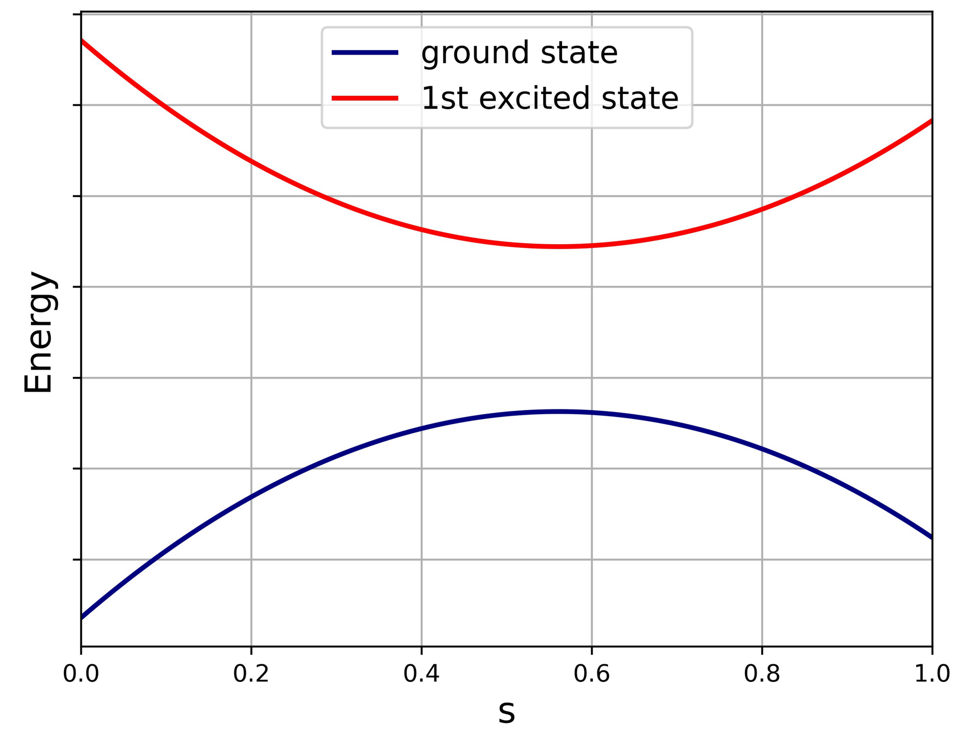
$$H_H = -t \sum_{\langle i,j\rangle,\sigma}^{L} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_{i}^{L} n_{i\downarrow} n_{i\uparrow}$$



How?

Use quantum annealing with a driving Hamiltonian (H_D)

$$H(s) = (1-s)H_D + sH_H$$



- With a good choice of the driving Hamiltonian, the aim is to find a spectrum of the evolving Hamiltonian that looks like the adjacent figure.
- An important characteristic of such a spectrum for a successful quantum annealing process is that the ground state should have gaps with the excited states at each point in the spectrum. To find a good annealing strategy means to find a good driving Hamiltonian that makes sure of this.
- We have tried 3 different driving Hamiltonians to see what the spectrum would look like for each of them as s goes from 0 to 1.

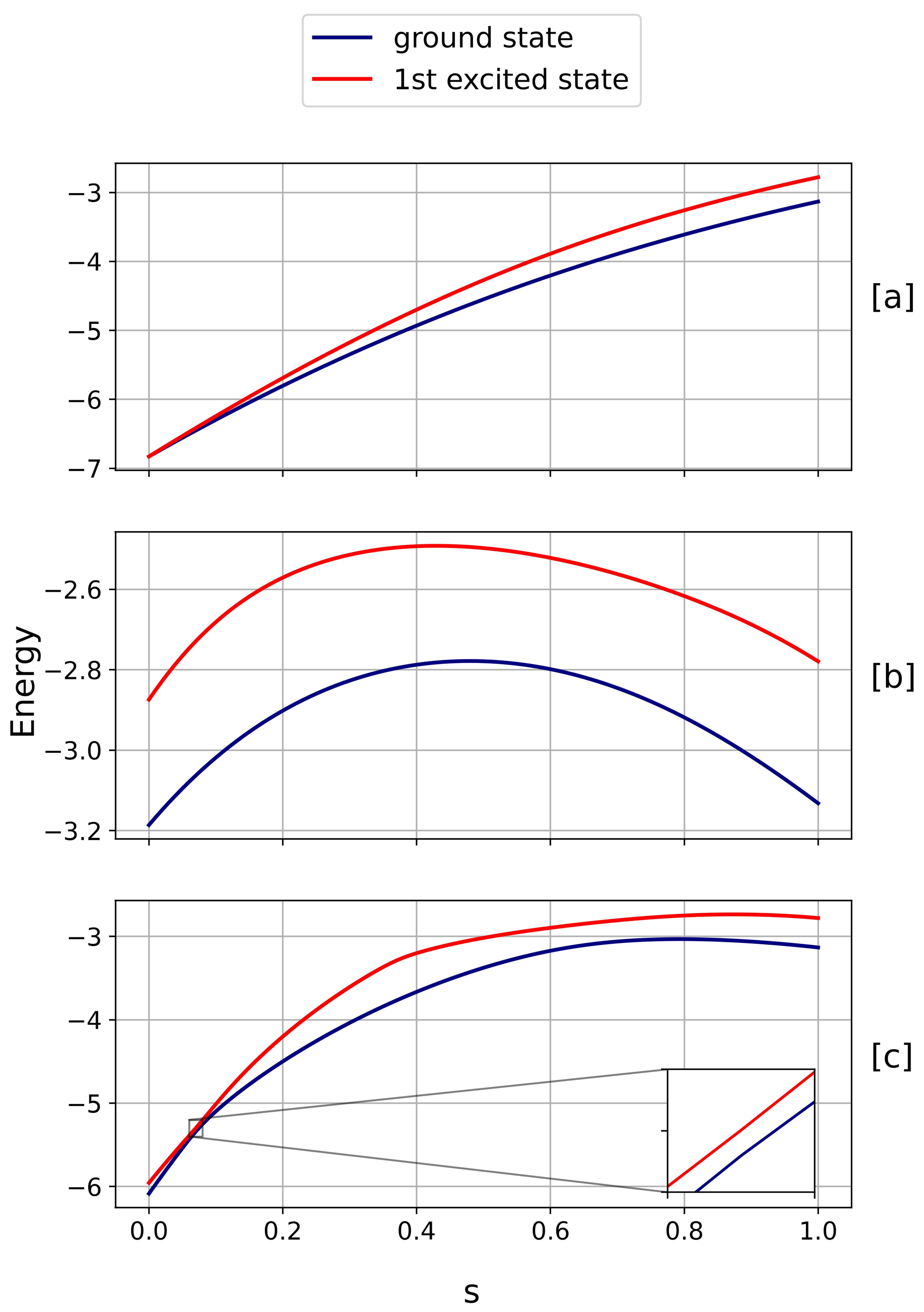
Results

$$H_D = -\sum_{i,j,\sigma}^{L} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma})$$

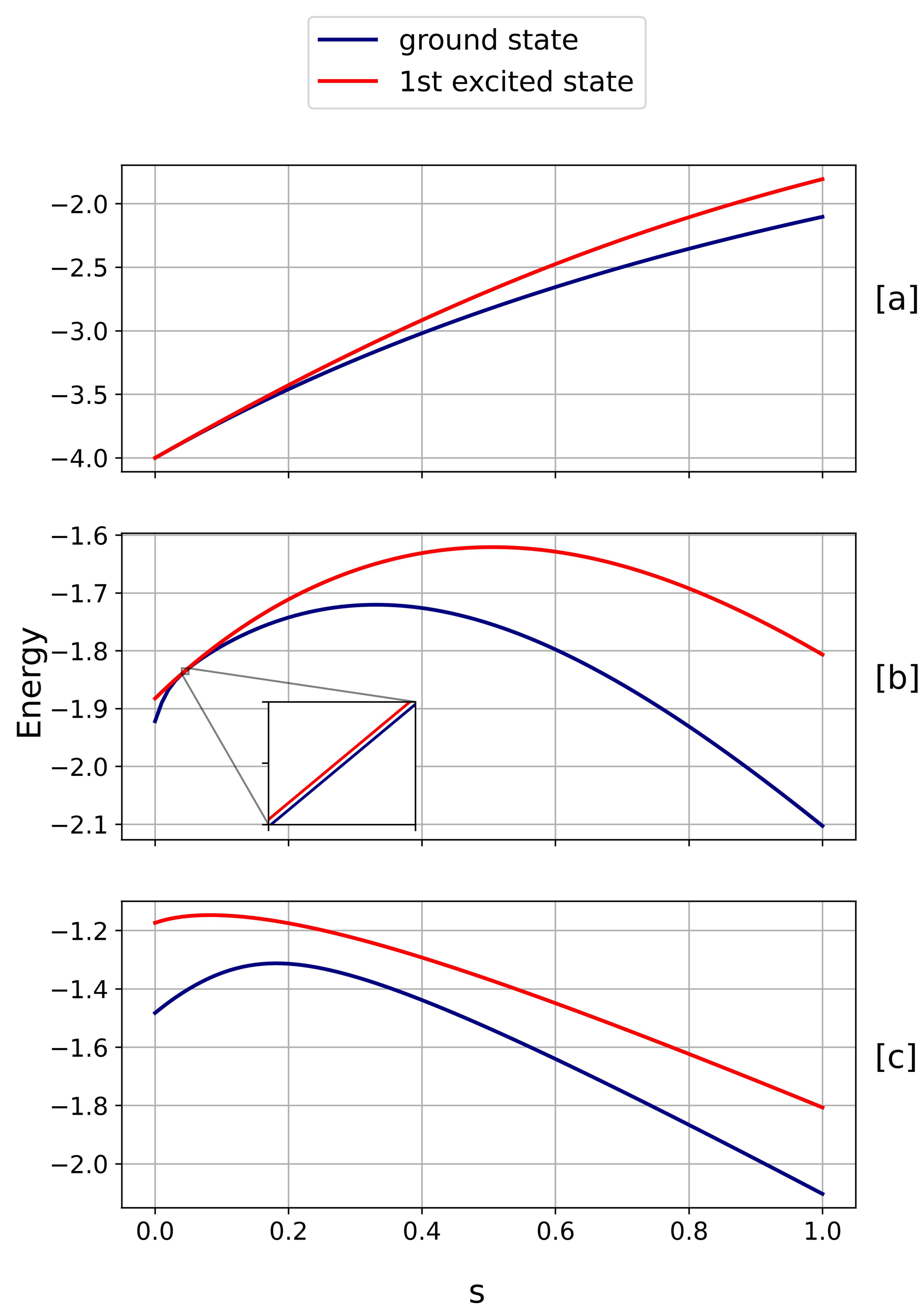
This is the kinetic energy term of the Hubbard Hamiltonian and all the driving Hamiltonians have this form with different hopping matrices

[a]	For the 1st driving Hamiltonian, the matrix includes nearest neighbor hoppings with value 1
[b]	For the 2nd case, the nearest neighbor hoppings are a random value between 0 and 1
[c]	For the 3rd driving Hamiltonian, all the hoppings are considered with their value between 0 and 1

$L = 8, \sum_i n_{i\uparrow}=6, \sum_i n_{i\downarrow}=2$



$L = 4, \sum_i n_{i\uparrow}=2, \sum_i n_{i\downarrow}=2$



The figures represent the spectrum for annealing Hamiltonians as s goes from 0 to 1

Observations

- With the 1st Hamiltonian, the spectrum has degeneracies at s=0 for the cases shown.
- With the 2nd and 3rd Hamilltonians, the gap opened up for these cases.
- The opening is more significant for the case on the left with the 2nd driving Hamiltonian, whereas it is more significant for the other case with the 3rd .

Further plans

- To inspect other driving Hamiltonians.
- Further, we plan to study the dynamics of quantum annealing by solving the Schrödinger equation.
- Finally, we want to investigate the effect of environmental temperature on the quantum annealing process. We plan to do this by coupling the annealing Hamiltonian to some bath and then study how the system evolves.

$$H = H(s) + H_B + H_I$$