DPG Frühjahrstagung

Sep 9, 2022

Quasiperiodic Nonlinear Capacitance

Dirac physics and charge localization

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Forschungszentrum Jülich

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Soon on arXiv

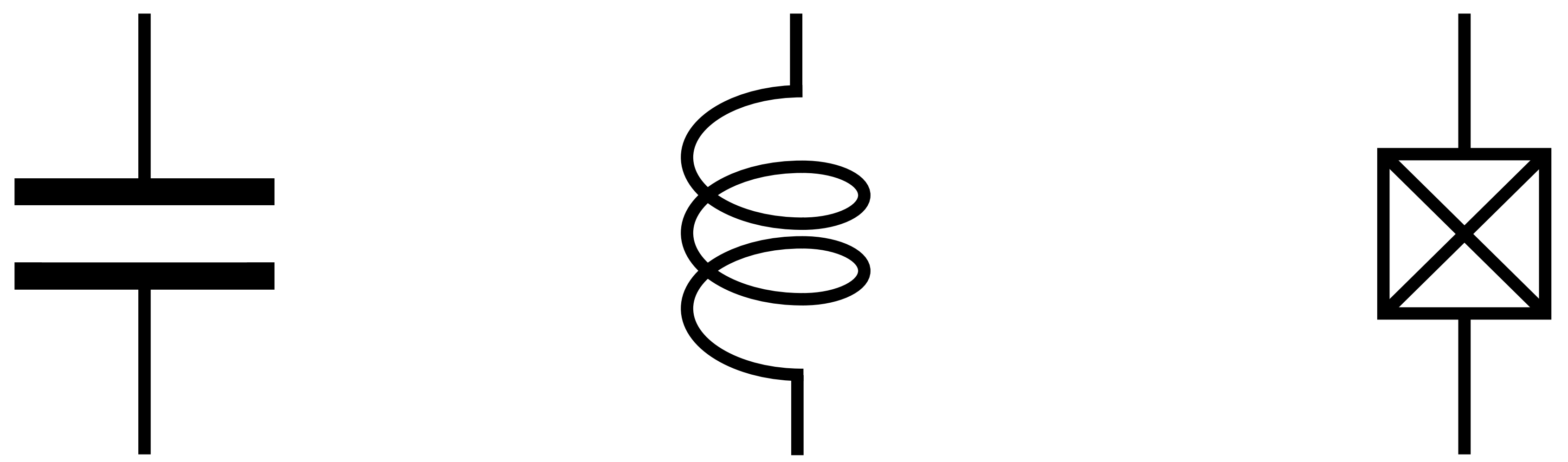
Why nonlinear capacitances?

$$\left[\widehat{\phi}, \widehat{N}\right] = i \quad \longleftrightarrow \quad [\widehat{x}, \widehat{p}] = i$$

Circuit elements $\longrightarrow$ shaping of energy profile $H(\phi, N)$

Why nonlinear capacitances?

Family of quantum circuit elements



$\sim N^2$ $\qquad$ $\sim \phi^2$ $\qquad$ $\sim \cos(\phi)$

Why nonlinear capacitances?

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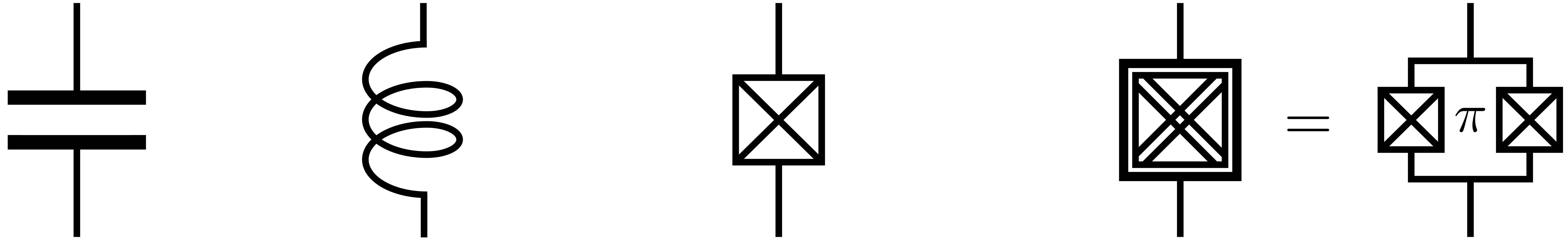
Cotunneling junction



$\sim N^2$ $\sim \phi^2$ $\sim \cos(\phi)$ $\sim \cos(2\phi)$

Smith et al.,
npj Quantum Inf (2020)

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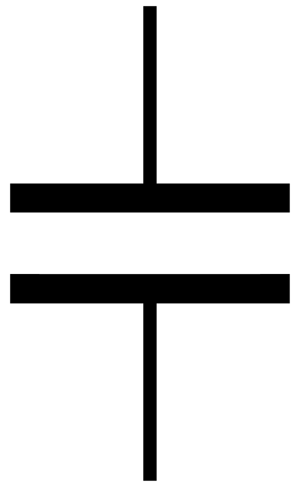
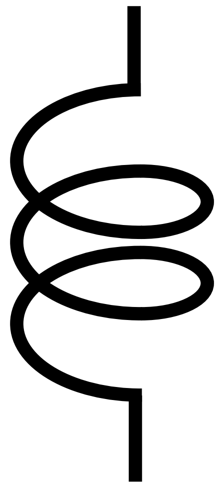


$\sim N^2$



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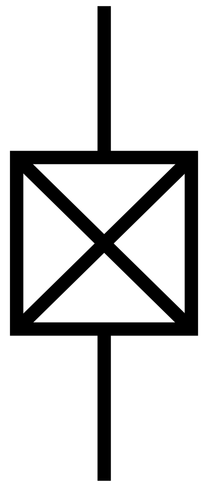

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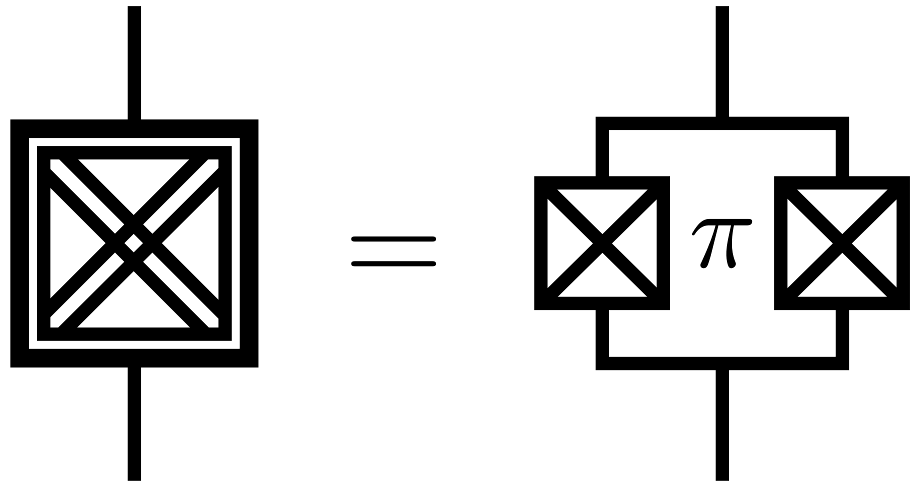

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Smith et al.,
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Majorana junction

Error 404

$\sim \cos(\phi/2)$

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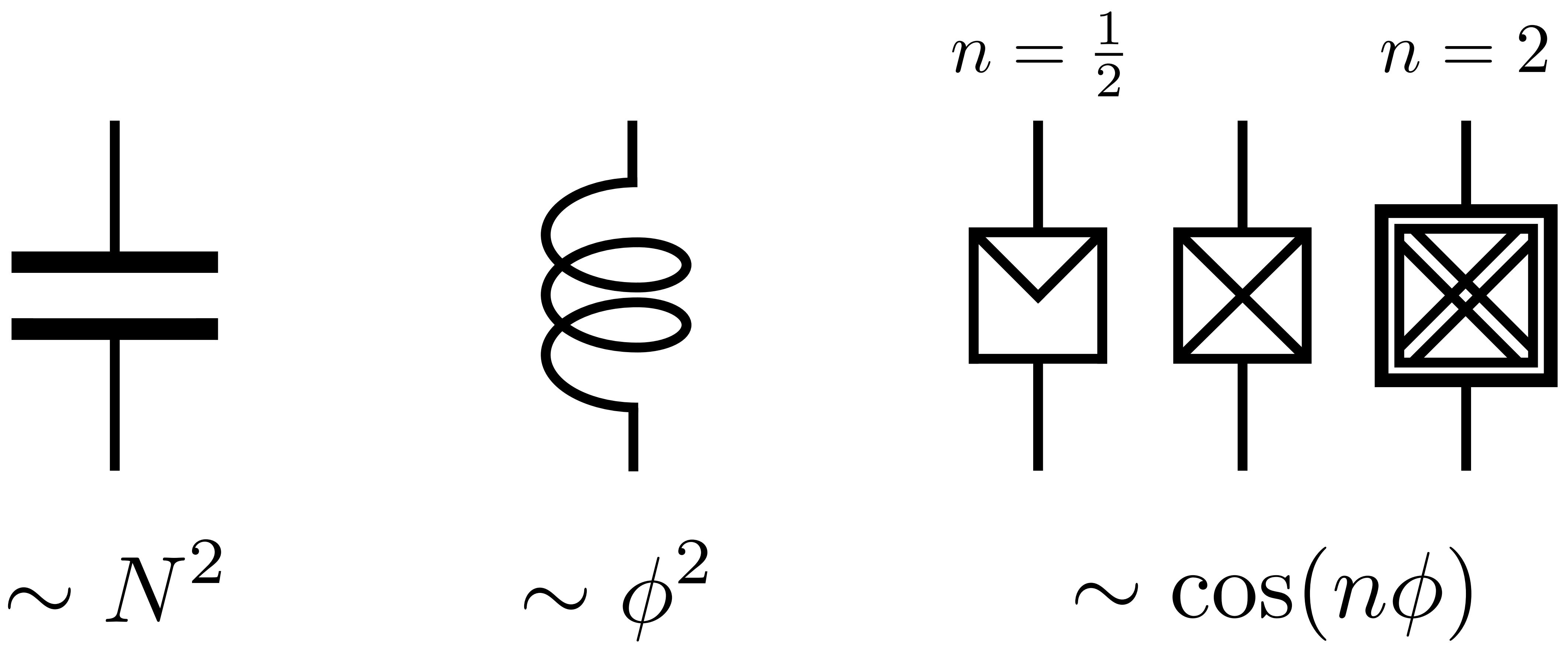

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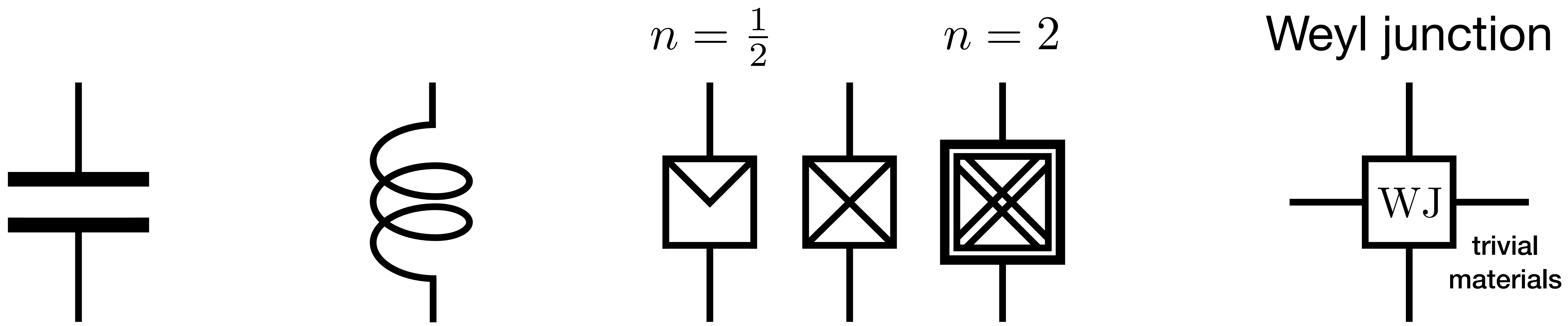

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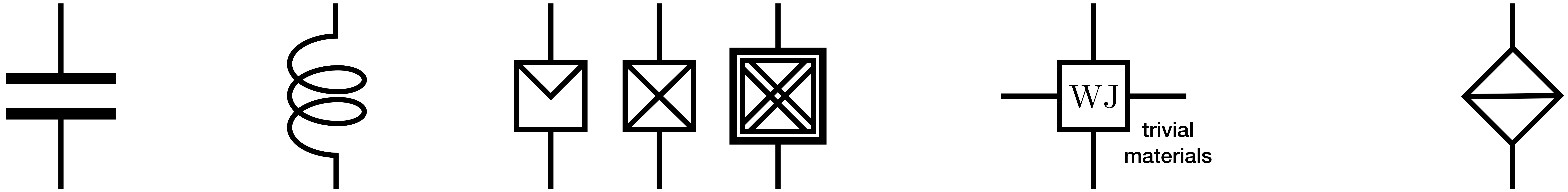
$n = \frac{1}{2}$ $n = 2$ Weyl junction Phase slip junction



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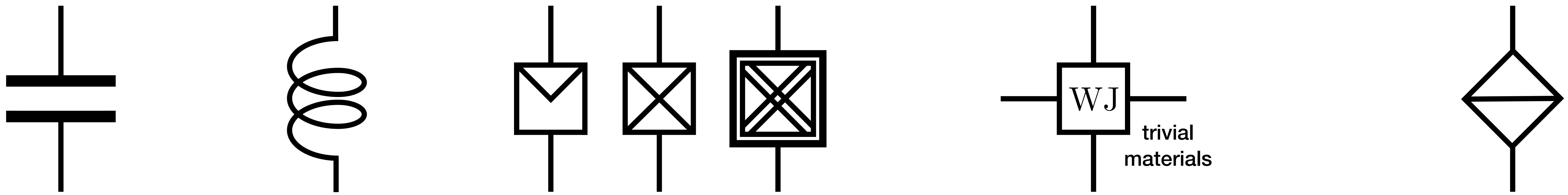

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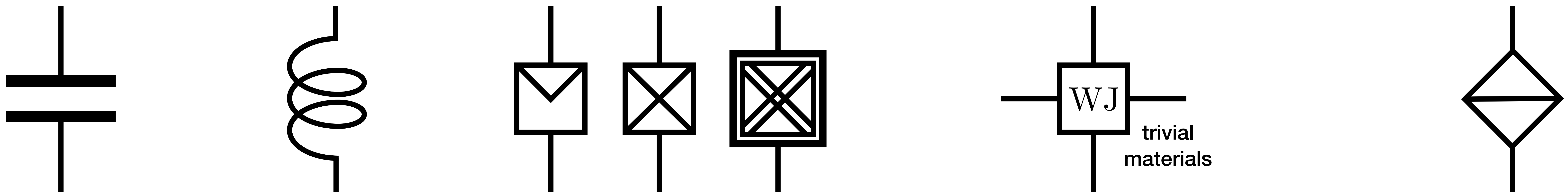

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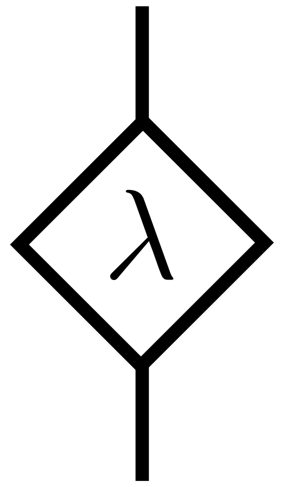
<u>New element</u> to shape energy profile in charge space N:



$$\sim \cos(2\pi\lambda N)$$

$$\lambda \in \mathbb{R}$$

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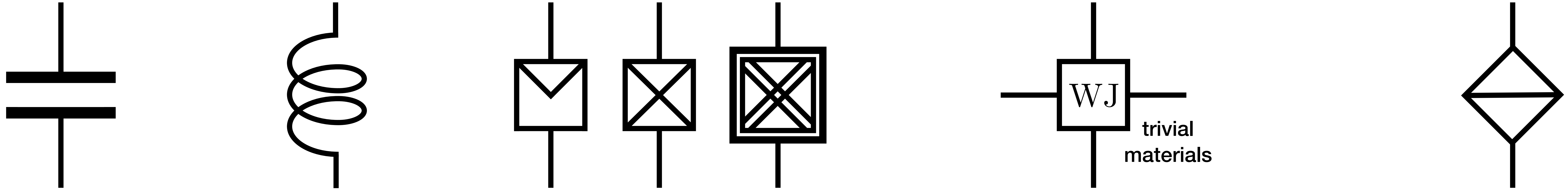

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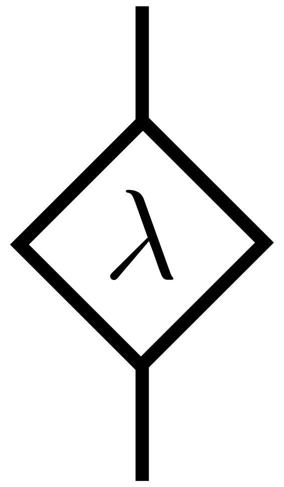

$\sim \cos(2\pi\lambda N) \longrightarrow$ Dirac physics and charge localization

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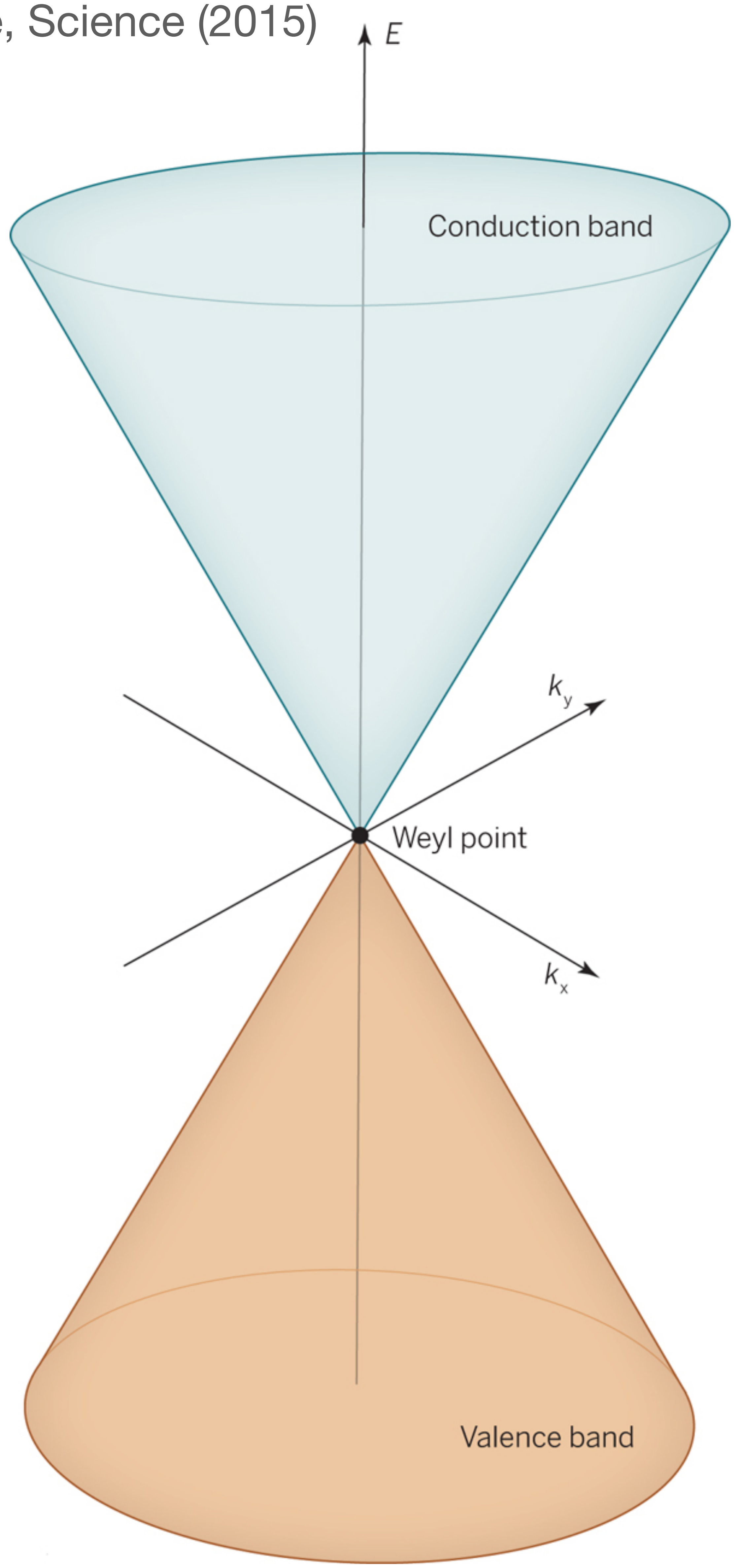
Dirac system

Stone, Science (2015)

$$\left[\widehat{x}_i, \widehat{k}_j\right] = \mathrm{i}\delta_{ij}$$



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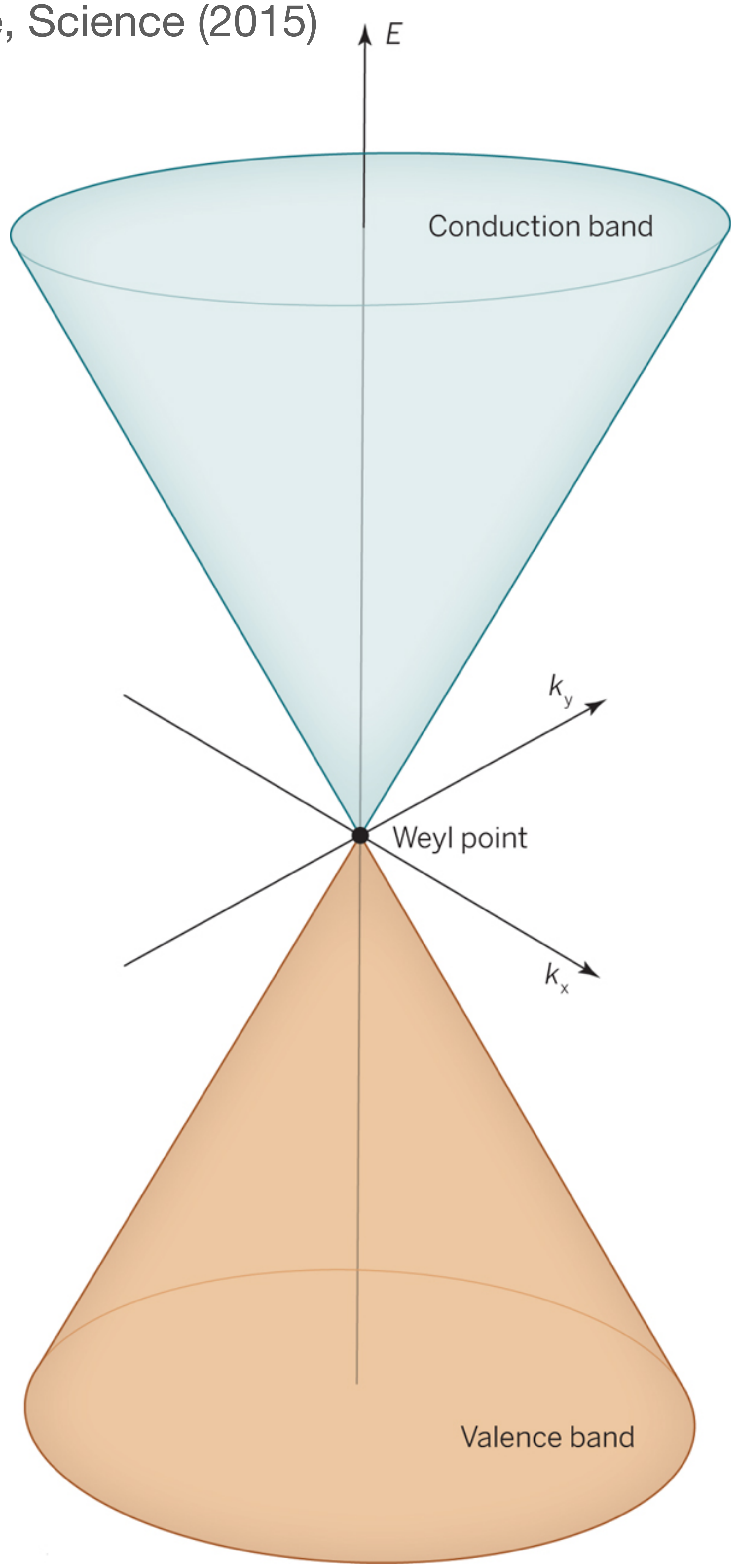

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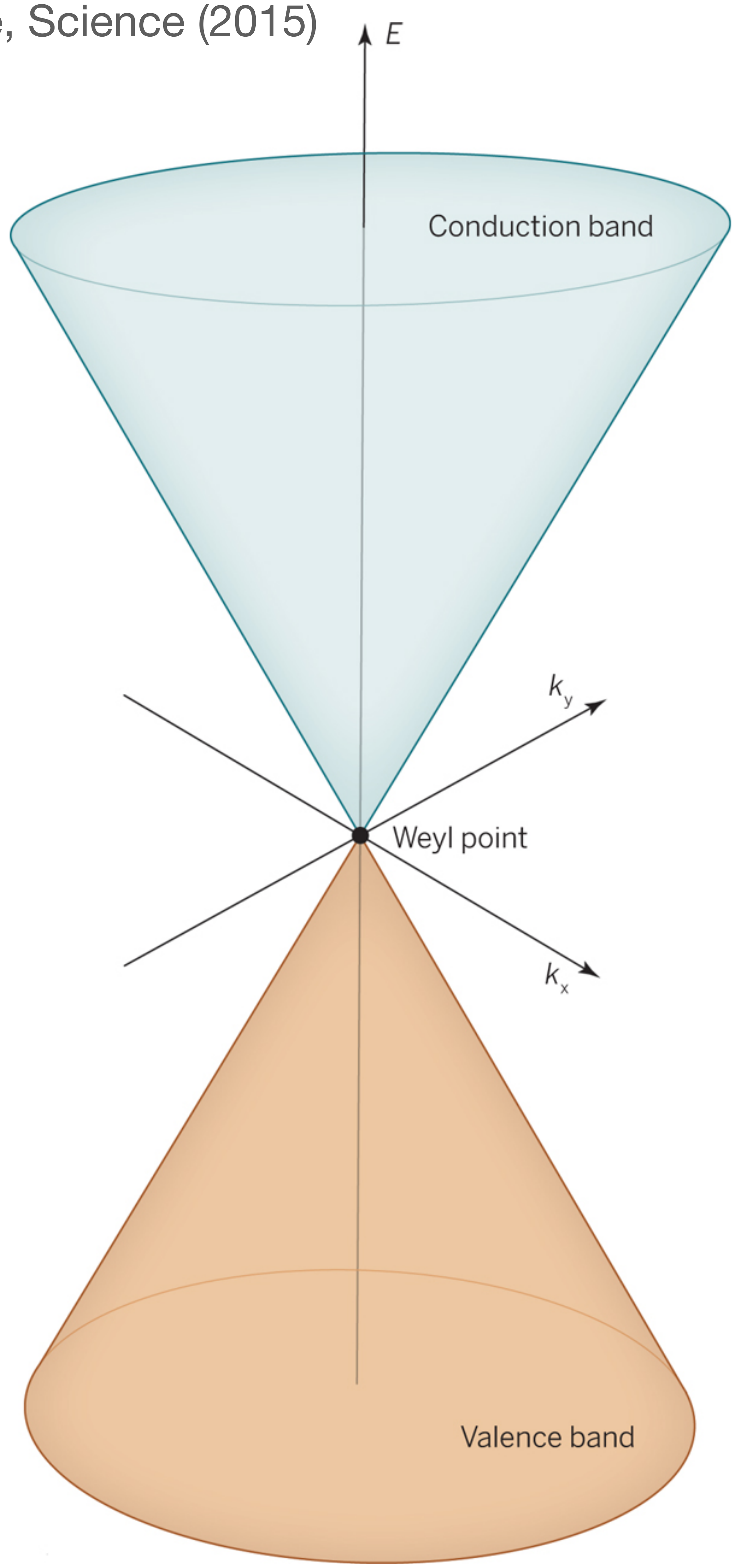

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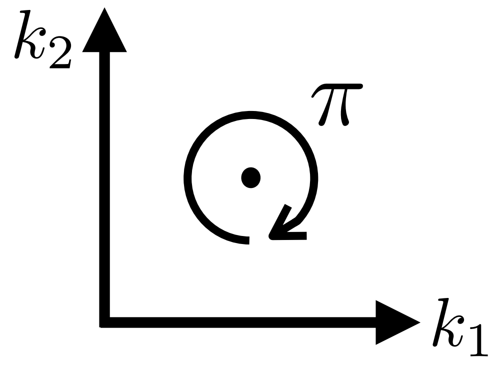

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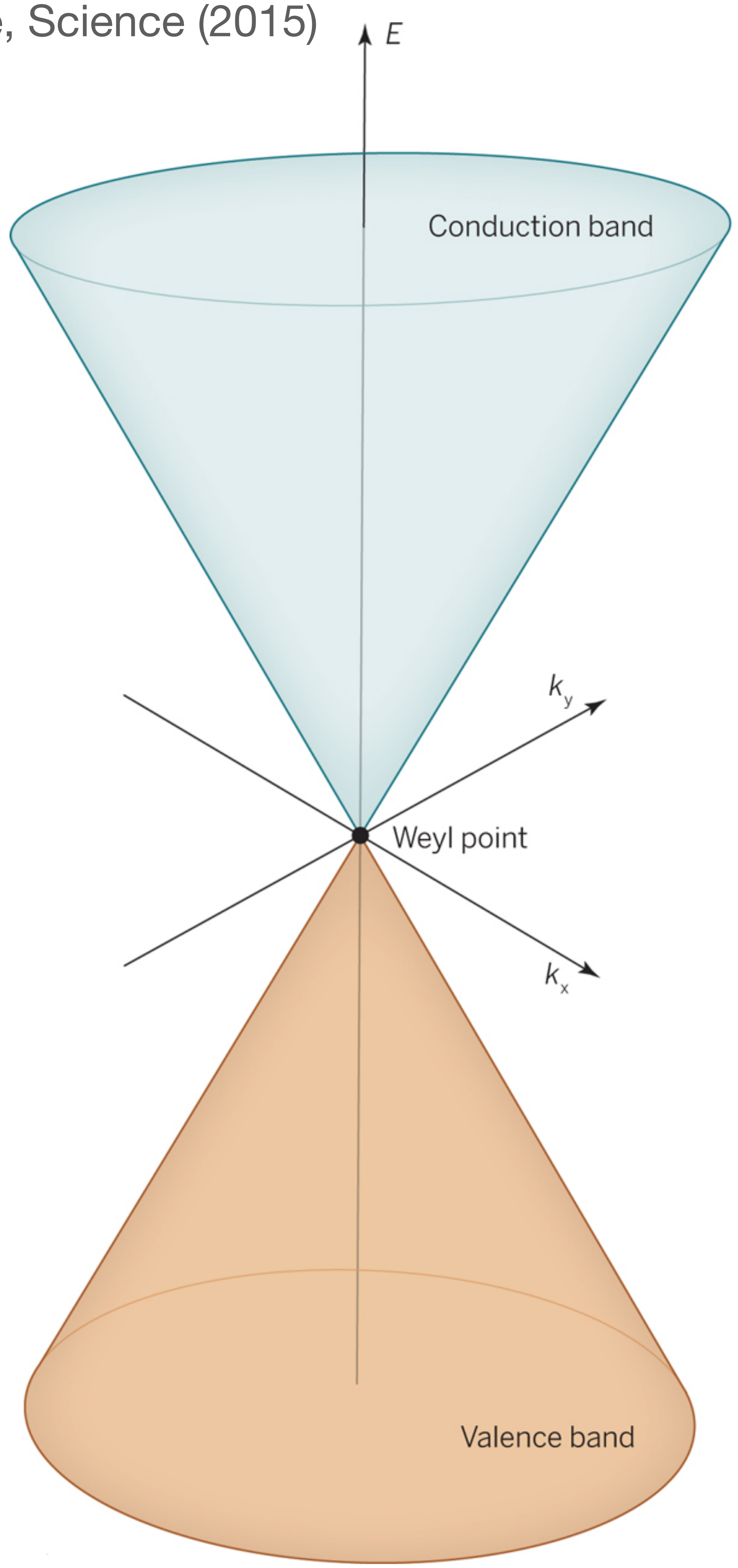

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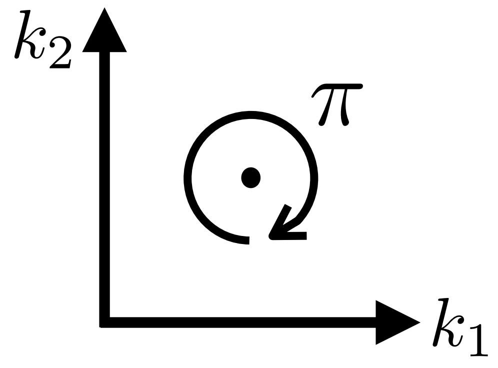

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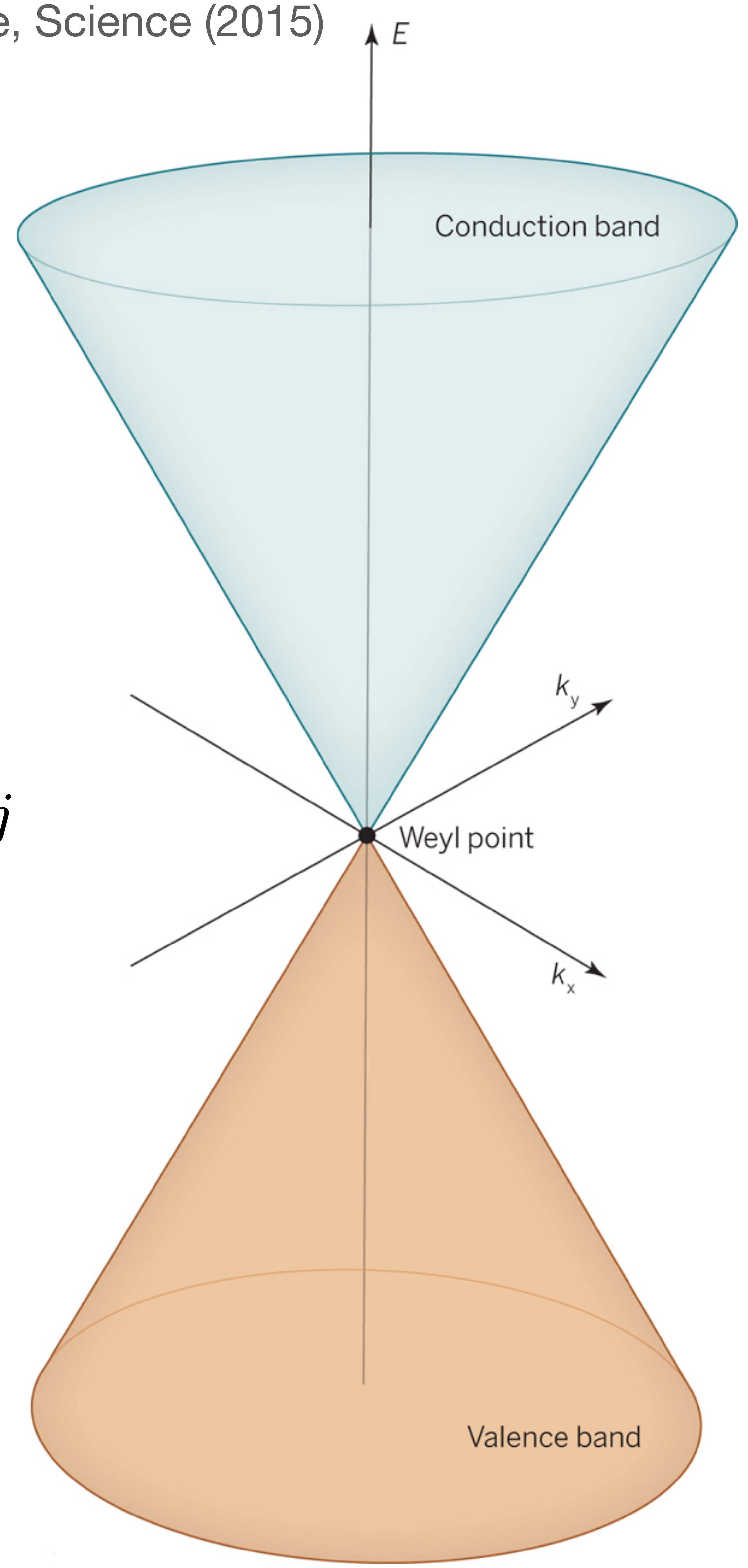

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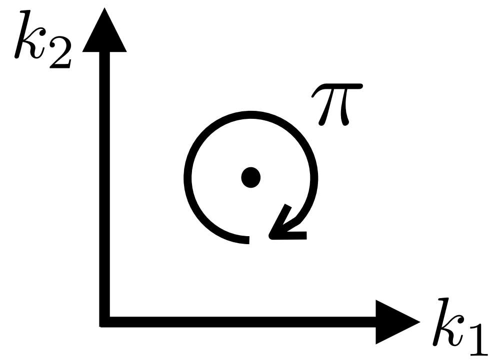

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Riwar et al., Nat. Commun. (2016)
Fatemi et al., Phys. Rev. Res. (2021)
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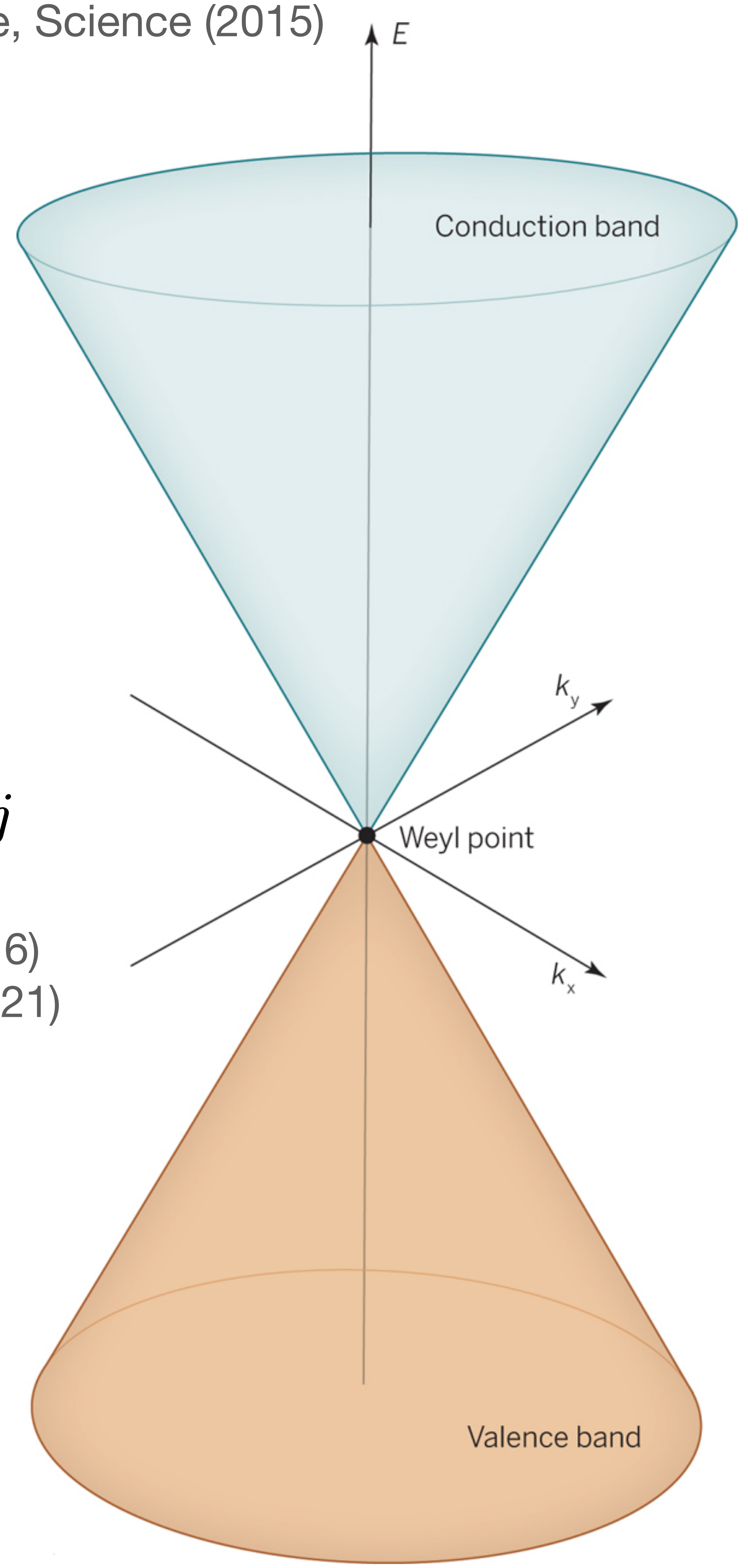

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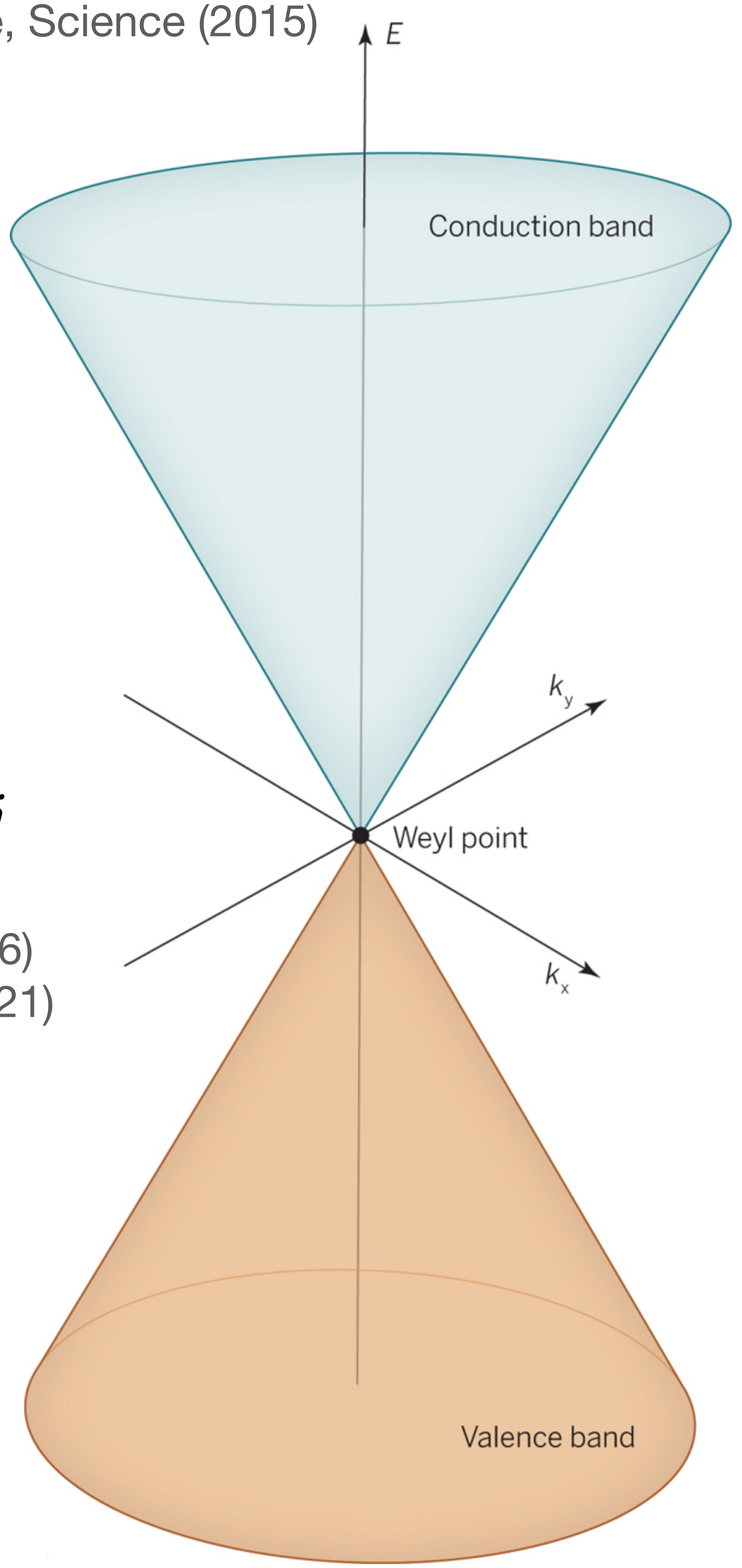

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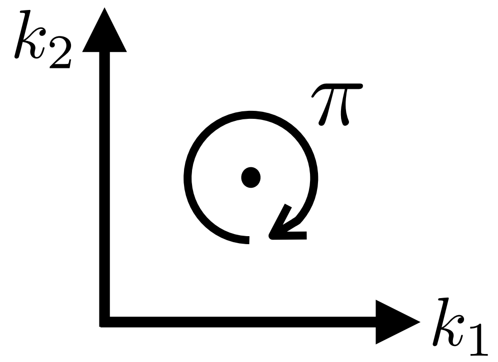

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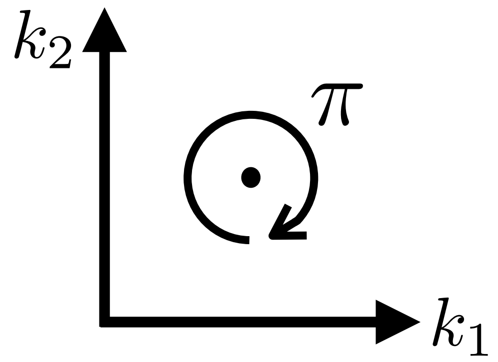

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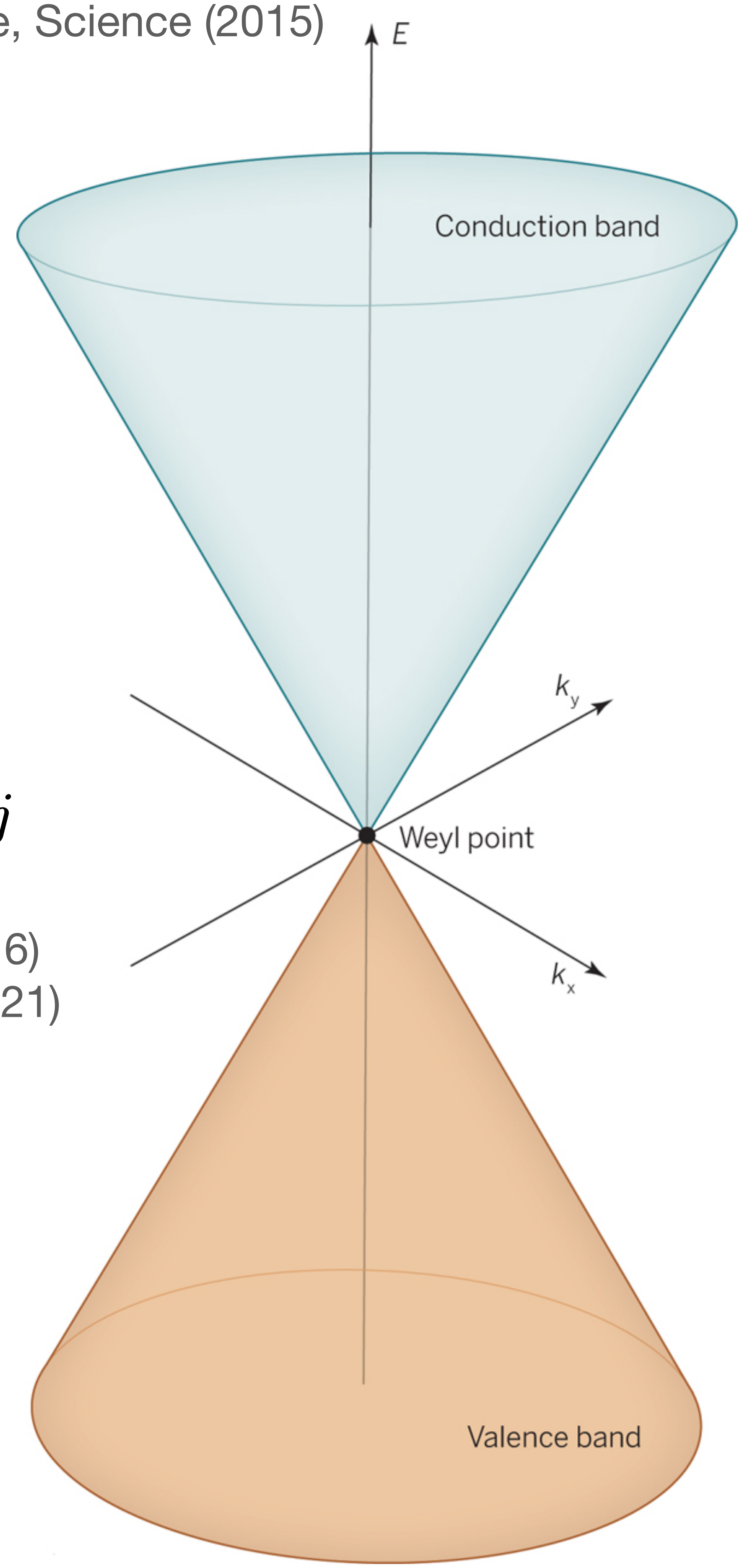
Stone, Science (2015)



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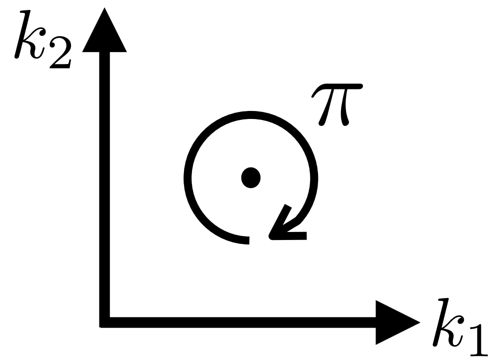

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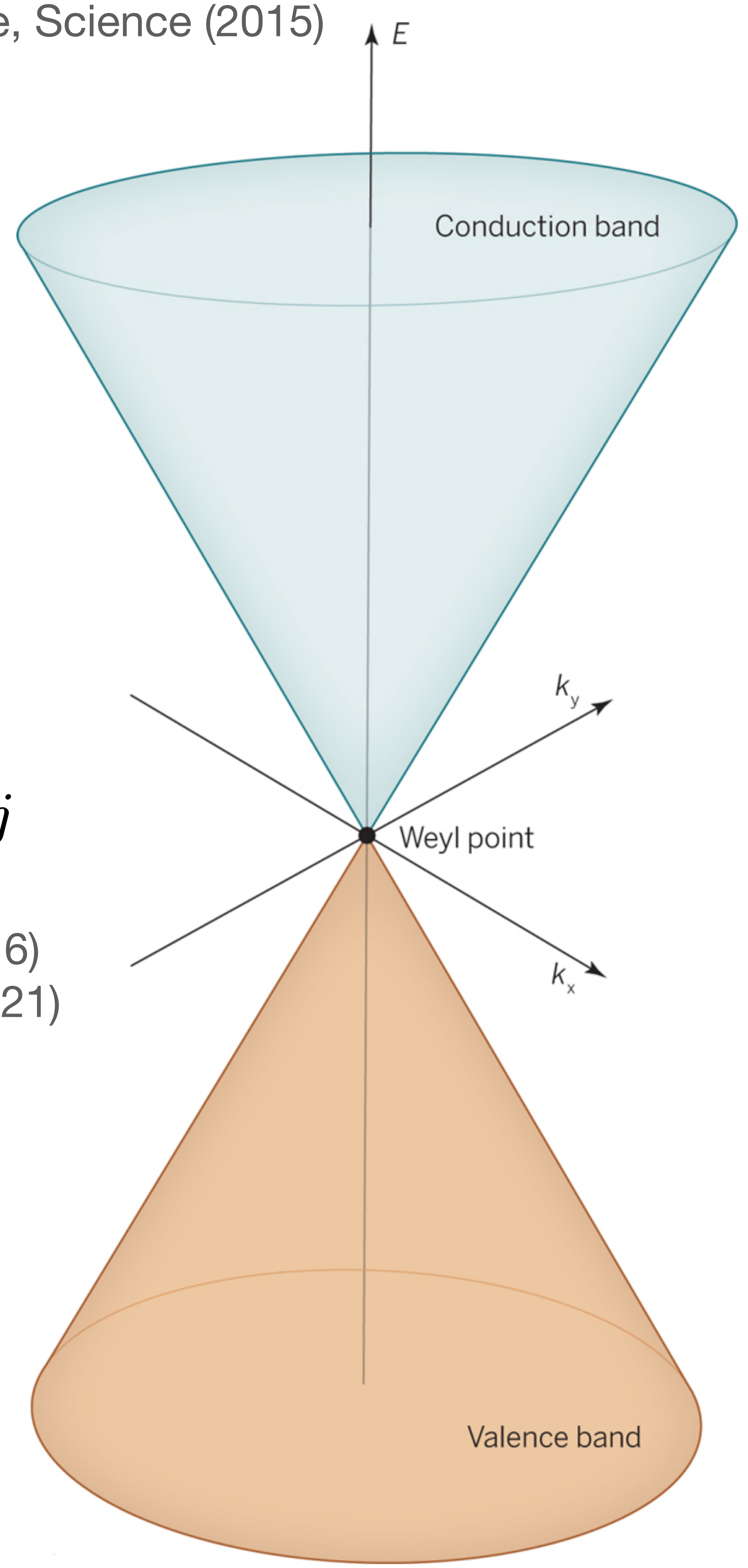
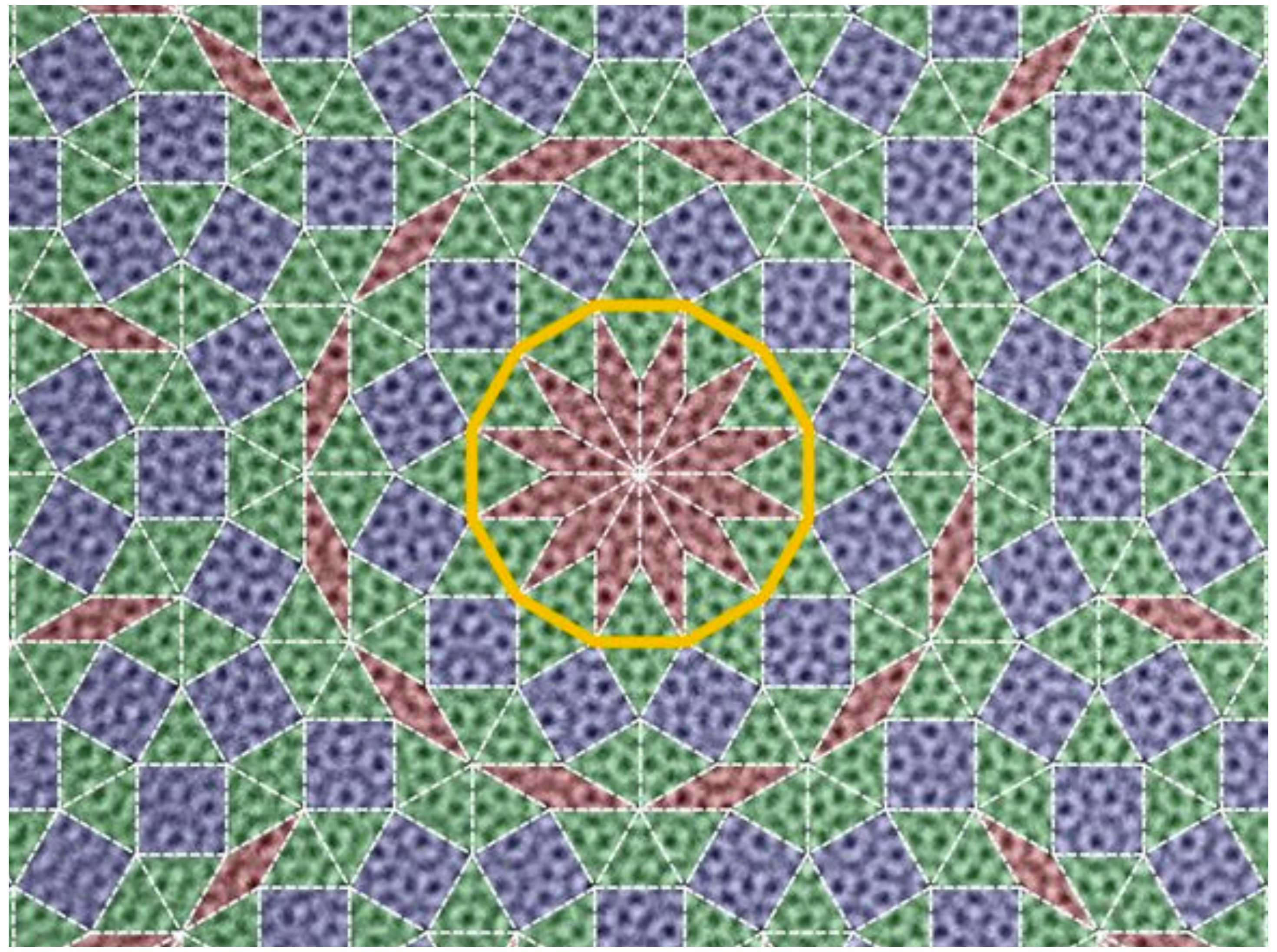


Quasiperiodicity and Anderson localization



[Ahn *et al*, Science, 2018]

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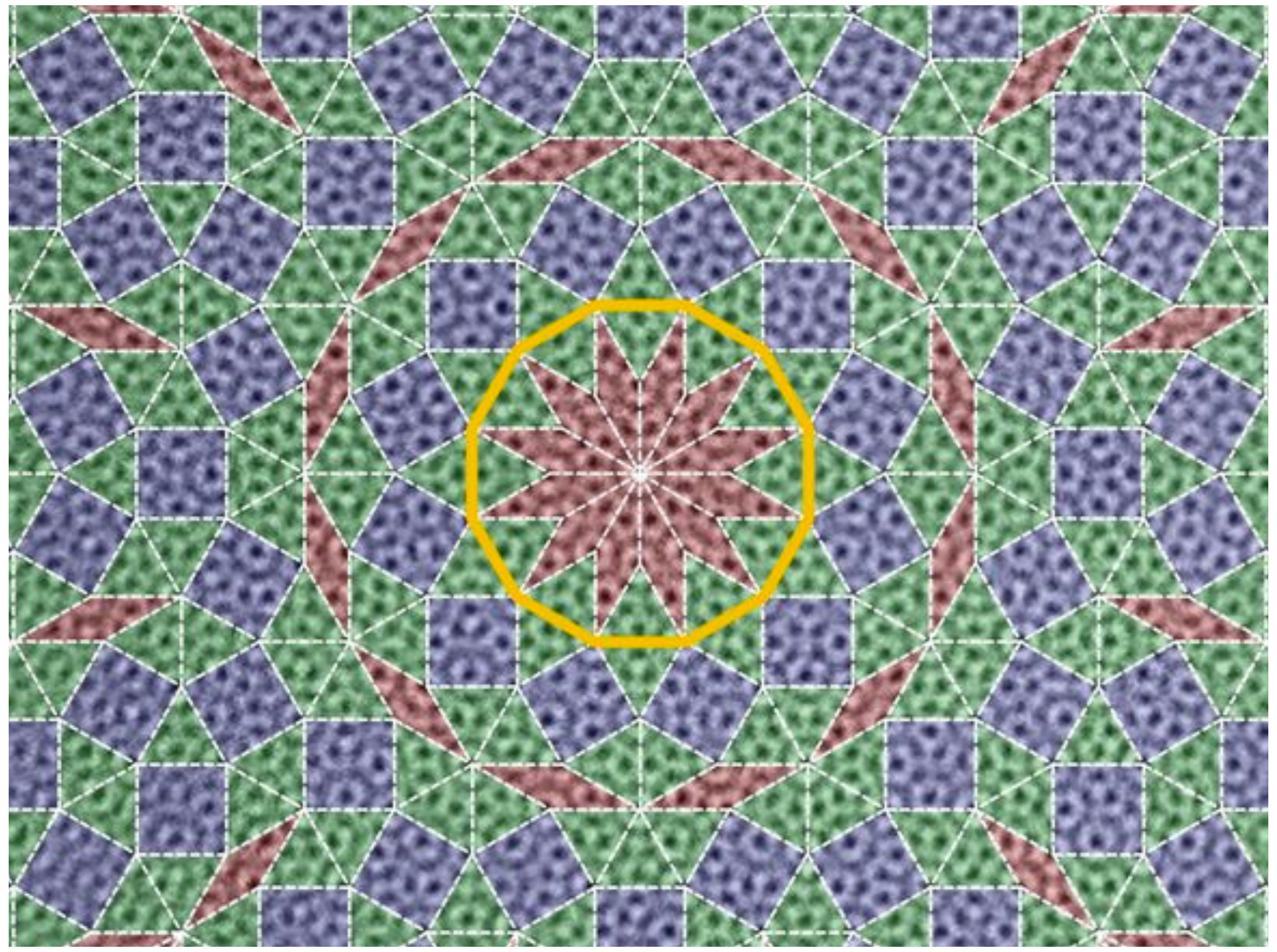

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Aubry-André model as toy model

$$\widehat{H} = W \sum_j \cos(2\pi\lambda j) |j\rangle \langle j| - t \sum_j |j\rangle \langle j-1| + \text{h.c.}$$

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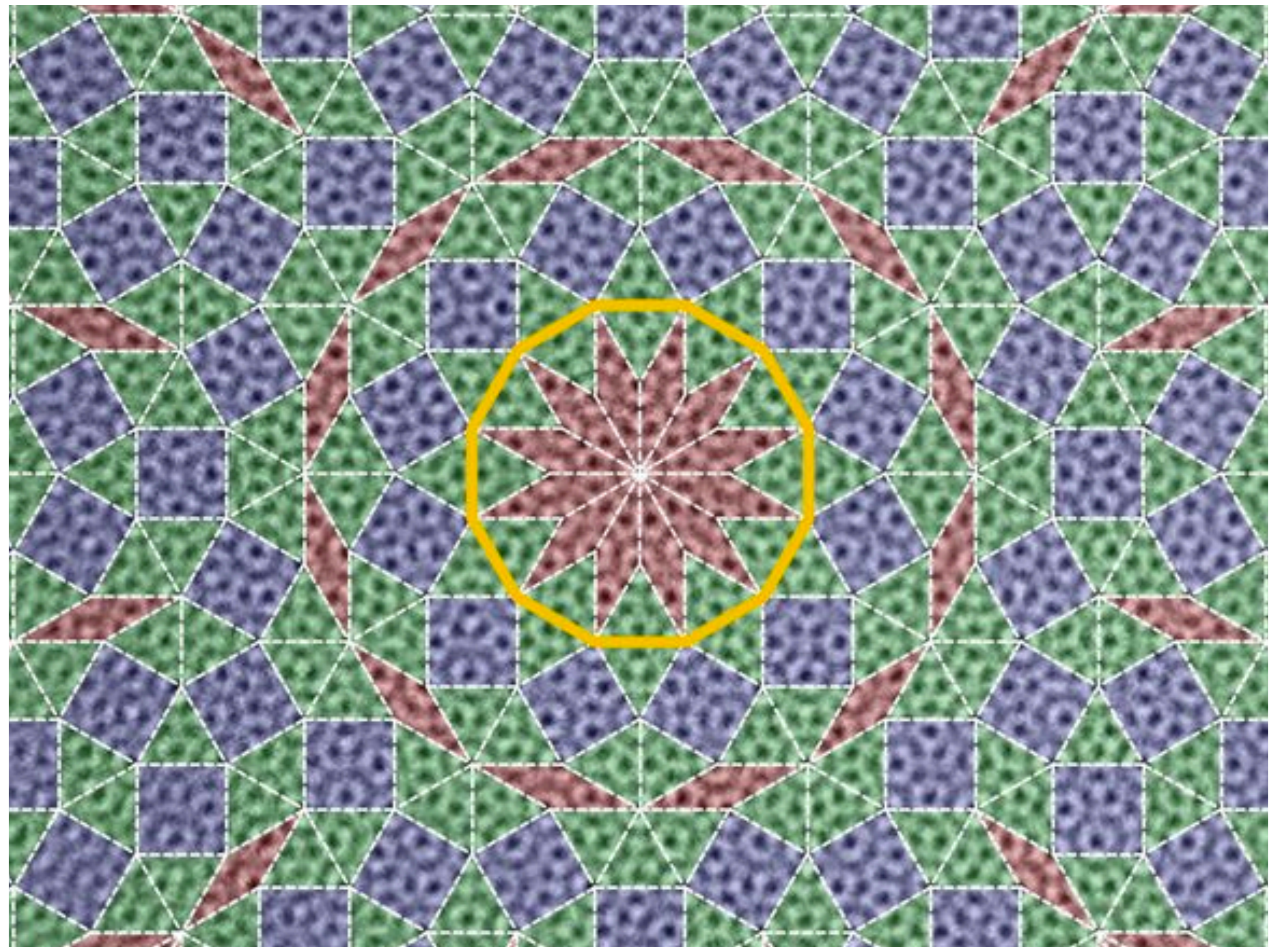

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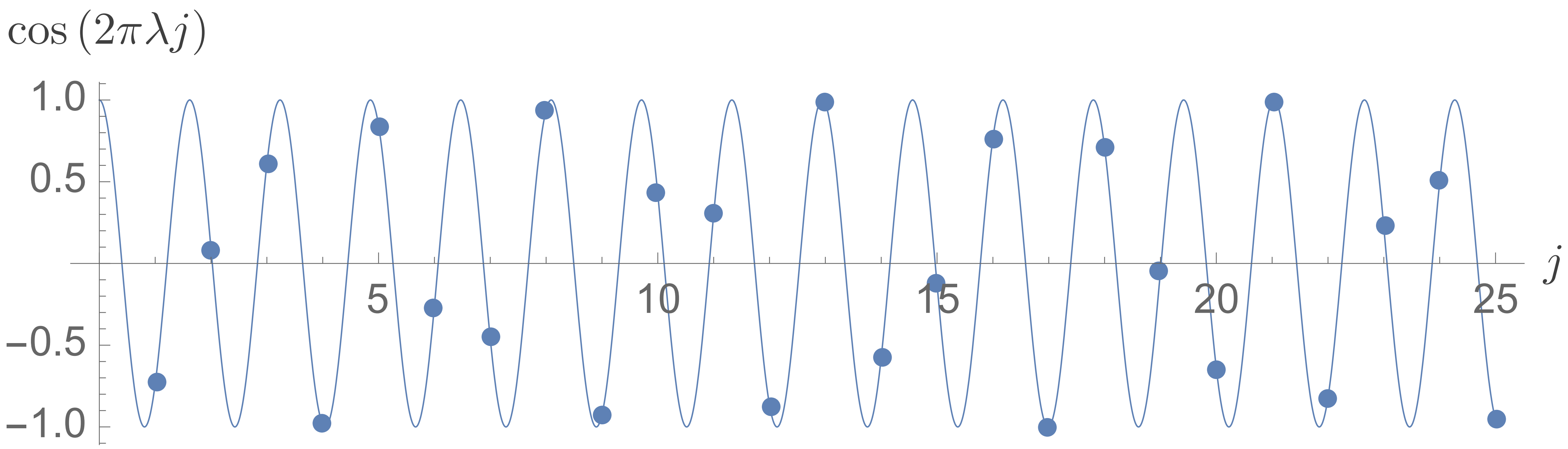

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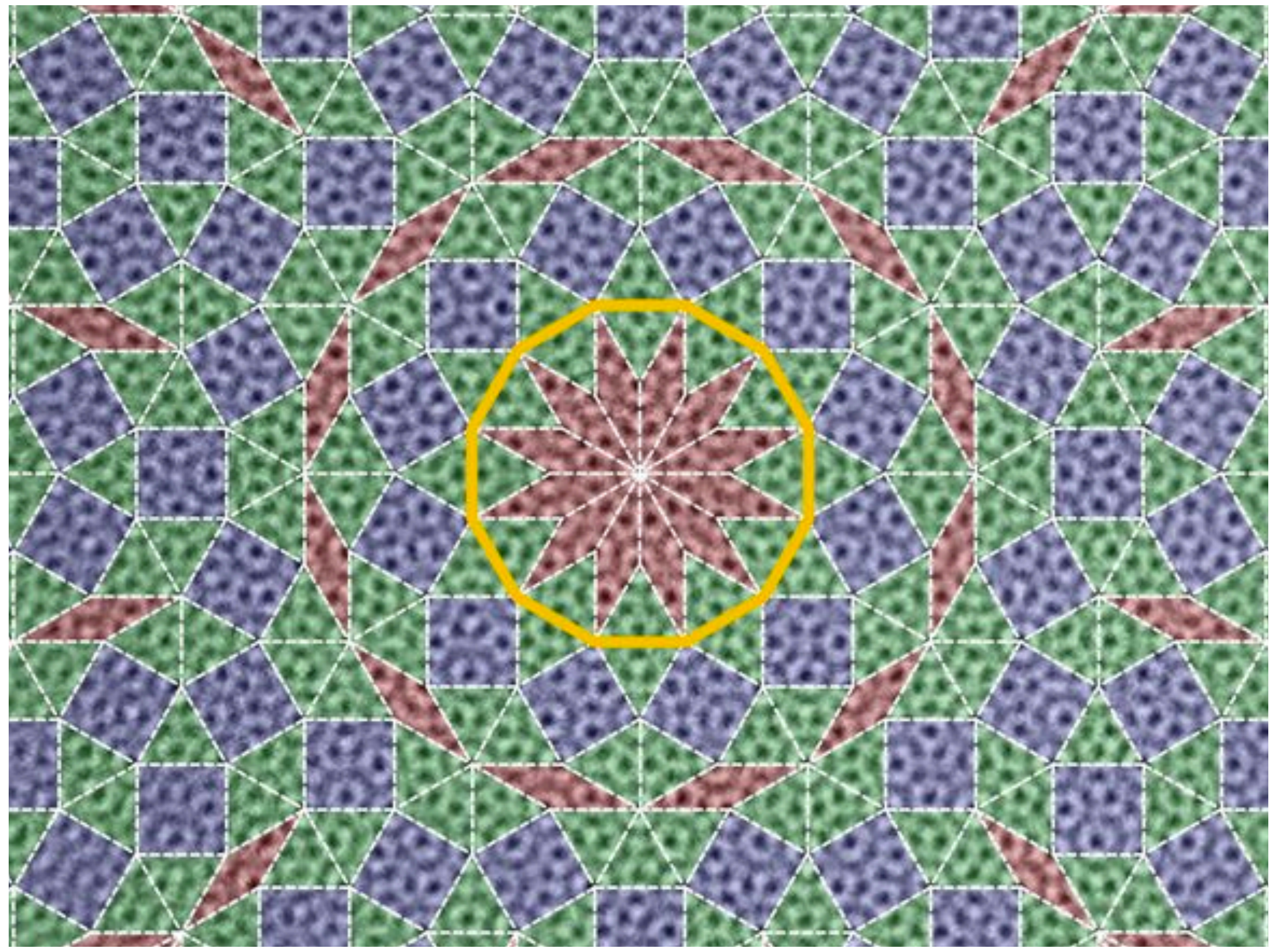

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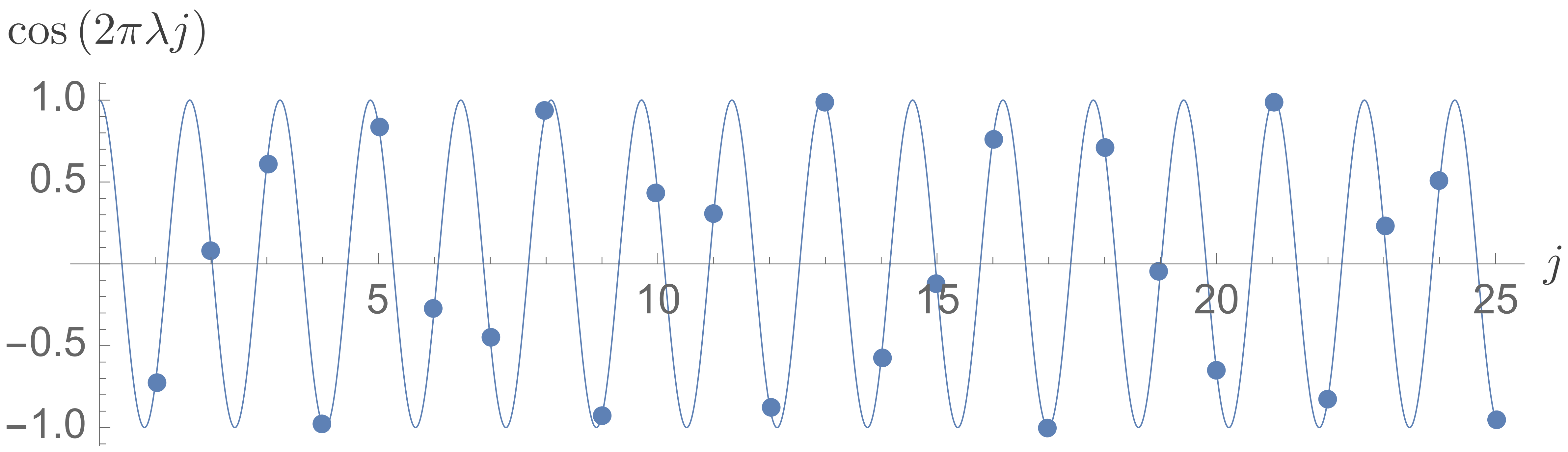


Phase transition:

Anderson-localized ($t \ll W$)
vs. delocalized ($t \gg W$) eigenstates

Implementation of the quasiperiodic capacitor

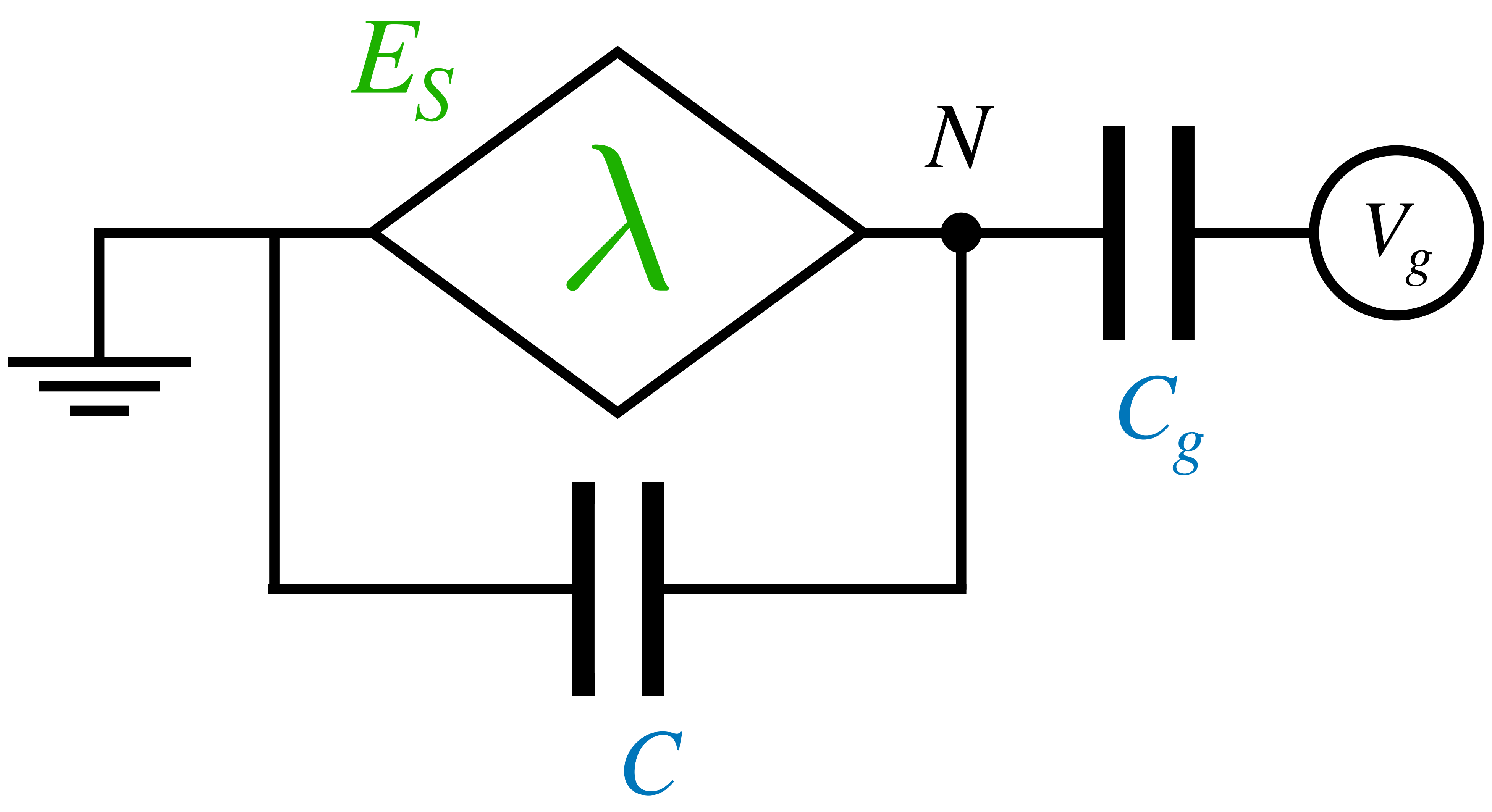
Dirac circuit

Charge localization in a quasiperiodic lattice

Effective nonlinear capacitance

$$H_\lambda = -E_S \cos\left[2\pi\lambda\left(N+N_g\right)\right]$$

$$+2E_C\left(N+N_g\right)^2$$



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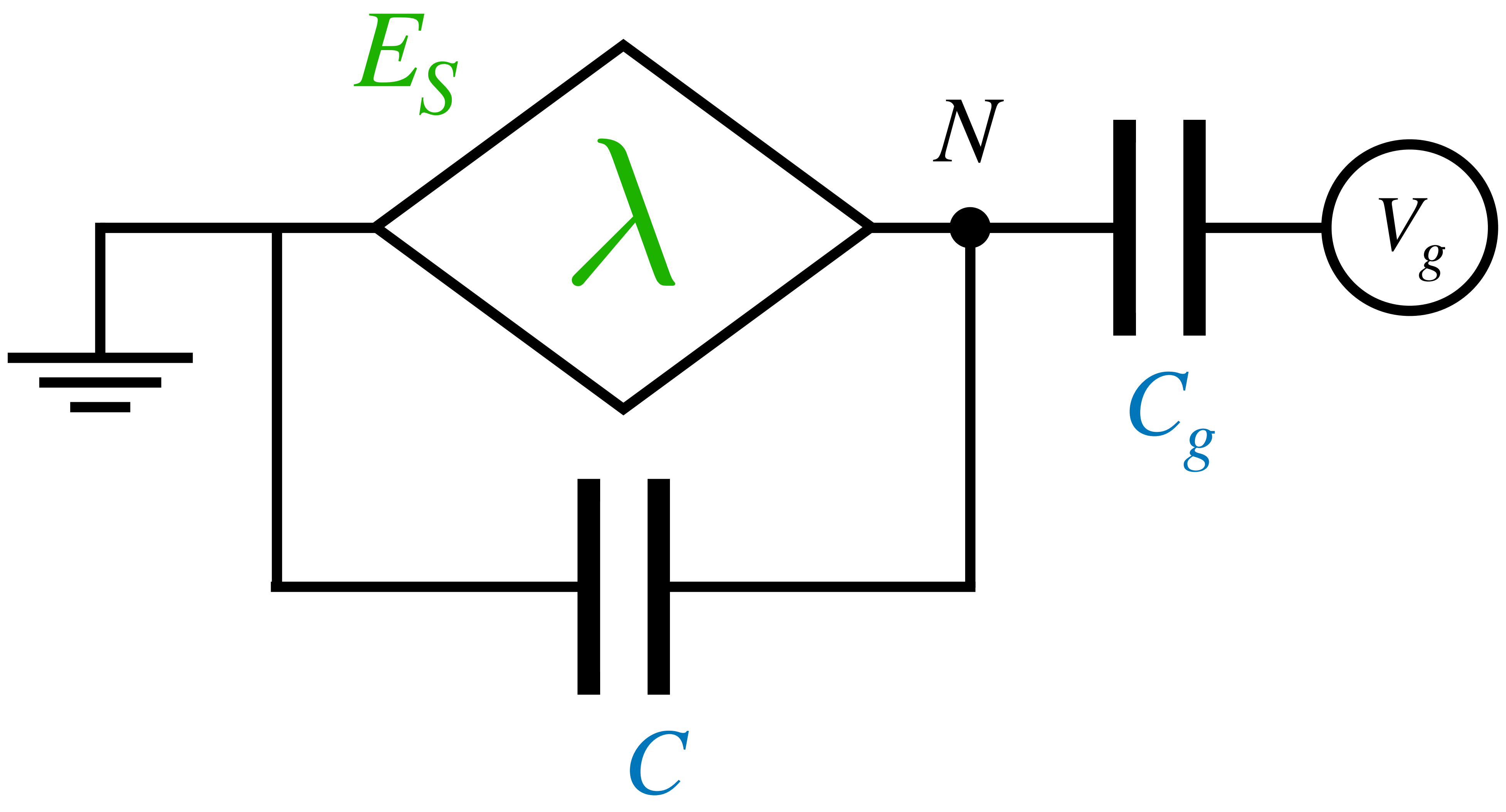

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$$\widehat{H}_\lambda = 2E_{C\lambda}\left[\widehat{N}_\lambda + \lambda\left(N + N_g\right)\right]^2$$

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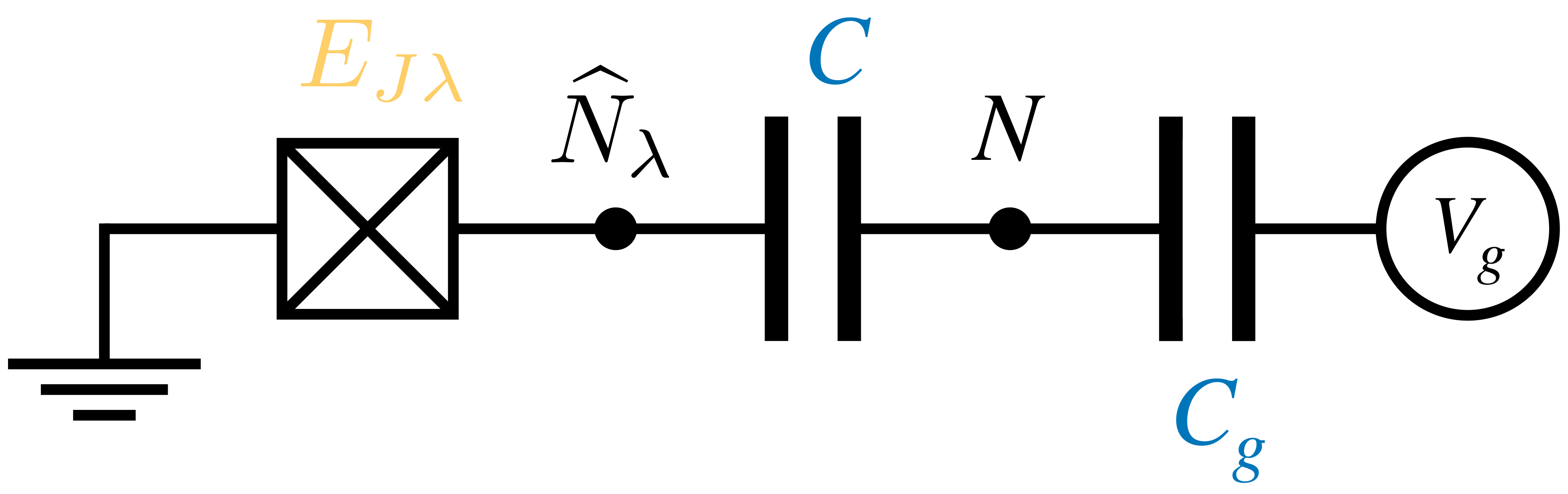

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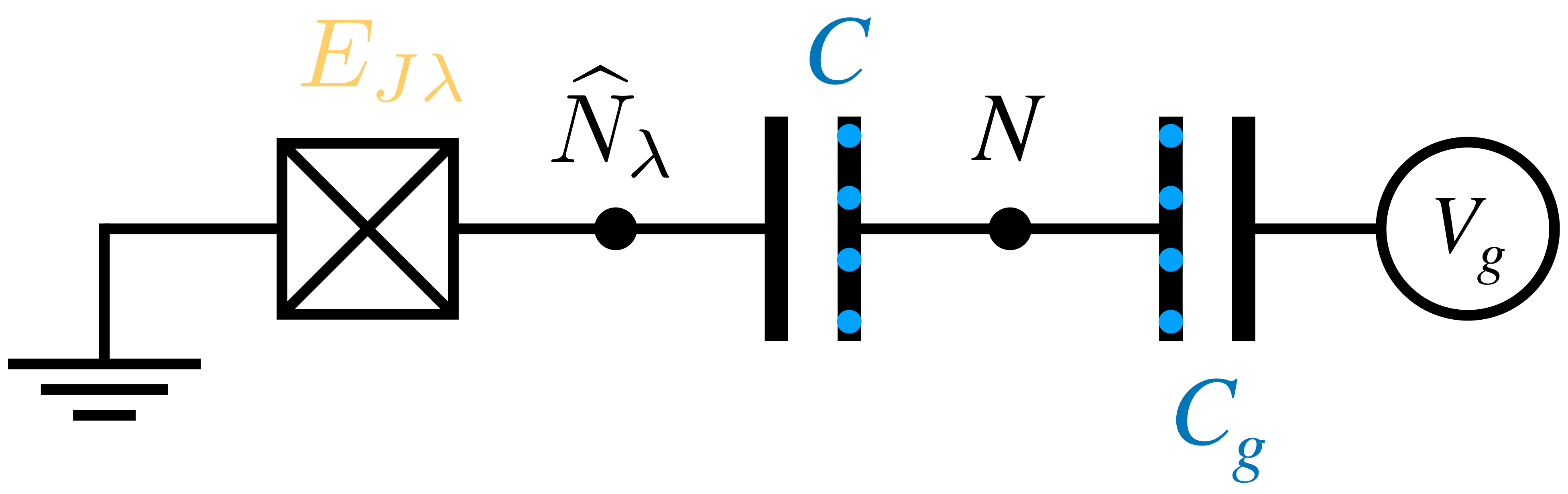

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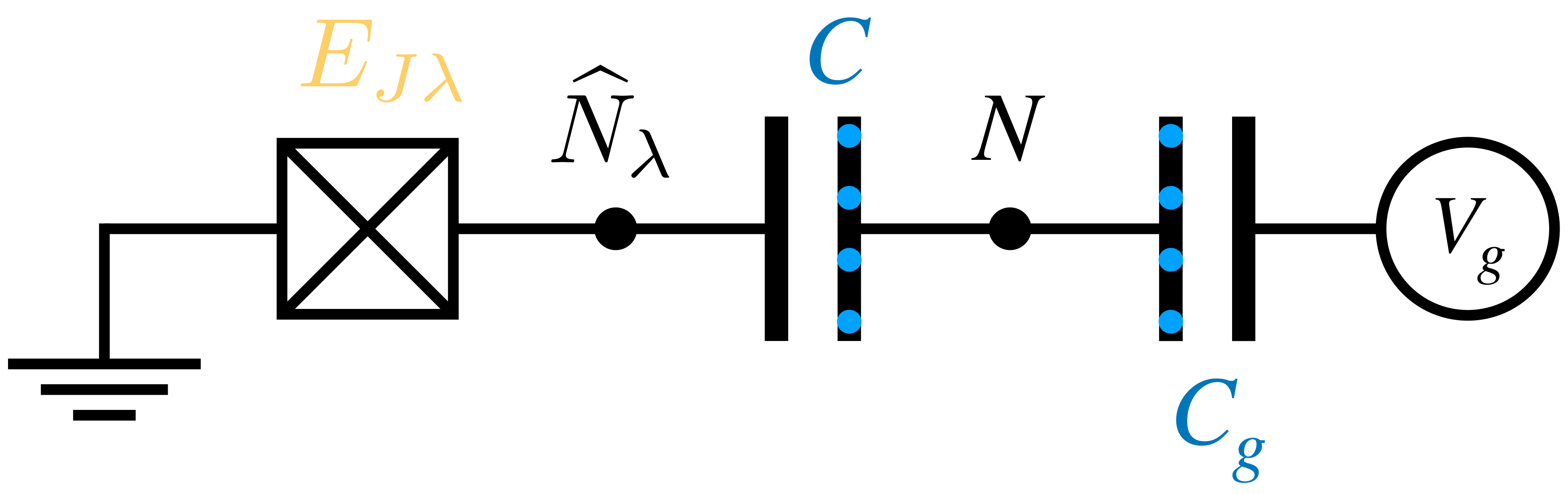

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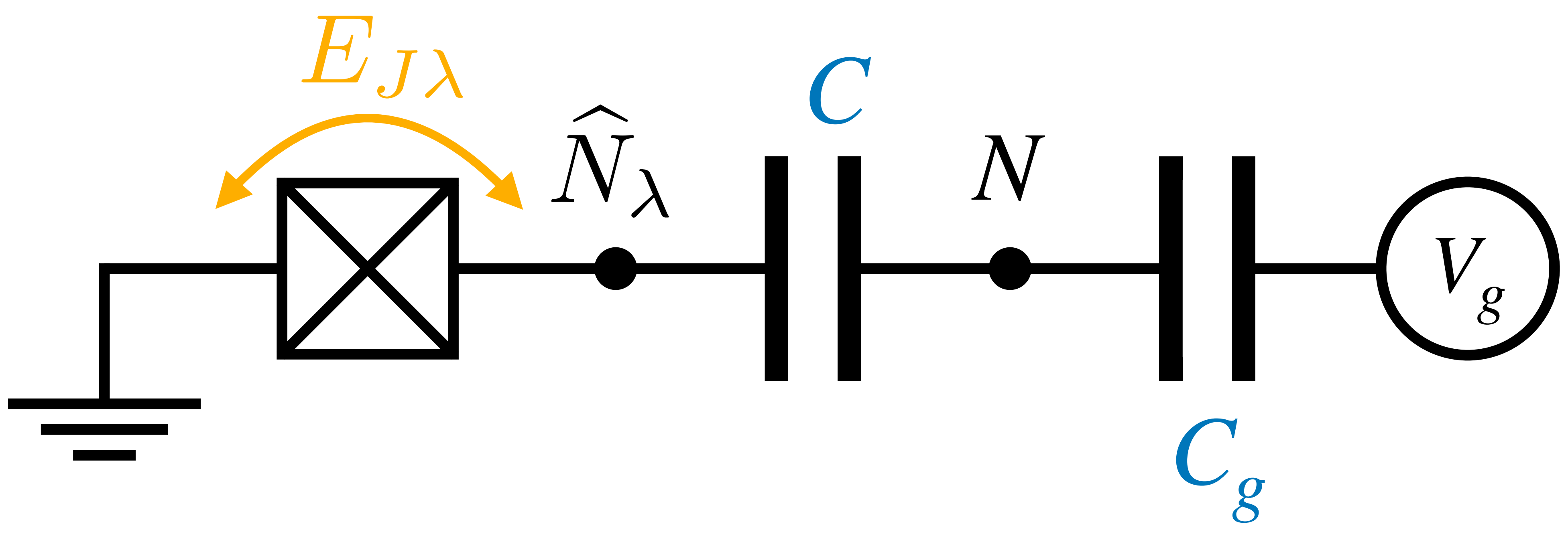

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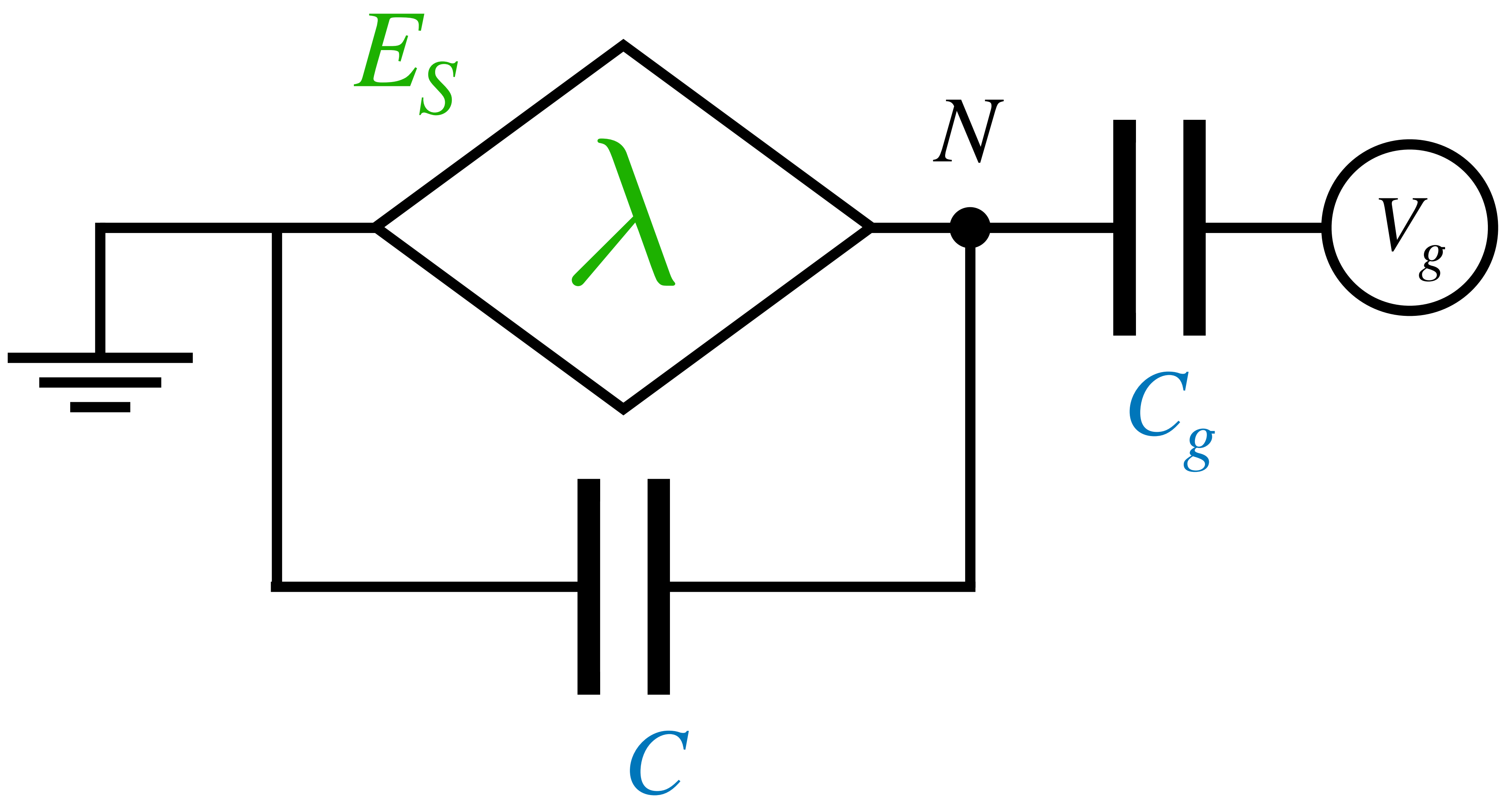

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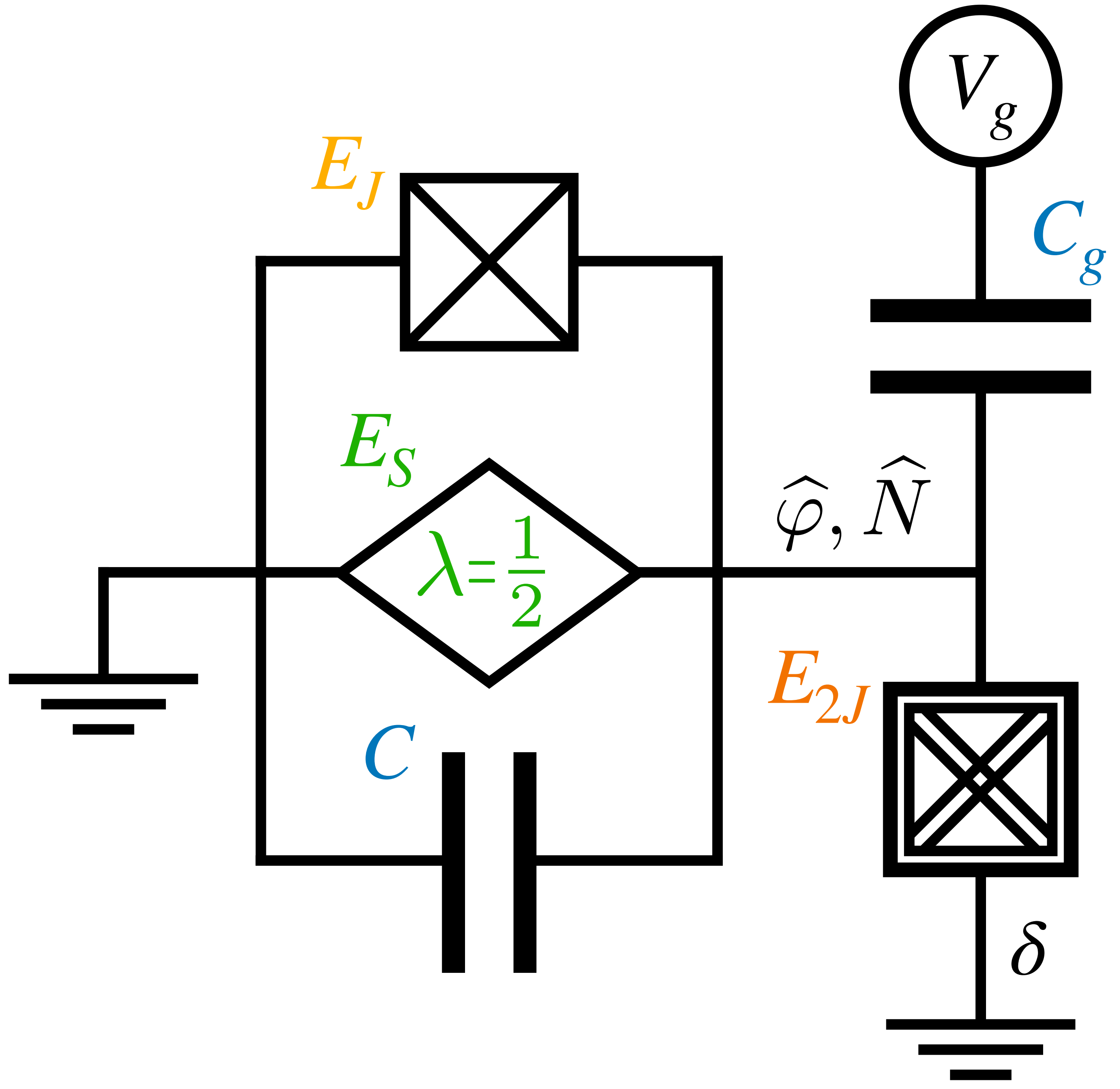
Dirac circuit

$$\boxed{\lambda = \frac{1}{2}}$$

dominant

$$\hat{H} = 2E_C\left(\hat{N}+N_g\right)^2 - E_{2J}\cos\left[2\left(\hat{\varphi}-\delta\right)\right] - E_S\cos\left[\pi\left(\hat{N}+N_g\right)\right] - E_J\cos\left(\hat{\varphi}\right)$$



Dirac circuit

$$\boxed{\lambda = \frac{1}{2}}$$

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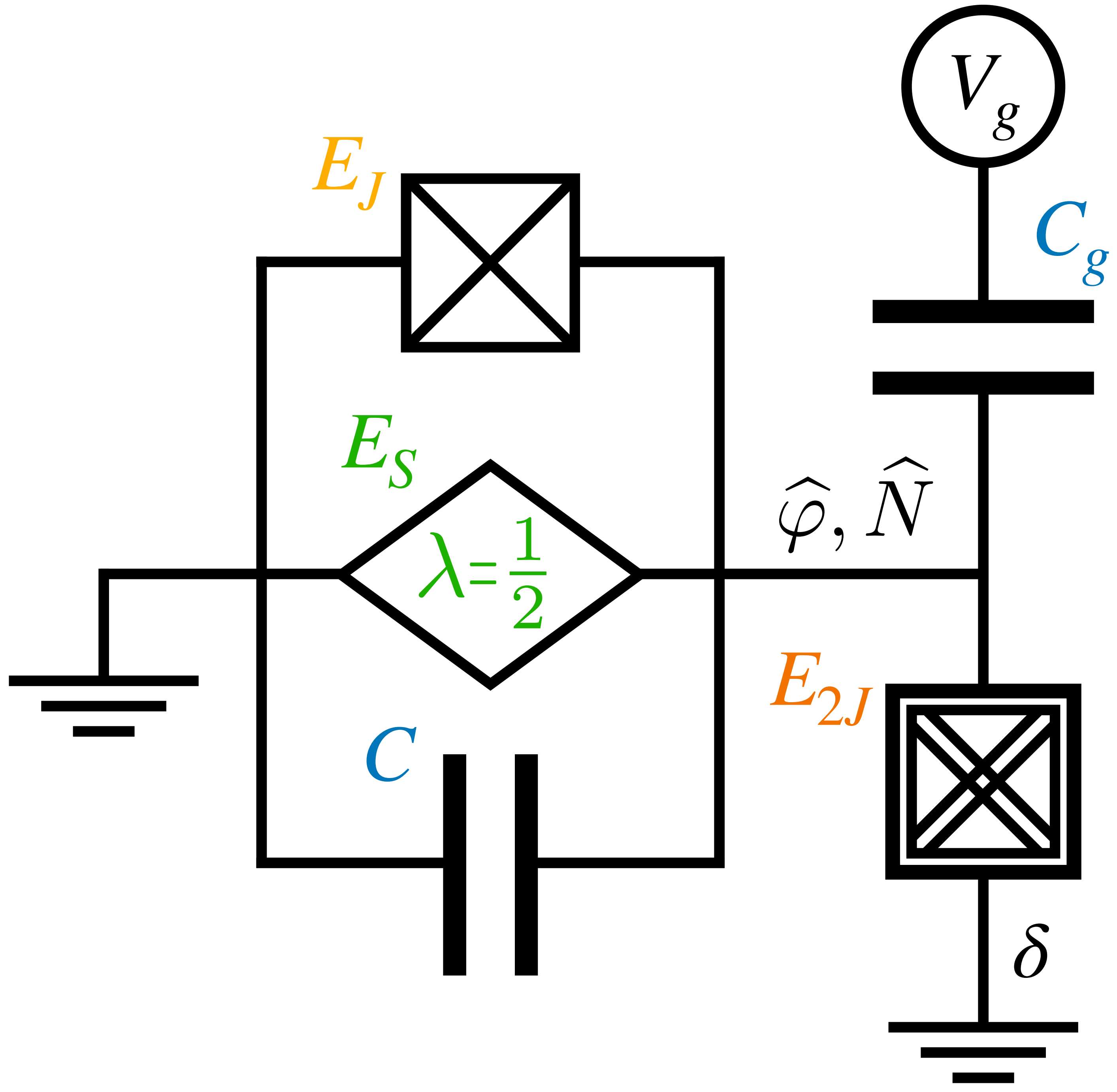

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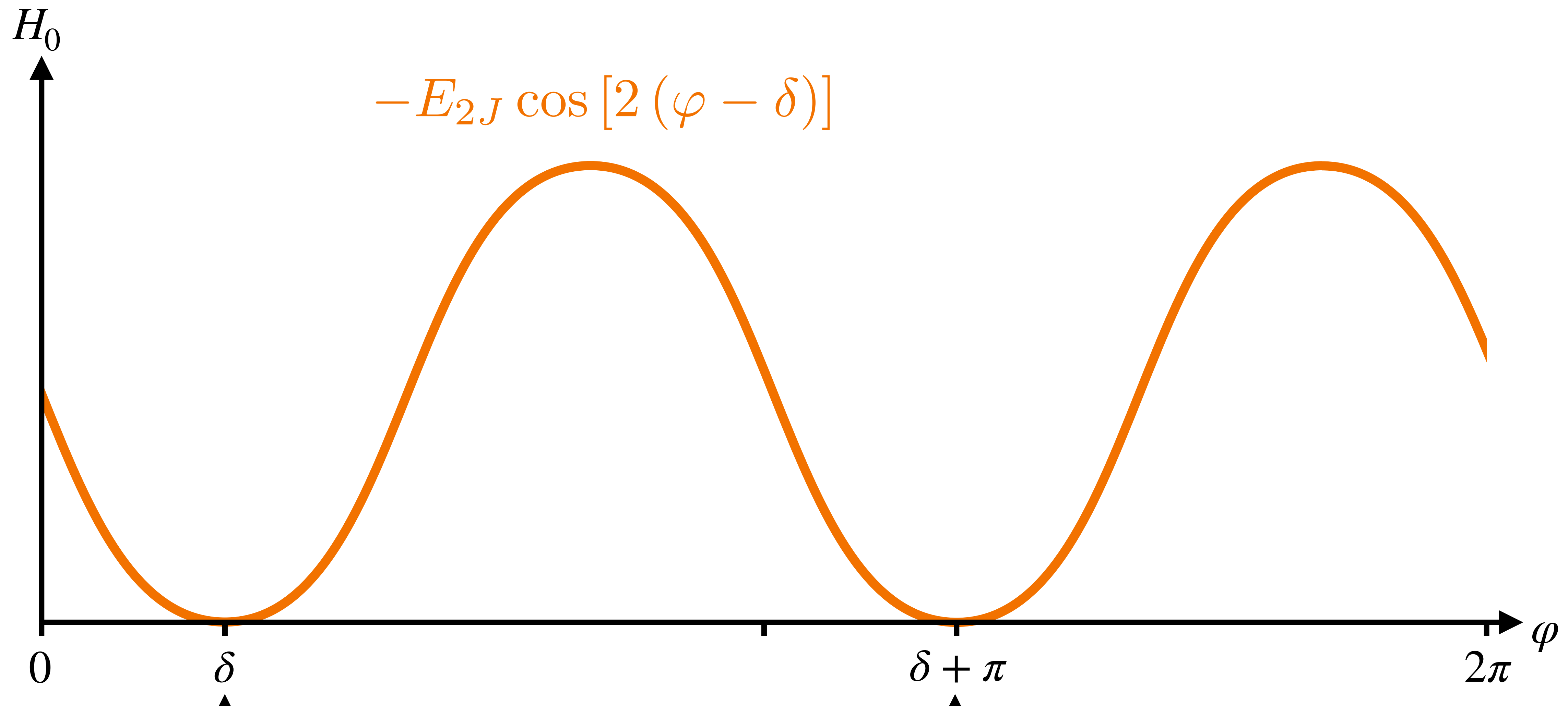
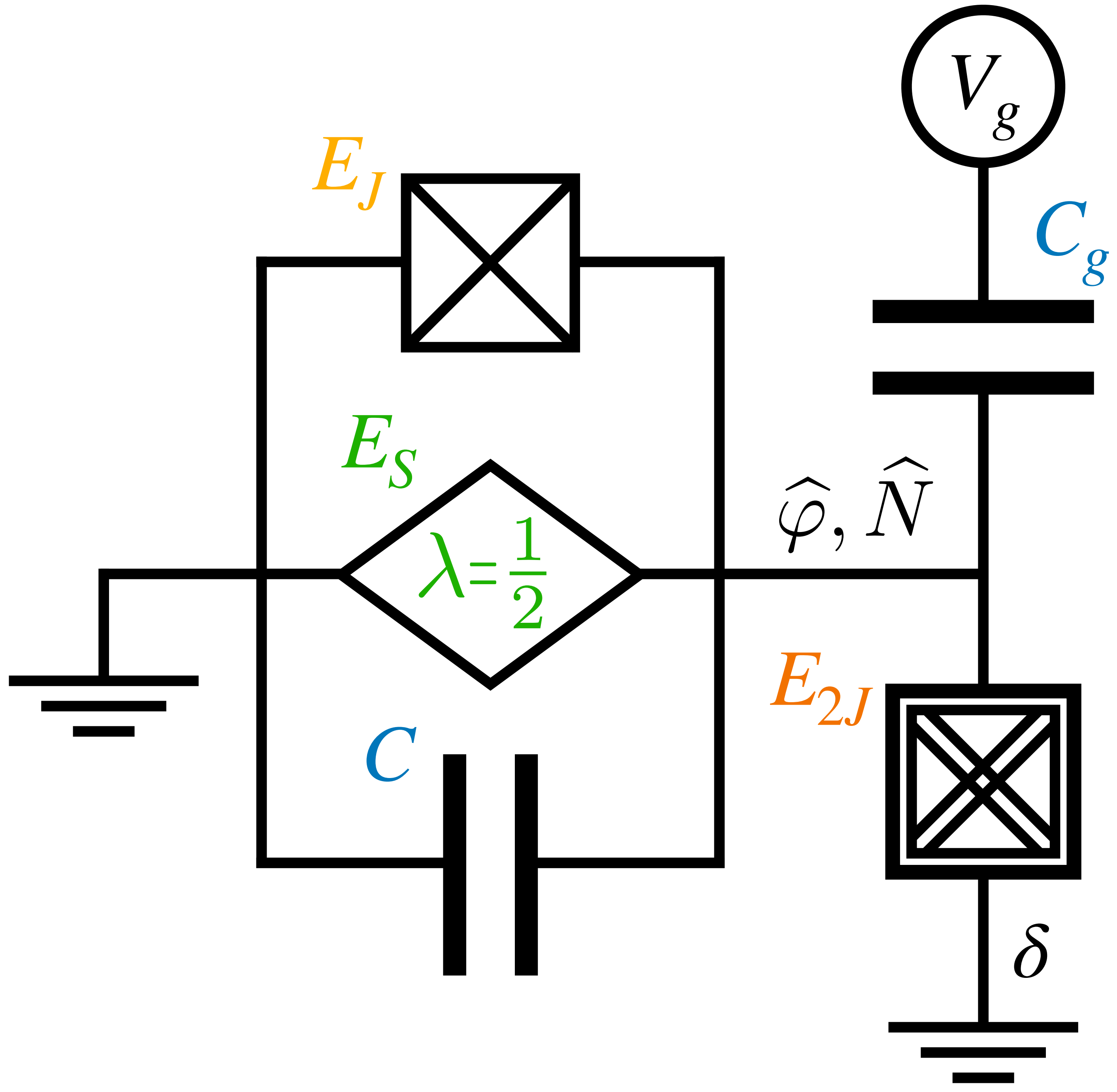


Doubly degenerate ground states as <u>new low-energy basis</u>



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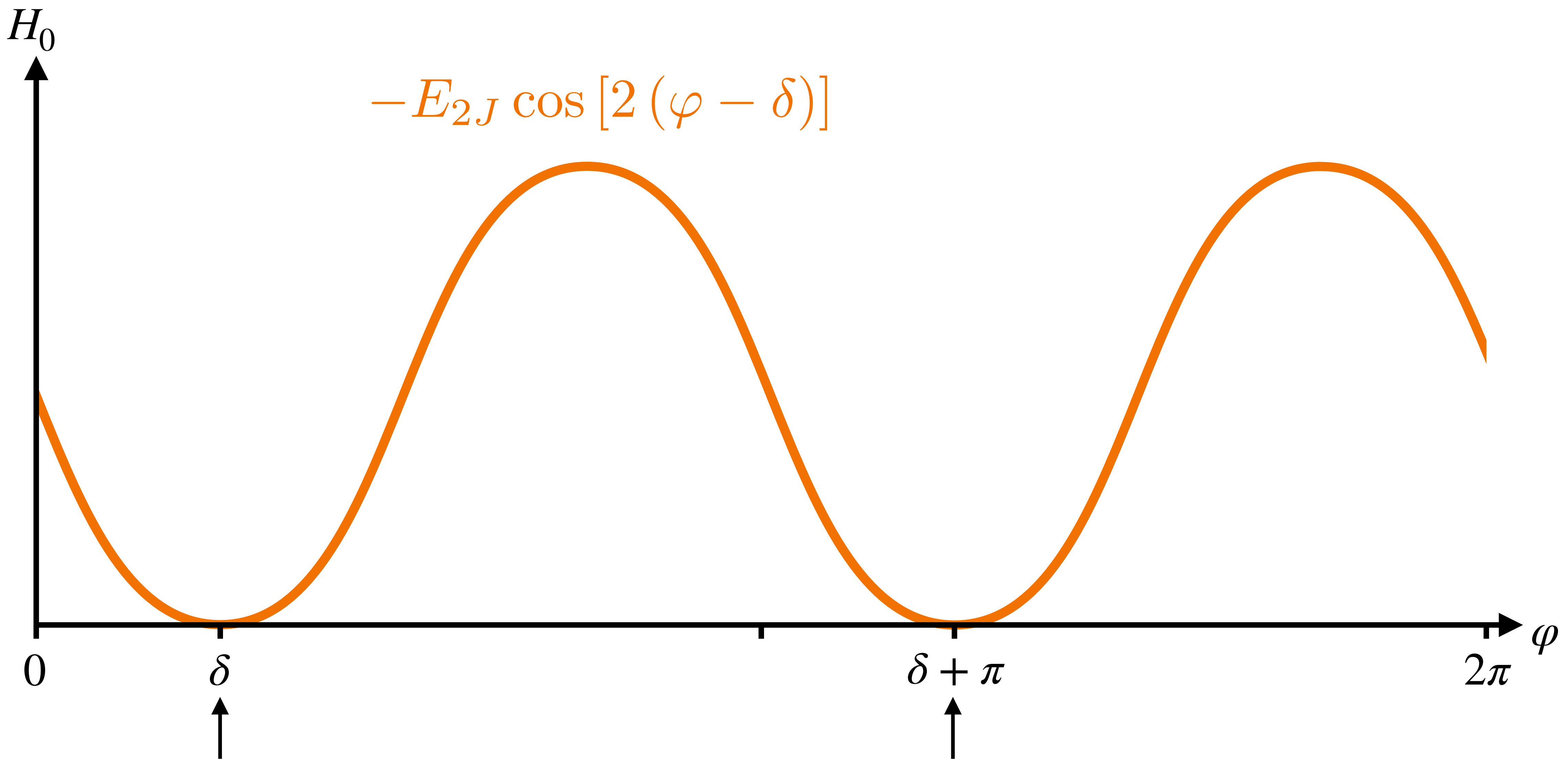

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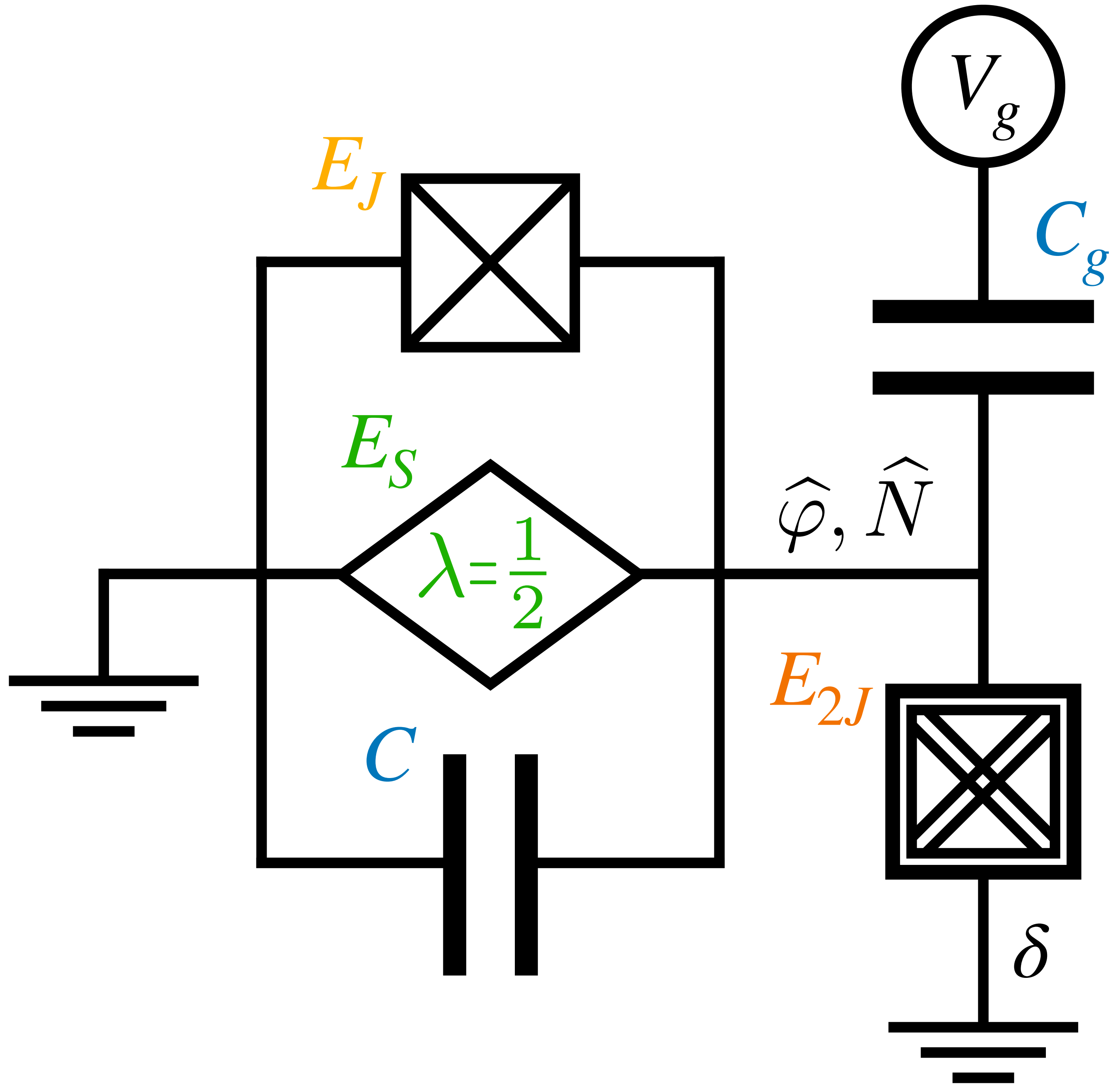

$$\hat{H} \approx -E_S\cos\left(\pi N_g\right)\hat{\sigma}_x - E_J\cos\left(\delta\right)\hat{\sigma}_z$$

Dirac circuit

$$\boxed{\lambda = \frac{1}{2} + \delta\lambda}$$

dominant

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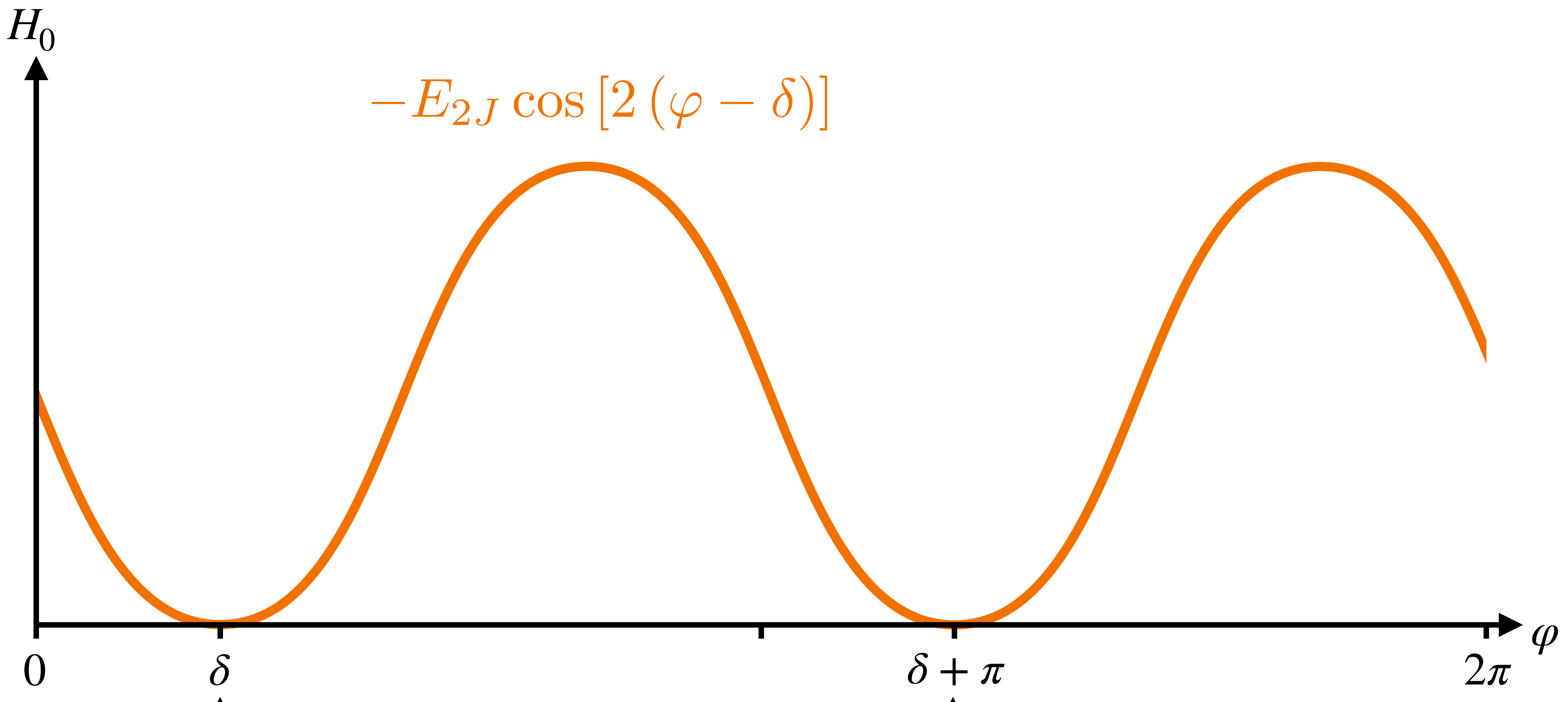

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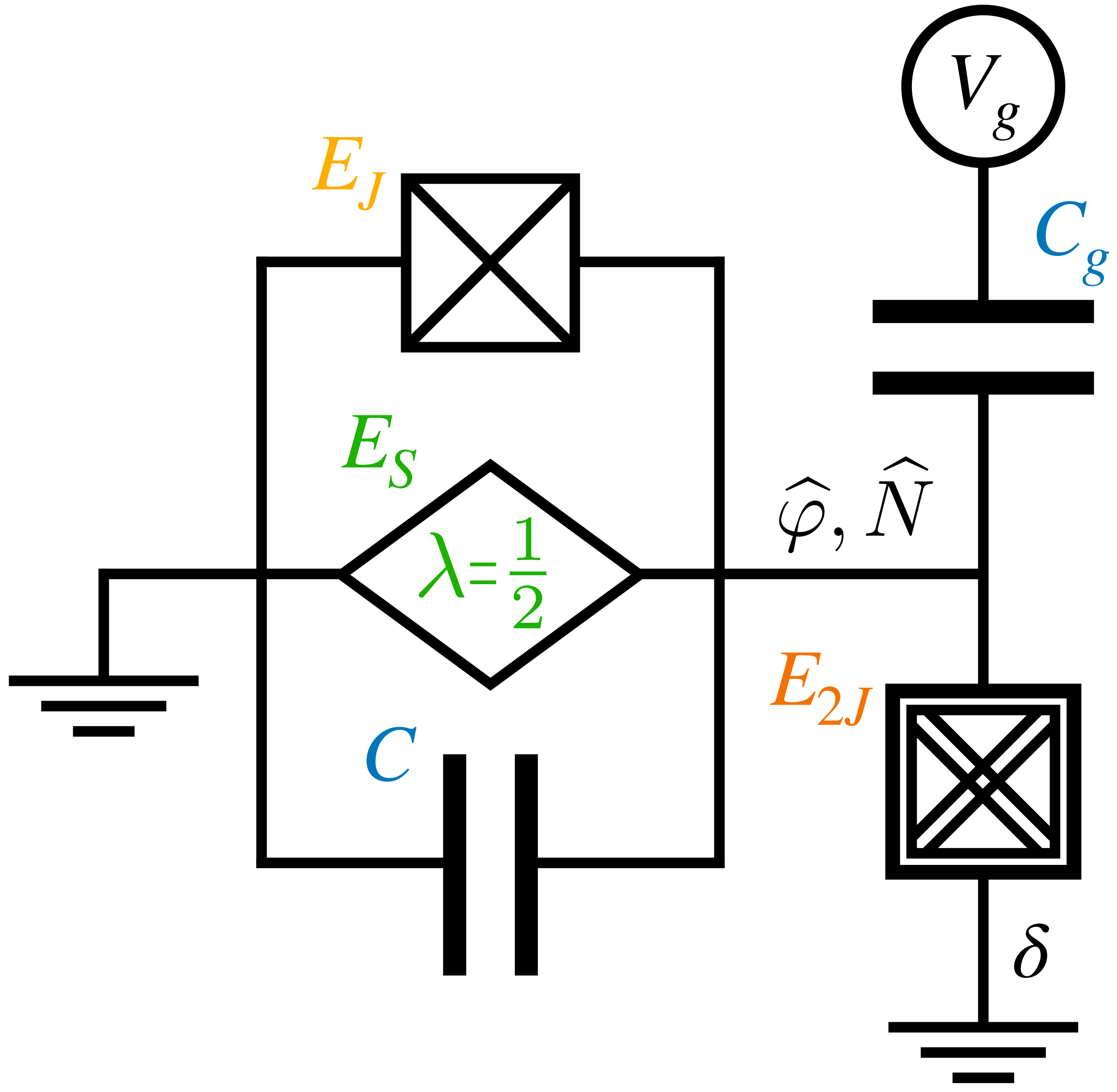

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Classical current noise spectrum

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    $$I = I_J + I_{\mathcal{B}}$$

    Herrig & Riwar, Phys. Rev. Res. (2022)

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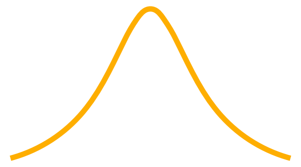
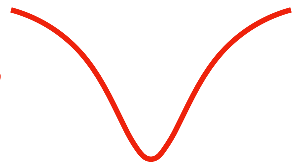
    $\sim$  $\sim$ 

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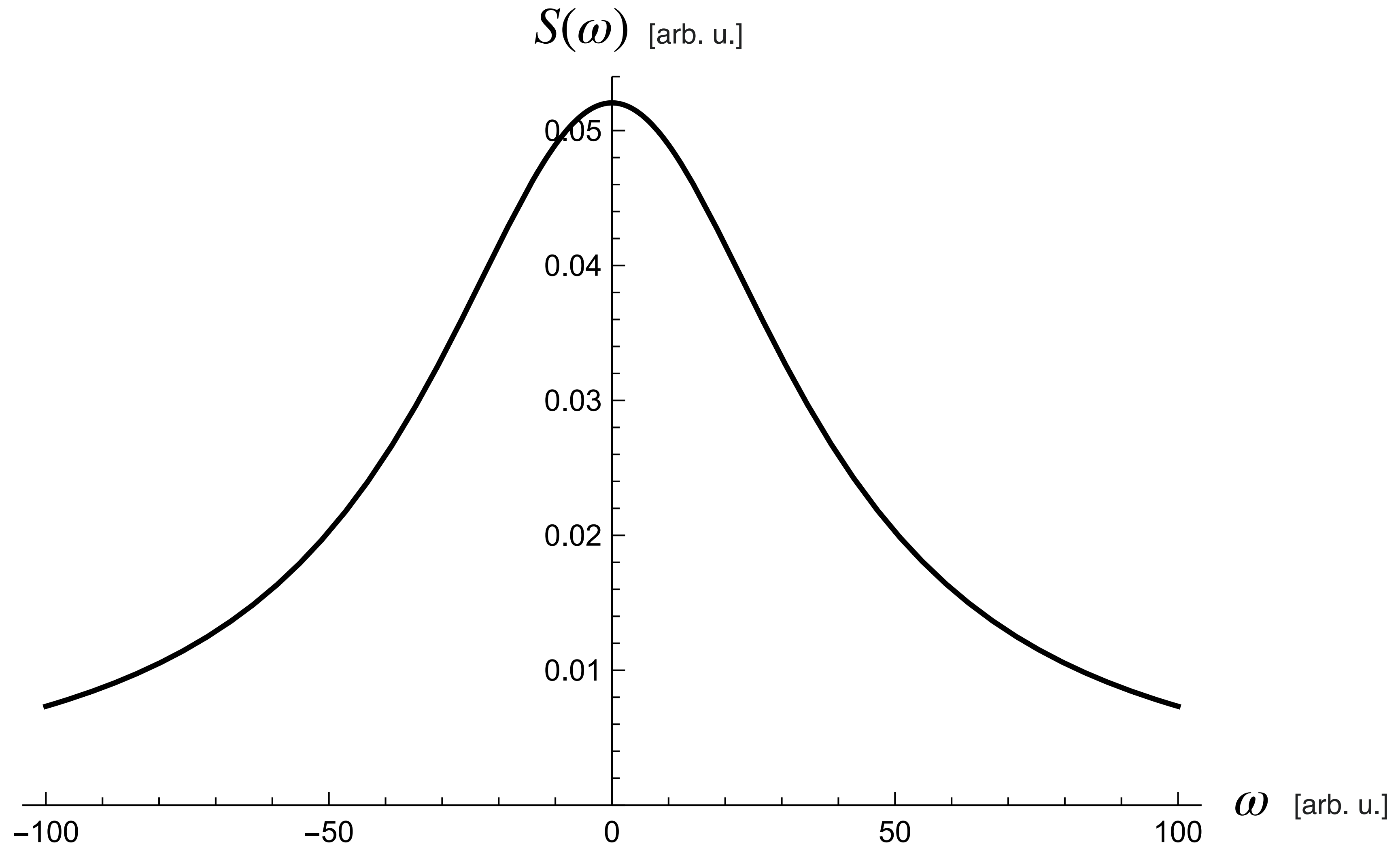

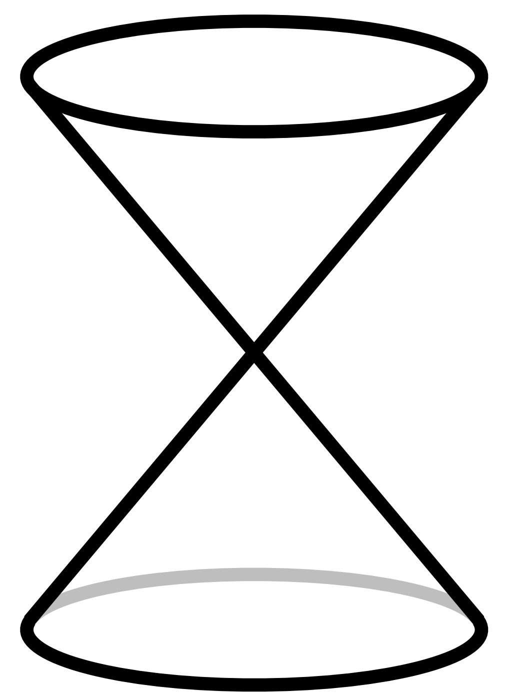

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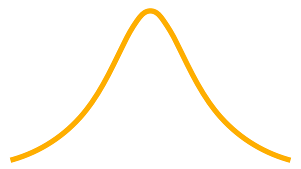
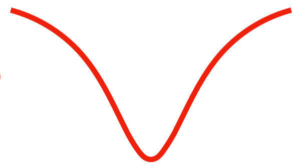
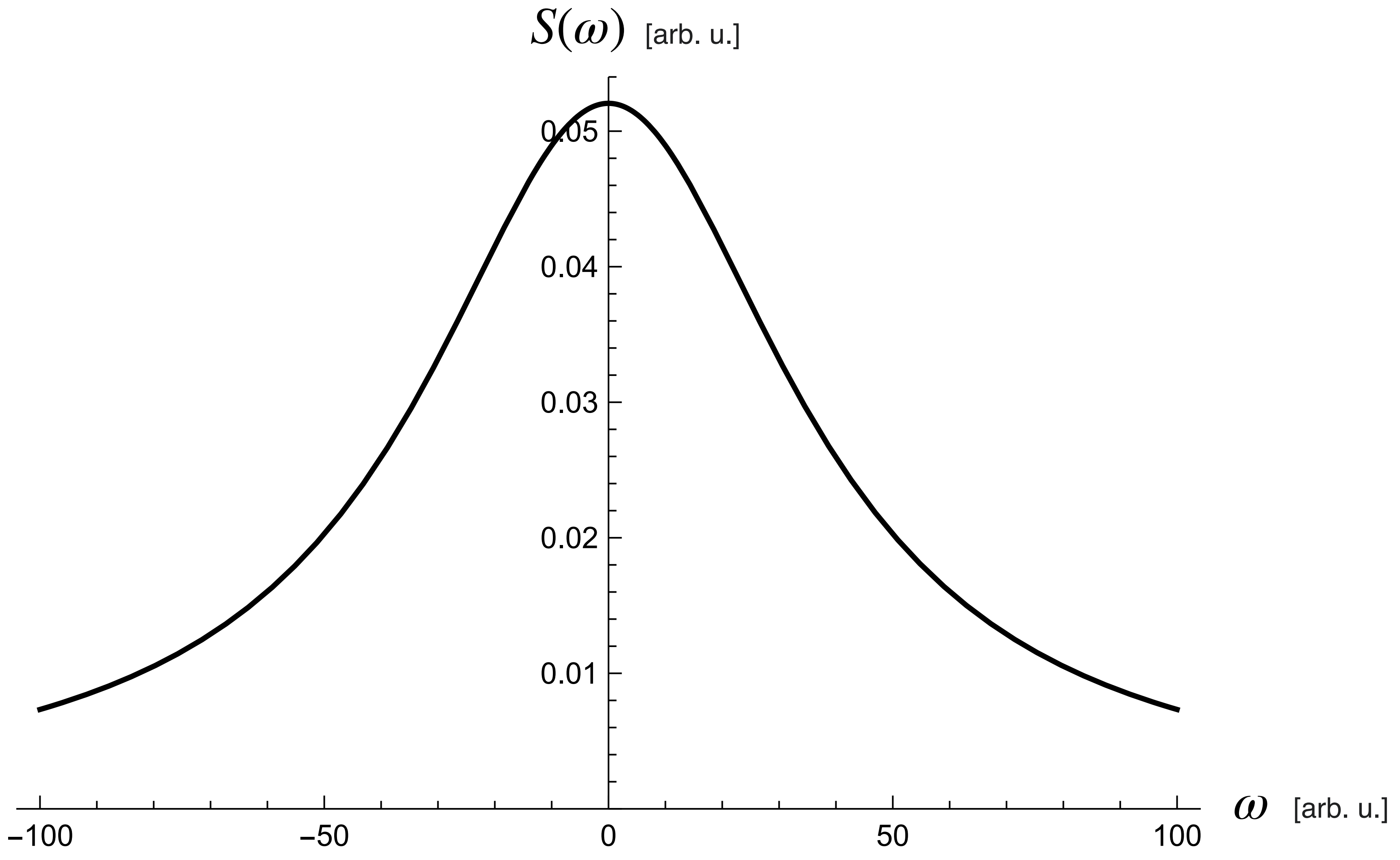
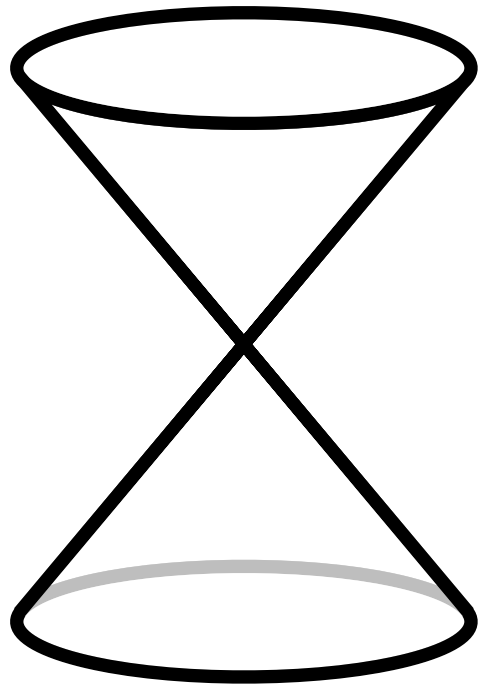
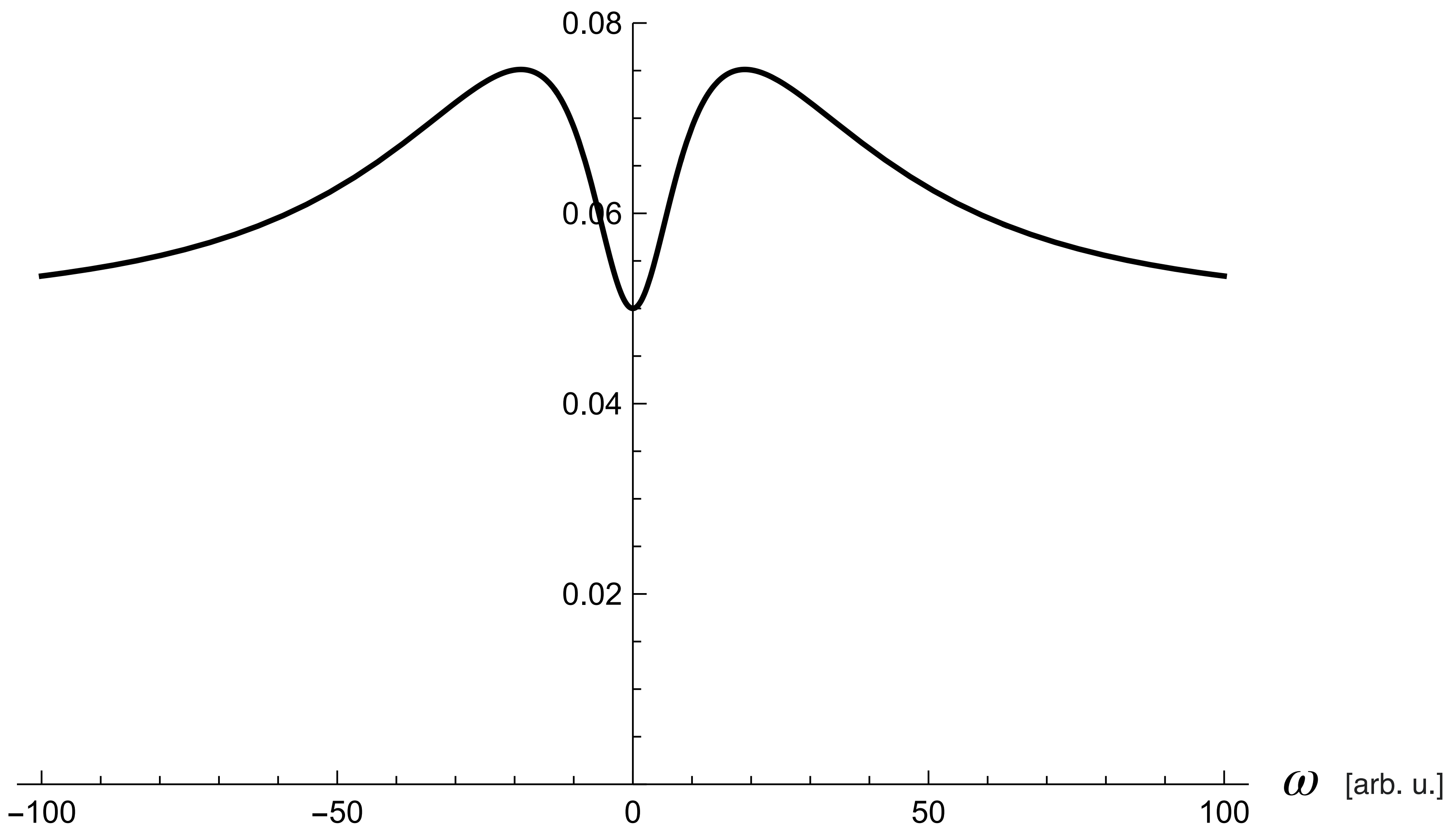
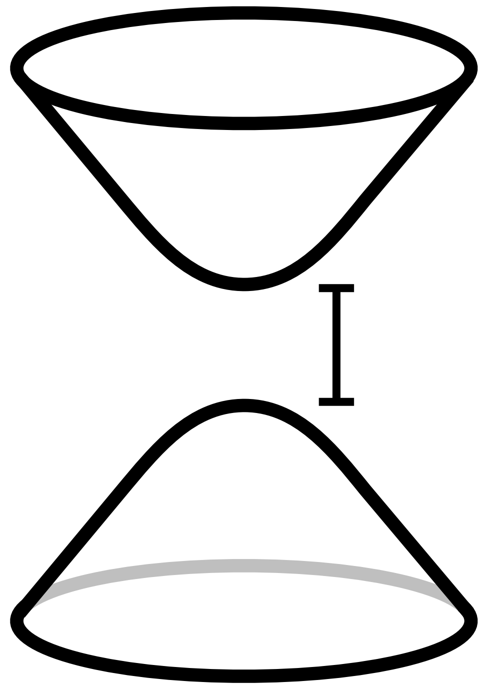
$\sim$  $\sim$ 









Implementation of the quasiperiodic capacitor

Dirac circuit

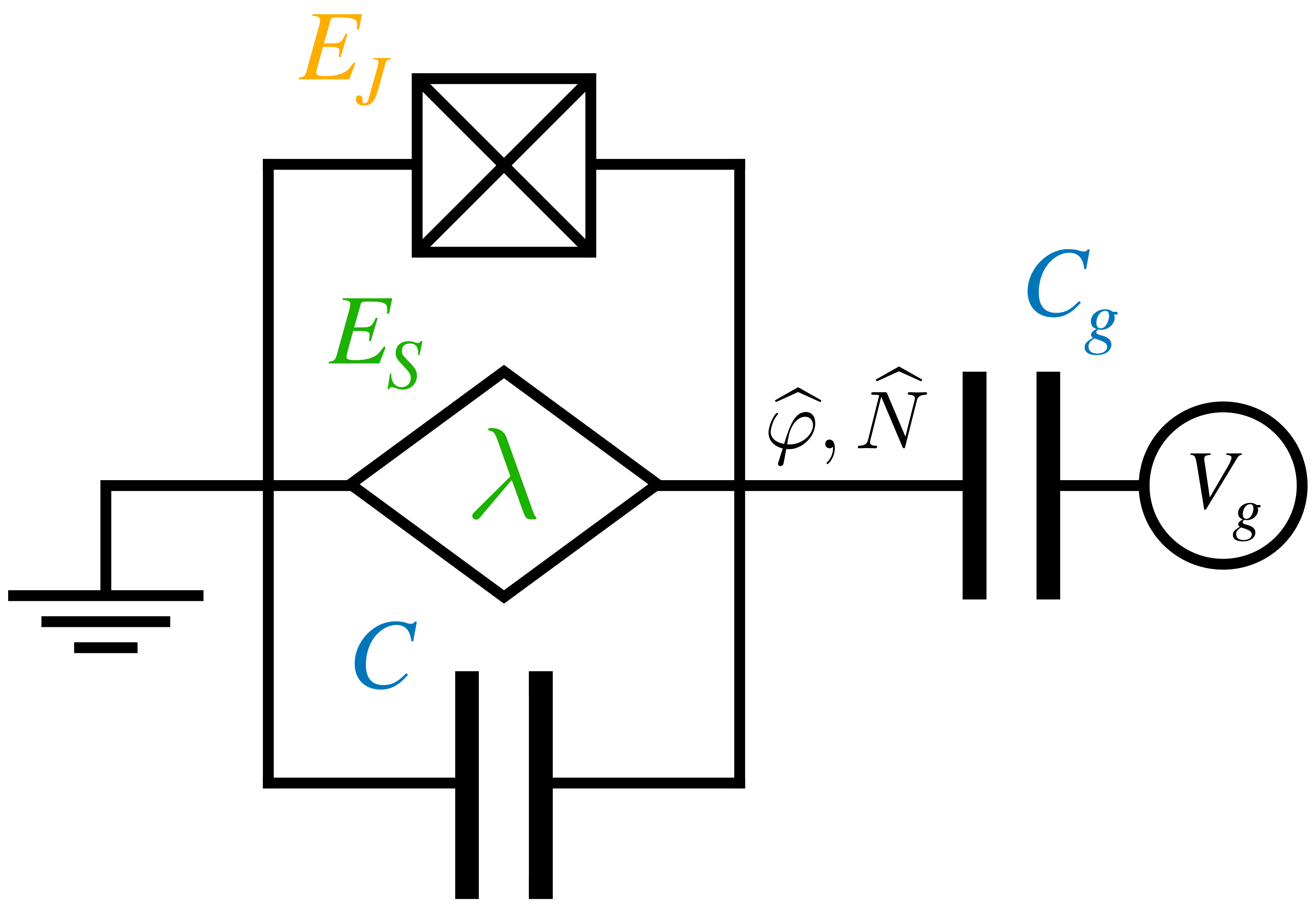
Charge localization in a quasiperiodic lattice

Charge localization

Now: Change perspective, think of charge space as lattice sites!

$$\sum_N \left(2E_C\left(N+N_g\right)^2 - E_S\cos\left[2\pi\lambda\left(N+N_g\right)\right]\right)|N\rangle\langle N| - E_J\sum_N |N+1\rangle\langle N| + \text{h.c.}$$

trapping potential   quasiperiodic potential



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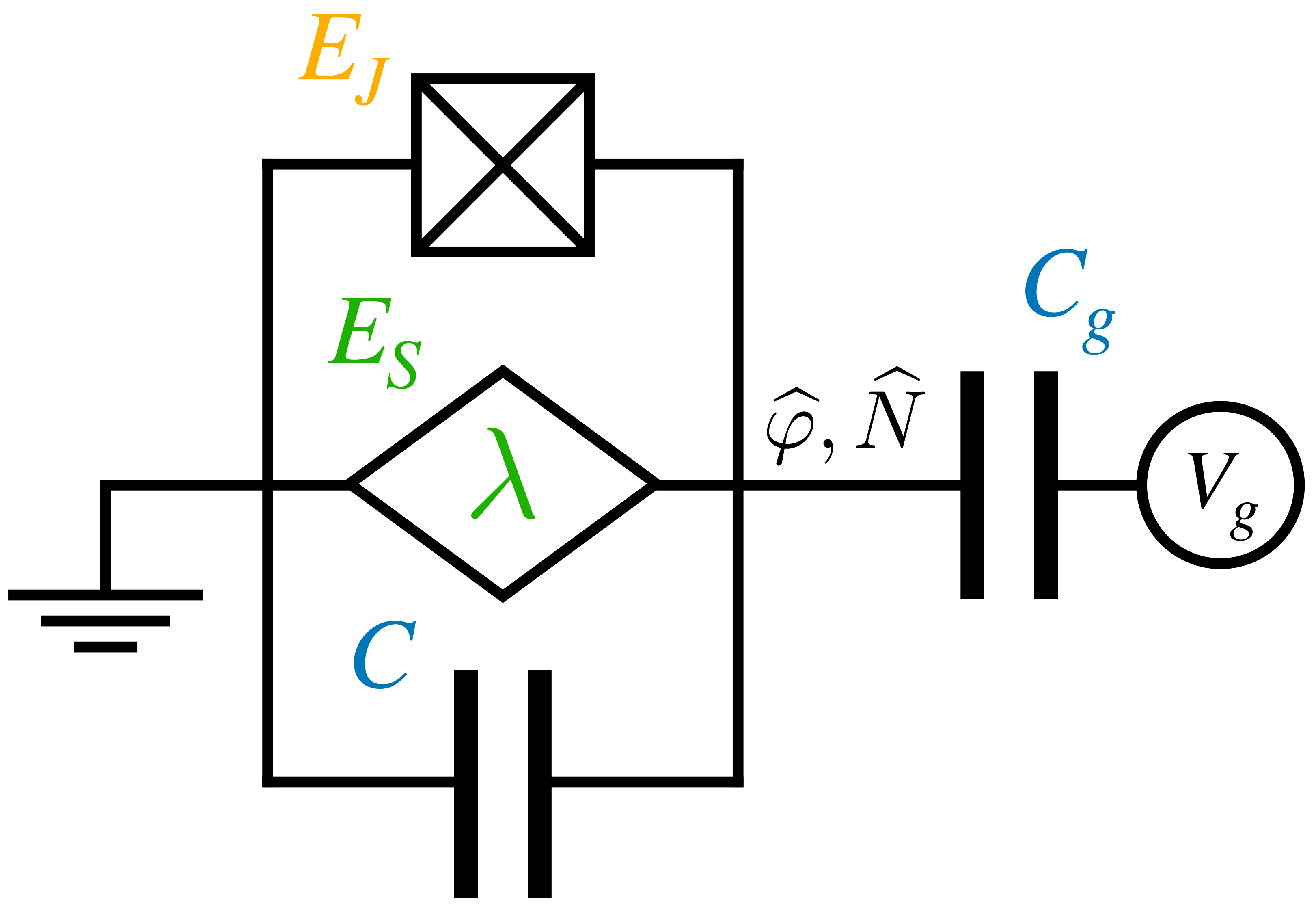
trapping potential quasiperiodic potential

$E_C \to 0 \implies$ Aubry-André model



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trapping potential quasiperiodic potential

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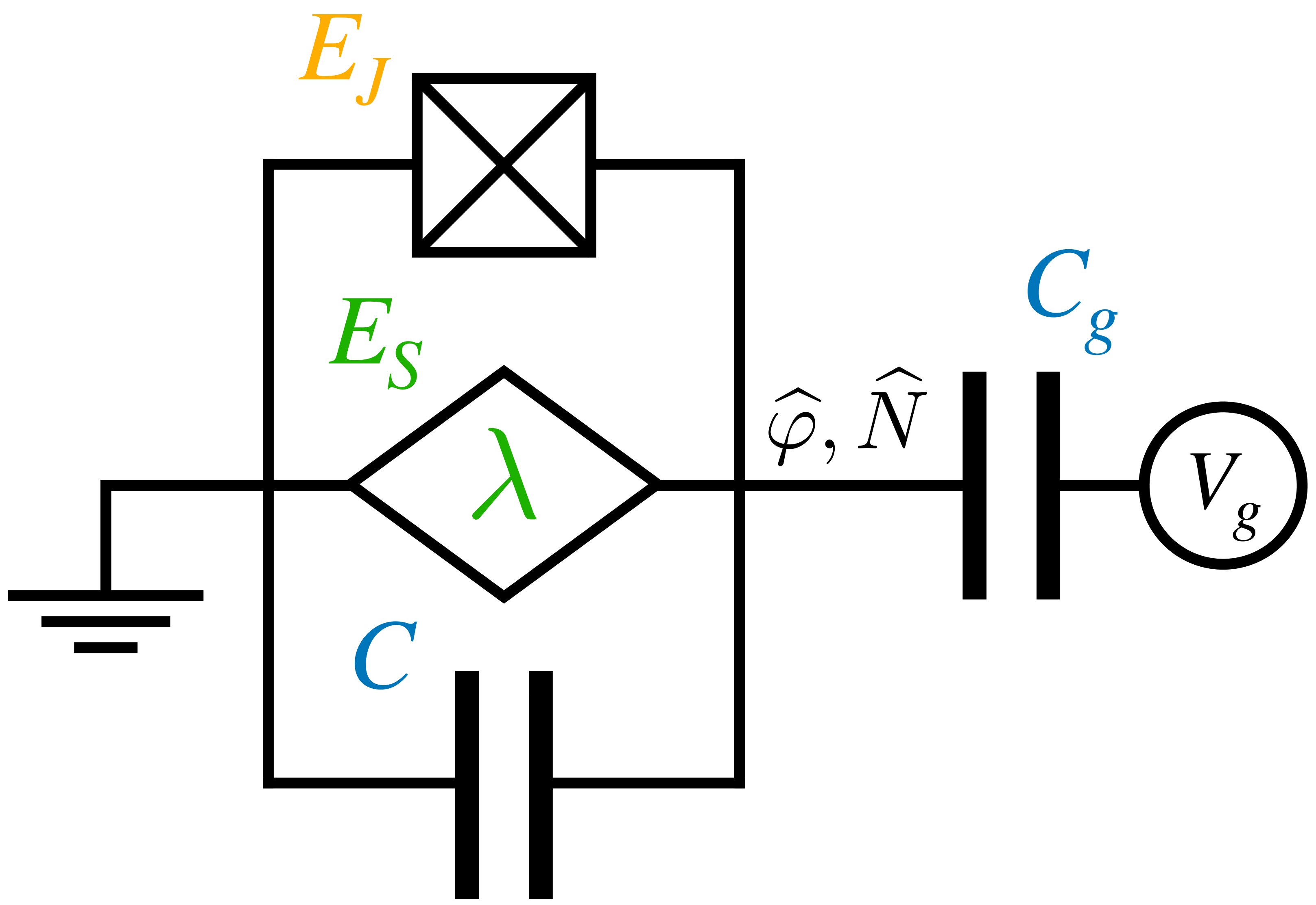
$E_C > 0 \implies$ trapped AA model



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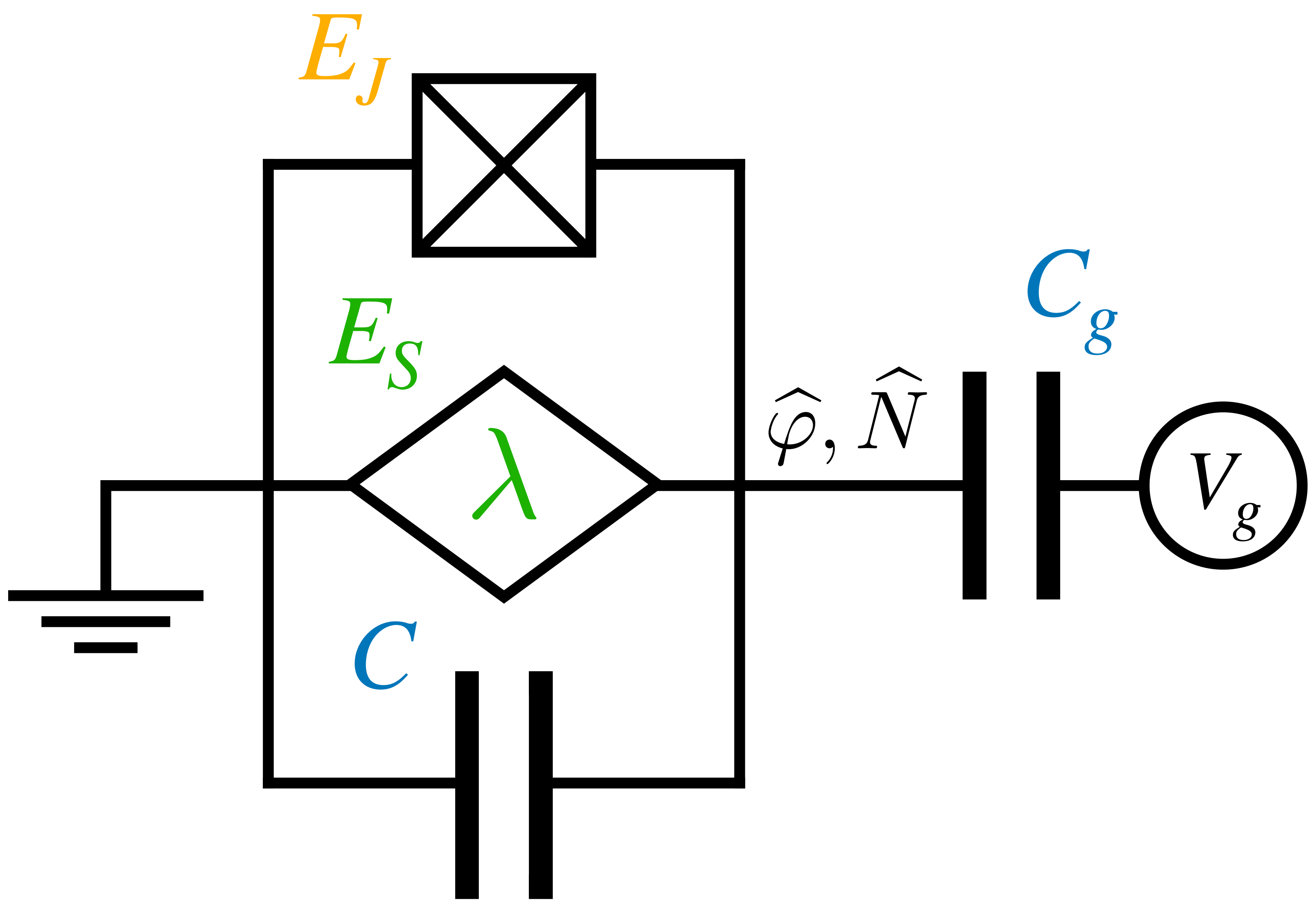
$\to$ Anderson localization as

Charge localization for $E_J \ll E_S$

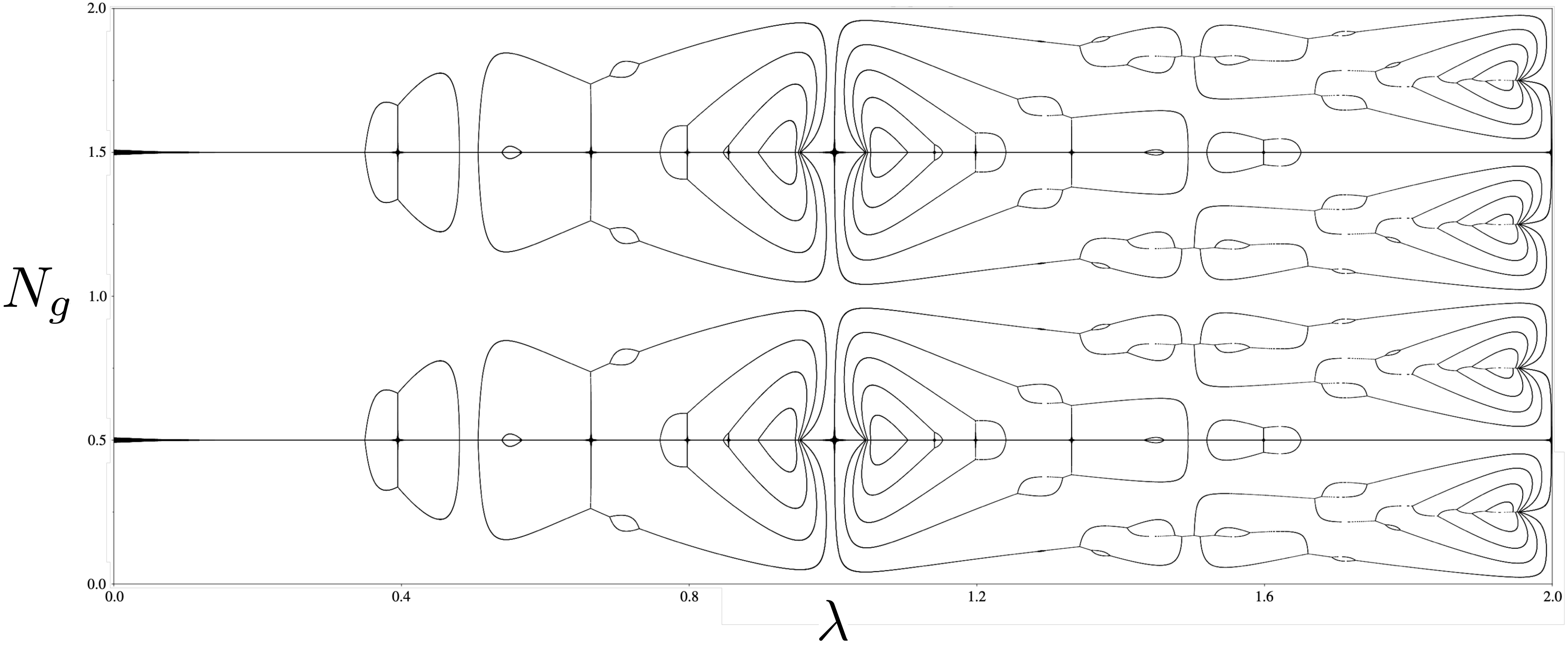
$$\left\langle \widehat{N}^2 \right\rangle - \left\langle \widehat{N} \right\rangle^2 = 0$$



Very soon on arXiv! email: t.herrig@fz-juelich.de

Dirac physics and charge localization due to quasiperiodic nonlinear capacitances

by T. Herrig, J. H. Pixley, E. König and R.-P. Riwar



Thank you for your attention!

Negative Capacitances