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Erratum: More on the flavor dependence of m_ϱ/f_π

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Tree-level correlators and decay constant

The m_ϱ/f_π ratio was computed in the chiral-continuum limit in SU(3) gauge theory coupled to various numbers of fermions in the fundamental representation via non-perturbative lattice simulations [1, 2]. None of the non-perturbative results are effected by this erratum.

The issue to be corrected here concerns the free theory which was used for illustration and comparison only. Clearly, in a free theory both m_ϱ and f_π first needs to be defined at finite fermion mass m and the chiral limit should be taken only for the ratio. Naturally, $m_\varrho = m_\pi = 2m$ where m is the fermion mass. In [3] the result $f_\pi = \sqrt{12}/L$ was obtained from lattice simulations extrapolated to the continuum, in finite volume $m_\pi L = 1$. The convention for the normalization of f_π used in [3] was not specified and it turns out it corresponded to 130 MeV in QCD, which differs from our convention by a factor $\sqrt{2}$. In any case, from the finite ratio m_ϱ/f_π in finite volume $m_\pi L = 1$, an incorrect conclusion was drawn in [1, 2], namely that m_ϱ/f_π is volume independent and the value obtained in [3] holds in infinite volume too. Furthermore, m_ϱ/f_π was misquoted in [1, 2] by a factor $\sqrt{2}$, beyond the $\sqrt{2}$ difference in conventions.

For completeness let us compute f_π directly in the continuum both in finite and infinite volume at tree level in Euclidean signature. It is enough to consider $N_c = 1$ and at the end restore the N_c-dependence by $f_\pi \to \sqrt{N_c}\, f_\pi$. The decay constant is defined from the large t behavior of the correlator at zero momentum,

$$C(x) = \langle \left(\bar\psi\gamma_5\psi\right)(x) \left(\bar\psi\gamma_5\psi\right)(0) \rangle$$
$$\int d^3x\, C(\mathbf{x},t) \sim \frac{m_\pi^3}{4m^2} f_\pi^2 e^{-m_\pi t} = 2m f_\pi^2 e^{-2mt} \qquad \text{for } t \gg \frac{1}{m}\,. \tag{1}$$

This normalization corresponds to $f_\pi = 92\, MeV$ in QCD as in [1, 2]. Now using the scalar and fermionic Green's functions,

$$G(x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ipx}}{m^2+p^2}$$
$$S(x) = (\gamma_\mu\partial_\mu + m)\, G(x) \tag{2}$$

we obtain in Fourier space,

$$\tilde C(p) = \int d^4x \mathrm{Tr}\left[S(x)\gamma_5 S(-x)\gamma_5\right] e^{-ipx} = 2\int d^4x e^{-ipx}\Box G^2 + 4\, G(0)$$
$$\tilde C(\mathbf{p}=0,p_4) = 2\int dt\, e^{-itp_4}\partial_t^2 \int d^3x G^2(\mathbf{x},t) + 4\, G(0)\,, \tag{3}$$

hence we need $\int d^3x\, G^2(\mathbf{x},t)$, which follows simply from (2),

$$\int d^3x\, G^2(\mathbf{x},t) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{-2t\sqrt{\mathbf{p}^2+m^2}}}{4(\mathbf{p}^2+m^2)}\,, \tag{4}$$

leading to the rather compact expression for $t > 0$,

$$C(t) = \int d^3x\; C(\mathbf{x},t) = 2\int \frac{d^3p}{(2\pi)^3} e^{-2t\sqrt{\mathbf{p}^2+m^2}}\,. \tag{5}$$

JHEP06(2022)032

The last expression holds in infinite volume, but in finite L^3 volume the momentum integral simply needs to be replaced by a momentum sum. The fermion fields are assumed to be periodic in all spatial directions.

Hence, in infinite volume, and positive time separation $t > 0$ we obtain,

$$C(t) = \frac{m^3}{\pi^2}\frac{K_2(2tm)}{2tm} \sim \frac{m^3}{4\pi^{3/2}}\frac{e^{-2tm}}{(tm)^{3/2}}\left(1 + O\left(\frac{1}{tm}\right)\right) \qquad \text{for } t \gg \frac{1}{m}\,, \tag{6}$$

with the Bessel function K_2. While in finite volume, mL fixed,

$$C(t) = \frac{2}{L^3}\sum_{\mathbf{n}=(n_1,n_2,n_3)} e^{-2t\sqrt{\left(\frac{2\pi}{L}\right)^2\mathbf{n}^2+m^2}} \sim \frac{2}{L^3}e^{-2mt} + \cdots \qquad \text{for } t \gg \frac{1}{m}\,, \tag{7}$$

where $\cdots$ refers to terms suppressed exponentially relative to the leading term e^{-2mt}. It is clear from (6) that the amplitude vanishes for $t \to \infty$ hence in infinite volume $f_\pi = 0$ even at finite m. In finite volume (7) shows that the amplitude is finite for asymptotically large time separations and we get, using (1),

$$f_\pi = \frac{m}{(mL)^{3/2}}\,, \tag{8}$$

which coincides with the continuum extrapolated result of [3] at the particular finite volume $mL = 1/2$ once it is multiplied by $\sqrt{3}$ since $N_c = 3$ and also by $\sqrt{2}$ to take into account the normalization conventions ($92\ MeV$ vs $130\ MeV$). As $L \to \infty$ at finite m, clearly $f_\pi \to 0$, consistently with the analysis directly in infinite volume.

Hence the ratio m_ϱ/f_π is divergent in infinite volume at tree-level. The tree-level result is relevant at the upper end of the conformal window, $N_f = 11N_c/2$. Hence presumably the non-perturbative result $m_\varrho/f_\pi = 7.85(14)$ with $N_c = 3$ from [1, 2] valid for $2 \le N_f \le 10$ *increases* towards $N_f = 16.5$ contrary to what was stated in [1, 2].

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JHEP06(2022)032